Ocean Engng. Vol. 8, pp. 17–24.
Pergamon Press Ltd. 1981. Printed in Great Britain

0029-8018/81/010017-08 $02.00/0

WAVE RESPONSE OF A SPAR BUOY WITH AND WITHOUT A DAMPING PLATE

LUIGI CAVALERI

Istituto per lo Studio della Dinamica delle Grandi Masse, C.N.R., S. Polo 1364, Venice, Italy

and

ERIK MOLLO-CHRISTENSEN

Department of Meteorology, Massachusetts Institute of Technology, Cambridge, MA 02139, U.S.A.

Abstract—Report on a computer analysis of wave response of a spar buoy that has been operated successfully in the Mediterranean over the past ten years, to find out how the presence of a large horizontal plate at the bottom affects its wave response. The calculations show that the addition of a damping plate decreases heave response for short waves but increases the response for very long waves.

INTRODUCTION

WHILE discus-shaped data buoys offer many operational advantages and conveniences, such as easy towing and relatively easy boarding for servicing in moderate weather, such buoys still represent stability problems in large size. For small discus buoys, where the natural pitch and roll frequencies are above the frequency of the dominant gale sea, the buoy will ride well, responding in phase with sea surface slope. A larger discus buoy, once it is large enough to have a pitch frequency below the frequency of a typical gale sea, will, in such a case, respond out of phase with sea surface slope, and risk tumbling into an approaching wave.

Even if spar buoys may be less convenient in setting and instrumenting, they do offer certain advantages due to their stability. Besides the point that they will not tumble when excited above pitch resonance frequency, they also provide a more stable antenna platform communicating with a geostationary satellite, for example. A conical antenna emission pattern tangent to the direction of the equatorial plane, would offer savings in antenna power. Yet another possible advantage of a spar buoy is its potential ability to duck under an ice floe, especially with a mooring attachment well below the water line.

The reason for equipping the spar buoy with a large horizontal plate at the bottom was to minimize heave by adding to the virtual mass of the buoy, and also to provide some damping through the drag of the plate. The numerical analysis of the planar motion of the buoy provides a basis for future improvements in design of spar buoys for instrument platforms.

BUOY DESCRIPTION

The buoy is shown in Fig. 1. The design is based on an earlier design reported by Mollo-Christensen and Dorman (1971), and was modified for the specific needs of the present user by Cavaleri and built by Greppi (Italy) in 1970. The buoy is now being used by the Consiglio Nazionale delle Ricerche of Italy in the Ligurian Sea.

A full description of the buoy can be found in Cavaleri (1974). The dimensions and characteristics of the buoy are given in Table 1.

TABLE 1. BUOY CHARACTERISTICS

Buoy dimensions	
Length (excluding the mast)	41.5 m
Tube diameter	0.609 m
Weight (including ballast)	11,300 kg
Reserve buoyancy	1800 kg
Design water line (below top of tube)	6 m
Ballast Metal	1000 kg
Ballast Water	3750 kg
Mast height above water line	13 m
Mooring attachment distance below water line	−23.5 m
Damping plate area (including tube)	4.9 m²

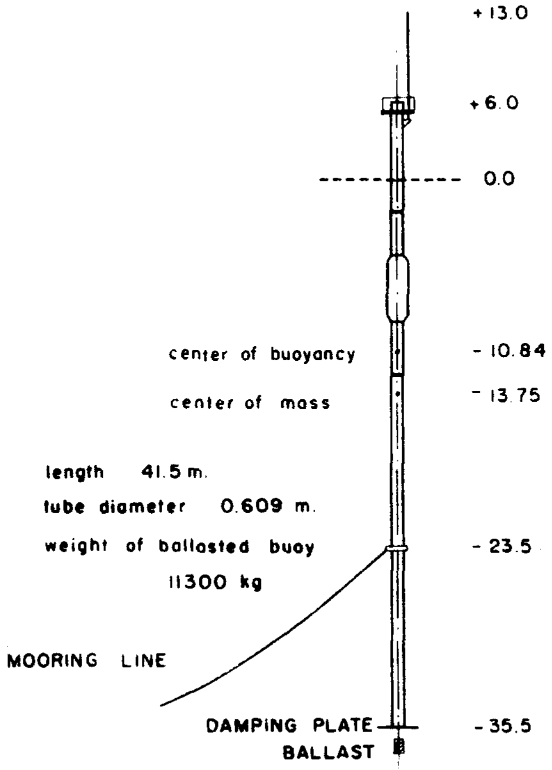

FIG. 1. Buoy configuration—main characteristics are given in Table 1, dimensions in meters.

The moorings used have been a slack line, consisting of 70 m of stainless steel wire attached to a line heavier than water next to the buoy and a line that floats further down, attached to a short chain next to the anchor, as shown in Fig. 2. This combination has worked well, and the buoy has been set in water of depth from 50 to 2600 m, and has survived for up to two years at anchor between overhauls. Wave crests at least 6 m above mean water line hit the buoy during the many gales in the operational period.

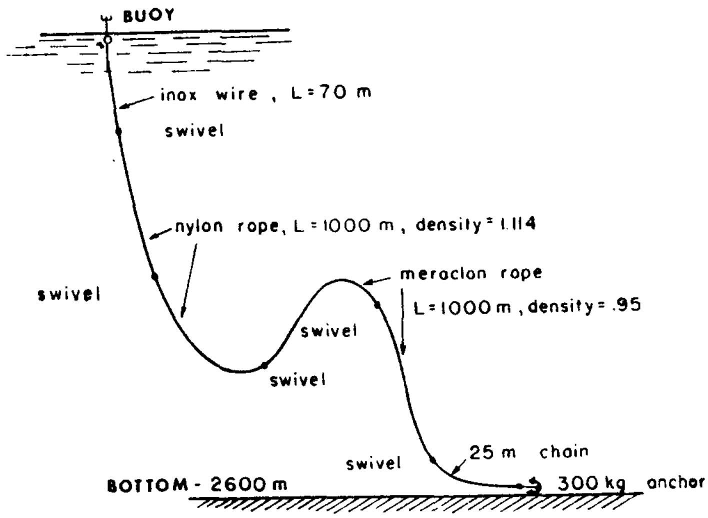

FIG. 2. Mooring arrangement in 2600 m depth.

EQUATIONS OF MOTION

Motion of the buoy is considered in a plane containing the buoy itself, the mooring line and the surface wave propagation vector. Three degrees of freedom are considered: vertical displacement x (heave), horizontal displacement y (surge) evaluated at the mass center of the buoy and rotation α (pitch).

The surface wave field is taken to be a simply harmonic wave of amplitude a and circular frequency σ. With k the wave number, in deep water conditions the velocity potential is given by (Lamb, 1945)

$$\varphi = \frac{ga}{\sigma} e^{kx_2} . \sin(kx_1 - \sigma t)$$

where t is time, g is gravity acceleration, x is horizontal coordinate and depth x_2 is considered positive downwards from the surface. The velocity vector is:

$$\mathbf{u} = \frac{gak}{\sigma} \cos(kx_1 - \sigma t),$$

$$\mathbf{v} = \frac{-gak}{\sigma} \sin(kx_1 - \sigma t),$$

with **u** and **v** horizontal components respectively.

For each of the three movements the following forces have been considered:

heave
- weight
- flotation
- buoy inertia
- forces from absolute fluid acceleration
- plate drag
- inertia from plate virtual water mass

surge
- buoy inertia
- force from absolute fluid acceleration
- force from relative transversal acceleration
- transversal drag

pitch
- weight
- flotation
- buoy inertia
- force from absolute fluid acceleration
- force from relative transversal acceleration
- transversal drag.

Drag is taken as proportional to the relative velocity square. Lateral drag has been neglected for heave because of small pitch angle. The horizontal and vertical mooring forces are taken to be constant. This avoids the necessity of considering also the mooring dynamics during the analysis of buoy motion.

Considering all the above mentioned forces leads to the three following equations for the three movements of the buoy:

$$
\begin{aligned}
&A_1\ddot{x} + B_1 x = \varphi_1, \\
&A_2\ddot{y} + B_2\ddot{\alpha} = \varphi_2, \\
&A_3\ddot{y} + B_3\ddot{\alpha} + (C_3 + D_3\varphi_3)\,\alpha - A_4\varphi_4\alpha x + B_4\,\alpha x^2 = C_4\varphi_5 - D_4\varphi_6.
\end{aligned}
$$

Here A_i, B_i, C_i, D_i are constants depending on the buoy characteristics. φ_i are functions of time and of amplitude and frequency of the exciting wave. The above system of three differential equations has been solved for different combinations of wave amplitude a and frequency $f = \sigma/2\pi$. The following values have been considered:

$a =$	1	1.5	2.5	3.5	5	m			
$f =$	0.027	0.031	0.035	0.041	0.049	0.061	0.070	0.081	0.098
	(36.9)	(32.8)	(28.7)	(24.6)	(20.5)	(16.4)	(14.3)	(12.3)	(10.2)
	0.122	0.163	0.244	Hz					
	(8.2)	(6.1)	(4.1)	s.					

Numbers in brackets show the period for the corresponding frequency. All the combinations of these a and f values have been tested. The results are reported in the following.

RESPONSE TO SINUSOIDAL WAVES

For the symmetrical excitation and response under consideration, the results of numerical solution are shown vs wave period for each wave amplitude. The calculations were done for a buoy with and without a damping plate. Figures 3 and 4 show the ratio of heave amplitude to wave amplitude. Note that the damping plate, installed to minimize heave, does not seem to do any good for higher and longer waves. For waves of 20 s period and 10 m height, the maximum vertical velocity due to the waves at the depth of the damping plate (− 35.5 m) is more than 1 m/s. The damping plate, making the buoy to respond in heave, does more harm than good. Eliminating the plate will also make it easier to tow the buoy in its horizontal position with the lower water ballast tank filled with air. Note the response near the heave resonance period of 23 s. Below this wave period, the response is out of phase with the waves, while for longer period excitation, the response is in phase. Note that the response ratio is less than unity, and for a wave period of 12 s, typical of a strong gale, the response ratio is close to 0.6. Therefore, for a steep wave of 12 s period, taking $2\pi a/\lambda = 0.015$ m, the heave amplitude will be 3 m; since the excitation frequency is above the heave resonance frequency, the immersion of the buoy will extend to 8 m above the mean water line, since the response is in opposite phase. In this respect, a discus buoy with its much larger reserve buoyancy, will be superior to a spar buoy, in the sense that it will ride the wave with the heave amplitude nearly equal to wave amplitude. But the crucial difference between the two kinds of buoys is in their pitch response. Figure 5 shows the pitch amplitude in degrees vs wave period and amplitude. While the angles are relatively small, because of the distance between the masthead and the center of gravity of the buoy, equal to 26.75 m, the amplitude of oscillation of the masthead due to pitch will, for $\alpha = 7°$, equal 3.3 m. This is of course well known to anyone who has been aloft on a ship in a seaway. In comparison with a discus-type buoy, the pitch response of the spar buoy is modest. For a very steep wave, the maximum sea surface slope is 23°. If the discus buoy is small, so that its natural pitch period is less than wave period, the pitch response will be in phase with the wave slope.

For a large discus-type buoy, with a weight distribution such that the pitch period exceeds the wave period, the pitch response will be opposed in phase to the wave slope. The geometry of the buoy will then be crucial as to whether the buoy can rise or be caught and topple in steep, short-crested seas of period less than the buoy pitch period. This is the risk that must be balanced against the operational convenience of discus-type buoys.

The horizontal translation (surge) response, expressed as horizontal amplitude/wave amplitude, is shown in Fig. 6. Note that for wave periods below 20 s, the response increases with increasing wave amplitude, while, in a very long swell of period 25 s, the smaller the wave amplitude, the larger the response ratio. We suggest that this is an effect of using a hydrodynamic drag proportional to the square of the velocity. This will give a damping that increases with wave amplitude. Again, to compare the spar buoy with a discus-type buoy, the latter, which tows much more easily than a vertical spar, may be more sensitive to mooring dynamics.

A spar buoy may well prove to have a larger surge amplitude than a discus buoy in moderate sea, but in large amplitude waves, the comparison depends crucially on the large amplitude pitch dynamics of the buoy.

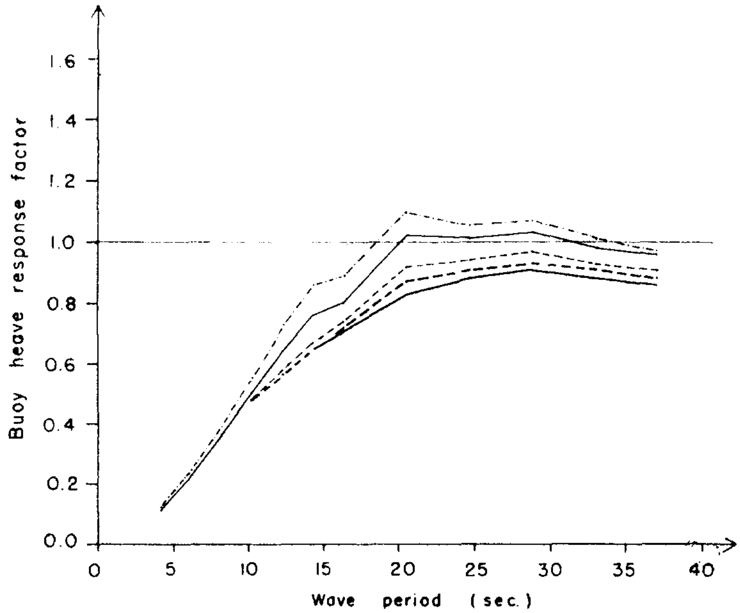

FIG. 3. Heave response factor (heave amplitude/wave amplitude) of the buoy, with damping plate. Wave amplitudes as indicated in Fig. 5.

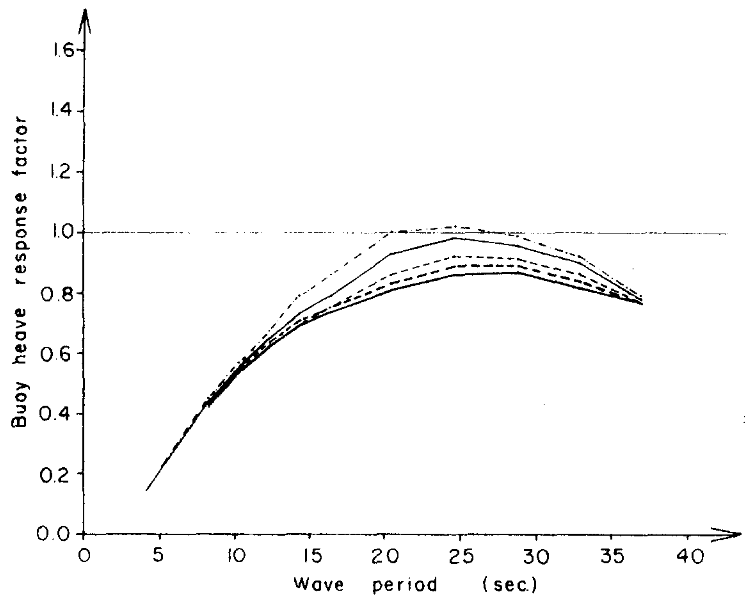

FIG. 4. As in Fig. 3, but without damping plate. Wave amplitudes as in Fig. 5.

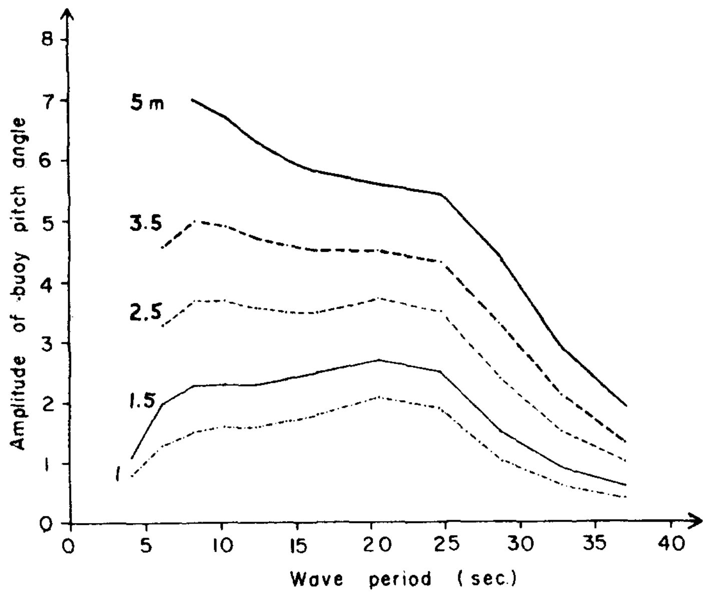

FIG. 5. Pitch response of the buoy. The presence of the damping plate does not affect the results. Wave amplitudes in meters are shown close to each line.

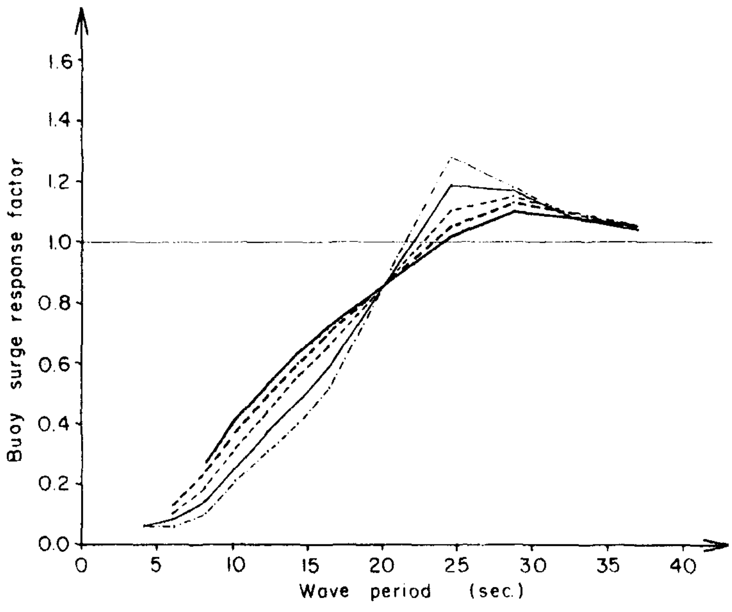

FIG. 6. Surge response factor (surge amplitude/wave amplitude) of the buoy. The presence of the damping plate does not affect the results. Wave amplitudes as in Fig. 5.

CONCLUSIONS

In spite of the awkward handling of spar buoys in hoisting, towing and for on-board stowage, their wave response and stability characteristics offer certain apparent advantages in that spar buoys will right themselves after being forced into horizontal position, as could happen during ice encounter. The buoy described has performed well over a number of years, and the mooring system and the buoy have proven themselves.

The numerical analysis shows that the presence of a damping plate at the lower end of the buoy strongly affects its heave response to wave excitation. In particular, while for higher frequencies and limited wave height there appears to be some advantage to a damping plate, at lower frequencies and larger wave heights, i.e. in the range where the buoy is heavily excited, the presence of the damping plate leads to an increased response in heave. There are no practical consequences for surge and pitch. We conclude that the decision to add a damping plate has to be based on the range of wave height and period expected. Leaving the plate off will make the buoy move more readily in really severe weather.

Acknowledgements—The original M.I.T. buoy, used in U.S. waters for air–sea interaction studies, was designed, built and operated with funding from the Office of Naval Research and the National Science Foundation. The CNR2 buoy, representing a modification and simplification of the previous design, was designed, built and operated under support of the Consiglio Nazionale delle Ricerche, Italy.

Piero Sguazzero, of the IBM Research Center, formerly of Venice, has helped in the mathematical aspect of the problem. The calculations have been run on the IBM computer 370/145 of the Center. We thank the director, Luciano Lippi, for his permission to use the machine.

REFERENCES

CAVALERI, L. 1974. *The Meteo-Oceanographic Buoy CNR-2*, TRN. 76, C.N.R. Istituto per lo Studio della Dinamica delle grandi Masse., S. Polo 1364, 30125, Venezia, Italy.

LAMB, Sir H. 1945. *Hydrodynamics*, Dover, 739pp.

MOLLO-CHRISTENSEN, E. and DORMAN, C. 1971. *A Buoy System for Air–Sea Interaction Studies*, Report No. 71-4, M.I.T., Department of Meteorology.