TABLE 6. RELATIVE AMOUNTS OF α_{II} AND β PHASES IN REGENERATED POLYMER A†

regeneration from formic acid

d.p.	evaporation 10 to 20° C	evaporation 80 to 100° C	precipitation 10 to 20° C
10	—	—	$\alpha_{II}\beta^{+++}$
30	$\alpha_{II}\beta^{++*}$	$\alpha_{II}\beta^{++}$	$\alpha_{II}\beta$
50	$\alpha_{II}\beta^{+}$	$\alpha_{II}^{+}\beta$	$\alpha_{II}\beta^{+}$
approximately 500	$\alpha_{II}\beta^{++}$	$\alpha_{II}^{+++}\beta$	$\alpha_{II}^{+}\beta$

regeneration from dichloroacetic acid

d.p.	evaporation 10 to 20° C	evaporation 80 to 100° C	precipitation 10 to 20° C
30	$\alpha_{II}^{++++}\beta$	$\alpha_{II}^{+++}\beta$	$\alpha_{II}\beta^{+}$
50	—	α_{II}	$\alpha_{II}^{+}\beta$
approximately 500	α_{II}	α_{II}	$\alpha_{II}^{+++}\beta$

* Spray dried below 8° C. † See footnote, p. 411.

An experimental study of the pressure variations in standing water waves

By R. I. B. COOPER AND M. S. LONGUET-HIGGINS

Department of Geodesy and Geophysics, University of Cambridge

(*Communicated by R. Stoneley, F.R.S.—Received* 9 *December* 1950)

Although the first-order pressure variations in surface waves on water are known to decrease exponentially downwards, it has recently been shown theoretically that in a standing wave there should be some second-order terms which are unattenuated with depth. The present paper describes experiments which verify the existence of pressure variations of this type in waves of period 0·45 to 0·50 sec. When the motion consists of two progressive waves of equal wave-length travelling in opposite directions, the amplitude of the unattenuated pressure variations is found to be proportional to the product of the wave amplitudes. This property is used to determine the coefficient of reflexion from a sloping plane barrier.

1. INTRODUCTION

In the well-known first-order theory of surface waves in deep water (Lamb 1932, chap. 9) the pressure fluctuations at a given point in any periodic wave motion decrease exponentially with the depth. Thus in a progressive wave of length $\lambda, = 2\pi/k$, of period $T, = 2\pi/\sigma$, and height $2a$ from crest to trough, the pressure fluctuations p_1 are given by

$$p_1 = \rho g a \, e^{-kz} \cos(kx - \sigma t), \tag{1}$$

where ρ is the density and g is the acceleration of gravity, t denotes the time and x and z are horizontal and vertical co-ordinates, z being measured downwards from the mean surface level. The wave-length and period are connected by the relation

$$\sigma^2 = gk \quad \text{or} \quad \lambda = gT^2/2\pi. \tag{2}$$

Therefore, at depths greater than about half a wave-length, the first-order pressure variations are very small. The work of Levi-Civita (1925) also shows that in a progressive wave the pressure variations are attenuated exponentially to all orders of approximation. Now, in the first approximation two progressive waves of equal wave-length and amplitude travelling in opposite directions combine to form a standing wave. The first-order pressure terms are of the form

$$p_1 = 2\rho g a \, \mathrm{e}^{-kz} \cos kx \cos \sigma t, \tag{3}$$

and are therefore attenuated exponentially as in a progressive wave. However, Miche (1944) recently showed that the second-order pressure variations in a standing wave do not tend to zero but to the value

$$(p_2)_\infty = 2\rho a^2 \sigma^2 \cos 2\sigma t. \tag{4}$$

Although this term is of second order it will become predominant over the first-order term at depths greater than about half a wave-length. Since it is independent of both x and z it represents a pressure variation applied uniformly over the whole fluid; it is of twice the fundamental frequency. A physical explanation of the existence of this term has been given (Longuet-Higgins 1950), and it has been shown that in the general case when two waves of equal length but of different amplitudes a_1 and a_2 encounter one another travelling in opposite directions the unattenuated pressure variation is given by

$$(p_2)_\infty = 2\rho a_1 a_2 \sigma^2 \cos 2\sigma t. \tag{5}$$

The standing wave is the special case where the two wave amplitudes are equal, the progressive wave the special case where one of them is zero.

The existence of second-order pressure variations of this type is not only of theoretical interest but may also be of importance in practice. Under suitable conditions the pressure variations in ocean waves may extend to considerable depths, and it has been suggested (Longuet-Higgins 1950) that these may be one cause of the small oscillations of the earth's crust which are generally known as microseisms. The present experiments were undertaken with the purpose of verifying the existence of these pressure variations on a model scale, and of showing how they may be used to determine the amount of wave reflexion from different types of obstacle.

2. Apparatus

The experiments were carried out in a wooden tank of length 970 cm., depth 50 cm. and width 24 cm. (see figure 1). At one end of the tank waves were generated by a paddle A consisting of a rectangular metal plate of width 23·5 cm. hinged at its lower edge and made to oscillate about a mean vertical position by a crank-shaft driven by a $\frac{1}{4}$ h.p. electric motor. By an adjustment to the arm of the crank the amplitude of the waves could be varied at will; the wave period, and hence the wave-length, was controlled through the speed of the motor. It was found convenient to use waves of period about 0·5 sec., the corresponding wave-length (about 40 cm.) being less than the depth of the tank but sufficiently large compared with the measuring apparatus.

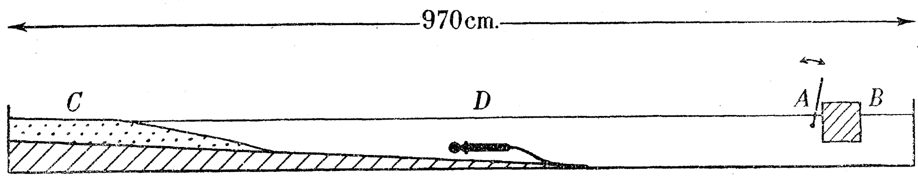

FIGURE 1. Vertical section of the wave tank, showing the relative positions of the paddle (A), wave-absorber (B), sand beach (C) and hydrophone (D).

Between the paddle and the near end of the tank was a honeycomb arrangement of metal sheets (B in figure 1) designed to destroy the wave energy on that side of the paddle. Waves travelled down the tank towards the far end, where they were either absorbed by a shallow sand beach (C), or reflected from a suitable reflector inserted in the tank. The regularity of the waves depended slightly on the depth at which the paddle was hinged. In the present experiments the hinge was at 10 cm., or about a quarter of a wave-length, below the surface, this giving a steady, sinusoidal wave profile. It was found that the wave surface could be made smoother by previously wetting the sides of the tank to a level higher than the wave crests.

Measurements of wave height were made with a hook-and-point gauge consisting of a thin wire (the point) extending vertically downwards into the water and fixed relatively to a similar inverted wire (the hook) extending upwards from below. Both wires were rigidly attached to a sliding vertical scale. The apparatus was initially adjusted so that the point, when lowered from above, first made contact with the still-water surface at the same scale reading as that at which the hook broke the surface when raised from below. To measure the wave height, the gauge was first raised until each crest just touched the point and then lowered until each trough just missed the hook; the difference between the two readings was taken to be the wave height $2a$.

The wave period was measured by timing, with a stop-watch, the passage of 20 successive crests past a fixed point.

The motor quickly accelerated from rest and reached an almost steady speed in about 2·5 sec. The speed was unaffected by the presence of the water, the energy imparted to the waves being in fact negligible.

The pressure was measured with a quartz-crystal hydrophone of a type constructed at the Admiralty Mining Establishment.* This consisted of a circular brass head, 9 cm. in diameter and 2 cm. thick, attached to a brass cylindrical body 6 cm. in diameter and 50 cm. long. The hydrophone head contained two circular quartz crystals cemented, back-to-back, to a central brass plate, the outer faces of the crystals being cemented also to two outer brass plates which were electrically

* The authors are indebted to the Admiralty for permission to describe this apparatus.

common but insulated from the body. Rubber sheet $\frac{1}{16}$ in. thick was cemented to the outer plates and clamped to the hydrophone head by a brass ring, making a water-tight seal. When pressure was applied, an electrical signal was developed between the inner and the two outer brass plates. The outer plates were connected to earth via a small resistance used for the purpose of calibration; the inner plate was connected to the grid of the first valve of a pre-amplifier contained in the body of the hydrophone. A cable, carrying power supplies and leads for the output signal and for calibration, entered the end of the body through a water-tight rubber gland. The electrical signal from the hydrophone was passed through an adjustable attenuator to a second amplifier, whose output operated an Esterline-Angus recording milliammeter. To reduce 'noise', series resistors and shunt capacities were incorporated between the stages, making the gain appreciable only between 1 and 10 c./sec. (Such an arrangement, however, necessarily introduces some phase distortion.)

Let K denote the piezo-electric constant of quartz, R the resistance of the crystals, A the area of the outer brass plates and C the capacity of the plates and their leads. Then it may be shown that an oscillatory pressure $p_0 \cos \sigma t$ applied to the plates of the hydrophone produces a voltage $V \cos \sigma t$ given by

$$V/p = KA/C,$$

provided that $\sigma^2 R^2 C^2 \gg 1$, a condition satisfied in the present case. The theoretical sensitivity of the hydrophone head, based on the above formula, was

$$1{\cdot}5 \times 10^{-5}\ \text{V/dyne-cm.}^{-2}.$$

To calibrate the apparatus a voltage equivalent to, say, 100 dynes-cm.$^{-2}$ and of the appropriate frequency was injected across the small calibrating resistor in the hydrophone head, and the deflexion of the milliammeter was then compared with that produced by the waves.

When the pressure was not uniform, it was assumed that the deflexion was the same as would be produced by a uniform pressure equal to the mean pressure over the plates of the hydrophone. Since the plates were circular, and the pressure p theoretically obeyed Laplace's equation $\nabla^2 p = 0$, this mean pressure was equal to the value of p at the centre of the plates.

The hydrophone was suspended in a horizontal position from two iron bars laid across the top of the tank, the head being away from the paddle (see figure 1). The plates of the hydrophone head were set in a vertical plane, in order to cause the least possible disturbance to the motion. Though the support was not rigid, the motion of the hydrophone itself was very small, and no difference in the measured pressure could be detected when the body was attached rigidly to the walls of the tank.

The estimated limits of accuracy of the different measurements are as follows; each figure represents the maximum error:

wave height $2a$:	$\pm 0{\cdot}02$ cm.
wave period T:	$\pm 0{\cdot}01$ sec.
depth z to centre of hydrophone:	$\pm 0{\cdot}05$ cm.

Thus for waves of height 1·0 cm. and period 0·5 sec. the maximum error in the theoretical first-order pressure variation $\rho g a\ e^{-\sigma^2 z/g}$ at a depth of 15 cm. is 13 %. The maximum error in the second-order pressure variation $2\rho a^2\sigma^2$ is 8 %.

The theoretical sensitivity of the hydrophone was known to within 5 %, and the voltage and frequency of the calibration signal could each be set to within 3 % of their desired values. The deflexion of the recorder was read to the nearest tenth of a division (the attenuator was adjusted so that the deflexion was normally about 1·5 divisions). A change of 3 % in the frequency of the calibration signal involved a change of not more than 6 % in the amplitude of the response. Hence the actual pressure could be measured with certainty to within about 20 %.

In each of the above estimates of the maximum error, allowance has been made for both a random and a systematic part. Though the rough analysis just given is sufficient for our purpose, we may notice that, since the systematic errors from independent sources do not necessarily tend in the same direction, the combined systematic errors are probably less than those given above. Also, because of the random errors, the accuracy of the measurements will be improved by taking the mean of several observations.

3. The progressive wave

When the motor was switched on, a train of waves advanced down the tank, the wave front travelling with approximately the theoretical group velocity $gT/4\pi$. The arrival of the wave front was preceded by several rather longer waves due perhaps partly to dispersion and partly to the initial acceleration of the paddle. In the steady state the wave height was found to diminish with distance from the paddle. The decrease was of the order of a few units per cent per metre, and was attributed mainly to friction at the sides of the tank. No significant variation in the wave height across the tank was observed, and, with the wave amplitudes used, the motion appeared to be entirely two-dimensional.

It will be convenient to denote by x and z respectively the horizontal distance of the centre of the hydrophone head from the paddle and its vertical distance below the mean surface level. A typical record of the pressure, taken at $x = 400$ cm. and $z = 8{\cdot}8$ cm., is shown in figure 2*a*. X_1Y_1 is a calibration signal equivalent to 100 dyne-cm.$^{-2}$ at the frequency of the waves, and TT' is a time trace operated by a clock which made and broke an electrical circuit once every half-second. The paddle motor was first switched on at O. Until the point A, the deflexion of the pen is negligible, indicating there is no signal due to vibration of the motor or paddle. On the arrival of the wave front the pressure variations increase quickly to a maximum and remain at a nearly constant amplitude. The theoretical time of arrival, supposing the front to travel with the group velocity, is denoted by G. In the steady state the pressure variations were of the same period as the waves.

A similar record, but taken at $z = 17{\cdot}3$ cm., is shown in figure 2*b*. The calibration signal X_1Y_1 represents 25 dynes-cm.$^{-2}$. It will be seen that the pressure variations are reduced to about one-quarter of their previous value.

To measure the amplitude of the pressure variations, the mean value was taken of twenty successive oscillations of the recorder pen in the steady state (for example, PQ in figure 2a). The period was deduced by timing the waves in the tank simultaneously, and a calibration signal of the appropriate frequency was afterwards injected. Three successive sets of observations were made for eight different values of z. The mean values of the three measured pressures at each depth are

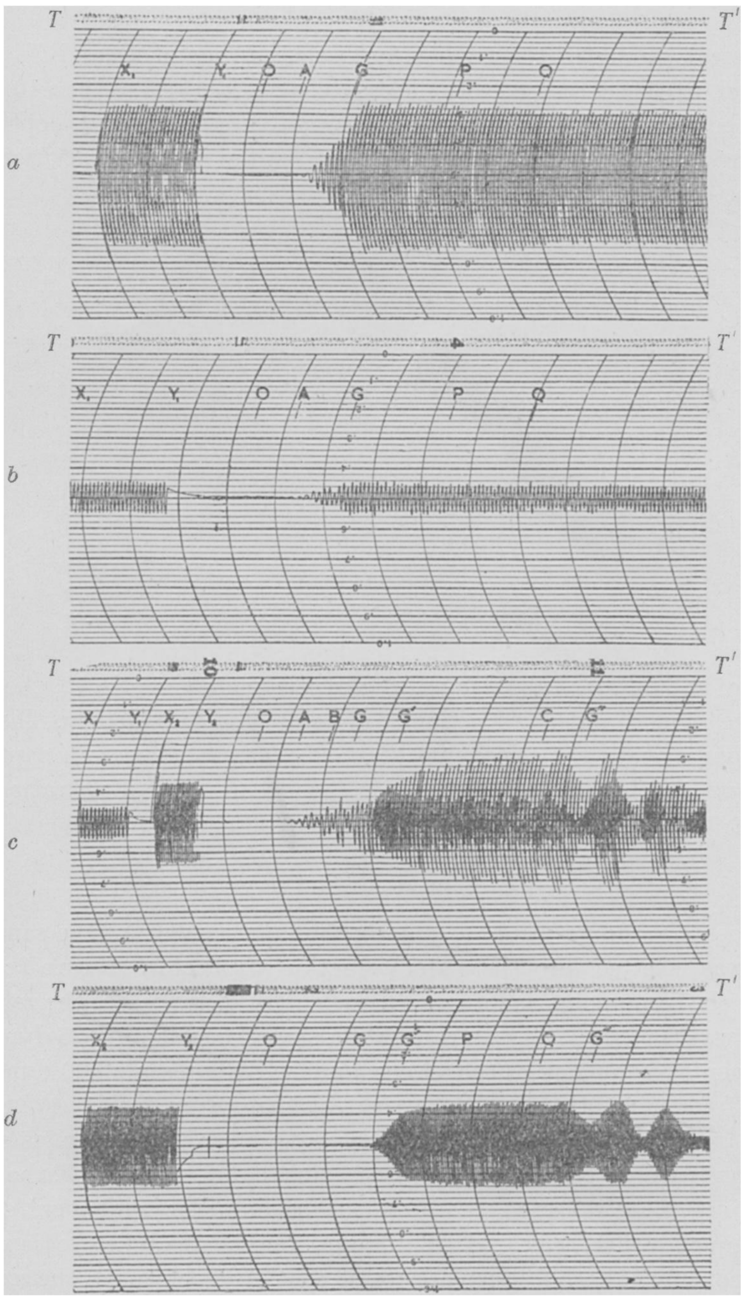

FIGURE 2. Typical records of pressure variations in progressive and reflected trains of waves. **a**, progressive wave, $z = 8 \cdot 8$ cm.; **b**, progressive wave, $z = 17 \cdot 3$ cm.; **c**, reflected wave, $z = 17 \cdot 3$ cm.; **d**, reflected wave, $z = 37 \cdot 0$ cm.

given in the second column of table 1. For each observation of the pressure a separate observation of the wave height $2a$ was made at a point immediately over the hydrophone head. The theoretical pressure, given in the third column of table 1, was calculated from the formula

$$p_{\text{th.}} = \rho g a \, \mathrm{e}^{-\sigma^2 z/g},$$

using the mean values of the twenty-four observations of $2a$ and T, which were 1·11 cm. and 0·453 sec. respectively. Pressure variations of less than 10 dyne-cm.$^{-2}$ could not be measured accurately; but it was verified that at depths between 29·0 and 36·8 cm. (the greatest value of z possible) the pressure variations were less than 5 dyne-cm.$^{-2}$.

TABLE 1. COMPARISON OF THE MEASURED AND THEORETICAL VALUES OF THE PRESSURE VARIATIONS AT DIFFERENT DEPTHS IN A PROGRESSIVE WAVE

$x = 400$ cm. $h = 41·6$ cm. $2a = 1·11$ cm. $T = 0·453$ sec.

z (cm.)	$p_{\text{meas.}}$ (dyne-cm.$^{-2}$)	$p_{\text{th.}}$ (dyne-cm.$^{-2}$)	$p_{\text{meas.}}/p_{\text{th.}}$
6·5	152	151	1·01
8·5	97	102	0·95
10·5	65	69	0·94
12·5	46·6	47·3	0·98
14·5	30·4	31·4	0·97
16·5	19·6	21·2	0·92
18·5	14·0	15·4	0·91
20·5	9·7	9·5	1·02

In view of the limits of accuracy of the experiments given in §2, the agreement between the second and third columns of table 1 is satisfactory. The measured pressure closely obeys the exponential law of decrease down to a depth equal to half the wave-length, and probably deeper. The mean value of the ratios in the last column is 0·96.

4. THE STANDING WAVE

To produce a standing wave, a smooth aluminium sheet, of the same width as the tank, was inserted into the water to act as a reflector. Figure 2*c* shows the pressure at the same point as in figure 2*b* when the distance l of the reflector from the paddle was 800 cm. $X_1 Y_1$ is a calibration signal representing 25 dynes-cm.$^{-2}$ at the fundamental frequency of the waves, and $X_2 Y_2$ represents 100 dynes-cm.$^{-2}$ at twice this frequency. On the first arrival of the waves (point A) the pressure variations are similar to those in the progressive wave. However, as soon as the reflected waves arrive (point B) the record begins to differ, and at G', the theoretical time of arrival of the reflected wave front, a second-order component of twice the original frequency has appeared. Later an almost steady state is reached in which components of both frequencies are present. At C, however, the record begins to degenerate owing to the arrival of the waves reflected a second time from the paddle. (G'' denotes the theoretical time of arrival of the twice-reflected wave front.)

In figure $2d$ the depth has been increased to 37·0 cm. The first-order pressure variations become very small, and the arrival of the first wave train can hardly be detected on the record. The only pressure variations are now of twice the fundamental frequency, which appear, as before, on the arrival of the reflected wave front. When the reflector was removed the pressure variations quickly became negligible.

To determine the amplitude of the second-order pressure variations, the reflector was placed at $l = 500$ cm. and the hydrophone at such a depth that the first-order pressure variations were small. It can easily be shown that the apparent effect of a small first-order term is to increase and decrease alternately the amplitude of the (second-order) oscillations by very nearly the same amount, leaving the mean amplitude practically unchanged. The amplitude was determined from forty successive oscillations (PQ in figure $2d$) in the steady state, before the record degenerated owing to the twice-reflected waves. Three sets of measurements were taken, at four different depths z. The mean values of the pressure at each depth are given in the second column of table 2. It was difficult to measure the height of the standing waves accurately during the short time that they remained regular. Accordingly, the height of the progressive wave in the absence of the reflector was first determined. Owing to the attenuation of the waves with distance along the tank the reflected wave was expected to be slightly smaller than the incident wave. To allow for this effect the wave height was measured at $x = 500$ cm.; for, on the assumption that the attenuation of the waves along the tank was exponential, the wave height at $x = 500$ cm. should be the geometric mean of the height of the incident and reflected waves at $x = 400$ cm. The mean values of the sixteen observations of wave height and period were 0·99 cm. and 0·461 sec. respectively. The theoretical pressure was calculated from the formula

$$p_{\text{th.}} = 2\rho a^2\sigma^2.$$

TABLE 2. COMPARISON OF MEASURED AND THEORETICAL PRESSURE VARIATIONS AT DIFFERENT DEPTHS IN A STANDING WAVE

$x = 400$ cm. $l = 500$ cm. $h = 41·2$ cm. $2a = 0·99$ cm. $T = 0·461$ sec.

z (cm.)	$p_{\text{meas.}}$ (dyne-cm.$^{-2}$)	$p_{\text{th.}}$ (dyne-cm.$^{-2}$)	$p_{\text{meas.}}/p_{\text{th.}}$
21·5	88	91	0·97
26·5	87	91	0·96
31·5	90	91	0·99
36·5	94	91	1·03

Table 2 shows that the difference between the measured and theoretical values is again very slight, and the measured values show no significant dependence on z. The mean value of the ratios in the last column is 0·99.

By carrying out similar observations at a constant depth but at distances from the reflector varying over a range of a quarter of a wave-length, it was verified that the unattenuated pressure variations were also independent of x. Because of the phase shift in the amplifier no attempt was made to determine the relative phases of first- and second-order pressure variations, but it was found that near a nodal plane the first-order variations were relatively small.

To determine the dependence of the unattenuated pressure variations on the wave height, similar observations were made with wave heights $2a$ lying between 0·65 and 1·75 cm. The results are given in table 3. Each figure represents the mean of three observations. It will be seen that the pressure variations increase very nearly as the square of the wave height. With waves of greater amplitude than about 2·0 cm., the wave crests in the standing wave became unstable, and a sideways oscillation was set up across the tank, destroying the two-dimensional motion.

TABLE 3. MEASURED AND THEORETICAL PRESSURE VARIATIONS IN THE STANDING WAVE, FOR DIFFERENT VALUES OF THE WAVE AMPLITUDE

$x = 400$ cm. $l = 500$ cm. $z = 35{\cdot}4$ cm. $h = 41{\cdot}3$ cm. $T = 0{\cdot}473$ sec.

$2a$ (cm.)	$p_{\text{meas.}}$ (dyne-cm.$^{-2}$)	$p_{\text{th.}}$ (dyne-cm.$^{-2}$)	$p_{\text{meas.}}/p_{\text{th.}}$
0·65	37	37	1·00
0·82	58	59	0·98
1·06	90	99	0·91
1·21	126	129	0·98
1·58	207	219	0·95
1·75	250	270	0·93

5. PARTIAL REFLEXION

To obtain interference between two opposite wave trains of known but unequal amplitude, the reflector used in §4 was replaced by another designed to reflect only part of the wave energy. This consisted of a rectangular metal sheet, of the same width as the tank, extending to a variable depth h' below the mean surface level. The reflexion coefficient from a vertical barrier of this kind has been obtained theoretically, for low waves, by Ursell (1947). Since the unattenuated pressure variations are in theory proportional to the height of the reflected wave, their amplitude should be given by $2\rho\beta a^2\sigma^2$, where a is the amplitude of the corresponding progressive wave at the point where the reflector is inserted, and β is the reflexion coefficient.

Three successive series of observations were carried out as for the standing wave, with values of h' lying between 2·0 and 10·0 cm. (Over this range β increases from 0·26 to nearly unity.) The measured and theoretical values of the pressure are compared in table 4. There is fair agreement, although for the smaller values of h' the measured pressure is rather lower than might be expected.

Let us consider whether the distance of the hydrophone from the reflector (100 cm.) is great enough for the reflected wave train to be regarded as infinite in length. If we suppose a uniform pressure variation p', given by

$$p' = \begin{cases} 2\rho\beta a^2\sigma^2 \cos 2\sigma t & (x < l), \\ 0 & (x > l), \end{cases}$$

to be applied to the upper surface of the water, then it may be shown that the pressure variation within the fluid is given by

$$p' = \left[1 - \frac{1}{\pi}\tan^{-1}\frac{z}{x-l} - \frac{1}{\pi}\sum_{h=1}^{\infty}\tan^{-1}\frac{(-)^n\,2(x-l)\,z}{4n^2h^2+(x-l)^2-z^2}\right]2\rho\beta a^2\sigma^2\cos 2\sigma t,$$

(neglecting surface displacements). Taking $(l-x) = 100$ cm., $z = 36{\cdot}6$ cm. and $h = 41{\cdot}5$ cm. we find that the expression in square brackets equals 0·98. Thus, the pressure variations should be reduced only by about 2 % owing to this cause.

TABLE 4. MEASURED AND THEORETICAL PRESSURE VARIATIONS IN A PARTIALLY REFLECTED TRAIN OF WAVES

$x = 400$ cm. $l = 500$ cm. $z = 36{\cdot}6$ cm. $h = 41{\cdot}5$ cm. $T = 0{\cdot}458$ sec. $2a = 0{\cdot}847$ cm.

h' (cm.)	β*	$p_{\text{th.}}$ (dyne-cm.$^{-2}$)	$p_{\text{meas.}}$ (dyne-cm.$^{-2}$)	$p_{\text{meas.}}/p_{\text{th.}}$
2·0	0·26	18	13	0·72
3·0	0·58	39	33	0·84
4·0	0·81	55	44	0·80
5·0	0·94	64	54	0·84
6·0	0·97	66	59	0·89
7·0	0·99	67	69	0·95
10·0	1·00	68	65	0·91

* From Ursell (1947).

From the last column of table 4 we see that the difference between the measured and the theoretical pressures is on the whole greater for the smaller values of h', that is, when the lower edge of the reflector is nearer the surface of the water. Hence it is reasonable to attribute the discrepancy to a loss of energy at the lower edge of the reflector, where the velocity theoretically becomes infinite. In practice we may expect that a thin boundary layer will be formed and that eddies will be thrown off alternately in either direction.

6. REFLEXION FROM A SLOPING BEACH

The measurement of second-order pressure variations provides a convenient method of determining by experiment the reflexion coefficient from a reflector of any given form; for since the theoretical amplitude is $2\rho\beta a^2\sigma^2$, β may be deduced if both a, σ and the actual amplitude are known. Alternatively, it may be assumed, consistently with the results of §4, that the reflexion coefficient from the smooth vertical reflector extending to the bottom equals unity. The reflexion coefficient in the given case may then be obtained simply as the ratio of the second-order pressure variations to those in the case of the vertical reflector. The latter method has the advantage that it is independent of the absolute sensitivity of the hydrophone and that it does not necessarily involve measurement of the wave height and period.

As an illustration of the second method a brief study was made of the reflexion coefficient from a plane barrier inclined at a varying angle α to the horizontal. Three measurements were taken at each position of the reflector, and between each observation a separate measurement was made with the reflector vertical. The mean values of the three corresponding ratios are given in table 5. It will be seen that between 90° and 45° the reflexion coefficient does not differ much from unity. After 45° there is a sharp decline, and at 15° the reflexion coefficient is less than 10 %. Thus at the lower angles nearly all the wave energy is absorbed at the barrier. There was no genuine 'breaking' of the waves; but at 15° the foremost edge of the wave began to be visibly turbulent.

TABLE 5. COEFFICIENT OF REFLEXION β FROM A SMOOTH PLANE BARRIER INCLINED AT AN ANGLE α TO THE HORIZONTAL

$x = 400$ cm. $z = 35{\cdot}6$ cm. $h = 41{\cdot}5$ cm. $l = 500$ to 630 cm. $2a = 1{\cdot}01$ cm.
$T = 0{\cdot}45$ sec. Temperature $= 13{\cdot}5°$ C.

α	β	α	β
90°	1·00	30°	0·72
75°	0·92	25°	0·56
60°	0·99	20°	0·31
45°	0·87	15°	0·08

CONCLUSIONS

The foregoing experiments have shown very clearly, for waves of half-second period, the difference in character between the pressure variations in progressive and in reflected trains of waves. In a purely progressive wave the pressure variations obey the exponential law of decrease down to a depth of at least half a wave-length, and below this depth are very small. However, if any of the wave energy is reflected, appreciable second-order pressure variations, proportional to the amplitude of the reflected wave, appear at all depths. Equations (1), (4) and (5) of §1 have been verified as accurate well within the limits of error of the experiments.

Since the second-order pressure variations increase as the square of the wave height they are relatively more important for waves of large amplitude; and they exert a considerable total force on the sides and bottom of the tank and on the reflector. For standing waves of height 1·8 cm. the second-order pressure variations were of the order of 320 dyne-cm.$^{-2}$, giving a total force on the reflector of $3{\cdot}07 \times 10^5$ dynes or about 0·31 kg. The force corresponding to the first-order pressure variations was only 0·13 kg.

The formulae of §1, which are independent of the viscosity, should hold equally well for waves on the same scale as ocean waves. The chief difference between the waves used in the present experiments and the ocean waves occurring in practice is that the latter are usually much less regular; a generalization of the simple formulae to the case of waves having a general type of frequency spectrum has been given elsewhere (Longuet-Higgins 1950).

The reflexion coefficient for a sloping plane reflector is nearly unity when the reflector is vertical, and decreases steadily with the inclination to the horizontal, as might be expected. Since, however, the reflexion coefficient in this case depends on the viscosity, the same values will not necessarily apply to waves of different period and wave-length. For ocean waves, some energy will be dissipated in breaking, which will also increase the turbulence present in the water. It would be useful to investigate in the laboratory the effect of wave height and period on the reflexion coefficient, on both sides of the breaking-point. It may also be possible to obtain information as to the amount of reflexion of ocean waves by comparing the frequency spectra of the pressure at points off a sloping beach and off a headland or harbour wall.

The authors are much indebted to Professor J. A. Steers and Mr W. V. Lewis for permission to use the wave tank in the laboratory of the Department of Geography, Cambridge.

References

Lamb, H. 1932 *Hydrodynamics*, 6th ed. Cambridge University Press.
Levi-Civita, T. 1925 *Math. Ann.* **93**, 264.
Longuet-Higgins, M. S. 1950 *Phil. Trans.* A, **243**, 1.
Miche, M. 1944 *Ann. Ponts Chauss.* **2**, 42.
Ursell, F. 1947 *Proc. Camb. Phil. Soc.* **43**, 374.