Comments on "Wave-induced Stress and the Drag of Air Flow over Sea Waves" and "Quasi-linear Theory of Wind-Wave Generation Applied to Wave Forecasting"

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31 March 1992 and 13 October 1992

In two papers P. A. E. M. Janssen (1989, 1991; hereafter J89 and J91) investigated the effect of gravity waves on the mean wind profile and the dynamical coupling between wind and waves. The approaches demonstrated in these articles are different, but since they are devoted to the same problem, it is convenient to discuss them jointly.

Janssen was the first to suggest the 1D hydrodynamical theory of the wave boundary layer (WBL) above sea waves taking into account directly the 2D wave spectrum. In principle, Janssen's equations may be derived by simplifications of 2D Reynolds equations of the WBL used in numerical modeling (Chalikov 1986). All previous work on this topic was devoted to the analysis of experimental data on the basis of more or less plausible hypothesis. Unfortunately, Janssen's approach does contain some inaccuracies, which will be discussed in this note.

The specific feature of the boundary layer above waves arises from the additional momentum flux produced by the wave fluctuations of velocity, pressure, and stresses. The author supposes that the "effect of waves is similar to the effect of molecular viscosity" and represents the equation for momentum balance in the form

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial z^2} + \frac{\partial \tau_t}{\partial z}, \qquad (1)$$

where the first term on the right side describes the effect of wave-produced momentum flux (WPMF) τ_w; the second, effect of turbulent momentum flux τ_t; and D, the "wave diffusion coefficient" derived from Miles' quasi-laminar theory.

First, it is doubtful that WPMF may be represented in common case by some sort of local diffusion hypothesis. The WPMF, unlike turbulence, is formed by a well-organized pressure field, and not by a random process. It may be directed to waves down or to air up depending on the ratio of wind and phase velocities [see experimental and numerical results by Donelan and Hui (1990) and Chalikov (1986), respectively]. This effect cannot principally be described with the help of a diffusion term. The reason for the discrepancy is that WPMF, in contrast to the turbulent momentum flux, is produced not by the wind itself but by an external factor, a moving water surface. In this case the simple connection between wind profile and WPMF is not so evident.

Second, if we consider the diffusion representation simply as an approximation, the final equation [(18) in J89 and (2) in J91] turns out to be uncommon because the viscosity coefficient is a function of height and the diffusion term cannot be represented in divergent form. As a result, to satisfy Newton's second law the author is forced to consider the stress, which depends on velocity and "wave diffusion coefficient" profiles in the layer above to infinity [(8) in J91]. When the phase velocity approaches the group velocity, the coefficient D goes to infinity. The situation is formally saved only due to the result of molecular viscosity, which cannot be the governing parameter for turbulent flow above rough surface. It should also be noted that it is incorrect to call this problem two-dimensional; this is a one-dimensional problem.

The fact that the author uses Miles' theory only for deriving the vertical profile of D, but not for the computation of momentum exchange seems doubtful because the vertical profile of the WPMF obtained with Miles' theory is not in agreement with its surface value computed with an empirical formula.

Furthermore, the author notes that the evolution of the wind profile is "determined by two competing mechanisms, namely air turbulence that attempts to maintain a logarithmic wind profile and the combination of diffusion due to viscosity and surface waves that tries to maintain a linear wind profile."

First of all, molecular viscosity does not influence the structure of the boundary layer above rough surfaces. Second, the wave diffusion coefficient is able "to maintain a linear wind profile" only because of inaccuracies in the diffusion term. In fact, neglecting the

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effect of turbulence, we can get the linear wind profile from Eq. (1), but its gradient is independent of D and even equal to unit. More importantly, we cannot neglect the second term in Eq. (1) even for qualitative considerations because it provides the true momentum balance in WBL. The author needs the linear wind profile because in this case the quasi-laminar theory formally prescribes the absence of interaction. But the Miles theory is not intended for the case of constant viscosity. In this case there also exists the flux of momentum and it can be easily computed. Obviously, the author used this artifact of the theory as an essential basis for consideration.

The WPMF is computed in J89 by the integration of elementary fluxes over frequencies from zero to infinity. If we extrapolate the linear dependence (Snyder et al. 1981) for the wind–wave interaction parameter to infinity (which is incorrect), the short waves will give a very large contribution, and this integral converges very slowly. Note that it is more reasonable to assume a quadratic form of the wind-wave interaction parameter at high frequencies (Donelan and Hui 1990). Then, for the case of the Phillips spectrum $S \propto \omega^{-5}$, the integral diverges, and introducing an upper-limit frequency and parameterization of the "tail-produced" momentum flux becomes more evident.

In J89 the disturbances considered are so small that they sink completely into the viscous sublayer (the height of such waves is smaller than 1 mm). Otherwise the author could not obtain the divergence.

Such small waves cannot be considered as waves at all because they have very short lifetimes and do not have dispersion relation. Besides, we do not know the spectrum in this range. Further, because the form of the diffusion term is incorrect, it works as a strongly increased molecular term that "tries to maintain" the wind profile desired. As a result, the integral for momentum formally converges but its numerical value is unpredictable.

Figure 1 in J89 represents the dependence of the surface value of WPMF on wave age. This dependence needs additional comments. It is easy to estimate that under moderate and strong wind the high-frequency surface disturbances are large enough, and the surface may be considered rough. This means that all of the momentum flux transfers by the pressure field, and if we had known the form of wind-wave interaction parameter and spectrum up to the very high frequencies, we could have obtained the total momentum flux by taking the integral type of (15) in J89 over frequencies. In the case of stationarity this momentum flux is equal to turbulent momentum flux outside the WBL. In the WBL, smaller and smaller waves approaching the surface contribute to the stress until it reaches the limiting external value, for example, turbulent stress outside the WBL. This raises the question: to what height may the curves in Fig. 1 (J89) be referred?

In J89 the contribution of waves does not turn out to be large enough, and Janssen parameterizes the tail-produced stress using the roughness parameter. Therefore, the effect of the smallest waves is taken into account twice: through turbulent tangential stress and directly through wave-produced stress. It would be more natural, for example, to restrict the integration of the wave fluxes by "cut" frequency and to use the character height of subgrid waves for estimation of the local roughness parameter.

It should also be noted that it is incorrect to use the Charnock relation for the calculation of the roughness parameter. The coefficient in Charnock's formula was derived from the data at sufficiently large heights, where the wave-produced drag is already negligibly small. The essence of this relation is that it gives the effective (total) roughness parameter outside the layer of direct wind-wave interaction (for a fully developed sea), which takes into account all the mechanisms of drag formation including the wave drag. Knowing this quantity is all we need; solving any equations in this case becomes irrelevant. So, the influence of the low-frequency part of the spectrum is also taken into account twice: directly through WPMF and indirectly through the Charnock relation for the total roughness parameter.

The physical sense of the total roughness parameter is very clear. If WPMF is directed downward, it increases the total stress (which is the sum of turbulent and wave-produced stress), and above the layer of direct wind-wave interaction the effective roughness parameter turns out to be larger than the local one. It is interesting that if the waves as a whole are faster than the wind, then the wave-produced momentum flux is directed opposite to the turbulent one and the total roughness parameter may be smaller than the local one. The very fast waves that are running in the wind direction may turn out to be very smooth.

Note that the spectral calculation of the WPMF profile in J89 and J91 was performed by the traditional method of using the wind velocity value at a fixed height for all frequencies. In reality, the energy and momentum input for each wave is formed in a layer whose height depends on the wave frequency. The introduction of friction velocity instead of wind velocity is only a reformulation of the same approach and cannot change anything.

In J91 the author mentioned an idea of dividing the wave spectrum into two parts: a low-frequency part, which is described with the WAM model, and a high-frequency subgrid part. But, in fact, this decomposition was not used. This approach was introduced by Chalikov (1978) and utilized in many following calculations (see the review by Chalikov 1985; Chalikov and Makin 1991; Chalikov and Belevich 1993).

In practice, we do not know the properties of high-frequency momentum exchange and we are restricted by the vertical resolution of the numerical model. This means that we introduce the "cut" frequency for wave spectrum and minimal height h_r above the surface where boundary conditions are prescribed. Because it is impossible to take into account all the waves, we are forced to parameterize the influence of the spectral "tail" in the form of local tangential drag. This problem is very close to the problem of parameterizing subgrid turbulence in the large-scale eddy simulation approach. In both cases the parameterizing of the high-frequency part of the spectrum is possible due to the universal structure of the high-frequency range.

The ratio of the WPMF and turbulent momentum flux depends on vertical and spectral resolution of the numerical model. The low-frequency part is taken into account in an integral type of (1) in J89. This part of the momentum transfers to the waves. It is assumed that the high-frequency tail (taken into account in formulation of the tangential stress law) produces the momentum flux directly to the current.

It seems that the ratio of these fluxes depends on arbitrarily chosen cut frequency, and the "true" relation between wave stress and turbulent stress is unknown.

Nevertheless, there is no contradiction in this uncertainty. In fact, the overwhelming part of momentum is transferred to all waves and surface disturbances by form drag. However, all waves vanish in the end, and their momentum passes to the currents. The lifetime of very short waves is small and that of long waves is so large that they can even reach a remote shore. So, the value of momentum flux to the currents obtained with WBL theory and dissipation function for sea waves depends only on time and space scales of the wave dissipation. In the scope of ocean–atmosphere coupled models, the correct description of these phenomena may be reached on the base of joined WBL, sea waves, and mixed-layer model.

Many equations and formulas from paper J89 are transferred in paper J91, where actually a much more simplified problem is solved. The author did not introduce the wave-produced momentum flux at all and took into account the wave's effects only through the modification of the roughness parameter. Janssen did write the finite-difference equations (31) for a nonstationary problem, but obviously did not solve it, because the truncation errors in this scheme must be larger than any desired wave effects. Of course it is more reasonable to use the quasi-stationarity assumption. The suggested formula for a wind profile is:

$$u(z) = \frac{u_*}{k} \ln \frac{z + z_1}{z_0 + z_1}, \tag{2}$$

where z_0 is the roughness parameter computed with the Charnock relation and z_1 is the additional parameter intended for describing the wave's effects. Then using the logarithmic wind profile (2), the turbulent stress at $z = z_0$ is found to be

$$\tau_t = (\tau_t + \tau_w)\left(\frac{z_0}{z_0 + z_1}\right)^2. \tag{3}$$

The combination of (2) and (3) gives a formula for z_1 which together with the log profile is presented as the solution of the problem.

The formula (2) may be considered to be pure approximation without any physical sense (because it is unlikely that roughness parameters are additive quantities). It seems more natural to suppose that the friction velocity at small heights (2) must be defined using the turbulent stress τ_t but not the total $\tau_t + \tau_w$. Introducing this supposition into (2), we obtain a contradiction. From a formal point of view we can operate only with the roughness parameter $z_0 + z_1$, and then we obtain the trivial identity. So, the formula for roughness parameter results from algebraic error. All connections of this type cannot be correct because WPMF is a strong function of height.

It is more important to state that such speculations with the roughness parameter are fundamentally incorrect because this quantity characterizes something that does not exist, for example, a parameter that we introduce in the wind profile to obtain the wind velocity needed. Close to the surface, this approximation is not valid, and a roughness parameter loses its sense. Simply speaking, operating with a roughness parameter is only possible at heights much larger than the roughness parameter itself.

The true 1D general solution for the wave boundary layer may be obtained from the equation

$$\frac{\partial}{\partial z}\left(K\frac{\partial u}{\partial z} + \tau_w\right) = 0. \tag{4}$$

The numerical calculation shows that influence of turbulent energy diffusion is negligible. Let us suppose that the local production of turbulent energy is equal to the local dissipation. The energy equation derived from (4) has the form

$$\frac{\partial}{\partial z}\left(uK\frac{\partial u}{dz} + u\tau_w\right) - \left(K\frac{\partial u}{\partial z} + \tau_w\right)\frac{\partial u}{\partial z} = 0. \tag{5}$$

The first term in this equation represents the vertical energy transfer by turbulence and by wave-produced motions. The second term is the rate of the transformation of energy of the mean motion and wave-produced motion into turbulent energy. Because the sum $K(\partial u/\partial z) + \tau_w = \tau$ is constant over height, the turbulent energy balance equation takes the form

$$\tau\frac{\partial u}{\partial z} - \frac{K^3}{l^4} = 0. \tag{6}$$

From (6) we can obtain a formula for the coefficient of turbulence

$$K = l^{4/3}\tau^{1/3}\left(\frac{\partial u}{\partial z}\right)^{1/3} \tag{7}$$

and turbulent momentum flux

$$\tau_t = l^{4/3}\tau^{1/3}\left(\frac{\partial u}{\partial z}\right)^{4/3}. \tag{8}$$

Equation (7) does not coincide with expression (3) in J91 because the author neglected the wave-produced production term, although it may be derived from Eq. (2) of J91. The momentum balance equation takes the form

$$l^{4/3}\tau^{1/3}\left(\frac{\partial u}{\partial z}\right)^{4/3} + \tau_w = \tau. \tag{9}$$

Assuming that the wave-produced momentum flux diminishes with height, and at a small height $z = z_r$ some local drag law is valid,

$$\left(K\frac{\partial u}{\partial z}\right)_r = \varphi(u_r), \tag{10}$$

we can obtain the formula for the wind profile

$$u = \varphi^{-1}(\tau - \tau_w) + \frac{\tau^{1/2}}{k}\int_{z_r}^{z}\frac{(1 - \tau_w/\tau)^{3/4}}{z}dz, \tag{11}$$

where φ^{-1} is an inverse function to φ.

More specified results may be derived from Eq. (11) using some assumptions on the shape of the wave-produced momentum flux profile. Equation (11) shows that if the wave-produced momentum flux is directed to the waves ($\tau_w > 0$), the deviation from the logarithmic wind profile is negative. A more detailed derivation of the equation similar to (11) was done in (Chalikov and Belevich 1993).

The interesting idea of coupled experiments with the one-point WAM model and a boundary-layer model must be realized in a more general approach than was done in J91 (and, of course, with an improved model of the WBL). In these calculations the wind velocity u_{10} was fixed, so that the coupling might influence only the stress. Also it is possible to fix the stress and to allow the wind to change. In these cases we obtain, of course, different results. The question arises, Which of them is right? The answer is—none. In real nonstationary coupling both the stress and wind profile change, and in order to calculate these changes it is necessary to introduce the boundary conditions at a sufficiently large height. The correct coupling may be reached only in a coupled ocean–atmosphere model where all mechanisms of momentum transformation are taken into account. If we consider the stationary problem, both boundary conditions are appropriate and the solution provides us with the relations between these variables. In any case, the energy input to the waves depends on wind and momentum flux profile. The existing schemes do not take into account this evident property.

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