Interaction Subrange Spectra of Turbulent Wind Over an Air-Water Interface[1]

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ABSTRACT

The nonlinear interaction of wave-induced air motion and the turbulent wind is examined in spectral space with the aid of a model developed through two averaging processes. The averaging periods are determined by the time scales of turbulent and of wave-induced air motion. It is shown that an interaction subrange spectrum characterized by a -1 slope may exist in the turbulent wind over the air-water interface. A method which involves the calculation of energy spectra in frequency and in wavenumber-frequency space is described. It permits an assessment of nonlinear interactions in these spectral domains. We are concerned particularly with interactions with the wave-induced motion and with the contribution which this may make to the total turbulent kinetic energy.

1. Introduction

The displacement and deformation of air caused by traveling water waves may cause the airflow in the vicinity of the air-water interface to have observable wave disturbance. The purpose of this paper is to examine the nonlinear interaction of wave-induced air motion and turbulent wind in spectral space. By transforming the energy equation into frequency and wavenumber-frequency spaces, a method is outlined to assess the contribution to the total turbulent kinetic energy of a particular component due to nonlinear interaction in the spectral domain. It allows one to examine in detail the effect of the wave-induced motion on the structure of turbulent wind.

The coupling between wind and waves has been formulated by Phillips (1966), Miles (1967) and recently by Yefimov and Pososhkov (1970). For the convenience of later discussion, this coupling model is derived here alternatively by defining two averaging processes, one over a time scale T of the wave-induced motion and the other over a time scale τ of the turbulent motion:

$$\left.\begin{aligned}\tilde{\eta}(\mathbf{x};T)&=\frac{1}{T}\int_{t-T/2}^{t+T/2}\eta(\mathbf{x};t_i)dt_i\\ \bar{\eta}(\mathbf{x};\tau)&=\frac{1}{\tau}\int_{t-\tau/2}^{t+\tau/2}\eta(\mathbf{x};t_i)dt_i\end{aligned}\right\}. \tag{1}$$

The time scale T which has the magnitude of the dominant period of water waves is generally much greater than τ, which may have the order of the integral time scale of turbulence and is still long enough to yield meaningful averages.

Let the instantaneous velocity and pressure fields be decomposed into the forms

$$\left.\begin{aligned}U_i&=\tilde{U}_i+U_{wi}+U_i'\\ P&=\tilde{P}+P_w+P'\end{aligned}\right\}, \tag{2}$$

where U_i, P are the total instantaneous velocity and pressure, $\tilde{U}_i$, $\tilde{P}$ the steady, time mean velocity and pressure, U_{wi}, P_w the wave-induced velocity component and pressure, and U_i', P' the turbulent fluctuations of velocity and pressure. It can be shown from the equations of motion and continuity that the energy equations for the wave-induced motion and the turbulent motion take the forms:

$$\begin{aligned}\frac{\partial}{\partial t}\left(\frac{U_{wi}^2}{2}\right)&\\ =-\Bigg[&U_{wi}\frac{\partial}{\partial x_j}\tilde{U}_i\tilde{U}_j+U_{wi}U_{wj}\frac{\partial \tilde{U}_i}{\partial x_j}+U_{wi}\frac{\overline{\partial U_i'U_j'}}{\partial x_j}\Bigg]\\ &+\nu\frac{\partial}{\partial x_j}\left[U_{wi}\left(\frac{\partial(\tilde{U}_i+U_{wi})}{\partial x_j}+\frac{\partial(\tilde{U}_j+U_{wj})}{\partial x_i}\right)\right]\\ &-\frac{\partial}{\partial x_j}\left[U_{wj}\left(\frac{\tilde{p}+p_w}{\rho}\right)+(\tilde{U}_j+U_{wj})\frac{U_{wi}^2}{2}\right]\\ &-\nu\left[\frac{\partial(\tilde{U}_i+U_{wi})}{\partial x_j}+\frac{\partial(\tilde{U}_j+U_{wj})}{\partial x_i}\right]\frac{\partial U_{wi}}{\partial x_j}+U_{wi}f_{Bi},\end{aligned} \tag{3}$$

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$$\frac{\partial}{\partial t}\left(\frac{\overline{U_i'^2}}{2}\right) = -\left[\overline{U_i'U_j'}\frac{\partial \tilde{U}_i}{\partial x_j}+\overline{U_i'U_j'}\frac{\partial U_{wi}}{\partial x_j}\right] +\nu\frac{\partial}{\partial x_j}\overline{U_i'\left(\frac{\partial U_i'}{\partial x_j}+\frac{\partial U_j'}{\partial x_i}\right)} -\frac{\partial}{\partial x_j}\left[\overline{U_j'\left(\frac{p'}{\rho}+\frac{U_i'^2}{2}\right)}+(\tilde{U}_j+U_{wj})\frac{\overline{U_i'^2}}{2}\right] -\nu\overline{\left(\frac{\partial U_i'}{\partial x_j}+\frac{\partial U_j'}{\partial x_i}\right)\frac{\partial U_i'}{\partial x_j}}, \quad (4)$$

where f_{Bi} is the external force.

The interaction or production terms, the first terms on the right of (3) and (4), are of primary interest. Since the variation of $\tilde{U}_i\tilde{U}_j$ in general is zero, (3) indicates that the wave-induced motion can draw energy from the mean flow through the wave-induced Reynolds stresses $U_{wi}U_{wj}\partial\tilde{U}_i/\partial x_j$ and that it can grow due to the work done by the turbulent Reynolds stresses $U_{wi}\partial\overline{U_i'U_j'}/\partial x_j$. While turbulence constantly draws energy from the mean flow through turbulent Reynolds stresses $\overline{U_i'U_j'}\partial\tilde{U}_i/\partial x_j$, the growth of wave-induced motion, which enhances the local velocity gradient through vortex stretching, will help the growth of turbulent kinetic energy through $\overline{U_i'U_j'}\partial U_{wi}/\partial x_j$ as shown in (4). This interaction process indicates that the rapidly varying turbulent motion could help the growth of the wave-induced motion while in return the wave-induced motion may enhance the rapidly varying turbulence.

2. Interaction subrange spectra

It is well known from the equation of total energy dissipation that the energy associated with wavenumbers not exceeding k may interact with the remainder of the spectrum in the forms of inertial transfer, shear production, and inhomogeneous diffusion. Several equilibrium subranges are thus obtained by considering the relative predominance of the processes.

As shown initially by Tchen (1953) and discussed in more detail by Gisina (1966), there can exist a -1 subrange in the energy spectra of turbulent shear flow. This occurs as a consequence of strong interaction between turbulence and a large mean velocity gradient inside the boundary layer. It results in a strong production of turbulence energy. For a turbulent wind over the air-water interface, this interaction mechanism of Tchen and Gisina may be enhanced through the appearance of wave-induced motion, which increases the local velocity gradient and produces additional interaction terms $\overline{U_i'U_j'}\partial U_{wi}/\partial x_j$. Consequently, the effects of wave-induced motion will show up in the turbulence not only as a spectral peak at the frequency of the water waves, but possibly as a modification of the spectrum over a broad range of frequencies. If the interaction mechanism of the motion induced by the waves and that due to mechanical turbulence can be considered as equivalent to the mechanism of Tchen, then one may expect to find a -1 region in a spectrum of turbulence over waves which is caused by this interaction.

To test the above hypothesis, some published experimental data were examined. Figs. 1a and 1b are the horizontal velocity spectra obtained by Kato and Sano (1969) in a wind-wave tunnel. In the case with mechanical waves (Fig. 1a), the spectra of $M-2$ measured at height $z=40$ cm from the mean water surface display a very dominant peak near 0.6 Hz corresponding to the frequency of the mechanical waves. The spectra at $z=6$ cm have only a small peak near $f=0.6$ Hz, which, according to Kato and Sano, is due to the fact the fluctuations induced by mechanical waves are fairly comparable with and rather hidden in the background turbulence which would exist without those mechanical waves. Both spectra, however, clearly reveal the existence of $a-1$ subrange which is bounded between the frequency of dominant water waves and the Kolmogoroff's $-5/3$ inertial subrange. Similar characteristics are observed in the turbulent spectra over pure wind waves (Fig. 1b). Although these data do not contradict our hypothesis of an interaction subrange, they cannot be considered as a conclusive proof since the -1 subrange in longitudinal turbulence results from the combined effects of the wave-induced motion and the overall mean velocity gradient.

In Fig. 2, two spectra of vertical velocity fluctuations are shown which were measured by Shemdin and Lai (1970) in a wind-wave tunnel. The spectral bump and -1 subrange observed in the longitudinal turbulence appear here also. Since there is no overall mean velocity in this direction, the -1 subrange must be a sole consequence of the nonlinear interaction between the wave-induced local velocity gradient and turbulence Reynolds stresses.

The results shown in Figs. 1 and 2 indicate that the laboratory surface waves perturb the air flow at low frequencies, while the inertial subrange of turbulence appears to be unaffected by the presence of surface waves. The turbulence in the atmosphere over the agitated sea surface may not show this behavior as clearly. Because of its much higher Reynolds number and much larger turbulence scale, atmospheric turbulence has a wide inertial subrange in its spectrum. The sea wave-induced motions, unlike those due to the laboratory-generated waves, generally have a dominant frequency which lies inside the inertial subrange of the

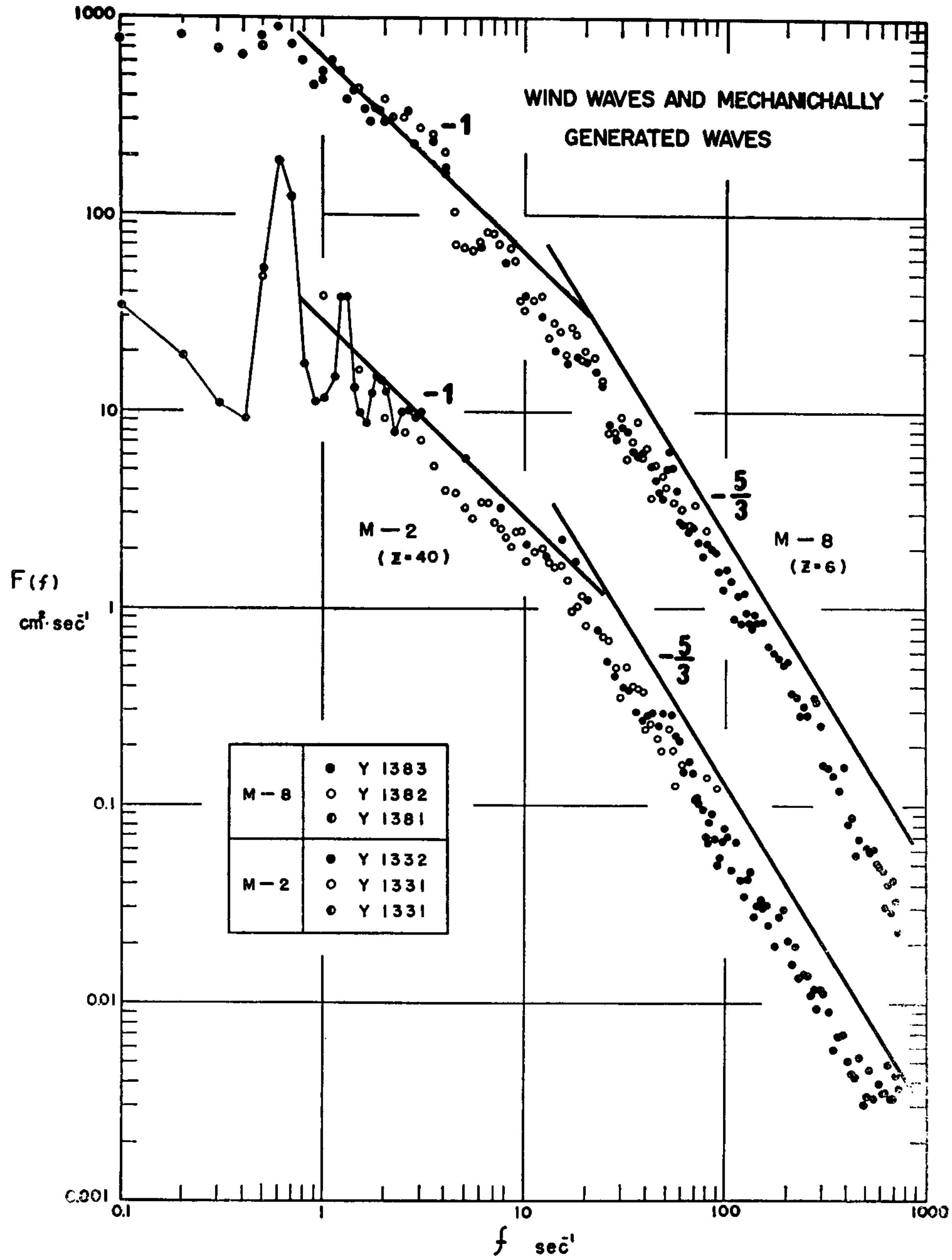

FIG. 1a. Horizontal air velocity spectra over wind waves and mechanically generated waves (from Kato and Sano, 1969).

atmospheric turbulence. The inertial subrange is persistent in preserving its similarity structure, so that wave-induced perturbations are localized in spectral space. The model developed in Section 1 under the assumption that wave-induced motions and background turbulence have distinctively different time scales does not apply in this case. The −1 interaction subrange may thus fail to show up in atmospheric spectra over the sea even though a clearly discernible wave-induced peak is obtained. These characteristics are observed in many atmospheric turbulence spectra above the sea such as those presented by Kitaigorodsky (1969), Volkov (1969) and Volkov and Mordukhovich (1971).

3. Nonlinear interaction in frequency and wave-number-frequency spaces

The previous discussions give only a qualitative picture of the effect of wave-induced motions on the structure of turbulent air. In order to estimate quantitatively

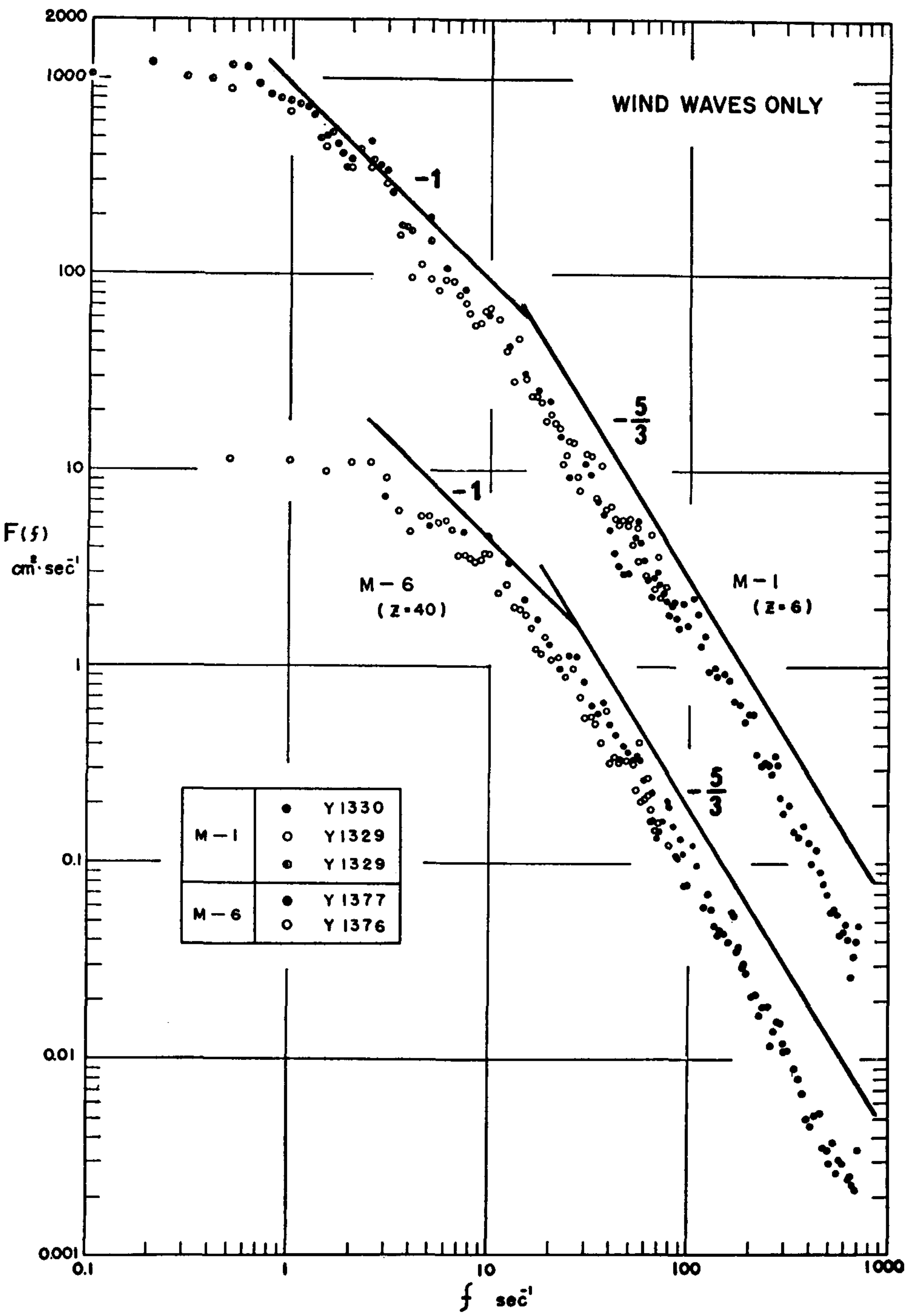

FIG. 1b. Horizontal air velocity spectra over wind waves only (from Kato and Sano, 1969).

how the wave-induced motions with the frequency of underlying water waves transfer energy in spectral space through nonlinear processes, one must analyze the motion in frequency and wavenumber-frequency spaces.

If $q(x,t)$ is a real, single-valued function which is piecewise differentiable, $q(x,t)$ may be transformed into wavenumber-frequency space or into frequency space for a long period of time at a certain point (Kao, 1968). The Fourier transforms and their inverse transforms for these two cases are as follows:

In frequency space only

$$Q(f)=\frac{1}{2\pi}\int_0^T q(t)e^{-ift}dt, \tag{5}$$

$$q(t)=\int_{-\infty}^{\infty} Q(f)e^{ift}df. \tag{6}$$

In wavenumber-frequency space

$$Q(k,f)=\frac{1}{4\pi^2}\int_0^T\int_0^L q(x,t)e^{-i(kx+ft)}dxdt, \tag{7}$$

$$q(x,t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} Q(k,f)e^{i(kx+ft)}dfdk, \tag{8}$$

where k and f represent wavenumber and frequency, respectively.

Considering another scalar function $s(x,t)$ with Fourier transform $S(k,f)$, it can be shown that:

In frequency space

$$\frac{1}{2\pi}\int_0^T s(t)q(t)e^{-ift}dt=\int_{-\infty}^{\infty} S(m)Q(f-m)dm, \tag{9}$$

where

$$S(f)=\frac{1}{2\pi}\int_0^T s(t)e^{-ift}dt.$$

In wavenumber-frequency space

$$\frac{1}{4\pi^2}\int_0^T\int_0^L s(x,t)q(x,t)e^{-i(kx+ft)}dxdt$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S(j,m)Q(k-j,\, f-m)dmdj, \tag{10}$$

where

$$S(j,m)=\frac{1}{4\pi^2}\int_0^T\int_0^L s(x,t)e^{-i(jx+mt)}dxdt.$$

The above convolution relations are useful for transforming the nonlinear terms in the equations of motion into frequency and wavenumber-frequency spaces. This is illustrated in the following by transforming the x-component equations of motion and energy for two-dimensional flow (x,z).

To obtain the energy equation in frequency and wavenumber-frequency spaces, the following notation for the Fourier coefficients is introduced:

$q(x,z,t)$	$u,\ w,\ p,\ f_{B1}$
$Q(k,z,f)$	$U,\ W,\ P,\ F_{B1}$

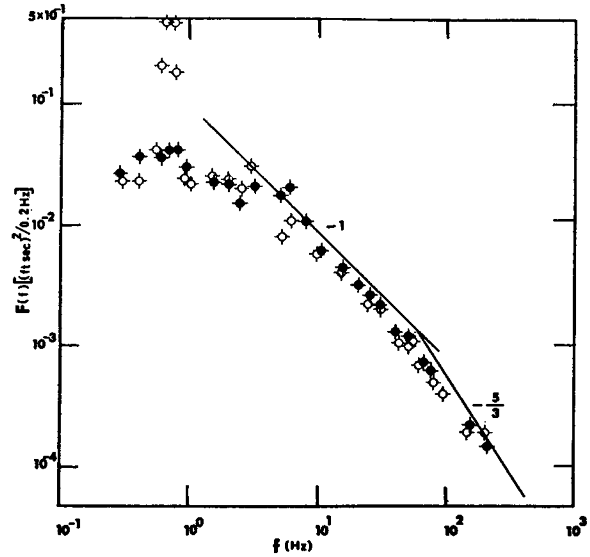

FIG. 2. Vertical air velocity spectra over an air-water interface (from Shemdin and Lai, 1970): solid symbols, over wind waves only; open symbols, over wind waves and mechanical waves.

The resulting energy equation can be shown to take the form:

In frequency space

$$|U(f)|^2=\frac{i}{f}\int_{-\infty}^{\infty}\{\underbrace{U_x(m)U(-f)U(f-m)}_{\text{(U1)}}$$

$$+\underbrace{U_z(m)U(-f)W(f-m)}_{\text{(U2)}}\}dm+\frac{1}{\rho}\frac{i}{f}P_x(f)U(-f)$$

$$-\frac{i}{f}\nu\{U_{xx}(f)+U_{zz}(f)\}U(-f)-\frac{i}{f}F_{B1}(f)U(-f) \tag{11}$$

In wavenumber-frequency space

$$|U(k,f)|^2$$

$$=\frac{i}{f}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\{\underbrace{\frac{i}{2}jU(j,m)U(-k,\,-f)U(k-j,\,f-m)}_{\text{(U1)}}\right.$$

$$\left.+\underbrace{U_z(j,m)U(-k,\,-f)W(k-j,\,f-m)}_{\text{(U2)}}\right\}dmdj$$

$$+\frac{1}{\rho}\frac{k}{f}P(k,f)U(-k,\,-f)-\frac{i}{f}\nu[-k^2U(k,f)$$

$$+U_{zz}(k,f)]U(-k,\,-f)-\frac{i}{f}F_{B1}(k,f)U(-k,\,-f). \tag{12}$$

The subscripts x and z indicate partial differentiation with respect to x and z, respectively, and $i=\sqrt{-1}$. The relation $Q^*(k,f)=Q(-k,-f)$, which denotes the complex conjugate of $Q(k,f)$, has been used to derive these equations.

The terms on the left-hand side of (11) and (12) represent the total spectral energy at a given frequency f, or at given wavenumber k and frequency f. The nonlinear terms represented by U1 in (11) and (12) stem from the transformed horizontal advection of the x-component kinetic energy while those represented by U2 arise from the transformed convection of z-component kinetic energy. The rest of the terms derive from the linear terms of the equations of motion.

The nonlinear terms in transform space involve sums of products of transforms for various wavenumbers and frequencies. In actual computations these are replaced by appropriate summation. For example, the trans-

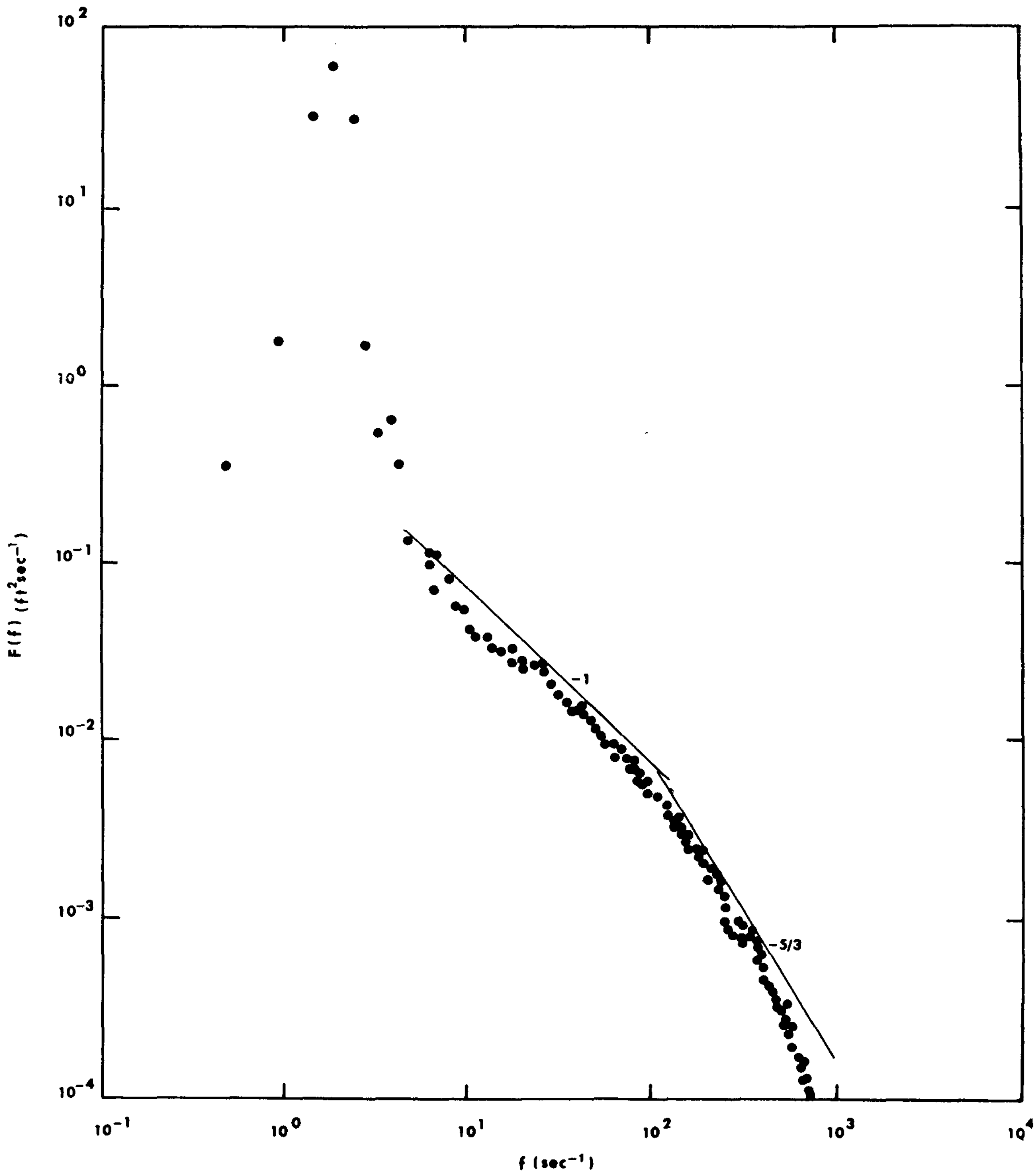

FIG. 3. Horizontal velocity spectrum in a periodic turbulent boundary layer with a dominant disturbing frequency of 1.88 Hz (from Cheng and Chang, 1971).

formed horizontal advection terms in the x-component equation in terms of summation is

$$\frac{i}{f}\sum_{m=-m_s}^{m_s} U_x(m)U(f-m), \tag{13}$$

where m_s is the upper and lower limit of frequency used in the computation. Each product may be considered as an interaction between frequencies represented by (m) and $(f-m)$. When (13) is multiplied by $U(-f)$, it becomes the term denoted by U1 in the energy equation (11) and represents a contribution to the spectral energy $|U(f)|^2$ for a given frequency f, through nonlinear interaction of various frequencies of motion. If m is allowed to vary over their entire range, the term U1 represents the total contribution from all possible interactions. The calculations of U1 and U2 may thus allow one to determine the contribution to the kinetic energy due to nonlinear interactions at a particular frequency, especially the portion associated with the wave-induced motion.

The technique outlined above has recently been employed by Cheng and Chang (1971) to investigate the effects of a periodic motion on turbulence in a periodic turbulent boundary layer. Since that problem and the one discussed in this paper have similar features, both involving the effect of a periodic motion on the characteristics of background air turbulence, one of their results in frequency space is given in the following section for the purpose of illustration.

4. Computation example of nonlinear interaction

The horizontal velocity spectrum obtained by Cheng and Chang (1971) in a periodic turbulent boundary layer (see Fig. 3) possesses a pronounced spectral bump at the disturbing frequency of 1.88 Hz followed by a well defined -1 subrange. This feature is very similar to that observed in the turbulent wind over an air-water interface as discussed in Section 2. To perform the nonlinear interaction calculation, they divided the frequency range of interest equally into ten intervals, I1 to I10, in which the frequency range I1 contains the dominant low-frequency wave and the frequency ranges I2 to I9 are inside the -1 subrange. The results of the interaction contribution to each frequency range are displayed in a block diagram form as shown in Fig. 4 for interval I3. In this diagram the important interaction combinations are designated by placing the frequency range symbols side by side with an asterisk separating them, (I3)*(I1), for example. The total contributions by the terms U1 and U2 are shown in the small blocks in the center of the diagram and labeled with the symbols for the terms they represent. The sum of the quantities U1 and U2 is displayed below the appropriate blocks. Under these sums is shown the sums of the spectral energy over the frequency range. The percentages indicate the energy of the nonlinear interaction terms as compared to the total energy, that is, $|U(f)|^2$, in each frequency range.

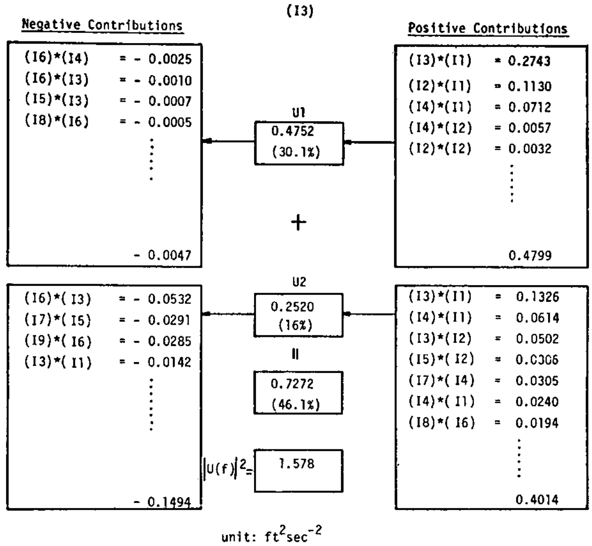

FIG. 4. Contributions of nonlinear interactions in frequency range (I3) which is inside the -1 subrange (from Cheng and Chang, 1971).

The larger blocks, connected with arrows to the small blocks labeled U1 and U2, contain positive and negative interactions which represent the major contributions to the totals in the small blocks. The figures at the bottom of the large blocks represent the total positive and negative contributions from all the interaction combinations. Since the spectral energy $|U(f)|^2$ is inherently a positive quantity, a negative value for an interaction combination for a given frequency range is interpreted as a transfer from the spectral band considered to some other band, while positive values represent a transfer of spectral energy to the band considered from some other band.

As the results shown in Fig. 4 for the interval I3, the most important nonlinear interaction in each frequency range within the -1 interaction subrange spectra is found to be always the interaction of the disturbing wave component I1 and the frequency range under consideration, e.g., (I3)*(I1). Furthermore, the percentage of the nonlinear interactions to the total spectral energy persistently remains high in the -1 subrange, ranging up to 50%. When the frequency range nears the end of the -1 subrange and enters the $-5/3$ subrange, the nonlinear interaction decreases gradually. Similar interaction processes are expected for turbulent air over the air-water interface. Since these computations give detailed information about the effects of wave-induced motion in spectral spaces, such an experimental investigation over the air-water interface is suggested.

In actual data collection over the air-water surface, two-point measurements along the x and z directions are sufficient to calculate U_x and U_z terms in frequency space. In the wavenumber-frequency space, on the other hand, one has to carry out several measurements along the x direction in addition to the two-point measurements along the z direction for each fixed x position. The number of x positions will depend on the wavenumber of interest. All the measurements can be achieved by using hot-wire or hot-film probes. In practice, because of the extremely large number of possible interactions, it is necessary for analysis purposes to make use of a system of classification which places the energy over specified groups of frequencies into particular frequency ranges (Cheng and Chang, 1971). One has to be careful in the selection of an upper limit of frequency m_s; generally this is a compromise between accuracy of computation and computer time.

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REFERENCES

Cheng, I. M., and P. C. Chang, 1971: Nonstationary turbulence in periodic turbulent boundary layers. Tech. Rept. CE 71-042, Dept. of Civil Engineering, University of Utah, Salt Lake City.

Gisina, F. A., 1966: The effect of mean velocity and temperature gradients on the spectral characteristics of turbulence. *Izv. Atmos. Oceanic Phys.*, **2**, No. 8, 487–491.

Kao, S. K., 1968: Governing equations and spectra for atmospheric motion and transports in frequency-wavenumber space. *J. Atmos. Sci.*, **25**, 32–38.

Kato, H., and K. Sano, 1969: Measurements of wind velocity fluctuations over waves in a wind-wave tunnel. *Repts. Port Harbor Res. Inst., Yokosuka, Japan*, **8**, 35 pp.

Kitaigorodsky, S. A., 1969: Small-scale atmosphere-ocean interactions. *Izv. Atmos. Oceanic Phys.*, **5**, 641–650.

Miles, J. W., 1967: On the generation of surface waves by shear flows. *J. Fluid Mech.*, **30**, Part 5, 163–175.

Phillips, O. M., 1966: *The Dynamics of the Upper Ocean.* Cambridge University Press.

Shemdin, O. H., and R. J. Lai, 1970: Laboratory investigation of wave-induced motion above air-sea interface. Tech. Rept. 6, Dept. of Coastal and Oceanographic Engineering, University of Florida, Gainesville.

Tchen, C. M., 1953: On the spectrum of energy in turbulent shear flow *J. Res. Natl. Bur. Stds.*, **50**, 51–62.

Volkov, Y. A., 1969: The spectra of velocity and temperature fluctuations in airflow above the agitated sea surface. *Izv. Atmos. Oceanic Phys.*, **5**, 723–729.

——, and M. I. Mordukhovich, 1971: Spectra of turbulent fluxes of momentum and heat over a wavy sea surface. *Izv. Atmos. Oceanic Phys.*, **7**, 11–16.

Yefimov, V. V., and V. L. Pososhkov, 1970: The dynamics of wave disturbances of atmospheric boundary layer by swell. *Izv. Atmos. Oceanic Phys.*, **6**, 358–362.