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Directional wave measurements using a slope array system

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Abstract

A directional wave measuring system developed at the Federal University of Rio de Janeiro, Brazil has been described. The system was operated by PETROBRAS (Brazilian Oil Company) as a part of a metocean station mounted in a jacket platform, CHERNE-1, at Campos Basin, Brazil. The directional wave spectra were determined from direct measurements of sea surface heave and slopes using resistive wave-staffs disposed in a square array. The wave meter performance was successfully compared with a pitch-and-roll buoy. The system promises to be a powerful tool for basic studies of wind waves. The low cost of manufacturing and maintenance allows its wide dissemination in less developed countries. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Directional wave spectra; Wave measurement; Wave staffs; Slope array method

1. Introduction

Many research institutes and oceanographic companies in developed countries specializing in wave measurement have invested intensively in autonomous systems and real time data transmission. Recent advances have been provided for different principle based equipment such as acoustic Doppler current profilers facing upward, microwave radars, current meters coupled with pressure sensor, buoys and satellite measurements from Synthetic Aperture Radars. However, the high operational costs of this level of technology have constrained their general use in less developed countries.

From October 1991 to December 1992, the Laboratory of Oceanographic Instrumentation (LIOc) of the Federal University of Rio de Janeiro (UFRJ) and PETROBRAS (Brazilian Oil Corporation) operated a metocean data acquisition system at Cherne-1 (PCH-1). It is a jacket oil platform sited in Campos Basin, Brazil. The experience has demonstrated that acquisition of environmental data from oil platforms significantly reduces the costs of installation, maintenance and transmission, and eliminates the risks of loss and damages of a mooring operation.

The greatest advantage of this system, however, was that it allowed the integration of one equipment under development phase. It was therefore possible to test the performance of a slope array directional wave meter, developed by LIOc. This instrument is relatively simple and solves the main problem of the wave directional measurement systems and their price [1]. Its installation and maintenance are at least 10 times lower than those of any other system available.

The slope array type wave meter was initially proposed by Ford et al. [2], and further developed by van Heteren and Keyser [3]. Carvalho [4] discusses its advantages and limitations under some operational situations based on numerical simulations of wave records with directional resolution [5] and comparisons with data simultaneously collected by a pitch-and-roll buoy.

In this paper, we describe the principle of operation of the wave meter and give details of operational procedures. We also discuss its performance, based on comparison with simultaneous wave measurements made by a 3 m discus buoy, the Marlim Oceanographic Buoy (MOB), and about its capability to determine the spreading parameter.

2. Theoretical formulation

2.1. The parametric estimation method

The formulation needed to obtain the directional spectrum is complex. It is given by Borgman [6] and Longuet-Higgins et al. [7]. It starts with a system of integral equations obtained from a function of space-temporal covariance between two independent properties of the wave. Their solution provides the directional spectral density for each

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frequency, $S(f, \theta)$, that can be approximated by:

$$S(f, \theta) = S(f)D(f, \theta), \tag{1}$$

where $S(f)$ is the unidimensional spectrum and $D(f, \theta)$ is the angular spreading function. The spreading function can be expanded as a Fourier series

$$D(f, \theta) = \frac{1}{2} + \sum_{n=1}^{\infty} [a_n(f)\cos(n\theta) + b_n(f)\sin(n\theta)], \tag{2}$$

where a_n and b_n are Fourier coefficients of order n.

Applying Eqs. (1) and (2) to the system of integral equations reduces them to an easily solved linear system. The unknown factors then become the Fourier coefficients, which are used to reconstitute the angular spreading function.

For a typical array of three sensors, the Fourier series is truncated at $n = 2$. Consequently, it is not sufficiently capable of satisfactorily reconstituting the angular spreading function. This can be overcome by imposing a shape for $D(f, \theta)$ based on empirical knowledge of the wave climate. The existing formulations are normally based on a mean direction of wave propagation and a spreading around this value. The widely used function, $\cos - 2s$, was proposed by Mitsuyasu et al. [8]. It is expressed by:

$$D(f, \theta) = G_0 \cos^{2s}\left(\frac{\theta - \theta_0}{2}\right), \tag{3}$$

where s is the spreading parameter[1] and θ_0 is the mean direction of a given frequency component. In order to preserve the energy we make:

$$\int_{\theta\min}^{\theta\max} D(f, \theta)\,\mathrm{d}\theta = 1, \tag{4}$$

by

$$G_0 = \left\{\int_{\theta\min}^{\theta\max} \cos^{2s}\left(\frac{\theta - \theta_0}{2}\right)\mathrm{d}\theta\right\}^{-1}. \tag{5}$$

The mean direction is given by the maximum $D(f, \theta)$. Therefore, the maximum values of each component (n) of Eq. (2) are in the abscissa

$$\theta_{0_n} = \arctan\left(\frac{b_n}{a_n}\right), \tag{6}$$

with ordinates

$$c_n = \pi(a_n^2 + b_n^2)^{1/2}. \tag{7}$$

From adjustments in the parameters of the $\cos - 2s$ function in a Japanese coastal survey, Mitsuyasu et al. [8] concluded that in order for Eq. (3) to be a good approximation to $D(f, \theta)$, s must satisfy the expressions

$$c_n = \frac{s(s-1)(s-2)\ldots(s-n)}{(s+1)(s+2)\ldots(s+n)} n = 1, 2, 3, \ldots \tag{8}$$

Consequently, the spreading parameter is

$$s = \frac{s_1 + s_2}{2}, \tag{9}$$

where:

$$s_1 = \frac{c_1}{1 - c_1}, \tag{10}$$

and

$$s_2 = \frac{1 + 3c_2 + (1 + 14c_2 + c_2^2)^{1/2}}{2(1 - c_2)}. \tag{11}$$

Due to technological limitations of the available sensors, however, the parameters of $\cos - 2s$ are usually estimated only by the first-order Fourier coefficients a_1 and b_1.

2.2. Slope array formulations

The wave properties used in the slope array method are the elevation (η) and the slope of the sea surface in the directions $x(\eta_x)$ and $y(\eta_y)$. We use a fixed right-hand-rule coordinate system, in which the positive z-axis is pointing up, with zero at the mean sea level. The wave number vector, $\mathbf{k}$, represents the wave, with positive direction measured counterclockwise from the positive x-axis (Fig. 1).

The first-order Fourier coefficients are determined from

$$a_1(f) = \frac{Q\eta\eta_x(f)}{k\pi S(f)}, \tag{12}$$

$$b_1(f) = \frac{Q\eta\eta_y(f)}{k\pi S(f)}, \tag{13}$$

where $Q\eta\eta_x$ and $Q\eta\eta_y$ are the quad-spectra of the cross spectrum between η and η_x and η and η_y, respectively. The wave number is determined as:

$$k(f) = \left(\frac{S\eta_x\eta_x(f) + Q\eta_y\eta_y(f)}{S\eta\eta(f)}\right)^{1/2}. \tag{14}$$

This equation is important, as it can be used to verify the

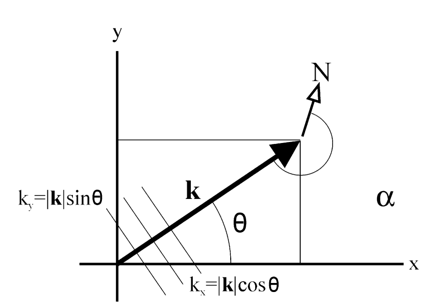

Fig. 1. Representation of the wave number vector and its projections onto the coordinate system. α is the direction of the wave referred to true North.

[1] The spreading parameter is commonly expressed as S while the directional wave spectra is expressed as $S(f, \theta)$ and the unidimensional wave spectra as $S(f)$. In order to avoid confusion we will use small letters for the spreading parameter.

operation of the directional system, since it cross-correlates three properties of the wave and may be compared to the values obtained from the dispersion relation based on linear theory:

$$\omega^2 = gk \tanh kh, \tag{15}$$

where $\omega = 2\pi f$ is the angular frequency, g the gravity acceleration and h the water depth.

3. The directional wave meter

The wave meter is one component of the Meteo-oceanographic Data Acquisition System of PCH-1. This system consists of a microcomputer with an analog to digital converter that receives the signals from the wave, wind and current sensors. The data acquisition, processing and registering are controlled by software.

3.1. Operation principle

The processing of the wave meter data from Cherne-1 is similar to that of a pitch and roll buoy. It differs in the way the time series are obtained. Sea-surface elevations and slopes are obtained through resistive wave staff sensors arranged over the vertices of a square 1.12 m side. One of these vertices is the origin of the coordinate system. The axes x and y pass through the adjacent vertices. The slopes are determined by finite difference as shown in Fig. 2.

If the resolution of an individual sensor is unlimited, the closer the staffs are disposed the more accurate is the slope estimation. However, the resolution is a restrictive factor. Carvalho [4] showed that the best distance between the sensors, for the resolution of 2.9 mm, varies from 1 m, for waves in the 7-s period, to 3 m, for waves in the 12-s period peak. Greater distances introduce spatial aliasing errors into the slope estimation.

The use of a fourth sensor not only serves as a spare, but also enables the reduction of geometric noise [4] caused by distortions in the orthogonal-symmetric disposing of the sensors, as they were not attached at the down extremity. The technique consists of averaging the directional parameters obtained from the four orthogonal sensors, each sensor taking a different place in the axes of the coordinate system, which is rotated 360°. The errors are therefore distributed and reduced by the average

$$\eta_x = \frac{n_2 - n_1}{L}, \tag{16}$$

and

$$\eta_y = \frac{n_3 - n_1}{L} \tag{17}$$

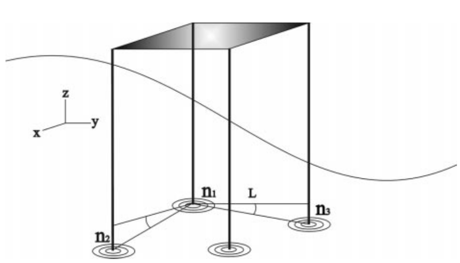

Fig. 2. Scheme of the directional wave meter.

3.2. Characteristics of resistive wave staffs

The sensors must ensure resolution, accuracy and must be strong enough to resist hostile sea states. In order to satisfy these three basic constraints, we manufactured a sensor composed of a nickel–chrome wire resistance ($\phi = 0.25$ mm and 12 Ω/m) attached as a spiral to a propylene cable ($\phi = 1.25$ mm) that sustained a 3.0 kg lead ballast. The sensor characteristics are: 12 m length; resistance of 168 Ω/m; pulsed CW excitation; and detection synchronized with excitation. The linearity is 1.7%. The resolution is unlimited (continuous resistance), but actual resolution depends on the capacity of digitalization of the A/D converter (12 bits) and on the length of the staff (12 m). The actual resolution is therefore 0.0029 m ($12/2^{12}$ m).

3.3. Calibration procedure

Calibration of the sensors is vital for the reliable operation of the directional wave meter. The linearity is verified in the laboratory and pre-adjustments are made. A fine adjustment is carried out using software, by applying a calibration curve generated through the acquisition of signals provided by an artificial sensor composed of identical series resistors.

4. Wave meter performance

In order to verify the behavior of the wave meter, we applied two techniques to the PCH-1 system: the average directional function (ADF) and a comparison with the oceanographic buoy of Marlim.

4.1. Average directional functions

The ADF method consists of checking the wave meter behavior during a specific quasi steady state situation using averaged spectral functions. We used hourly data from 8 a.m. on 18th December to 4 p.m. on 20th December 1992. At this time, the sea was almost constant. Waves approached from the Northeast with a 7-s spectral peak period while the significant wave heights slightly decreased from 3 to 1 m.

Fig. 3a–d shows the frequency-dependant functions of the unidimensional spectrum, spreading parameter, direction and wave number averaged over the whole 56 h. The probable error is plotted in continuous dashed lines in order to show the variability of the functions during the period.

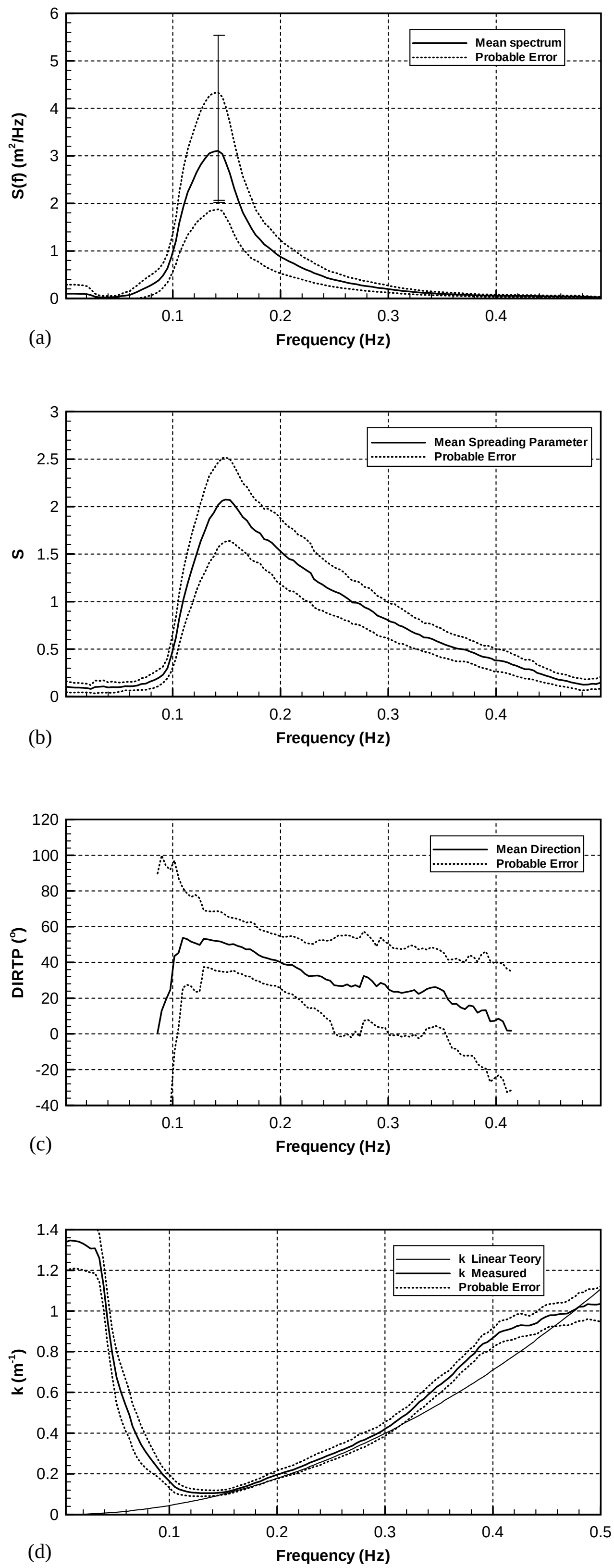

Fig. 3. (a) Power spectrum. The error bar shows the 90% confidence limit at the peak frequency. (b) Angular spreading parameter. (c) Direction. (d) Wave number.

The spectrum parameters were determined by applying the FFT algorithm over 1024-s length, 1.0 Hz sampling rate records. Smoothing was provided by applying a Hann window over the frequency domain assuring 22 degrees of freedom.

At the frequency band where the energy is significant (from 0.1 to 0.3 Hz) all the averaged functions show well-behaved patterns. The unidimensional spectrum and spreading parameter average function show the largest variability at the peak frequency. This is supposed to be related to the continuous decreasing of the wave heights over the sample period.

The probable error of the wave direction was up to 16.9°. This value is not large when compared to the results obtained by Carvalho [4]. It reports ADFs for 50 synthetic time series of η, η_x, and η_y, generated by numerical simulation. For constant input parameters, the direction function presented a probable error up to 10° for the same frequency band. Otherwise, we have to consider that at the beginning of the sample period a decreasing southeasterly swell of 10-s period peak is still existing. It could introduce some additional uncertainties to the direction, as the Parametric Estimation Method is not reliable for computing the direction of waves with same frequencies in a crossed sea.

The wave number function showed values consistent with the linear theory between 0.1 and 0.3 Hz. Its variability was very similar to the ones, obtained with a pitch-and-roll buoy, described by Allender et al. [9]. It reports results of the WADIC project, which inter-compared several systems of directional wave measurement operating simultaneously for 3 months close to an offshore platform in the North Sea.

4.2. A comparison with the oceanographic buoy of Marlim

The MOB is part of the PETROBRAS's Deep Water Capacitation Program (PROCAP). It was installed in March 1991 in order to supply directional wave information necessary for constructing of the platforms that would operate in water depths greater than 600 m. It is a 3 m Discus Buoy that measures surface elevation and slope with a HIPPY-120 sensor. The data were acquired every 3 h with a 1.0 Hz sampling rate and 1024 s record length. Internal processing provides the wave main direction versus frequency and all parameters generated by the time domain analysis.

The distance (Fig. 4) between the two systems does not ensure ergodicity, mainly at the high frequencies associated with the local wind. Therefore, a rigorous comparative analysis involving statistical criteria cannot be applied. However, there is no problem in comparing temporal histories generated by both systems in order to show the reliability of the wave meter.

Fig. 5a–c shows the temporal histories of significant height (HM0), spectral peak period (TP) and direction

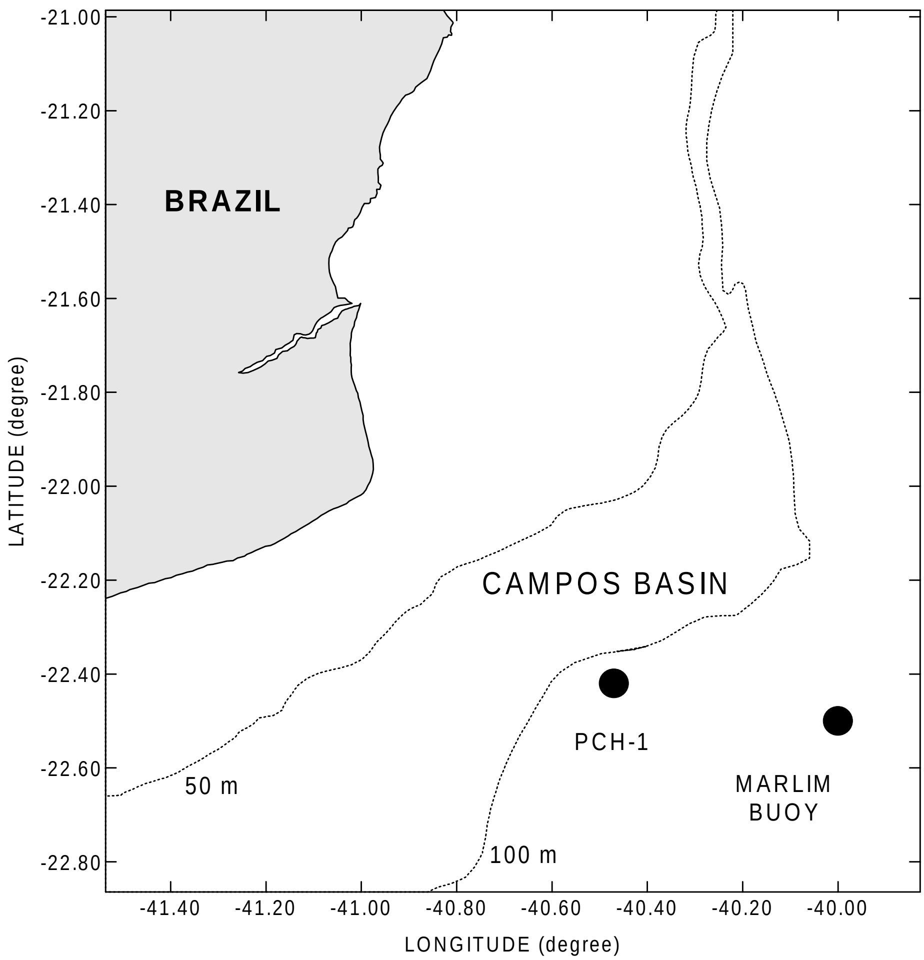

Fig. 4. Localization of the stations.

associated with the peak period (DIRTP), obtained using the two systems from 16th to 21st December 1992. During this period, two types of sea states occurred. Both are very common in the Campos Basin. From 16th to 17th December 1992 there was a Southeasterly (DIRTP = 120°) swell with a TP = 10 s and HM0 = 2 m. From 18th December, till the end of the period, there was a predominance of a Northeasterly sea (DIRTP = 65°), TP = 7 s and HM0 ranging from 1 to 3 m.

There was good agreement between the HS and TP values obtained by both systems. For DIRTP, the values were almost identical in the swell situation, although they differed by about 30° during Northeasterly waves. This was probably caused by the different position of each station relative to the South Atlantic High Pressure Center, which is responsible for the generation of the Northeasterly winds in the region.

4.3. The spreading parameter

Fig. 3b, the averaged values of the spreading parameter as a function of frequency, shows similarity with the results observed by Mitsuyasu [8] using a cloverleaf buoy (acceleration, slope and curvature) on the coast of Japan. It presented the angular spreading as a function of dimensionless spectrum frequency $\tilde{f}$ in which its maximum value s_m coincided with the dimensionless spectrum peak frequency ($\tilde{f}_m$). This suggests, the following relationship:

$$\frac{s}{s_m} = \left(\frac{\tilde{f}}{\tilde{f}_m}\right)^{-2.5} \qquad \text{for } \tilde{f} \geq \tilde{f}_m, \tag{18}$$

$$\frac{s}{s_m} = \left(\frac{\tilde{f}}{\tilde{f}_m}\right)^{5.0} \qquad \text{for } \tilde{f} \leq \tilde{f}_m, \tag{19}$$

where

$$s_m = 11.5\tilde{f}_m^{-2.5}, \tag{20}$$

$$\tilde{f} = 2\pi\frac{fU}{g} = \frac{U}{C}, \tag{21}$$

$$\tilde{f}_m = 2\pi\frac{f_m U}{g}, \tag{22}$$

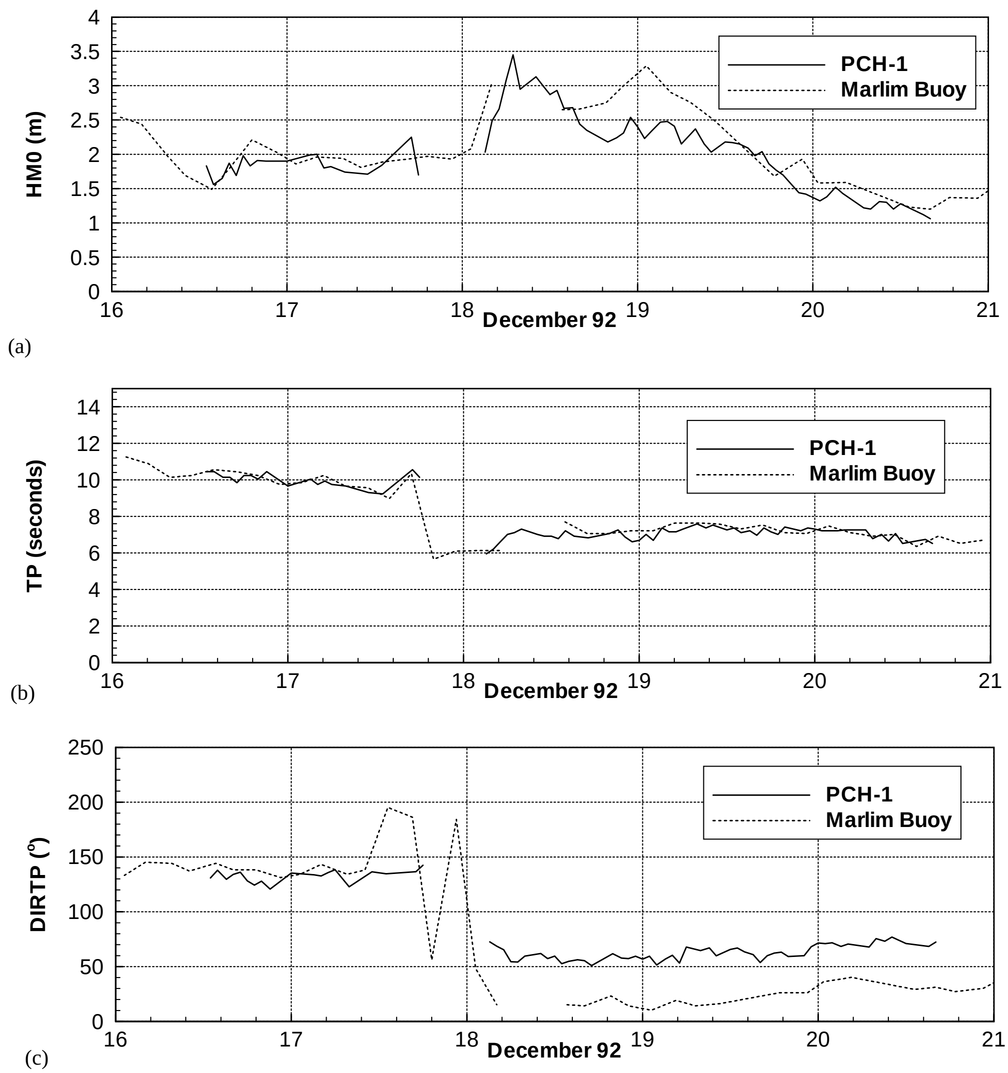

Fig. 5. (a) Time history of significant wave height. (b) Time history of spectral peak period. (c) Temporal history of peak direction.

and for fully developed seas [10]

$$\frac{f_{\mathrm{m}}U}{g} = 0.13, \tag{23}$$

where U is the wind velocity measured at 10 m above the sea surface and g is the acceleration of gravity.

Fig. 6 shows a comparison between the dimensionless spreading parameter obtained at Cherne-l and the formulation described in Eqs. (18) and (19) for completely developed seas. The curves of the measured absolute s and the estimated formulation of Mitsuyasu [8] for a wind of 8.0 m/s, the average wind speed during the experiment, is shown in Fig. 7. Although there is a certain similarity in both functions' shape, the spreading parameter in the PCH-1 system presented values lower by one order of magnitude than the estimated value. Possibly, it was caused by adoption of only first-order Fourier coefficients to parameterize the angular spreading function. Additionally, Brissette and Tsanis [11] concluded that the Direct Fourier Transform Method is not adequate to reconstruct the entire spreading while the Maximum Likelihood Method gives a more robust estimate of it. Hence, conclusive analyses of the magnitude of this parameter would require a field experiment especially designed for this purpose. This experiment would require control of all the restrictive factors of the sensors as well as monitoring of all environmental variables.

Nevertheless, the results show that the wave meter is able to provide reliable estimates of the s parameter. There are few directional wave systems available what can accomplish this characteristic, since the angular spreading parameter is extremely noise sensitive, especially in directional buoys due to the interference of the mooring-hull system.

In order to obtain high-order Fourier coefficients a fifth sensor could be installed at the center of the square array. It would allow the introduction of a new parameter in the directional spectra procedures: the surface curvature (η_{xx} and η_{yy}). Additionally, it would provide four extra slope

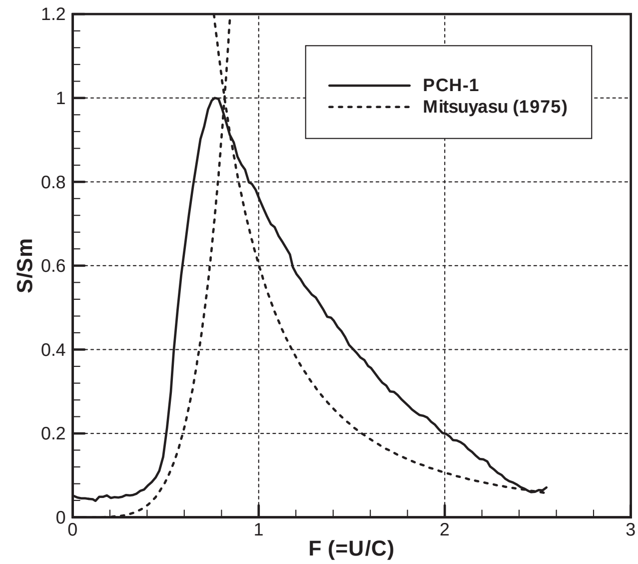

Fig. 6. Overlap of the curve of the spreading function obtained in PCH-1 and the Mitsuyasu [8] one.

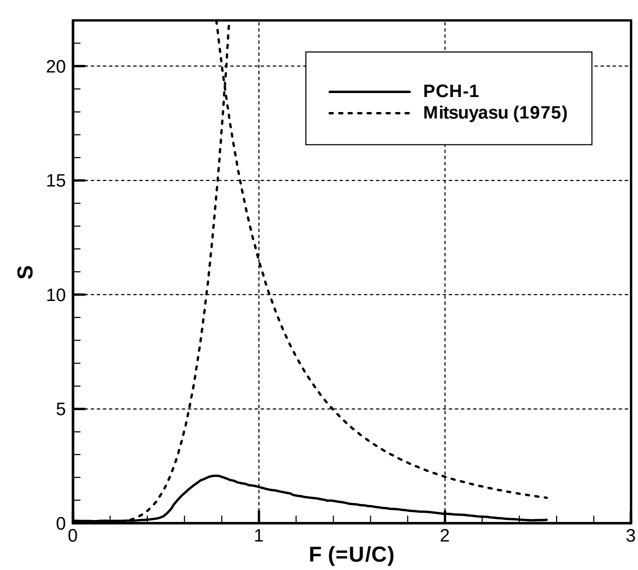

Fig. 7. Curves of the absolute spreading function from PCH-1 and of the estimated formulation of Mitsuyasu [8] for an average wind speed of 8.0 m/s.

time series with 0.707*L* side to reduce geometrical noise and to avoid spatial aliasing.

5. Conclusions and final remarks

The results showed that the use of resistive wave staffs to determine time series of surface elevation and slope is a promising technique. It can provide good estimates of the directional spectral functions, particularly the angular spreading function, which is difficult to determine in many other systems. The low cost of the equipment allows its dissemination on several fixed structures such as oil platforms, harbors, fishing piers. This would represent a great advance in filling the information gap concerning waves in the coasts of underdeveloped countries.

First, it is necessary to improve the reliability of the wave meter and enhance the sensors useful life. Some experiments at shallow water conditions should be provided. Other configurations should be tested in order to improve the accuracy and further reduce the noise. Additionally, the surface time series could be determined perfectly by other distance sensors, based on the optical or acoustical principle. Other methods for estimating the directional wave spectra, such as the Maximum Likelihood Method, should be tested in order to improve the accuracy of the estimate.

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