ON THE DEPENDENCE OF THE EQUILIBRIUM RANGE OF SEA WAVE SPECTRA WITH WAVE AGE

GERARD CAUDAL
Centre d'étude des Environnements Terrestre et Planétaires (CETP), 10-12 Avenue de l'Europe, 78140 Vélizy, France

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Abstract. Donelan and Pierson have proposed a semiphysical model of the equilibrium sea wave spectrum, based upon a parameterization of wave growth and dissipation terms. Their model is applicable for fully developed seas only. In the framework of Donelan and Pierson's approach, this paper explores the dependence of the equilibrium spectrum upon wave age. To this end, we examine how the dissipation through wave breaking is expected to vary with wave age, according to the approach proposed by Longuet-Higgins in 1969. The constraint imposed by Longuet-Higgins' theory requires an increase of the equilibrium spectrum $F(k,0)$ in the wind direction with increasing inverse wave age U/C_p. This is in accordance with Banner's empirically deduced statement that $F(k,0)$ is proportional to $(U/C_p)^{0.5}$ in the equilibrium range. Our inferred $F(k,0)$ tends to increase more or less linearly with U/C_p (we find $F(k,0)$ proportional to $1+0.25(U/C_p-0.83)$), rather than through a power law. If a power law is fitted we obtain $F(k,0)$ approximately proportional to $(U/C_p)^{0.35}$ for the range $0.83 < U/C_p < 5$. Finally, the roughness length of the air-sea interface is inferred from our modelled spectrum through integration of the form drag over wave number under rough conditions. This shows a wave age dependence that is compatible with measurements of wind stress performed in the field at various wave ages.

Key words: Sea waves, Wave breaking, Spectrum, Fetch

1. Introduction

Phillips (1958) assumed the existence of a universal equilibrium range in the spectrum of wind-generated gravity waves and suggested that the spectral density within this range is determined primarily by dissipation through wave breaking. Then, on dimensional grounds, he obtained the following forms for the frequency spectrum $\Phi(\omega)$ and the omnidirectional wave number spectrum $f(k)$ in the equilibrium range

$$\Phi(\omega) = \alpha_P g^2 \omega^{-5}, \qquad f(k) = \beta_P k^{-4} \tag{1}$$

where ω is angular frequency, k is wave number, g is the gravitational acceleration, and α_P, β_P are the Phillips equilibrium range constants.

More recent observational studies (Toba, 1973; Kawai et al., 1977; Kahma, 1981; Forristall, 1981; Battjes et al., 1987), however, have brought an increasing body of support for an equilibrium frequency spectrum of the form

$$\Phi(\omega) = \alpha_T g u_* \omega^{-4} \tag{2}$$

from near the peak up to a certain non-dimensional upper frequency bound. Here u_* is friction velocity and α_T is the Toba constant. Leykin and Rozenberg (1984)

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argue that the spectral slope transition from ω^{-4} to ω^{-5} occurs at about $3.2\omega_p$ (where ω_p is the angular frequency at the peak), which corresponds to k equal to 10 times the peak wave number k_p.

Banner (1990) performed a compilation of existing wave measurements in the equilibrium range, and proposed a link between those two spectral domains by considering the directional wave number spectrum instead of the frequency spectrum. He showed that the transition of the frequency spectrum from ω^{-4} to ω^{-5} above $\omega/\omega_p \approx 3.5$ could be understood as a consequence of the decreasing directionality of the wave field with increasing wave number. At the same time, the directional wave number spectrum in the dominant wave direction (azimuth $\varphi = 0$) could be simply modelled through the following expression involving the inverse wave age (U/C_p),

$$F(k, \varphi = 0) = 0.45 \times 10^{-4}(U/C_p)^{0.5}k^{-4} \tag{3}$$

from near the spectral peak up to the short gravity-wave scales. Here U is the 10-m wind speed and C_p is the wave phase speed at the spectral peak.

Such a dependence of the equilibrium spectrum as $(U/C_p)^{0.5}$ was deduced notably from Donelan et al.'s (1985) measurements of fetch-limited wind wave growth obtained for $1.5 < \omega/\omega_p < 3.5$, and the model was also shown not to contradict observations made at higher frequencies.

Far above the spectral peak, however, most measurements of wave spectra at sea are performed in the frequency, rather than wave number, domain. At high frequencies (relative to the peak), observations of frequency spectra are difficult to interpret in terms of the underlying wave number spectra, since the measured frequencies are a complex mix of intrinsic wave frequencies and Doppler shifts occasioned by wind drift and orbital velocities of longer waves. Direct measurements of the directional wave spectrum in the spatial domain are therefore desirable. The spatial domain measurements performed by Donelan et al. (1985), based on arrays of height gauges, were able to document the wave spectrum up to $k \approx 10k_p$. At higher wave numbers, measurements were performed in the spatial domain by Banner et al. (1989), based on stereophotogrammetric analysis. Their results tend to show no wave-age dependence in the wavelength range 0.2–1.6 m, contrary to the form of Equation (3). Also, a study performed in the frequency domain by Battjes et al. (1987) tends to indicate no dependence of the high frequency spectrum with wave age. The wave-age dependence of the high-wave number spectrum is thus a controversial question. Makin et al. (1995) showed, however, that the drag of developing seas on the atmosphere depends crucially on the form of the wave spectrum in the equilibrium range. While several experiments performed in the field have allowed measurements of the drag coefficient (or the roughness length) of the sea surface as a function of wave development (Hsu, 1974; Smith et al., 1992; Donelan, 1990; Donelan et al., 1993), measurements of the high wave number spectrum constitute a considerable technical challenge (Banner, 1990).

Further observational and theoretical wave number-frequency spectral determinations are thus needed to provide further direct confirmation of Banner's (1990) model, particularly for wave numbers distant from the spectral peak. Donelan and Pierson (1987) have proposed a semiphysical model of the equilibrium sea wave spectrum, based upon a parameterization of wave growth and dissipation terms. That model, however, is applicable for fully developed seas only. In the framework of Donelan and Pierson's approach, the purpose of this paper is to explore the wave age dependence of the wave number spectrum in the equilibrium range. To this end, we examine how the dissipation through wave breaking is expected to vary with varying wave age, under the assumptions of Longuet Higgins (1969) concerning the rate of energy loss through wave breaking. The case of fully developed seas according to Donelan and Pierson's approach will be presented in Section 2, and the case of fetch-limited situations will be considered in Section 3. As an assessment of the inferred wave age dependence of the equilibrium spectrum, the drag of the sea surface consistent with this spectrum will be compared with observations in Section 4.

2. The Case of Fully Developed Seas

2.1. Donelan-Pierson's semiphysical approach for fully developed seas

Donelan and Pierson (1987) (hereinafter referred to as DP) have proposed a model for the equilibrium range of fully developed seas, which will be briefly reviewed in this section. These authors assume that in the high-frequency portion of the equilibrium range the spectrum results principally from a balance between wind forcing and dissipation terms, ignoring the non-linear resonant interactions. The effect of the terms involved in the equilibrium is described by means of the exponential growth rate β of wave components, which is defined as $\beta = (1/F)(\partial F/\mathrm{d}t)$, where $F(k,\varphi)$ is the two-dimensional (2-D) wave spectrum as a function of wave number k and azimuth φ. β is the sum of a wind input term β_w, a dissipation term through wave breaking β_b, and a viscous dissipation term β_v.

There are several ways to account for the wind input term. Plant (1982) proposed that β_w is proportional to u_*^2, while others proposed that β_w in the wind direction is proportional to some power of $(U(z)/C(k) - 1)$, where $U(z)$ is the wind velocity at some height z and $C(k)$ is the wave phase speed (Snyder et al., 1981; Hsiao and Shemdin, 1983; Al Zanaidi and Hui, 1984). Arguing that the appropriate reference wind should be at some height above the roughness elements that is related to their scale, DP took the following expression for the wind input term in the wind direction (obtained from a fit to Larson and Wright's (1975) data),

$$\frac{\beta_w}{\omega}(k, \varphi = 0) = 0.194 \frac{\rho_a}{\rho_w} \left(\frac{U(\pi/k)}{C(k)} - 1 \right)^2 \tag{4}$$

where ω is angular frequency of sea waves, ρ_a and ρ_w are air and water mass densities, and $U(\pi/k)$ is the mean wind speed at height π/k.

The viscous dissipation term is taken classically as (e.g., Lamb, 1932)

$$\frac{\beta_v}{\omega} = -\frac{4\nu k}{C(k)} \tag{5}$$

where ν is the kinematic viscosity of water.

Finally, the dissipation term through wave breaking is taken by DP as

$$\frac{\beta_b}{\omega} = -\alpha(k^4 F(k,\varphi))^n \tag{6}$$

where α and n are functions of wave number k.

DP assumed that the total growth rate at equilibrium vanishes at equilibrium

$$\beta_w + \beta_b + \beta_v \approx \frac{1}{F}\frac{\partial F}{\partial t} = 0, \tag{7}$$

and obtain immediately the following spectrum of the short waves in the wind direction,

$$F(k,\varphi = 0) = k^{-4}\left[\frac{0.194}{\alpha}\frac{\rho_a}{\rho_w}\left(\frac{U(\pi/k)}{C(k)} - 1\right)^2 - \frac{4\nu k}{\alpha C(k)}\right]^{1/n}. \tag{8}$$

The way $F(k,\varphi)$ is determined for $\varphi \neq 0$ is explained in detail by DP and will not be reproduced here for the sake of brevity.

The dependence of the functions $\alpha(k)$ and $n(k)$ upon k is based on the idea that the breaking process is expected to be different depending upon whether the waves are dispersive or non-dispersive. DP therefore propose two extreme values for the pair (α, n). The first one (α_1, n_1) (dispersive waves) thus corresponds to the limit where k is far from the gravity-capillary transition. DP take $n_1 = 5$ in order that the spectral slope be close to k^{-4}, to conform to observations beyond $\approx 10k_p$ (where k_p is the peak wave number). The constant α_1 is adjusted to fit the spectral levels observed at $10k_p$ by Donelan et al. (1985) at full development, and therefore DP take $\ln\alpha_1 = 22$. For the other extreme values (α_2, n_2), DP take $n_2 = 1.15$ and $\ln\alpha_2 = 4.6$ in order to conform to radar backscatter observations at the Ku band. The transition between those extreme values is obtained by expressing $n(k)$ and $\alpha(k)$ through,

$$n(k) = (n_1 - n_2)\left|2 - \frac{g + 3\gamma k^2}{g + \gamma k^2}\right|^b + n_2 \tag{9a}$$

$$\ln\alpha(k) = (\ln\alpha_1 - \ln\alpha_2)\left|2 - \frac{g + 3\gamma k^2}{g + \gamma k^2}\right|^b + \ln\alpha_2 \tag{9b}$$

where γ (= 74.4 cm^3 s^{-2}) is the surface tension/density ratio for water. In the above expressions, the constant b monitors the sharpness of the transition between both regimes. From radar observations, DP take $b = 3$. For example, expression (9a) implies that $n(k)$ differs from n_1 by less than 5% of the total amplitude of variation $(n_1 - n_2)$, as long as k remains smaller than $\approx$ 0.3 cm^{-1} (which corresponds to a wavelength of 20 cm). As assessed by comparing Equations (9a) and (9b), the behaviour of $\ln \alpha(k)$ is quite similar. The transition to n_2 (respectively $\ln \alpha_2$) occurs between $k = 0.3$ and $k = 3$ cm^{-1}. Furthermore, DP restrict the range of their modelled equilibrium spectrum to $k \geq 10k_p$, a restriction that may be regarded as a consequence of the neglect of wave-wave interactions in their analysis. For those reasons, we will restrict ourselves to the domain $10k_p \leq k \leq 0.3\,\text{cm}^{-1}$ in this paper. This high-frequency domain of the equilibrium range will be characterised by $\alpha(k)$ and $n(k)$ being very close to the constants α_1 and n_1.

2.2. Verification of Longuet-Higgins' constraint of energy loss through wave breaking

Longuet-Higgins (1969) made a semi-theoretical calculation of the spectral level in the equilibrium range for a given shape of the wave spectrum (see also Kahma, 1981). Longuet-Higgins first noticed that in a progressive Stokes wave whose limiting form at the crest is the Stokes 120° angle, the maximum acceleration is $g/2$. On this basis, he assumed that whitecaps will appear whenever the vertical acceleration at the crest approaches $-g/2$. For a linear monochromatic wave with amplitude a and frequency ω, this implies that the condition for breaking will be $a = (g/2\omega^2)$. Based on these grounds, he assumes, for a natural sea, that wave breaking occurs when the wave amplitude a exceeds the limiting value

$$a_0 = \frac{g}{2\bar{\omega}^2} \tag{10}$$

where $\bar{\omega}$ is the mean frequency, given by

$$\bar{\omega}^2 = \int_0^\infty \omega^2 \Phi(\omega)\ \mathrm{d}\omega/\sigma^2 \tag{11}$$

where $\Phi(\omega)$ is the frequency spectrum and $\sigma^2 = \int_0^\infty \Phi(\omega)\ \mathrm{d}\omega$ is the height variance of the sea waves.

When the wave amplitude exceeds a_0, wave breaking reduces it back to a_0. The mean loss of energy through wave breaking per average cycle $T = 2\pi/\bar{\omega}$ is

$$E_d = \int_{a_0}^\infty \frac{1}{2}\rho_w g(a^2 - a_0^2)p(a)\ \mathrm{d}a \tag{12}$$

where ρ_w is water density and $p(a)$ is the probability distribution function of wave amplitudes a.

Using the Rayleigh distribution for $p(a)$ Longuet-Higgins found that

$$E_d = E\,\exp(-E_0/E) \tag{13}$$

where,

$$E_0 = \frac{1}{2}\rho_w g a_0^2 \tag{14a}$$

and,

$$E = \rho_w g \sigma^2. \tag{14b}$$

Longuet-Higgins applied these results to the case where the spectrum has the Phillips form with a low frequency cut-off

$$\begin{aligned} \Phi(\omega) &= \alpha_p g^2 \omega^{-5} && \text{if } \omega > \omega_1 \\ &= 0 && \text{if } \omega > \omega_1. \end{aligned} \tag{15}$$

As discussed by Longuet-Higgins, Equation (10) for determining a_0 may be questioned, since the value of a_0 is deduced by considering a linear wave, while wave breaking is a nonlinear process. As a matter of fact, Equation (10) is only approximate and a value expected to be more realistic will be proposed below (Equation (20)). Another question that may be discussed in Longuet-Higgins' approach is the time scale associated with the loss of energy by wave breaking. Longuet-Higgins implicitly assumed that the appropriate time scale of the process of breaking is one average wave period. Observations in tank and in nature suggest that a typical active whitecap exists for a time between 1/4 and 1/2 of a wave period (Donelan and Yuan, 1994). Also, a restriction of Longuet-Higgins' approach was to assume that the wave spectrum may be regarded as narrow banded and, correlatively, it gives only a spectrally integrated rate of energy loss through wave breaking, rather than a dissipation damping rate β_b versus wavenumber such as the one given in Equation (6). In this paper, Longuet-Higgins' model will serve as a constraint, which must be fulfilled by integral quantities of the spectrum, and an integral evaluation of energy loss will be sufficient for this purpose. Some support for the relevance of Longuet-Higgins' statements is given by Kahma (1981) who successfully compared saturation levels of the sea wave spectrum inferred from Longuet-Higgins' approach with the results from several field experiments.

Here we shall apply Longuet-Higgins' approach to the case of DP's modelled spectrum presented in the preceding section. In that case the rate of energy loss through wave breaking, E_d, is particularly simple to determine since the functional form of the dissipation term through wave breaking is known (Equation (6)). This gives

$$E_d = \frac{2\pi}{\bar{\omega}}\rho_w g \int_0^\infty \int_{-\pi}^{\pi} \omega\alpha(k^4 F(k,\varphi))^n F(k,\varphi) k \,\mathrm{d}k\,\mathrm{d}\varphi. \tag{16}$$

Combining Equations (13) and (16) gives

$$\int_0^\infty \int_{-\pi}^{\pi} \omega\alpha(k^4 F(k,\varphi))^n F(k,\varphi) k \, \mathrm{d}k \, \mathrm{d}\varphi = \frac{\bar{\omega}}{2\pi}\sigma^2 \exp\left(-\frac{g^2}{8\bar{\omega}^4\sigma^2}\right). \tag{17}$$

Here the expression of the dissipation term due to wave breaking (Equation (6)) has been assumed to hold throughout the spectrum, including the vicinity of the spectral peak. Of course, at low frequencies, the simple equilibrium between wind input and dissipation proposed by DP does not hold, as the effect of the non-linear resonant energy transfer becomes important, especially in the vicinity of the peak. However, since Equation (16) involves an integration over the entire spectrum (from $k = 0$ to infinity), the resonant interactions among the groups of 4 wave numbers do not contribute to the dissipation, since they conserve total action, energy and momentum (e.g., Hasselmann et al., 1994).

DP proposed a composite spectral model by merging the equilibrium spectrum (Equation (8)) with the low frequency observed spectral form of Donelan et al. (1985) for a fully developed wind-generated sea, setting the transition at $k = 10k_p$. We have tested the consistency between DP's composite spectral model and Longuet-Higgin's constraint in the following manner. Let us denote the left-hand and right-hand sides of Equation (17), A and B respectively. From DP's composite model, we have computed both quantities A and B separately, for various wind conditions. The resulting quantities A and B exhibit similar behaviours with wind velocity, but differ from each other by a factor of more than 10. The reason for the large discrepancy resides in the exponential term in expression B, which results in large variations as soon as a small error on a_0 occurs. As discussed by Longuet-Higgins, the criterion for breaking (Equation (10)) is approximate, because it was obtained through a linearized approach, whereas it concerns an obviously non-linear effect. Also, laboratory tests performed by Ochi and Tsai (1983) indicate that the wave breaking criterion for irregular waves is lower than for regular waves, and may be estimated as

$$H \geq H_0, \quad \text{with } H_0 = 0.020\, gT^2 \tag{18}$$

where H is the crest-to-trough height, and T is the time interval between positive wave maxima. In terms of wave amplitude $a = H/2$ and wave angular frequency $\omega = 2\pi/T$, this would give

$$a \geq \frac{0.79g}{2\omega^2}. \tag{19}$$

Compared to Equation (10), these results lead us to introduce a slight modification to a_0, replacing it by

$$a_0' = 0.79a_0 = \frac{0.79g}{2\bar{\omega}^2}. \tag{20}$$

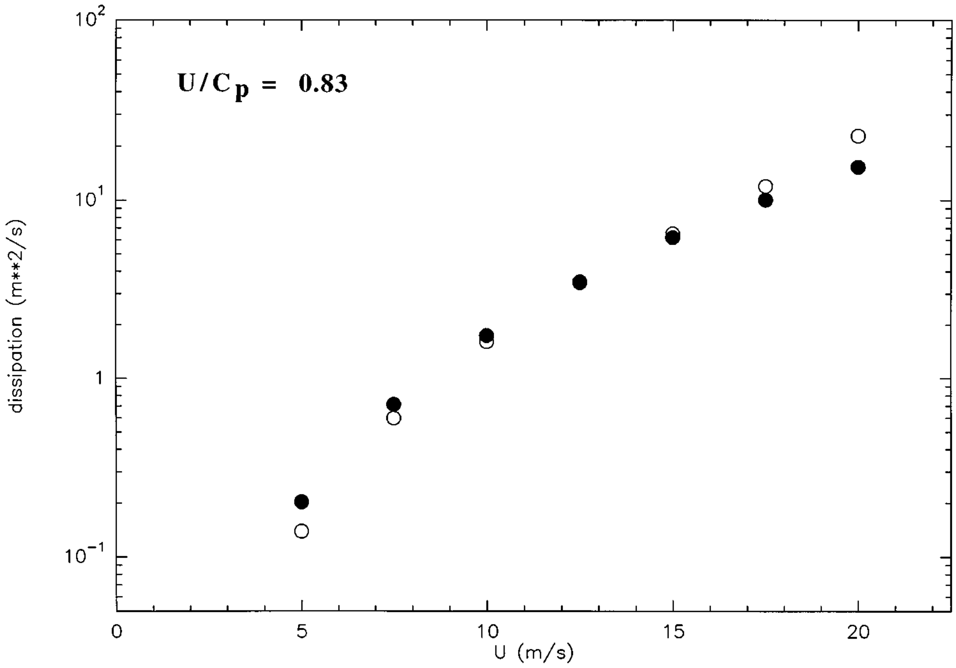

Figure 1. Comparison between quantities A (= left-hand side of Equation (17), solid circles) and B (= right-hand side of Equation (17), open circles), consisting of two independent computations of the dissipation rate through wave breaking. A fully developed wind-generated sea is assumed $(U/C_p = 0.83)$.

With this re-scaled limiting amplitude, the quantities A and B are fairly similar, and their variations as a function of wind speed are displayed in Figure 1. Even though they vary over almost two orders of magnitude between $U = 5$ m s^{-1} and $U = 20$ m s^{-1}, we see that both quantities A and B remain within a factor of 1.5 of each other.

3. The Case of Fetch-Limited Wind Generated Seas

3.1. General Comments

We now wish to generalise DP's equilibrium spectrum to the case of fetch-limited situations. In this case, local spectral equilibrium requires that wave components in the equilibrium range should satisfy the following equation, instead of Equation (7)

$$(\beta_w + \beta_b + \beta_v)F \approx \mathbf{C_g}.\nabla F. \tag{21}$$

Here $\mathbf{C_g}$ is the wave group velocity vector. A calculation was performed to assess the order of magnitude of $\mathbf{C_g}.\nabla F$ in comparison with the magnitude of the wind input source term $\beta_w F$. The relation between U/C_p and fetch was taken from Donelan et al. (1985), β_w from Hsiao and Shemdin (1983), and the spectrum from Banner (1990). For $k \geq 10k_p$, and $0.83 \leq U/C_p \leq 5$, $\mathbf{C_g}.\nabla F$ is found to be at least three orders of magnitude smaller than the wind input term in the wind direction, and is therefore insignificant. As a consequence, it is still allowable to express the equilibrium range of developing seas through Equation (8).

An important aspect of DP's approach is that they neglected nonlinear wave-wave interactions in their balance equation for fully developed seas, for $k > 10k_p$. The question arises as to whether this approach is justified during active wave growth. Janssen et al. (1994) have discussed the spectral energy balance in a growing sea, in the light of results of numerical computation. They stress that a small deviation from the equilibrium shape would give rise to a large nonlinear source term that will drive the spectrum back to its equilibrium shape. Thus, the nonlinear wave-wave source term will provide an important relaxation process toward equilibrium. However, they found that the nonlinear wave-wave source term at equilibrium is quite small in the 'high'-frequency range of a young wind-generated sea (see their Figure 3.9 and discussion therewith). As in Section 2, we will thus restrict ourselves to analysing the spectral equilibrium in the range $10k_p \leq k \leq 0.3$ cm^{-1}, and will neglect the nonlinear wave-wave interactions in the spectral balance.

3.2. Wave breaking as a coupled or uncoupled process

In the expression for the wave breaking term (Equation (6)), it is noticeable that $\beta_b(k)$ is related only to the wave spectral level at wave number k, independently of the spectral level at other wave numbers. This means that DP have made the assumption that the whitecapping process is essentially local in wave number space (uncoupled process). A different approach has been taken by Hasselmann (1974) and Komen et al. (1984). These authors proposed an expression for β_b involving integral quantities of the whole wave number spectrum (rms height and mean frequency).In such a situation the quantities α and/or n in Equation (6) would need to be wave-age dependent. The differences between these approaches have been described in detail by Donelan and Yuan (1994). The consequence of DP's assumption that the breaking process is uncoupled is that the resulting modelled equilibrium spectrum (Equation (8)) exhibits virtually no wave-age dependence.

Makin et al. (1995) have briefly reviewed a number of models of the wave-age dependence of the wave spectrum in the high frequency part (i.e., well above the spectral peak). They tested two hypotheses, one with the high frequency spectrum given by unaltered DP's model (our Equation (8)), and therefore virtually independent of wave age, and one in which a $(U/C_p)^{2/5}$ dependence was introduced. They showed how these hypotheses influenced the drag coefficient, but recognized that the actual dependence of the high wave number spectrum with wave age is still a controversial question. We shall see in the next sections how the constraint on the wave breaking energy permits us to consider that question.

3.3. Longuet-Higgins' criterion if wave breaking is taken as an uncoupled process

Let us for a while make the assumption that $\alpha(k)$ and $n(k)$ do not depend upon the stage of wave development. This hypothesis corresponds to assuming that the breaking process is uncoupled (since β_b in Equation (6) then depends on $F(k, \varphi)$ only). If this hypothesis is correct, it is thus possible to construct a model of the sea spectrum for fetch-limited situations by merging DP's high wave number spectrum (Equation (8)) with a model of fetch-limited spectrum at low wave number. Several empirical models of the low wave number spectrum $(k < 10k_p)$ are available. Here we used the parameterization proposed by Donelan et al. (1985), which was given for inverse wave ages U/C_p ranging from 0.83 (fully developed sea) to 5.

In a similar way as done in section 2.2, we have then tested the ability of this fetch-limited composite model spectrum to fill Longuet-Higgin's constraint of energy loss through wave breaking. The re-scaled limiting amplitude a'_0 determined from the full development case (Equation (20)) was used. In order to determine $F(k, \varphi = 0)$ from Equation (8) for given 10-m wind speed U, one needs to know the wind profile $U(\pi/k)$. The latter may be modelled as the logarithmic profile provided that the drag coefficient is known. Here the drag coefficient was estimated by means of Caudal's (1993) model based on integration of the wave drag over the spectrum (that model is strictly valid for $U \geq 7$ m s^{-1} only but, even though the results presented below might be considered as rather tentative for $U < 7$ m s^{-1}, the effect of an inaccurate estimation of the drag coefficient was found to be very weak indeed).

An example of the results, obtained for $U/C_p = 3$, is given in Figure 2. One sees that quantities A and B now differ by a factor of 10 to 22 over the whole range of wind speeds from 5 to 20 m s^{-1}. The discrepancy increases with U/C_p, and the ratio between A and B reaches $\approx$ 65 to 150 (depending upon wind speed) for $U/C_p = 5$. There seems to be no way to reconcile our hypothetical fetch-limited composite spectrum with Longuet-Higgins' constraint, except by accepting that wave breaking does not behave as a local process in wave number space.

3.4. Results when wave breaking is taken as a coupled process

From the discussion above it follows that a semi-physical model of the equilibrium spectrum based on DP's approach should be wave age dependent. In the following, we shall thus allow the wave breaking term β_b at a given wave number to depend not only on the spectral level at this wave number, but also on the sea state, i.e., on integral quantities of the wave spectrum (e.g., rms height, mean frequency). Here we shall not discuss which integral quantities are the most suitable, but we shall explicitly introduce a dependence of β_b upon wave age. In order to obtain a

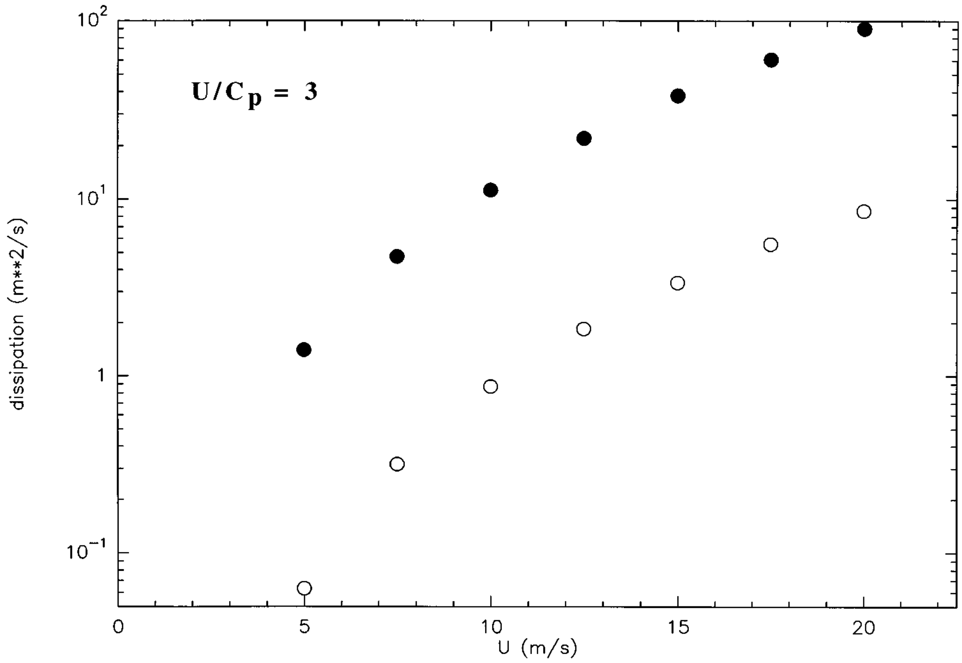

Figure 2. Same as Figure 1, but a fetch-limited wind generated sea is assumed, with $U/C_p = 3$.

first-order approach of the fetch dependence of the equilibrium spectrum, we shall simply express the fetch limited equilibrium spectrum in the wind direction above $k = 10k_p$ as

$$F(k, \varphi = 0) = F_{fd}(k, \varphi = 0) \cdot f(U/C_p) \tag{22}$$

where F_{fd} is the equilibrium spectrum in the fully developed case (i.e., when $U/C_p = 0.83$), and the function $f(U/C_p)$ is to be determined.

In a first step, we shall assume that Equation (22) holds for any wave number k above $10k_p$. This includes our selected interval $10k_p \leq k \leq 0.3$ cm^{-1}, but also the extreme high tail above 0.3 cm^{-1}. Since F_{fd} is given by Equation (8), it is easy to see that this assumption corresponds to taking the dissipation parameters α and n to be related to wave age according to,

$$n = n_{fd} \tag{23a}$$

$$\alpha = \frac{\alpha_{fd}}{[f(U/C_p)]^n}, \tag{23b}$$

where subscript 'fd' refers to the fully developed case, determined by Equations (9a) and (9b).

For a given wind U and wave age U/C_p, a composite spectrum was taken, consisting of Donelan et al.'s (1985) model at low wave number $(k < 10k_p)$, merged with DP's high frequency model in which the functions α and n were given by Equations (23a) and (23b). The quantities A and B of Section 2.2 were then computed and the parameter f was adjusted in such a way that A and B coincide in order to fit Longuet-Higgin's constraint. Since we wish here to explore the behaviour of the spectrum with wave age, for each value of U we have first performed a fine adjustment of the limiting amplitude a_0' in such a way that $f(U/C_p = 0.83) = 1$ exactly (fully developed case). The quantity f was thus determined for various wind speeds and wave ages. In Section 2.1 we have selected the domain of wave numbers k for which $10k_p < k < 0.3$ cm^{-1}. Our domain is thus defined only if $10k_p$ is smaller than 0.3 cm^{-1}. Therefore only the combinations of U/C_p and U filling this requirement have been considered here. The condition is $g(U/C_p)^2/U^2 < 3$ m^{-1}, and excludes very young seas at low wind speed. The results are displayed in Figure 3. For every wind speed, one notices a very consistent increase of the equilibrium range spectral density $F(k, \varphi = 0)$ with inverse wave age. For very young seas $(U/C_p = 5)$, the spectral density is about twice its value at full development. There is an extensive litterature concerning the Phillips coefficient's dependence on wave age (see, e.g., Phillips, 1980). Those studies involve the omnidirectional spectra, and their results cannot be related directly to $F(k, \varphi = 0)$. Banner (1990) has studied the behaviour of directional spectra in the equilibrium range, and proposed a wave-age dependence of $F(k, \varphi = 0)$ as $(U/C_p)^{0.5}$. A similar trend is found here, although the exponent of the power law expression is ≈ 0.35 rather than 0.5. However, the simple linear dependence $f(U/C_p) = 1 + 0.25(U/C_p - 0.83)$ would provide a better fit to our results.

In order to test the robustness of our results, we have also examined the case where the wave age affects the spectrum in the interval $10k_p \leq k \leq 0.3$ cm^{-1} only, excluding the high wave number capillary tail. This was done by replacing Equation (23b) by

$$\alpha_1 = \frac{\alpha_{1_{fd}}}{[f(U/C_p)]^{n_1}}, \tag{24a}$$

$$\alpha_2 = \alpha_{2_{fd}}. \tag{24b}$$

The resulting function $f(U/C_p)$ (not displayed here) is very similar to that of Figure 3, the difference being less than 2% everywhere.

Finally, we have also tested the effect of the assumed azimuthal dependence of the wave spectrum on our results. For that purpose, DP's spreading function above $10k_p$ has been replaced by the one suggested by Banner (1990) (see his Equation (2.9b)). Here again, the behaviour of the function $f(U/C_p)$ is little affected by this assumption, the difference with Figure 3 being less than 2% again.

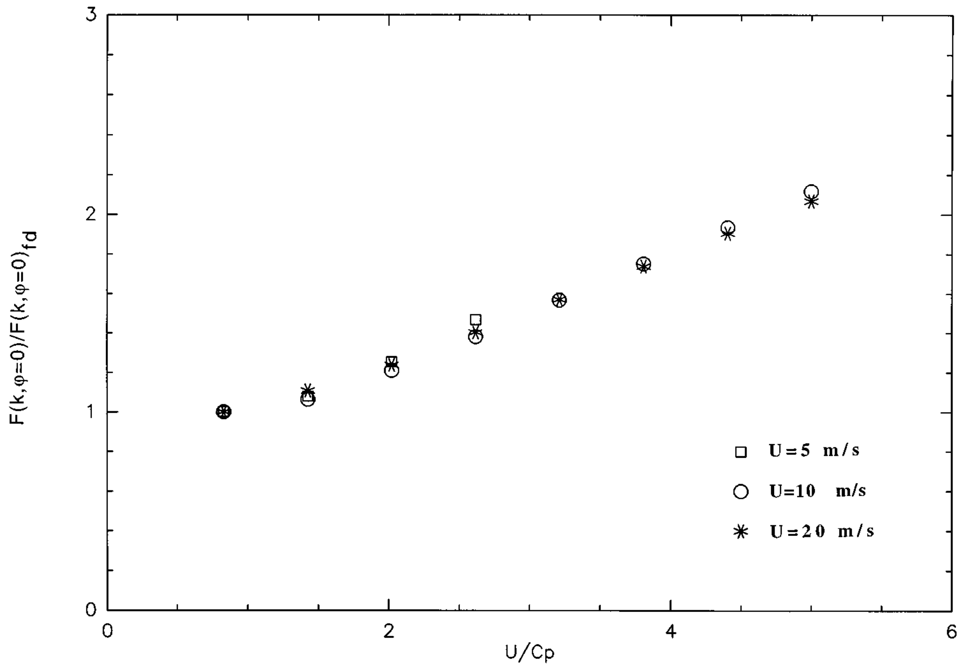

Figure 3. Quantity f plotted versus U/C_p, for different values of wind speed ($U = 5$, 10, or 20 m s^{-1}). The quantity f, which is defined by Equation (22), represents the ratio of the equilibrium range spectral density in the wind direction obtained at inverse wave age U/C_p, to the same quantity obtained at full development.

4. Roughness Length as a Function of Fetch

Given the wave spectrum and a model for the wind-induced wave growth parameter β_W, it is possible to determine the drag of the sea surface in a self-consistent way by an iterative computation of the friction velocity u_* (Janssen, 1989; Caudal, 1993; Makin et al, 1995). As noted by Makin et al. (1995), the drag of developing seas depends crucially on the wave spectrum in the decimetre to metre range, while the actual dependence of the spectrum with wave age in that range of wave numbers is still a controversial question. In the previous sections, we have obtained a semi-physical model for the spectrum in the equilibrium range as a function of wind speed and wave age. In order to check the relevance of the inferred wave-age dependence of our spectral model, it is therefore tempting to check the extent to which the drag of the sea surface computed from this spectrum is consistent with the drag observed in the field. Here, to compute the drag of the sea surface from the spectrum, we used the method described by Caudal (1993), which is expected to be valid for a fully rough air-sea interface ($U > 7$ m s^{-1}, typically).

For given U and U/C_p, the results of the computation of the sea drag may be expressed in terms of friction velocity u_*, or roughness length z_0, since both

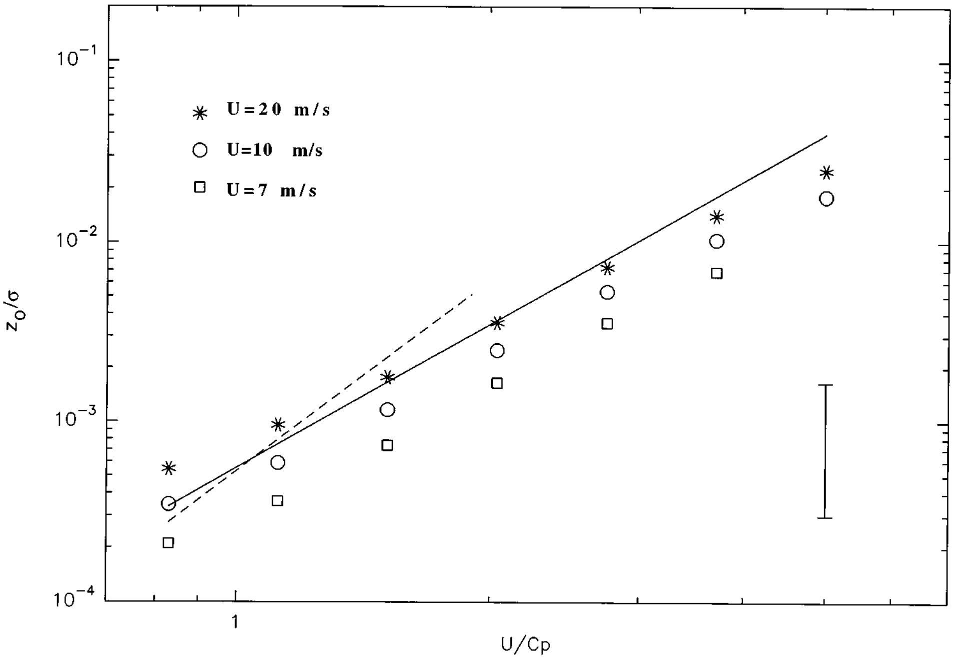

Figure 4. Quantity z_0/σ (where z_0 is roughness length and σ is rms wave height) inferred by integrating the wind stress over our spectral model (squares: $U = 7$ m s^{-1}; circles: $U = 10$ m s^{-1}; stars: $U = 20$ m s^{-1}. Dashed line: experimental regression line of z_0/σ obtained by Smith et al. (1992). Solid line: experimental regression line obtained by Donelan (1990). The error bar relates to Donelan's (1990) data.

quantities are related through the logarithmic profile of wind velocity with height (taking $z = 10$ m)

$$U(z) = \frac{u_*}{0.41} \ln(z/z_0). \tag{25}$$

In Figure 4, the quantity z_0/σ inferred from the model spectrum is displayed as a function of inverse wave age for various wind speeds above 7 m s^{-1}, where σ is the rms wave height (obtained through integration of the wave number spectrum). The regression line

$$\frac{z_0}{\sigma} = 5.53 \times 10^{-4} \left(\frac{U}{C_p}\right)^{2.66} \tag{26}$$

obtained by Donelan (1990) in Lake Ontario is also shown, as well as the one obtained by Smith et al. (1992) from the HEXOS data set

$$\frac{z_0}{\sigma} = 5.32 \times 10^{-4} \left(\frac{U}{C_p}\right)^{3.53}. \tag{27}$$

As seen in Figure 4, the fit between the computed and measured values of z_0/σ is not perfect, especially at low wind speed ($U = 7$ m s^{-1}) where the computed values tend to be systematically lower. One should keep in mind, however, the large scatter of measured z_0/σ about the regression line, visualised by the error bar (from Donelan's (1990) data). The rate of increase of z_0/σ with increasing U/C_p is satisfactorily reproduced at all wind speeds, although the computed z_0/σ tends to increase slightly more slowly. For moderate to strong winds ($U = 10$ or 20 m s^{-1}), the computed z_0/σ are well within the error bar of the measurements. This provides some support for our inferred increase with U/C_p of the equilibrium range spectral level in the wind direction.

As in Section 3.1, we have also examined the case where the wave age affects the spectrum in the equilibrium range only, and not in the high wave number tail (i.e., α given by Equations (24a) and (24b)). As mentioned by Makin et al. (1995), waves with wavelength less than 5 cm play a minor role in the momentum transfer from wind to waves, and therefore a modification of the high wave number spectrum close to the capillary-gravity transition should not affect the wave drag significantly. As a matter of fact, the inferred z_0/σ is modified by no more than 7% under that assumption, over the range of winds and wave ages probed. Similarly, the modification of the azimuthal spreading function according to Banner's (1990) approach modifies the inferred z_0/σ by less than 10% under the same conditions.

5. Conclusion

Within the framework of existing models for the rate of dissipation of wave energy by wave breaking (Longuet-Higgins, 1969; Donelan and Pierson, 1987), we have explored the dependence of the sea wave spectrum with wave age in the high-frequency portion of the equilibrium range. Our results show an increase of the equilibrium spectrum in the wind direction $F(k,0)$ with increasing inverse wave age U/C_p. This is in accordance with Banner's (1990) empirically deduced conclusion that $F(k,0)$ is proportional to $(U/C_p)^{0.5}$ in the equilibrium range. If a power law is fitted to our results, we obtain a slightly less pronounced wave age effect, with $F(k,0)$ approximately proportional to $(U/C_p)^{0.35}$ for $0.83 < U/C_p < 5$. However, a better fit is obtained by taking a linear relationship, in which case $F(k,0)$ is found to be proportional to $1 + 0.25(U/C_p - 0.83)$. Our results are little affected by the assumption made for the azimuthal spreading function of the wave number spectrum.

Correlatively to the preceding results, it also follows from this analysis that wave breaking cannot be considered as a local process in wave number space, but rather as a coupled process. This means that the dissipation rate through wave breaking at a given wave number should depend not only on the wave spectrum at this wavenumber, but also on integral quantities involving the wave energy elsewhere in the spectrum.

As concerns the assumed behaviour of the wave spectrum close to and above the gravity-capillary transition ($k > 0.3$ cm^{-1}), it is found to have little effect on the fulfilment of Longuet Higgins' constraint, and correlatively it does not influence our inferred dependence of the equilibrium range spectrum ($k < 0.3$ cm^{-1}) with wave age. Since, in DP's approach, the parameters describing wave breaking in the equilibrium range and in the gravity-capillary transition are modelled separately, our approach is thus unable to predict the behaviour of the high wave number tail close to and above the gravity-capillary transition, since any assumption for the wave age dependence of that high wave number tail is allowed. Our results are expected to be applicable for $10k_p < k < 0.3$ cm^{-1}. This domain does not cover the wave numbers probed by wind scatterometers, which are usually closer to the gravity-capillary transition (the Bragg wave number is of the order of $k \approx 1$ cm^{-1} for a C-band radar, or $k \approx 3$ cm^{-1} for a Ku-band radar). Therefore, the wave age dependence of the scatterometer signal may not follow the law inferred here for the equilibrium range. Glazman (1987) reported an error bias in the Seasat scatterometer wind measurement at Ku-band, with a tendency for the scatterometer to overestimate the wind speed at long fetch, and underestimate at low fetch. He invoked an additional component of surface scattering due to electromagnetic-wave diffraction at the crests of steep wavelets at long fetch, as well as a broadening of the (azimuthally averaged) wave number spectrum with increasing fetch. The decrease of the equilibrium range spectral density in the wind direction with increasing fetch does not exclude the possibility of an increase of the capillary component with increasing fetch, as suggested by Glazman's observations.

Finally, the roughness length of the air-sea interface inferred from our modelled spectrum, through integration of form drag over wave number under rough conditions, shows a wave age dependence that is compatible with measurements of wind stress performed in the field at various wave ages.

References

Al-Zanaidi, M. A. and Hui, W. H.: 1984, 'Turbulent Air Flow Over Water Waves – A Numerical Study', *J. Fluid Mech.* **148**, 225–246.

Banner, M. L.: 1990, 'Equilibrium Spectra of Wind Waves', *J. Phys. Oceanog.* **20**, 966–984.

Banner, M. L., Jones, I. S. F., and Trinder, J. C.: 1989, 'Wavenumber Spectra of Short Gravity Waves', *J. Fluid Mech.* **198**, 321–344.

Battjes, J. A., Zitman, T. J., and Holthuijsen, L. H.: 1987, 'A Reanalysis of the Spectra Observed in JONSWAP', *J. Phys. Oceanog.* **17**, 1288–1295.

Caudal, G.: 1993, 'Self-Consistency Between Wind Stress, Wave Spectrum, and Wind-Induced Wave Growth for Fully Rough Air-Sea Interface', *J. Geophys. Res.* **98**(C12), 22743–22752.

Donelan, M. A.: 1990, 'Air-Sea Interaction', in B. Le Méhauté and D. M. Hanes (eds.), *The Sea*, J. Wiley and Sons, New York, pp. 239–292.

Donelan, M. A., Hamilton, J., and Hui, W. H.: 1985, 'Directional Spectra of Wind Generated Waves', *Philos. Trans. R. Soc. London, Ser. A* **315**, 509–562.

Donelan, M. A. and Pierson, Jr., W. J.: 1987, 'Radar Scattering and Equilibrium Ranges in Wind-Generated Waves with Application to Scatterometry', *J. Geophys. Res.* **92**(C5), 4971–5029.
Donelan, M. A., Dobson, F. W., Smith, S. D., and Anderson, R. J.: 1993, 'On the Dependence of Sea Surface Roughness on Wave Development', *J. Phys. Oceanog.* **23**(9), 2143–2149.
Donelan, M. and Yuan, Y.: 1994, 'Wave Dissipation by Surface Processes', in G. J. Komen, L. Cavaleri, M. Donelan, K. Hasselmann, S. Hasselmann, and P. A. E. M. Janssen (eds.), *Dynamics and modelling of ocean waves*, Cambridge University Press, Cambridge, pp. 143–155.
Forristall, G. Z.: 1981, 'Measurements of a Saturated Range in Ocean Wave Spectra', *J. Geophys. Res.* **86**, 8075–8084.
Glazman, R. E.: 1987, 'Wind-Fetch Dependence of Seasat Scatterometer Measurements', *Int. J. Remote Sens.* **8**, 1641–1647.
Hasselmann, K.: 1974, 'On the Spectral Dissipation of Ocean Waves Due to Whitecapping', *Boundary-Layer Meteorol.* **6**, 107–127.
Hasselmann, K., Janssen, P. A. E. M., and Komen, G. J.: 1994, 'Wave-Wave Interaction', in G. J. Komen, L. Cavaleri, M. Donelan, K. Hasselmann, S. Hasselmann, and P. A. E. M. Janssen (eds.), *Dynamics and Modelling of Ocean Waves*, Cambridge University Press, Cambridge, pp.113–143.
Hsiao, S. V. and Shemdin, O. H.: 1983, 'Measurements of Wind Velocity and Pressure With a Wave Follower During MARSEN', *J. Geophys. Res.* **88**(C14), 9841–9849.
Hsu, S. A.: 1974, 'A Dynamical Roughness Equation and Its Application to Wind Stress Determination at the Air-Sea Interface', *J. Phys. Oceanog.* **4**, 116–120.
Janssen, P. A. E. M.: 1989, 'Wave-Induced Stress and the Drag of Air Flow Over Sea Waves', *J. Phys. Oceanog.* **19**, 745–754.
Kahma, K. K.: 1981, 'A Study of the Growth of the Wave Spectrum With Fetch', *J. Phys. Oceanog.* **11**, 1503–1515.
Kawai, S., Okuda, K., and Toba, Y.: 1977, 'Field Data Support of Three Seconds Power Law and $gu * \omega^{-4}$ Spectral Form for Growing Wind Waves', *J. Oceanogr. Soc. Japan* **33**, 137–150.
Komen, G. J., Hasselmann, S., and Hasselmann, K.: 1984, 'On the Existence of Fully Developed Wind-Sea Spectrum', *J. Phys. Oceanog.* **14**, 1271–1285.
Lamb, H.: 1932, *Hydrodynamics*, 6th. ed., Cambridge University Press, Cambridge, reprinted by Dover Publications, New York, 1945, 738 pp.
Larson, T. R. and Wright, J. W.: 1975, 'Wind-Generated Gravity-Capillary Waves: Laboratory Measurements of Temporal Growth Rates Using Microwave Backscatter', *J. Fluid Mech.* **70**, 417–436.
Leykin, I. A. and Rozenberg, A. D.: 1984, 'Sea-Tower Measurements of Wind-Wave Spectra in the Caspian Sea', *J. Phys. Oceanog.* **14**(1), 168–176.
Longuet-Higgins, M. S.: 1969, 'On Wave Breaking and the Equilibrium Spectrum of Wind-Generated Waves', *Proc. Roy. Soc. London* **A310**, 151–159.
Makin, V. K., Kudryavtsev, V. N., and Mastenbroek, C.: 1995, 'Drag of the Sea Surface', *Boundary-Layer Meteorol.* **73**, 159–182.
Ochi, M. K. and Tsai, C.-H.: 1983, 'Prediction of Occurrence of Breaking Waves in Deep Water', *J. Phys. Oceanog.* **13**, 2008–2019.
Phillips, O. M.: 1958, 'The Equilibrium Range in the Spectra of Wind-Generated Waves', *J. Fluid Mech.* **4**, 426–434.
Phillips, O. M.: 1980, *The Dynamics of the Upper Ocean*, 2nd. ed., Cambridge University Press, Cambridge, 336 pp.
Plant, W. J.: 1982, 'A Relationship Between Wind Stress and Wave Slope', *J. Geophys. Res.* **87**(C3), 1961–1967.
Smith, S. D., Anderson, R. J., Oost, W. A., Kraan, C., Maat, N., DeCosmo, J., Katsaros, K. B., Davidson, K. L., Bumke, K., Hasse, L., and Chadwick, H. M.: 1992, 'Sea Surface Wind Stress and Drag Coefficients: The HEXOS Results', *Boundary-Layer Meteorol.* **60**, 109–142.
Snyder, R. L., Dobson, F. W., Elliot, J. A., and Long, R. B.: 1981, 'Array Measurements of Atmospheric Pressure Fluctuations Above Surface Gravity Waves', *J. Fluid Mech.* **102**, 1–59.
Toba, Y.: 1973, 'Local Balance in the Air-Sea Boundary Processes. Part III: On the Spectrum of Wind Waves', *J. Oceanog. Soc. Japan* **29**, 209–220.