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THE EXTRACTION OF GEOPHYSICAL PARAMETERS FROM RADAR ALTIMETER RETURN FROM A NONLINEAR SEA SURFACE

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1. INTRODUCTION

Radar altimeters have been flown on a number of satellites (GEOS-3, SEASAT, GEOSAT) and been found to give useful geophysical information about the ocean. The basic altimetric measurement is the distance between the satellite and the sea surface, which allows the determination of mean sea surface slopes and hence surface currents, under an assumption of geostrophy. In addition the shape of the return pulse has been found to give information on significant waveheight, while the backscattered power has been used to estimate surface wind speeed.

In order to extract this information, models of the radar return, based on specular reflection from the sea surface, have been proposed by Barrick (1972) and Brown (1977). These assume that waves on the sea surface are linear and therefore the corresponding statistics of surface elevations and slopes are Gaussian. Such models have, for example, allowed good estimates of significant waveheight to be obtained (see Webb, 1981).

In practice, however, it is known that surface waves are nonlinear and this will affect the radar return. Jackson (1979) analysed the effects of nonlinearity on the radar return for the case of undirectional waves. Here we generalise his analysis to allow for directional spreading in the wavefield. Allowing for nonlinear wave effects in the model introduces two more parameters that can be estimated from the radar return, the skewness of the sea surface, which is a measure of the nonlinearity of the wavefield, and a "cross-skewness" parameter, which enables corrections to be made for "sea-state bias" in the altimetric height measurement.

Having derived a model for the radar return we describe how the geophysical parameters (altimetric height, significant waveheight, skewness, cross-skewness and backscattered power, the latter being related to wind speed) can be estimated from an actual return. In order to obtain unbiased estimates of the parameters we employ maximum likelihood estimation, which is known to give asymptotically unbiased estimators with minimum variance. This approach also allows us to calculate the variance-covariance matrix for the estimators and thus determine the accuracy of the estimates.

2. A MODEL OF THE RADAR RETURN

Following Brown (1977) we note that the radar altimeter return from the sea surface $P_r(t)$ may be written as the following three-fold convolution

$$P_r(t) = P_{FS}(t) * q(\zeta) * P_{PT}(t) \qquad (1)$$

where P_{FS} is the flat surface response
P_{PT} is the point target response
and q is the probability density function of the elevations of specular points on the sea surface

Here t is the time, measured such that t=0 corresponds to the mean level of the specular points. The height ζ is converted to time via

$$\zeta = \frac{-ct}{2} \qquad (2)$$

where c is the speed of light.

Brown (1977) shows that (1) may be reduced to the following expression

$$P_r(t) = \int_0^\infty \int_{-\infty}^\infty \left(\frac{c}{2}\right) P_{PT}(t-\tau)\, q\left(\frac{c}{2}(\tau-\hat{\tau})\right) d\tau\, d\hat{\tau} \times \begin{cases} P_{FS}(0) & \text{for } t<0 \\ P_{FS}(t) & \text{for } t\geq 0 \end{cases} \qquad (3)$$

where

$$P_{FS}(t) = \frac{\alpha\,\sigma^0}{h^3} \exp\left\{-\frac{4}{\gamma}\sin^2\xi - \frac{4c}{\gamma h} t \cos 2\xi\right\} . I_0\left(\frac{4}{\gamma}\sqrt{\frac{ct}{h}} \sin 2\xi\right) . \qquad (4)$$

Here α and γ are constants depending on the radar parameters (see Brown, 1977), σ^0 is the backscattered power and h is the height of the satellite. ξ is the pointing angle of the antenna, ideally equal to zero. In practice the antenna does not always point at nadir so we allow for this effect in our analysis and note that generally $\xi<0.5°$. The point target response is given by (Brown, 1977)

$$P_{PT}(t) = \eta P_T \exp\left\{-\frac{t^2}{2\sigma_p^2}\right\} \qquad (5)$$

where η is the pulse compression ratio
P_T is the peak transmitted power
and σ_p is a measure of the pulse width.

Finally from Srokosz (1986) we have for the distribution of specular points

$$q(\zeta) = \frac{1}{\sqrt{2\pi}\,\sigma_s} \exp\left\{-\frac{\zeta^2}{2\sigma_s^2}\right\} . \left\{1 + \frac{1}{6}\lambda H_3(\zeta/\sigma_s) - \frac{1}{2}\delta H_1(\zeta/\sigma_s)\right\} \qquad (6)$$

where σ_s is the standard deviation of sea surface elevation
λ is the skewness of the sea surface
and δ is a "cross-skewness" parameter related to the normalised expectation of the elevation and slope squared (see Srokosz, 1986, for details).

The H_n are Hermite polynomials with

$$\left.\begin{aligned} H_3(x) &= x^3 - 3x \\ H_2(x) &= x^2 - 1 \\ H_1(x) &= x \;. \end{aligned}\right\} \qquad (7)$$

The probability density function of specular points (6) is obtained from a weakly nonlinear dynamical model of the surface waves due to Longuet-Higgins (1963). He obtained the probability density function for the surface elevation

$$p(\zeta) = \frac{1}{\sqrt{2\pi}\,\sigma_s} \exp\left\{-\frac{\zeta^2}{2\sigma_s^2}\right\} \cdot \left\{1 + \frac{1}{6}\lambda H_3\left(\frac{\zeta}{\sigma_s}\right)\right\} \qquad (8)$$

as a Gram-Charlier series (modified Gaussian). Longuet-Higgins (1963) also considered the joint distribution of slopes $p(\zeta_x, \zeta_y)$, while Jackson (1979) considered the joint distribution of elevation and slope $p(\zeta, \zeta_x)$. Srokosz (1986) has extended these results to obtain the probability density function of elevation and slopes $p(\zeta, \zeta_x, \zeta_y)$, from which (6) is obtained.

To obtain the form of the return it is necessary to evaluate the double integral in (3) (denoted by $I(t)$). The inner integral may be considered as the convolution of two probability density functions and so may be easily evaluated by adding the cumulants of the two functions (or directly by integration) to yield

$$I(t) = \int_0^\infty \frac{\eta P_T \sigma_p}{\sigma} \left[1 + \frac{1}{6}\lambda_3 H_3\left(\frac{t-\hat{\tau}}{\sigma}\right) + \lambda_1 H_1\left(\frac{t-\hat{\tau}}{\sigma}\right)\right] \cdot \exp\left\{-\frac{(t-\hat{\tau})^2}{2\sigma^2}\right\} d\hat{\tau} \qquad (10)$$

where

$$\sigma^2 = \sigma_p^2 + 4\sigma_s^2/c^2$$
$$\lambda_1 = \delta\left(\frac{\sigma_s}{c\sigma}\right) \qquad (11)$$
$$\lambda_3 = -\lambda\, 8\left(\frac{\sigma_s}{c\sigma}\right)^3 .$$

The remaining integral (10) can now be evaluated by a change of variable $v = (t - \tau)\ \sqrt{2}\,\sigma$ and use of standard results for Hermite polynomials (Abramowitz & Stegun, 1965, chapter 22) to yield

$$I(t) = \frac{\eta P_T}{2}\sqrt{2\pi}\,\sigma_p \left[1 + \mathrm{erf}\left(\frac{t}{\sqrt{2}\,\sigma}\right) - \frac{1}{\sqrt{2\pi}} e^{-t^2/2\sigma^2}\left\{2\lambda_1 - \frac{1}{3}\lambda_3 H_2\left(\frac{t}{\sigma}\right)\right\}\right] . \qquad (12)$$

Together with (3) this specifies the form of the radar altimeter return from the sea surface.

From equations (3),(4),(11) and (12) it can be seen that the return depends on three wave parameters σ_s (the significant waveheight $H_s = 4\sigma_s$), λ and δ together with σ^o, ξ and h. We note that for $\lambda = \delta = 0$, that is the linear case, the result reduces to that given by Brown (1977).

3. EFFECTS OF WAVE NONLINEARITY ON THE RETURN

To obtain accurate measurements of the height of the mean sea level, to be used in calculating geostrophic currents, it is necessary to allow for "sea-state bias". This bias in the altimetric height measurement is due to nonlinear wave effects. For example, the mean level of the sea surface is given from (8) by

$$\int_{-\infty}^{\infty} \zeta p(\zeta)\, d\zeta = 0 \qquad (13)$$

while the mean level of the specular reflectors is given from (6) by

$$\int_{-\infty}^{\infty} \zeta q(\zeta)\, d\zeta = -\frac{\delta}{8} H_s \qquad (14)$$

In the linear case $\delta = 0$, so the two results are identical. Fig. 1 illustrates this result diagrammatically.

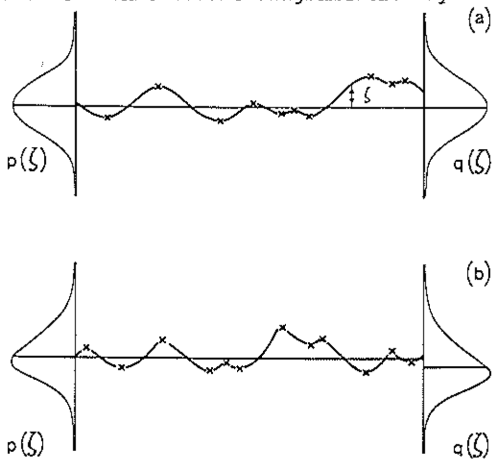

Fig. 1 Schematic diagram showing the difference between the pdfs of surface elevation $p(\zeta)$ and specular points (x) $q(\zeta)$ for (a) linear waves and (b) nonlinear waves. Note that for the nonlinear case the mean level of the specular points differs from that of the surface elevation.

The need for a "sea-state bias" correction arises because the "on-board" trackers used to determine height from satellite altimeter returns make no allowance for nonlinear wave effects. Therefore the trackers incorrectly determine the mean level of the sea surface. Most trackers (for example, those on GEOS-3, SEASAT and GEOSAT) determine the mean level from the position in time of the half power point of the return. This corresponds to determining the median of the distribution of specular points. In the Gaussian case ($\lambda = \delta = 0$) this is the same as the mean and hence the same as the mean level of the sea surface (see (13) and (14)). In general, however, the median and the mean of the specular reflectors differ and they both differ from the mean level of the sea surface. Thus tracking the half-power point of the return does not give the mean sea surface position. From the results given in Srokosz (1986) it can be seen that this leads to errors in the height measurement of order 10 cm, which is the same as the accuracy necessary to estimate geostrophic surface currents.

The method described in the following sections, for extracting geophysical parameters from the altimeter return, avoids these problems as it fits the theoretical return model to the actual return rather than using the half-power point to estimate the mean level of the sea surface.

4. EXTRACTION OF GEOPHYSICAL PARAMETERS

In order to obtain geophysical information from the return we must estimate the six parameters σ_s, λ, δ, t_0, σ° and ξ. The time origin t_0 must be included as it is unknown with respect to the time of transmission of the pulse and it gives the height of the satellite. In developing the model of the return we referred all measurements in time to the mean level, which is one of the parameters we are attempting to estimate. This may be done by replacing t by $(t - t_0)$ in the model return and estimating.

Typically the altimeter tracker will produce estimates of the return power at a number of gates M (for SEASAT and for ERS-1 M = 63) situated in time such that the middle gate falls approximately on the mid-point of the leading edge of the return. The model of the return developed in section 2 assumes that the reflections from the sea surface are all in phase, in practice this is not so and it can be shown (Ulaby, Moore and Fung, 1982) that the return power $\hat{g}_i$, from a single pulse, measured at the i^{th} gate has a negative exponential distribution with mean equal to the theoretical return power g_i, thus

$$f(\hat{g}_i) = \frac{1}{g_i} e^{-\hat{g}_i/g_i} . \qquad (15)$$

A reasonable assumption to make if the altimeter moves at least the diameter on the antenna between transmitting pulses and if adjacent gates in the receiver do not overlap is statistical independence of the return power in each gate.

In practice N pulses are averaged together before data is transmitted from the satellite to ground (for SEASAT N = 100 and for ERS-1 N = 50, with a pulse repetition frequency of 1000 Hz). From (15) the average of N pulses will have a gamma, or chi-squared distribution

$$f(\hat{g}_i) = \frac{N^{N-1}\hat{g}_i^{\,N-1}}{N!\; g_i^{\,N}} \exp\left(-N\hat{g}_i/g_i\right) \qquad (16)$$

where g_i is the return form given in section 2 (that is, $P_r(t)$) and depends on six parameters σ_s, λ, δ, t_0, σ°, ξ, which will be denoted by θ_j (j=1,..,6) for convenience.

Maximum likelihood estimation gives, asymptotically, minimum variance unbiased estimators of the parameters and so we will use this method to estimate the θ_j. From (16) and the assumption of independence the likelihood of an averaged pulse is given by

$$L = \prod_{i=1}^{M} \frac{N^{N-1}\hat{g}^{N-1}}{N!\; g_i^{\,N}} \exp\left(-N\hat{g}_i/g_i\right) \qquad (17)$$

and the log likelihood by

$$LL = \sum_{i=1}^{M} \left\{ (N-1)\ln N + (N-1)\ln \hat{g}_i - N\hat{g}_i/g_i - N\ln g_i - \ln N! \right\}. \qquad (18)$$

Now g_i is a function of n parameters θ_j (j=1,..n) which we wish to estimate (here n=6, but for generality we will develop the theory for arbitrary n). To derive the maximum likelihood estimators we take derivatives of (18) with respect to θ_j and set the resulting expressions to zero. Thus

$$\frac{\partial LL}{\partial \theta_j} = N \sum_{i=1}^{M} \left(\frac{\hat{g}_i}{g_i^2}\right) \frac{\partial g_i}{\partial \theta_j} - N \sum_{i=1}^{M} \frac{1}{g_i} \frac{\partial g_i}{\partial \theta_j} \quad j=1,\ldots,n \qquad (19)$$

and so

$$\sum_{i=1}^{M} \left(\frac{\hat{g}_i - g_i}{g_i^2}\right) \frac{\partial g_i}{\partial \theta_j} = 0 \quad j=1,\ldots,n \qquad (20)$$

are the n simultaneous equations that need to be solved for the maximum likelihood estimators of θ_j.

It is also possible to derive the variance-covariance matrix $\underline{\underline{V}}$ of the estimators. This is the inverse of the Fisher Information matrix $\underline{\underline{F}}$ and so

$$\underline{\underline{V}} = \underline{\underline{F}}^{-1} \qquad (21)$$

where

$$\underline{\underline{F}} = \left\{ E \left(\frac{\partial LL}{\partial \theta_j} \cdot \frac{\partial LL}{\partial \theta_k}\right) \right\} \quad j,k=1,..,n. \qquad (22)$$

Here E denotes the expectation operator (see Cox and Hinckley, 1974, for details both of maximum likelihood estimation and Fisher information).

From (19) we obtain

$$\frac{\partial LL}{\partial \theta_j} \cdot \frac{\partial LL}{\partial \theta_k} = N^2 \sum_{i=1}^{M} \sum_{l=1}^{M} \left\{ \frac{(\hat{g}_i - g_i)}{g_i^2} \frac{(\hat{g}_l - g_l)}{g_l^2} \frac{\partial g_i}{\partial \theta_j} \frac{\partial g_l}{\partial \theta_k} \right\}$$

while from the properties of the gamma distribution and our assumption of independence between gates and pulses we have

$$E[(\hat{g}_i - g_i)(\hat{g}_l - g_l)] = \frac{g_i^2}{N} \delta_{il}$$

where δ_{il} is the Kronecker delta. Together with (22) these give the following result for $\underline{\underline{F}}$

$$\underline{\underline{F}} = \left\{ N \sum_{i=1}^{M} \frac{1}{g_i^2} \frac{\partial g_i}{\partial \theta_j} \frac{\partial g_i}{\partial \theta_k} \right\} \quad j,k=1,\ldots,n . \qquad (23)$$

From this the inverse may be calculated numerically to obtain the variance-covariance matrix $\underline{\underline{V}}$.

In the above we have described the theoretical basis of maximum likelihood estimation of the geophysical parameters from the return. We have also shown that it is possible to obtain the variance-covariance matrix of the estimators, which gives a measure of the possible error in our estimates. Although the mathematical theory is elegant its implementation is rather more complicated due to complex form of the theoretical return derived in section 2. It is therefore necessary to implement the estimation procedure numerically. This has been done and some results will be presented in the following section.

5. RESULTS AND DISCUSSION

In order to test the method of estimation return pulses have been simulated from the model return given in section 2 together with their statistics described in section 4. Fig. 2 gives an example of both the mean return P_r and the average of 50 individual returns for a specific set of parameters (those of the ERS-1 altimeter; for comparison with an actual SEASAT return see Webb, 1981). The noise on the return can be reduced by averaging a larger number of pulses. In practice it is necessary to balance the number of pulses averaged to reduce noise, against the distance travelled by the satellite in acquiring the data, as the geophysical parameters may vary along the satellite ground track (typically the satellites travel at 7 km per second over the ground).

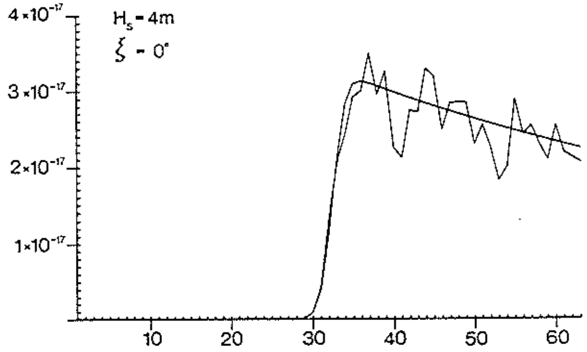

Fig. 2 The mean return P_r (normalised by the transmitted power) plotted against gate number of the ERS-1 altimeter, together with the average of 50 returns to illustrate the departure from the mean of an actual return. (The gate spacing is 3 nanoseconds.)

Fig. 3 illustrates the effect of antenna mispointing on the return pulse. Ideally the antenna should point at nadir but in practice drag on the satellite and differential heating due to solar radiation leads to a small amount of mispointing (for SEASAT about 0.2°) which varies slowly on the time scale of a satellite orbit (approximately 100 minutes). The main effect of the mispointing of the antenna is to reduce the backscattered power and a secondary effect is to change the slope of the trailing edge of the return. As a direct consequence of this it is clear that it is not possible to simultaneously estimate the backscattered power σ° and the pointing angle ξ from the return. This becomes even clearer when we calculate the variance-covariance matrix for a number of cases and hence find that the correlation between σ° and ξ is greater than 0.9999. Furthermore the variance of the estimators of σ° and ξ is large and tends to infinity as ξ tends to zero. This occurs because it is not possible to determine ξ from the amplitude of the return so the only remaining information in the return on ξ is the slope of the trailing edge, but as ξ tends to zero the slope of the trailing edge is almost constant, so it cannot be used to determine ξ. In fact the Fisher Information matrix becomes singular at ξ=0 showing that there is no information on ξ in the return.

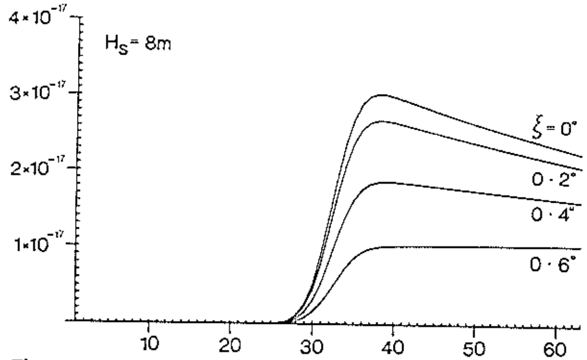

Fig. 3 The mean return P_r (normalised by the transmitted power) plotted against gate number, for various values of the pointing angle ξ. (The gate spacing is 3 nanoseconds.)

This result has implications for the extraction of wind speed information from σ°. If variations in backscattered power due to variations in wind speed and those due to pointing angle cannot be separated then clearly it will not be possible to determine the wind speed accurately from the return. Various authors (Brown, 1977; Barrick and Lipa, 1985) have suggested that the pointing angle can be estimated from the slope of the

trailing edge of the return but have failed to note the strong correlation between σ° and ξ. It would seem that to use the backscattered power to obtain accurate wind speed estimate necessitates the independent determination of the pointing angle ξ. Alternatively a biased estimation procedure or use of information from successive returns (rather than the single return considered here) may allow ξ to be estimated, albeit in a non-optimal fashion. Whether these approaches would give adequate wind speed information from the return remains to be investigated.

The results described above, although preliminary, show the power of mathematical and statistical models to give insight into practical problems. A further area that is currently under investigation, using the techniques described above, is that of the estimation of the nonlinear wave parameters λ and δ and the correction of the sea-state bias in the height measurement described in section 3. If geostrophic surface currents are to be obtained from altimetric height measurements a good understanding of this problem is necessary. It is hoped to report results on this topic in a future paper.

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THE ASSIMILATION OF SATELLITE ALTIMETER DATA INTO OCEAN MODELS

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1. INTRODUCTION

Despite the problems that still exist in interpreting the data, satellite measurements of the sea surface are quietly changing the way in which we study the ocean. Two important examples of the way in which GEOS and Seasat altimeter data has been used is the work of Woodworth and Cartwright (1986) in mapping the world wide distribution of tides and that of Cheney, Marsh and Beckley (1983) in mapping the distribution of the synoptic-scale eddy field. During the next five years further ocean surface surveillance satellites are due to be launched which should give more accurate measurements of the surface topography and the wind stress. The latter is important because it drives most of the near surface circulation. Improved radiometer measurements will also give information on surface temperatures and on the radiative heat fluxes which drive the deep thermo-haline circulation of the ocean.

In the U.K., in addition to the work of Woodworth, there has also been an interest in using the altimeter data to study the surface current field. At Imperial College, Marshall (1985) has investigated the problem of distinguishing the effects of ocean currents from those due to errors in the geoid. At Oxford University interest has centred on the oceanography of equatorial regions, where the high speed of planetary waves in the ocean helps to simplify the analysis of the altimeter data (Anderson and Moore 1986).

At I.O.S. we have a special interest in mid-latitude oceans where the Coriolis effects are large. In such regions the surface slope of the ocean is, to a first approximation, balanced by the Coriolis force acting on the surface currents of the ocean. This means that the radar altimeter measurements