On the Southward Motion of Mediterranean Salt Lenses

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ABSTRACT

A new mechanism is proposed to account for the southward motion of salt lenses in the Canary Basin. It relies on the active mixing that is observed at the periphery of such eddies between the warm salty Mediterranean waters of the core and the surrounding fresher and cooler Atlantic waters. Intrusions driven by this thermohaline contrast feed on the eddy potential energy and drain off the eddy volume anomaly. Submitted to the continuous entrainment of its own fluid by the frontal turbulence, the salt lens suffers a velocity divergence at its periphery, induced by a geostrophic adjustment process. This causes in turn a squashing of the density surfaces and the salt lens must then move southward to conserve its potential vorticity, a possibility offered by the spherical nature of the earth. It appears that mixing may lead to the coherent latitudinal displacement of an ensemble of fluid parcels on a β plane, while at the same time dissipating the energy of the structure itself.

1. Introduction

It is well known that the Mediterranean outflow of saline water at Gibraltar does not produce a smooth spreading plume in the Atlantic Ocean. Hydrography has shown that "mesoscale" lenses of anomalous water can persist for a long time and be found far away from their regions of formation. Recently, direct observations by Lagrangian techniques (SOFAR floats) have been made confirming the bodily motions of the so-called "meddies" (Richardson et al. 1989; RI hereafter). The floats seeded inside the core of the meddy remain there for several years, providing the path of the whole structure. In the Canary Basin these meddies move equatorward at speeds near 2 km day^{-1}. Meridional motion on a rotating spherical earth is severely constrained, and in this particular case one wonders immediately about its cause. Kinematic information is by itself insufficient, but fortunately, dedicated hydrographic cruises by Armi et al. (1989; AR hereafter) have provided a wealth of information about a persistent structure at several stages of its life cycle, and the goal of this note is to provide a plausible mechanism that could account for the observed southward motion. The first idea, which was suggested by RI, is to have the general circulation advect the feature. There are two difficulties with this: first, the circulation associated with the southward motions of the upper central waters in this part of the subtropical gyre does not extend beyond 800 m, the depth that represents the upper part of the meddy. How then could the bottom of the eddy lying near 1500 m be entrained? Geostrophic computations referred to 3000 m indeed show no signs of a southward flow at such depths (Maillard 1986) in the region of interest. The second argument against advection by the general circulation is provided by an extensive coverage of the region by 1000-m SOFAR floats (RI). Although the float density in this experiment is probably not sufficient to estimate the mean circulation accurately, the data show unambiguously that meddies move through the water as statistics of mean float displacements differ markedly if floats in meddies are included or not. Therefore, it appears that meddies are more likely self-propelled than advected by a local mean flow. However, Hogg and Stommel (1990) have expounded an interesting idea that resolves this dilemma by postulating a second anticyclonic vortex on top of the primary meddy. The role of this upper-layer vortex, itself advected southward by the ambient upper-layer mean flow, is to lock to the lower-layer vortex, a locking that occurs under certain favorable circumstances depending on mean flow shear, layer depths, etc. The model makes use of the dynamics of point vortices on an f plane and neglects the advection of planetary vorticity. Additional velocity observations in the vertical could confirm the existence of this secondary vortex.

Finally, in addition to advection, control of the meddies by the mean circulation could also be exerted through the deformation of the large-scale potential vorticity. However, the mapping of McDowell et al. (1982) does not reveal significant distortions of the potential vorticity contours from zonal latitude circles near the level of the Mediterranean water ($\sigma_\theta = 27.6$) in the Canary Basin, and we therefore consider in what

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follows the evolution of meddy-type structures in an ocean at rest.

The dynamics of nonlinear coherent structures on a β plane have aroused much interest in recent years, with the literature concentrating on steady, conservative solutions (see McWilliams 1985, for a review). Because such coherent structures (much as meddies) carry water with them, they are robust, solitary features that can somehow defeat the tendency of β to disperse them. As first pointed out by Flierl et al. (1980), steadily translating isolated eddies are constrained to move along latitude circles because, when potential vorticity is conserved, net meridional motion would necessarily change the relative vorticity or thickness of the feature. Killworth (1986) extended the result with more appropriate shallow-water theory predicting speeds of westward motion of meddies of about 1 cm s^{-1}. If these ideal, steady solutions fall short of explaining the numerous observations of southward-moving meddies in the Canary Basin, several numerical experiments appear more relevant. Rather early, Bretherton and Karweit (1975) observed in a quasi-geostrophic ocean model that an anticyclonic baroclinic vortex moved to the southwest under the combined influence of nonlinearity, β, and small friction. Such a vortex is initially associated with an anomaly of negative relative potential vorticity q (relative vorticity plus stretching terms). Northward (southward) moving fluid parcels on the western (eastern) side of the eddy gain (lose) planetary vorticity, inducing respectively additional negative and positive anomalies of q, whose net effect is to produce a correcting velocity field moving the primary vortex in a southwest direction. Such solutions are not steadily translating, and the net meridional motion of the eddy center is associated with the β effect, which tries to deform and disperse the initial anomaly as Rossby waves. With an initialization more appropriate for meddies, such quasigeostrophic experiments have also been carried out more recently by Beckmann and Käse (1989). In this simulation the eddies appear to move to the southwest at a speed of 0.8 cm s^{-1} with considerable variations of circular symmetry being generated by instability events. The processes at work in such transient but nearly conservative experiments might therefore account for about half the observed southward translation. However, the significant variations of circular symmetry that are implied are not conspicuous in observed data as floats appear to move circularly around meddies for very long periods of time (RI).

The extremely useful observations of the particular meddy described in AR suggest additional lines of investigation. The striking fact in this dataset is that over the course of its life, the meddy evolves considerably, slowly losing its water mass characteristics by mixing with the surrounding water. Could this transient phase of spindown somehow be connected with the southward coherent motion of the eddy?

Let us review first the temporal evolution of the meddy observed and described by AR and RI. Named Sharon by AR and meddy 1 by RI, it was observed first at 32°N, 22°W in October 1984 and sampled three more times before being lost around 22°N, 22°W two years later. The implied mean southward velocity is about 1.8 cm s^{-1}. The anticyclonic rotary motion occurs at a faster speed of about 11 cm s^{-1}. The lens extends vertically from 600 to 1400 m, and it is possible to identify in the horizontal plane a central core of 20-km radius, filled with Mediterranean water ($S = 36.2$ psu), which is surrounded by an annular transition region 15 km wide. The core rotates as a solid body with an initial relative vorticity that amounts to one-third of the Coriolis frequency and decreases by about 10% by survey 3, a year later. Because the anticyclonic structure is vertically trapped, a vertical section of density through the core shows the shape of a convex lens. As the lens appears to collapse over the course of a year, its thickness varies considerably in time, decreasing by more than 20% at the eddy center (Fig. 1). At a radius larger than 20 km, the azimuthal velocity decreases rapidly, creating a large horizontal shear in an annular outer region that is dominated by active mixing in the form of thermohaline intrusions or interleaving. It is in this region that Ruddick and Hebert (1988) estimated a diffusivity of 0.4 $m^2 s^{-1}$ to quantify the lateral exchange of warm salty water with cooler fresher water of Atlantic origin through intrusive mixing. The overall effect of mixing was also estimated since heat and salt content were computed for each survey. Each of them decayed nearly exponentially with a time constant of about 300 days. Although thermohaline convection was active at the top of the meddy and salt finger at the bottom, it appears that lateral thermohaline intrusions at the sides were responsible for the largest fraction of the net mixing. As the eddy moves along, the mixing processes are akin to a slow erosion of the feature with fragments of warm salty water being gradually left behind.

2. Discussion

As they move southward on the rotating earth, meddies lose planetary vorticity. Since this motion is also accompanied by a large variation of the thickness, it seems judicious to start with a cursory examination of the potential vorticity field. It can be crudely estimated at the eddy core during each of the four surveys, using values given by AR. The vertical thickness H at the core was obtained from density sections and measured between values of σ_1 equal to 31.8 and 32.25. The values of the total potential vorticity $(\xi + f)/H$ at the eddy center are shown in Table 1 for the first three surveys spanning the course of one year.

As f decreases by about 15% between surveys 1 and 3, H decreases by about 20% and the relative vorticity increases by 4% in percentage of f. Therefore, both

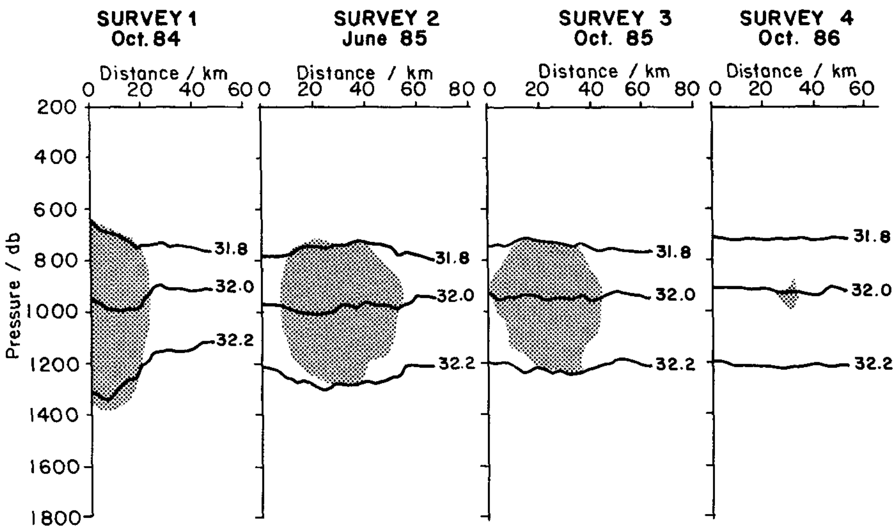

FIG. 1. The four vertical density sections carried out by Armi et al. (1983). The shaded region indicates salinity in excess of 35.8 psu.

thickness and relative vorticity have the right evolution to counterbalance the variation of f, and the potential vorticity, which changes by only 7%, is more conserved than its contributing terms, the thickness showing the largest variations. This crude evaluation leads to the idea that the meddy core dynamics might be ideal in the sense of local potential vorticity conservation. This idea is also supported by the solid-body rotation and the lack of vertical shear of the core region, two characteristics that inhibit turbulent transfer to smaller scales. Accepting this hypothesis of a free core, we turn our interest to the outer shell, the area of active mixing, to try to parameterize its effects on the free core in a simple way. We first neglect mixing effects occurring at the top and bottom of this outer shell because the largest net mixing effect is attributed to the lateral thermohaline intrusions, which occur along the lateral surface of the shell. In this area, momentum, mass, heat, salt, and buoyancy might all be exchanged between the core and the surrounding waters. Let us examine these possibilities in turn. First, frictional turbulent torques can be expected to occur because of the presence of the large horizontal shear in the annular region. Second, the variations of the net buoyancy (hence the fluxes) can be estimated because the net heat and salt loss have been measured between each survey. If Q and H denote the heat and salt loss, the buoyancy loss (or gain) is governed simply by the quantity $B = -\alpha Q/C_p + \beta H$, which has units of kilograms per second, and where α and β are respectively the thermal and haline expansion coefficients and C_p is the heat capacity of seawater. Taking mean values from the bounds given by AR produces the results of Table 2 after choosing $\alpha = 2 \times 10^{-4}\,^{\circ}\mathrm{C}^{-1}$, $\beta = 0.77$, and $C_p = 4 \times 10^3$ J kg^{-1}, all values appropriate for a 1000-m depth.

This simple calculation shows that the residual buoyancy loss or gain is probably not different from zero, given the error estimates on the heat and salt fluxes. This indicates that the mixing of temperature and salinity anomalies occurs nearly isopycnally, a picture that is in agreement with the model of unstable intrusions first put forward by Stern (1967). From this evidence, we accept that the lateral mixing is mostly isopycnal. If the buoyancy exchange across the annular frontal region is so limited, it is helpful to use simple ideas developed in the context of the study of turbulent boundary layers (see Tennekes and Lumley 1972). The boundary that separates the region of irrotational flow from that of active turbulence is usually highly convoluted, and turbulence practitioners have used the

TABLE 1. Values of the total potential vorticity at the eddy center.

Survey	ξ ($\times 10^{-5}$ s^{-1})	f ($\times 10^{-5}$ s^{-1})	$\xi + f$ ($\times 10^{-5}$ s^{-1})	H (m)	$\frac{f}{H}$ ($\times 10^{-7}$ m^{-1} s^{-1})	$\frac{\xi + f}{H}$ ($\times 10^{-7}$ m^{-1} s^{-1})
1 Oct 84	−2.66	7.75	5.09	744	1.04	0.68
2 Jun 85	−2.50	7.02	4.52	632	1.11	0.71
3 Oct 85	−2.35	6.73	4.38	600	1.12	0.73

TABLE 2. Heat, salt, and buoyancy loss.

	Survey 1–2	Survey 2–3
$Q/10^{11}$ W	7.7	3.95
$H/10^4$ kg s^{-1}	5.45	2.15
$B/10^4$ kg s^{-1}	+0.34	−0.29

concept of an "extrainment velocity" to quantify the rate of advance of the mean position of the interface separating the quiet from the turbulent fluid. Such an entrainment velocity is defined as the average flow of fluid per unit area normal to the mean position of the boundary and is mostly the result of the action of the largest eddies. Oceanographers have used the concept of "entrainment" in the integral models of the surface mixed layer to parameterize the deepening action of local turbulence. Herein, the turbulence that is active in the frontal region slowly erodes the turbulence-free core of the meddy, and an entrainment velocity u_e is introduced to quantify that erosion. As the frontal turbulence is due to a mixture of shear-flow instabilities and interleaving intrusions, it is expected that the ratio of the entrainment over the rms turbulent velocity will depend upon the density ratio, the Rayleigh number and the Reynolds number. However, that dependence is not yet known, and we have to rely upon the observations of the frontal turbulence itself to deduce an order of magnitude of this entrainment velocity u_e. The mean position of the front r^* defined as the position of maximal salinity or relative vorticity gradients is known and can be used to define a boundary between a turbulence-free core and its turbulent wake at the eddy periphery. The local source of the turbulence being due to the existence of these sharp gradients, this boundary advances into the eddy interior at a certain rate, which is the sought entrainment velocity (Table 3).

Quite logically, the entrainment rate must be related to the overall mixing associated with the heat and salt losses that originate mostly from lateral turbulent fluxes on the sides. Lateral turbulent diffusion coefficients can be estimated as:

$$K_H = \frac{\partial}{\partial t}\int\int\int S dv \Big/ \oint \left[\frac{\partial}{\partial r}\int S dz\right] dl,$$

TABLE 3. Entrainment and divergent velocities.

Surveys	1		2		3		4
r^* (km)	35		27		22		15
$u_e = dr^*/dt$ (cm s^{-1})		3.5×10^{-2}		4×10^{-2}		2.3×10^{-2}	
K_H (m^2 s^{-1})		16		9.5			
u_r (cm s^{-1})		8.5×10^{-3}		5.3×10^{-3}		1.8×10^{-3}	

where the line integral at the denominator is taken around the core boundary defined previously. This expression is easily evaluated with the data from AR, and gives lateral diffusion coefficients K_H of order 10 m^2 s^{-1} (Table 3). Such values allow us to quantify the average displacement of the front as $(K_H t)^{1/2}$, a result broadly consistent with the entrainment velocity of Table 3. (Between survey 1 and 3, spanning 400 days, the advance of the front computed as $(10 \text{ m}^2 \text{ s}^{-1} \times 400 \text{ days})^{1/2}$ reaches 18 km, a value that compares favorably with the 13-km displacement inferred from Table 3). The much weaker diffusion coefficients (0.4 m^2 s^{-1}) of Ruddick and Hebert (1988) relate to mixing at the smaller scale of the interleaving features themselves. When this mixing operates in the presence of a strong circulatory flow, there is little doubt that the overall mixing is considerably enhanced by the lateral shear of the eddy, providing for the much higher values inferred from the variations of the heat and salt content.

3. A conceptual model of meddy evolution

The previous discussion suggests some ideas that will allow us to link in a simple way the observed mixing to the dynamics. We first consider a conservative core region that is rotating sufficiently rapidly to trap fluid particles and is surrounded by a free boundary along which turbulence is acting (Fig. 2). We assume further that this boundary moves inward toward the core at a known rate given by an entrainment velocity and we then look for the dynamical consequences of such a turbulent erosion. To illustrate the processes in the most simple fashion, a quasigeostrophic approximation is made. This cannot be justified rigorously, because the centrifugal forces are not small, the Rossby number being about 1/3. A better model looking at the finite Rossby number situation will be desirable when more accurate comparisons are needed (see Appendix). Because the β effect is small at the scale of a meddy, we look at the problem in two steps: the first step is a rapid geostrophic adjustment to the varying boundary conditions at the core periphery, while the second specifically considers the effect of β on a longer time scale.

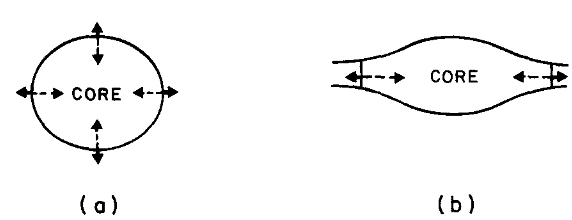

FIG. 2. The conceptual model of a meddy core in the horizontal (a) and vertical plane (b). The contour bounding the core is lying in the frontal region, where interleaving occurs. We picture the eroding effects of the frontal turbulence as an entrainment (dotted arrows) that geostrophically induces a divergence (solid arrows) that slowly drains the eddy volume anomaly, with little buoyancy exchange.

a. The geostrophic adjustment

Quasigeostrophic theory on the f plane readily gives the consequences of the turbulent erosion. As the front moves inward, geostrophic equilibrium is perturbed and the core must adjust its shape to restore the equilibrium. A well-known result of geostrophic adjustment is that the aspect ratio of the structure H/L should scale as the ratio of Coriolis over Brunt–Väisälä frequency. So that when the lateral scale L is forced to decrease, the thickness H must decrease as well. Can we follow this idea and relate the divergence in the core to the inward motion of the front? To do so, consider a 1½ layer model of the vertical structure of a meddy that consists of one active middle layer of density ρ_2 and depth $H + h$, embedded within two upper and lower layers at rest of respective densities ρ_1 and ρ_3. When the perturbation height h is small compared to the mean height H (small Rossby number), the geostrophic and continuity equation read respectively:

$$f\mathbf{k} \times \mathbf{u} = -g'\nabla h$$
$$\frac{\partial h}{\partial t} + H\nabla\cdot\mathbf{u} = 0, \qquad (1)$$

where $\mathbf{k}$ is a vertical unit vector and g' is the reduced gravity

$$g\frac{(\rho_2 - \rho_1)(\rho_3 - \rho_2)}{\rho_2(\rho_3 - \rho_1)}.$$

Appropriate boundary conditions must be appended to parameterize the erosion process at the boundary, the displacement of which is imposed. What is chosen here is to impose a zero perturbation pressure (or zero thickness) at the boundary. Consequently, the motions vanish outside the boundary, which is effectively a vortex sheet. Physically, such a singularity is needed because it is the turbulence generated by the instabilities of that sheet that, in turn, drives the erosion process. To explore the dynamical consequences of this boundary condition, we need an initial state to characterize the eddy core. One outstanding feature of the observations made by AR is the axisymmetry of the core and its solid-body rotation. In view of the strength of the swirling flow, such constant relative vorticity may not be too surprising. The Prandtl–Batchelor theorem (derived for a homogeneous fluid) tells us that such states are necessary consequences of high Reynolds number, closed-streamline situations (Batchelor 1956). That it may find an application here is probably a consequence of the "equivalent barotropic" vertical structure of the flow.

We can now solve for the perturbation height h, given the constant relative vorticity ξ and the side boundary conditions (that are slowly evolving in time). With the use of geostrophic velocities, the problem reduces to

$$\left.\begin{aligned} (g'/f)\frac{1}{r}\frac{\partial}{\partial r}(rh_r) &= \xi \\ h = 0 \quad \text{at} \quad r &= r^*(t) \end{aligned}\right\}. \qquad (2)$$

The regular solution of (2) is simply:

$$h/H = \frac{\xi}{4f}(r^2 - r^{*2})/\lambda^2, \qquad (3)$$

where λ is the internal Rossby radius $(g'H)^{1/2}/f$. Being given a functional form for r^*, this parabolic solution allows us to find the leakage of the core volume V, which is necessary to maintain the geostrophic equilibrium. With the volume V given by $\int_0^{r^*} h2\pi r dr$, one finds:

$$V = -2\pi\frac{\xi f}{16g'}r^{*4}.$$

To ensure continuity, the rate of change of V must then be balanced by an outward radial velocity u_r at the periphery r^*, which is $[(-1/2\pi r^* H)(dV/dt)]$ or

$$u_r = \frac{\xi}{4f}\frac{r^{*2}}{\lambda^2}\frac{dr^*}{dt}. \qquad (4)$$

This is the desired result, which relates the peripheral ageostrophic divergence with the imposed turbulent entrainment dr^*/dt.

We calibrate the model at the time of the first survey by the observed values of the core radius (35 km), its vorticity (-2.5×10^{-5} s^{-1}), and its thickness at the center ($\sim$300 m). Expression (3) then gives

$$g'\left(= \frac{\xi f r^{*2}}{4h(r = 0)}\right) = 1.78 \times 10^{-3} \text{ m s}^{-2}.$$

There is more freedom to choose the unperturbed height H, and it was determined by fixing the Rossby radius λ to 20 km in rough agreement with the observed spatial variations of properties. With this choice, H turns out to be 1100 m, or about four times the initial perturbation height—a value consistent with the Rossby number of the flow. The solution (3) is shown in Fig. 3 at the time of the four surveys. It illustrates readily the geostrophic adjustment of the structure when subject to erosion. The corresponding values of the ageostrophic outward velocity u_r at the core boundary, responsible for the volume loss, are shown in Table 3. Viewed from the meddy core, this ageostrophic radial velocity may be seen as a drain of the nonturbulent fluid to the outside, driven by the pressure gradient, and its values must therefore be compatible with the rate of collapse of the lens observed previously at the center. In order of magnitude, continuity imposes that the collapse $H^{-1}dh/dt \sim u_r P/S$, where P and S are the perimeter and surface of the contour enclosing the core region and h the height taken at the core center. Approximating this contour by a circle of radius r gives $P/S = 2/r$. Between surveys 1–2 and 2–3, respectively, $H^{-1}dh/dt$ is (7.3×10^{-9}, 4.6×10^{-9} s^{-1}) and $u_r P/S$

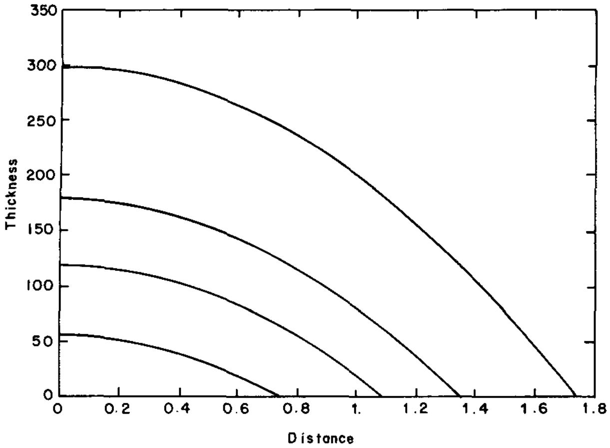

FIG. 3. The thickness (m) of the meddy core is calculated from the geostrophic model at the times of the four surveys described by AR. The calculation assumes a constant relative vorticity and a condition of zero thickness at the varying edge of the core. The horizontal distance is scaled by the internal Rossby radius $(g'H)^{1/2}/f$ (=20 km).

is (5.5×10^{-9}, 4.2×10^{-9} s^{-1}), so that the orders of magnitude of the estimates found for the ageostrophic leakage are indeed consistent with the macroscopic observations of the lens collapse. This analysis reconciles two conflicting aspects of the data: the meddy core boundary moves inward, suggesting convergence, and the core collapses, indicating divergence. The present analysis shows that the former is a kinematical entrainment-erosion process that causes, through a geostrophic adjustment, the dynamically more important collapse of the structure.

b. The β-plane effects

In the above modeling of the erosion process on an f plane, the potential vorticity of the core increases since ξ is constant and h decreases. On the β plane, however, and as suggested by the observations (Table 1), the possibility exists to conserve the potential vorticity of the core in the presence of the above adjustment. The displacement of the front toward the core and the small outward radial velocity that transports the anomalous core water across the front are admittedly a simplification of the mixing processes, which imply true exchange of water between the warm salty core and the cooler, fresher outside waters. However, their consequences on the β plane are worth exploring. Because the meddy core carries fluid particles as it moves meridionally, we need to express dynamical statements in a Lagrangian fashion. The central idea proposed in this paragraph is to consider the frontal boundary as an averaged material contour across which a known radial velocity, computed as in (4), is acting. As the erosion process slowly works its way inward, the radial velocity component of fluid parcels is then entirely determined along the boundary. From this vantage point, Bjerknes' circulation theorem can be used to infer the dynamics in a rather general way with minimal scaling assumptions.

The rate of change of the circulation along a closed material contour in a rotating stratified fluid is given by (see, for instance, Pedlosky 1979):

$$\frac{d}{dt}\oint \mathbf{u}d\mathbf{l} = -\oint 2\mathbf{\Omega} \times \mathbf{u}\cdot d\mathbf{l} + \iint \frac{\nabla\rho \times \nabla P}{\rho^2}\mathbf{n}dS + \oint \frac{\mathbf{F}}{\rho} d\mathbf{l}. \quad (5)$$

In the above, the first term of the right-hand side is the Coriolis torque and the last is a frictional torque representing possible momentum losses from the core. We now rely upon the weakness of cross-isopycnal fluxes to adapt this theorem to the situation of a material contour enclosing the meddy core and lying in a density surface. In this case, the second baroclinic term on the right-hand side of (5) vanishes identically. The Coriolis torque can be rewritten as

$$-\oint 2\mathbf{\Omega} \times \mathbf{u}d\mathbf{l} = -2\Omega\frac{dA}{dt},$$

where A is the equatorial projection of the area S enclosed by the contour. Because the isopycnal surfaces are nearly normal to the gravity vector, and because the eddy is so small compared to the earth radius, we may approximate A by $S\sin\theta$, where θ is the latitude.

This allows us to rewrite $2\Omega(dA/dt)$ as

$$2\Omega\frac{dA}{dt} = 2\Omega\sin\theta\frac{dS}{dt} + \frac{2\Omega\cos\theta S}{R_T}\frac{d}{dt}(R_T\theta),$$

where $d(R_T\theta)/dt$ is the northward component C_y of the translation velocity of the feature. After proper identification and simplification, (5) now reads:

$$\underset{\text{A}}{\frac{d}{dt}\oint \mathbf{u}d\mathbf{l}} = \underset{\text{B}}{-f\frac{dS}{dt}} - S\beta C_y + \underset{\text{C}}{\oint\frac{\mathbf{F}}{\rho}d\mathbf{l}}.$$

This relation shows that meridional motion may come from a combination of transient effects (A and B) and dissipation (C). The nearly ideal numerical experiments described previously involve the β dispersion process to provide for nonzero values of A and B. In the mechanism proposed here, the term B, which results indirectly from peripheric mixing activity, has the central role, and we intend to show that the observations of AR and RI apparently support that conclusion. The following expression for the meridional translation velocity of the material contour enclosing the eddy core is then discussed:

$$c_y = \underset{\text{A}'}{-\frac{1}{\beta S}\frac{d}{dt}\oint \mathbf{u}d\mathbf{l}} \underset{\text{B}'}{- \frac{f}{\beta}\frac{1}{S}\frac{dS}{dt}} + \underset{\text{C}'}{\frac{1}{\beta S}\oint\frac{\mathbf{F}}{\rho}d\mathbf{l}}. \quad (6)$$

Signs and order of magnitude of the two terms A′, B′ on the right-hand side of (6) can be evaluated from the observations, assuming axisymmetry. The first, A′, the transient contribution, is $\sim\beta^{-1}d\xi/dt$, where the vorticity ξ has been estimated in the core, and the second, B′, proportional to the relative rate of change of the surface enclosed by the contour, is therefore simply estimated by the divergence due to the outward radial velocity u_r given by (4). This second term, B′, is therefore negative and of order $\sim 2fu_r/\beta r$. The last term, C′, the momentum frictional loss, is not easily estimated but for its sign, which must be such as to spin down the anticyclonic core so that C′ is a positive term. The results of the present discussion of expression (6) parallel at the level of rate of change that of potential vorticity given previously, with the important addition that the horizontal divergence is calculated from the geostrophic adjustment of the core when subject to lateral erosion.

As summarized in Table 4, the rate of spindown of the circulation (A′) induces southward velocity of about 0.5 cm s^{-1}. This decrease of relative vorticity (in absolute values) may be the result of tongues of low relative vorticity intruding in the core region, a process formally included in the action of the dissipation **F** and (or) the β-induced dispersion effect. These two processes appear insufficient by themselves to explain the data. The positive dissipative term C′ cannot account for the southward translation, and we are led to accept that an important term that contributes to southward velocity of the right order of magnitude is the baroclinic term B′. Acted upon by a peripheric volume loss due to the adjustment–entrainment induced by the frontal turbulence, the solid-body rotating core attempts to retain its potential vorticity by moving south at a speed whose order of magnitude is therefore the outward radial velocity u_r multiplied by a large amplification factor, which is the ratio of earth radius R_T over eddy radius r^*:

$$C_y = -u_r \frac{2R_T}{r^*} \tan\theta$$

or, after using (4):

$$C_y = -\frac{\xi}{2\beta r^*} \frac{r^{*2}}{\lambda^2} \frac{dr^*}{dt}. \quad (7)$$

In this last expression, the meridional translation velocity is shown to be much larger than the entrainment velocity, dr^*/dt, because, in the case of meddies, $\xi/\beta r^*$, is about 30 while the radius r^* is of the order of the internal Rossby radius λ.

TABLE 4. Rate of spindown of the circulation.

	Survey 1–2	Survey 2–3	Survey 3–4
A' (cm s^{-1})	−0.35	−0.70	
B' (cm s^{-1})	−2.	−1.34	−0.55

Expression (7) is the central result of this note. It shows that if the mixing effect can be reduced to an entrainment, which then induces an isopycnal drain of the eddy volume anomaly, the order of magnitude of the turbulent mixing is consistent with the southward motion of the meddy on a β plane. Although the central meddy core appears ideal by conserving its potential vorticity, the origins of the variations of the "thickness" are due to lateral mixing, which feeds on the eddy energy. That turbulent mixing may lead to self-propelled, meridional motion of coherent structures on a β plane is an idea that may be a viable alternative to other schemes invoking transport by the mean circulation. To transform the present idea into a more quantitative, predictive theory beyond expression (7) will require an explicit consideration of the centrifugal nonlinear terms (see Appendix) and, more importantly, an understanding of the relationship between the net entrainment and the thermohaline mixing at the interleaving scales. In this context, the finite amplitude instabilities examined by Stern (1987) are likely to play a role. If the present idea is correct, the observation of the bulk motion of a meddy may offer, in the end, an important check in the difficult determination of the small-scale thermohaline fluxes themselves.

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APPENDIX

Effects of the Centrifugal Terms

It is not difficult to include the effects of the centrifugal terms in the calculation leading to expression (3) in the simple case of a solid body rotating core. The azimuthal velocity u_θ is equal to $(\xi/2)r$ and the momentum balance becomes:

$$\frac{u_\theta^2}{r} + fu_\theta = g' \frac{\partial h}{\partial r}.$$

The thickness obtained after integration of the above equation becomes:

$$h/H = \xi/4f(1 + \xi/2f)(r^2 - r^{*2})/\lambda^2$$

Hence formula (3) has to be corrected by a factor $(1 + \xi/2f)$ when the centrifugal terms are taken into account. In an anticyclonic flow the centrifugal acceleration opposes the Coriolis force and the correction is a negative one. It merely decreases the geostrophic estimates of the thickness h, the divergent velocity u_r, and the southward velocity C_y given in (7) by about 15% since in the analysis of this particular structure, ξ/f is about 0.3 in the meddy core.

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