A comparison of the geodetic reference levels of England and France by means of the sea surface

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In order to relate the levelling zeros of England and France, an analysis is made of the differences in mean sea level across the English Channel due to the dynamics of water movement, and applied to a 723-day period of tidal and other data recorded between Ramsgate and Dunkerque during 1957 and 1958. The principal factor affecting the transverse slope of the surface is found to be the Coriolis stress due to the effect of the earth's rotation on the longitudinal currents, as expected. The mean longitudinal currents were estimated from the electrical potential across a submarine cable across the Dover Strait (Faraday effect), but the associated slopes were found to be less than would be expected from the assumption of a uniform current across the section. The concentration of current in the deeper parts of the channel and the mean effect of bottom friction probably account for this. The respective levelling zeros are found to differ by 19·6 cm with standard error 1·5 cm. A less reliable estimate of 25 cm with standard error 3 cm is deduced from 2 months' records from Shoreham and Dieppe during 1953.

1. Introduction

The growing precision of geodetic levelling methods and the recent efforts to achieve a 'United European Levelling Network'† have called geodesists' attention to the fact that the mean level of the sea is not a geopotential surface. Because of this it is not possible to equate the datum levels of the British and French levelling networks, which are based on mean sea level at Newlyn and Marseille respectively. Oceanographic methods exist in principle for estimating the slope of the sea surface (Bowden 1956; Doodson 1960; Proudman 1960; Rossiter 1960; Vantroys 1958), but the available data for temperature, salinity, currents and wind, etc., are not voluminous enough to provide an accurate estimate between the Mediterranean Sea and the English Channel. If, however, we consider the slope across such a channel of water as the region of the Dover Strait, factors such as density are insignificant, and although the currents are strong we have means of keeping them and other relevant variables under fairly close observation. A study of the levels and movements in this channel is therefore a convenient method of relating the two levelling networks (an idea probably first suggested by Proudman 1953, p. 58). An application of this method is described in the present paper.

Two other methods of levelling across the Channel are theoretically possible, but at present impractical. One of these, sighting across the 34 km between South Foreland and Cap Gris Nez, is inaccurate because of atmospheric refraction. It has been estimated that at best an accuracy of about $\frac{1}{2}$ m could be achieved, using all known corrections; this is considerably greater than the probable difference between the two zero levels. The other method would be to lay a pipeline across the Channel, fill it with water, and observe the surface levels at each end. Pipeline levelling has

† Discussed, for example, in several articles in *Bulletin Géodésique*, no. 55, March, 1960.

been carried out successfully across distances of order 10 km between Danish and Dutch islands (Nørlund 1945, 1946; Waalewijn 1959). The main advantage of using a filled pipe rather than the sea itself is the virtual absence of tidal currents, waves and wind stresses. However, it would be very expensive, and according to Waalewijn there may be difficulties in eliminating all the air from the pipe. On the whole, the present method appears to have the best combination of convenience and accuracy.

2. Hydrodynamic theory

We shall consider perpendicular axes x and y lying in a true horizontal (i.e. geopotential) surface, such that the y axis joins the two places whose difference in level is to be estimated, and x is in a direction making a right-handed system with a third axis z which is vertically downwards. In the present case, with the y axis pointing from England to France, the x axis points roughly along the channel into the North Sea, for example as in figure 1. Since the geopotential surface is not plane, this co-ordinate system is strictly curvilinear, but the distances and depths of water are so small compared with the earth's radius that for hydrodynamical purposes we may consider the system as if rectilinear without introducing any appreciable error. If u and v are components of water velocity in the x and y directions respectively (the vertical velocities are obviously very small, and can be shown to be quite negligible), and t is the time, the dynamic equation for acceleration in the direction y is

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=fu+F+\frac{1}{\rho}\frac{\partial \tau}{\partial z}-\frac{1}{\rho}\frac{\partial p}{\partial y}. \qquad (1)$$

On the right-hand side of this equation, f is the Coriolis parameter $2\Omega\sin\lambda$, where $\Omega = 2\pi \div$ the earth's rotational period, and $\lambda =$ latitude north of the equator. F represents the body forces other than those due to the earth's rotation, resolved in the y direction, the tide-raising force γ per unit mass, and wave-induced stresses. τ is the shearing stress, which takes the values τ_a (due to wind) at the surface, and τ_b (due to friction) at the bottom, considered, for example, by Lacombe (1949). The last term, in which p is the pressure, and ρ is water density, can be expressed in terms of surface elevation ζ above $z = 0$ by equating the vertical acceleration to zero, thus

$$0=f'u'+g-\frac{1}{\rho}\frac{\partial p}{\partial z}. \qquad (2)$$

Here, $f' = 2\Omega\cos\lambda$, and u' is the component of horizontal velocity normal to the meridian; since their product can be shown to be of order $10^{-5}g$, where g is gravitational acceleration, for the strongest possible current in the region, it can be ignored in this equation. On integrating with respect to z from the surface downwards and differentiating with respect to y, equation (2) then leads to

$$\frac{1}{\rho}\frac{\partial p}{\partial y}=\frac{1}{\rho}\frac{\partial p_a}{\partial y}+g\frac{\partial \zeta}{\partial y}, \qquad (3)$$

where p_a is atmospheric pressure.

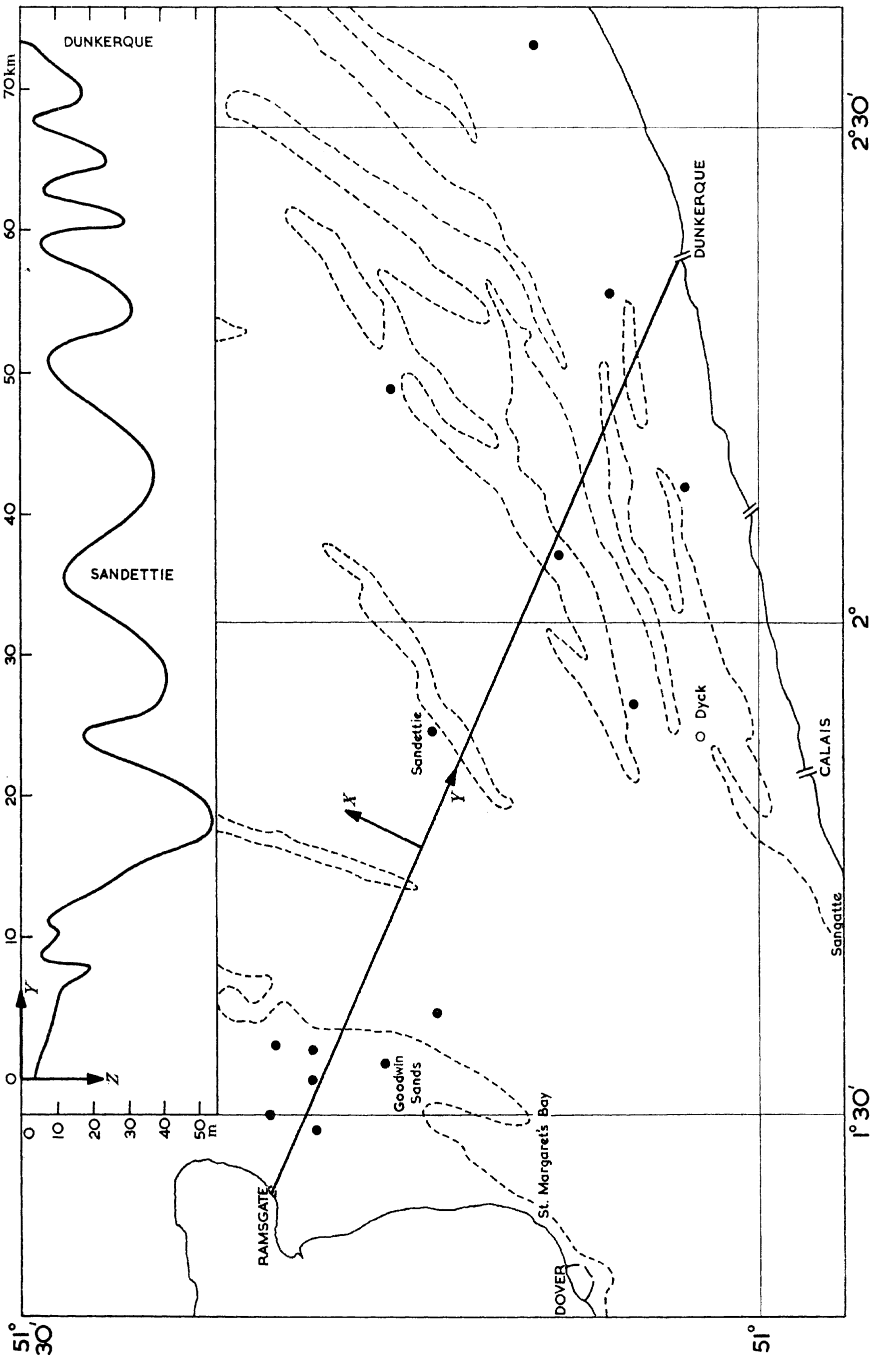

FIGURE 1. Map of sea area between Ramsgate and Dunkerque (Southern Bight of North Sea). Dashed line is 10 fathom contour (chart datum). Black spots are positions where tidal current data are known. Inset: profile of depth in metres below mean sea level along section (y axis).

We now substitute (3) in (1), and first average over the depth $h(y)$ and then integrate with respect to y over the width L of the channel, and obtain the following expression for difference in surface elevation:

$$\Delta\zeta = -\frac{1}{\rho g}\Delta p_a - \frac{1}{2g}\Delta(\overline{v^2}) + \frac{L}{\rho g h'}(\tau_a + \tau_b) + \frac{L\gamma}{g} + \frac{1}{g}\int_0^L \left[f\bar{u} - \frac{\partial \bar{v}}{\partial t} - \overline{\left(u\frac{\partial v}{\partial x}\right)}\right] \mathrm{d}y. \quad (4)$$

In equation (4) Δ denotes the total difference of a quantity across the width of the channel in the direction of increasing y, and a bar denotes an average value over local depth. $(1/h')$ is the mean reciprocal depth, defined as

$$\frac{1}{h'} = \frac{1}{L}\int_0^L \frac{\mathrm{d}y}{h(y)}.$$

3. Examination of terms in equation (4)

Equation (4) is the fundamental relation we shall use in determining levels across the channel, so we shall now consider the magnitude of each term and how best it can be measured. $\Delta\zeta$ is of course the difference in departure of the water levels at each end from the geopotential surface, which is what we really want to know. If the section is taken between two ports where tide gauges are installed, then the difference between the mean sea levels as measured by the gauges from their respective reference datums, adjusted for $\Delta\zeta$, is the required estimate of the difference between the true geopotential levels of the reference datums. This strictly holds true at every instant of time, but as will be seen later it is more convenient to take time averages over periods long enough to eliminate tidal movements.

The difference in atmospheric pressure across the channel, Δp_a, is easily and accurately obtained from meteorological records. It contributes very little on the average, being always well within ± 4 cm of water, but it is worth including because of its ease of measurement.

The term $\Delta(\overline{v^2})$ can be ignored, because the end points of the integration are at tide gauges within sheltered harbours, ideally not at the mouth of a river which would allow strong tidal currents up and down stream. If the tide gauges were exposed to the off-shore currents this would not necessarily be the case, because the channel's shores are not necessarily perpendicular to the cross-section, so that currents up and down the channel may have a considerable v component. With the y axis from Ramsgate to Dunkerque the tidal currents just off Dunkerque would contribute a mean value $\frac{1}{2}(\overline{v^2}) = 400\ \mathrm{cm^2/s^2}$ approximately, corresponding to a persistent mean level difference of 0·4 cm. This increment would not be entirely negligible, but we are fairly safe in assuming it is reduced to zero within the harbour.

The stress τ_a on the water surface due to the wind has been the subject of many experimental investigations, but results of various investigators differ. The measurements which would appear most applicable to our case are those of Darbyshire & Darbyshire (1955) who actually measured surface slopes due to wind over Lough Neagh, whose dimensions are similar to the width of the English Channel near Dover. They found that the component of stress along the line of slope measurement could be fairly well approximated by the usual formula

$$\tau_a = C\rho_a W^2 \cos\theta_w, \quad (5)$$

where ρ_a is air density, W is wind speed at about 10 m height and θ_w its direction relative to the line. The coefficient C varied, but $C = 2\times10^{-3}$ is a reasonable average over all conditions, and agrees roughly with results of other workers (Charnock 1956). Formula (5) was therefore adopted for the present work, with the use of values of W and θ_w recorded at local lightships. Although it occasionally accounts for level differences as large as 10 cm in 50 km the variation of θ_w, and the fact that W^2 is practically zero during long periods of the year make the average effect of wind stress over a year very small. Therefore there appears to be no need to search for a more elaborate formula for the present purpose.

A wind across the channel will also generate waves to break on the windward shore. Longuet-Higgins & Stewart (1962) have shown theoretically that the waves themselves can thereby produce a steady rise or fall of mean sea level near the shoreline. Approaching the shore, the level falls slightly as far as the breaker zone and then rises beyond it. Dorrestein (1962) studied some measurements of wave set-up on an actual beach. The changes in level are attributable to the changing 'radiation stress' of the waves as their height changes in shoaling water, and this stress should therefore strictly be included in equation (4). However, in practice the empirical values used for τ_a probably already contain a typical wave radiation stress, being derived from actual measurements of set-up, and so it is difficult to separate the two. Pure wave set-up may however occur when the wind is absent but swell waves propagate into the channel and break preferentially on one shore. For example, swell propagating southwards through the North Sea may affect the sea level at Dunkerque but not that at Ramsgate, which is somewhat sheltered. However inspection of sea-level differences on days when such swell was present as reported by Dutch lightships did not reveal any consistent effect, and so it was ignored in the present application. The multiple sand bars off Dunkerque (figure 1) would in any case make the theoretical evaluation of wave set-up very difficult because of their complicated effect on breaking.

The stress τ_b on the bottom due to the frictional drag of currents is usually supposed to be of similar form to (5), namely

$$\tau_b = -C'\rho(u^2+v^2)\cos\theta' = -C'\rho(u^2+v^2)^{\frac{1}{2}}v, \tag{6}$$

where $\theta' = \tan^{-1}(u/v)$ is the direction of the current relative to the y axis. The drag coefficient C' varies with bottom roughness, but an average value from the work of Allard (1952), Bowden & Fairbairn (1956) and Charnock (1959) is $C' = 3\times10^{-3}$. As an idea of the possible magnitude of the effect, a longitudinal current with $u = 0$, $v = 10$ cm/s, would produce a tilt of about 0·7 cm in 50 km. For a more realistic example, consider the main part of v as oscillatory with tidal periods, and roughly in phase with the transverse tidal flow u, say $v \doteqdot \alpha u$. Then we have as a good approximation,

$$\tau_b \doteqdot -C'\rho\alpha(1+\alpha^2)^{\frac{1}{2}}u\,|u|. \tag{7}$$

If we now let u have a small mean drift u_0 in addition to a sinusoidal component of amplitude u_1, the mean value of (7) over a tidal cycle can be shown to be approximately

$$-(4/\pi)\,C'\rho\alpha(1+\alpha^2)^{\frac{1}{2}}u_0u_1. \tag{8}$$

Thus τ_b can be regarded as on the average nearly proportional to the mean drift u_0. Its effective value should of course be an average across the section, and will vary with u_1 from spring to neap tides; a rough estimate for the Ramsgate–Dunkerque section gives a level difference of about 1·3 cm for a mean drift $u_0 = 10$ cm/s. It will be seen later that the considerably larger effect of Coriolis stress, which is also proportional to u_0, was estimated empirically by correlation of $\Delta\zeta$ with voltage fluctuations. Therefore, any changes in $\Delta\zeta$ due to bottom friction of type (8) will on the whole be included in the allowance for Coriolis stress. Otherwise, it would be rather difficult to make a direct estimate of the net effect of bottom friction over a tidal cycle, but it is certainly rather small when averaged over a long period.

The tide-raising force γ can be shown to be quite negligible. Its largest possible instantaneous value, at the time of an eclipse, is $1{\cdot}22 \times 10^{-7}$ g, equivalent to a slope of 0·6 cm in 50 km. This is just appreciable, but the variations of daily or monthly means, with which we are concerned, give slopes of order 10^{-2} cm in 50 km, well within the expected order of accuracy. The seasonal variations in sea level, which are usually much larger than the long-period astronomical tides, are mainly due to large-scale meteorological and steric effects, and do not affect the surface slope across such a small area. Earth-tides affect our measurements in so far as they move the supports of the tide gauges, but these too are quite negligible in daily or monthly means.

We come now to the kinematic terms at the end of equation (4), of which that involving fu proves to be on average the most important contributor to $\Delta\zeta$ in the equation. It was in fact the only term mentioned by Proudman (1953) in a brief reference to the levelling problem, though Lacombe (1949) has studied all the kinematic terms in relation to the tidal currents in the central part of the English Channel. In the region of the Dover Strait a current, $\bar{u} = 10$ cm/s produces a transverse slope of 5·8 cm in 50 km. A typical tidal current produces an oscillatory slope with amplitude of some 50 cm per 50 km, but the residual over several tidal cycles is of smaller order. We are here fortunate in having a continuous measure closely related to $\int \bar{u}\,\mathrm{d}y$ in the voltage generated in a cross-channel telegraphic cable by the movement of water through the earth's magnetic field (Longuet-Higgins 1949; Bowden 1956; Cartwright 1961). Apart from various calibration constants, considered in the next section, the voltage signal effectively measures the total flux of water defined by

$$\Phi(t) = \iint u \,\mathrm{d}y\,\mathrm{d}z, \tag{9}$$

which with a possible short time delay is invariant across any section of the channel. However, Φ differs somewhat from $\bar{h}\int \bar{u}\,\mathrm{d}y$, where $\bar{h}$ denotes the mean depth of the section (not necessarily equal to h'), because the ratio

$$\beta = \int_0^L h(y)\,\bar{u}\,\mathrm{d}y \bigg/ \bar{h}\int_0^L \bar{u}\,\mathrm{d}y \tag{10}$$

is not in general equal to 1 unless either the flow $\bar{u}(y)$ or the depth $h(y)$ is uniform across the section. Although the flow coefficient β was only a little greater than 1 for ordinary semi-diurnal tidal flows, it was shown to have the considerably greater value of 2·1 for residual flow between Ramsgate and Dunkerque. This appears to indicate that residual (non-tidal) flow through that section tends to be concentrated in the deepest sections of the channel, whose complicated profile is seen in figure 1, though there may be some modification due to bottom friction, as explained earlier. In any case, a fairly satisfactory value of β can usually be found, as explained in more detail later, and this enables the flux Φ to be used to determine the Coriolis term continuously in time.

The acceleration $\partial\bar{v}/\partial t$ may also be a very important term instantaneously, particularly across sections such as that in figure 1 for which the major axes of the tidal current ellipses are not everywhere normal to the section. However, its average value over a time interval $0 \leqslant t \leqslant T$ is fundamentally

$$T^{-1}[\bar{v}(T)-\bar{v}(0)],$$

which soon becomes negligibly small, especially after a complete number of tidal cycles.

The chief reason for considering the acceleration term is as a warning against making an instantaneous estimate of level at a time of tidal slack water. It is fairly easy to find times when the tidal current is practically zero everywhere across the section, and winds and pressure gradients are negligible, so that nearly all the terms vanish on the right of (4). But these are just the times when $\partial\bar{v}/\partial t$ is large and causes a considerable gradient. Measurements at Ramsgate and Dunkerque at times of slack water and no wind showed differences as great as 150 cm between values before ebb and before flood at spring tides, due to accelerations with amplitude of order 10^{-2} cm/s^2. Averaging over successive pairs of slack water times tends to reduce the effect, but it is well known that such averages are unreliable, particularly when as here shallow-water tidal components are present. It is clear that the only way to eliminate the effect of acceleration is to take true average values with respect to time, using constant time intervals of not more than 3 h, or preferably less.

Finally we consider the term $\overline{u\,\partial v/\partial x}$. Omitting the bar for convenience, we first note the alternative expression

$$u\frac{\partial v}{\partial x} = v\frac{\partial u}{\partial x}-(u^2+v^2)\frac{\partial \theta}{\partial x}, \tag{11}$$

which follows by differentiating the relation $u = v\tan\theta$. By the equation of continuity, the term $v\,\partial u/\partial x$ may be equated to $-v\,\partial v/\partial y$, whose contribution Δv^2 to the integral may be ignored, as shown above, so the principal term on the right-hand side of (11) is the product of the squared absolute current and the curvature of flow. This essentially centrifugal effect on sea levels was discussed by Courtier (1933) in application to the rapid currents around the Cherbourg peninsular, and also by Lacombe (1949) (see also Stewart 1957). Although in the present application it is rather small, it is the only kinematic contribution to the set-up (apart strictly from $\Delta\overline{v^2}$) which has a mean non-zero value for a pure tidal cycle without residual drift. Its mean value can therefore be estimated from the constants of the tidal currents in the region if they are known in a sufficient number of places. This estimation was carried out in the present case, and proved to be the only method which yielded a satisfactory result.

Another method of estimating this second-order term is to search for long-period variations corresponding to the difference-frequencies of large amplitude tidal harmonics, principally M_2, S_2, N_2. As discussed in more detail later, the cross-products produce sum-frequencies of quarter-diurnal period which are largely suppressed in running daily averages, and also periods of several days essentially in phase or anti-phase with the spring tides, as is obvious from other considerations. In order to analyze these long-period variations it is necessary to deal with averaged measurements taken at intervals of 6 h at most in order that residual semi-diurnal components do not appear as 'pseudo' or 'alias' long-period components, and the analysis must extend over many multiples of the long periods involved, in order that the harmonic components should stand out significantly above 'background noise' effects. In the present application with 6-hourly 25 h means extending over 2 years, the long-period amplitudes were still too small to give a significant result. However, the method might prove valuable in regions where the currents are more curved.

4. Practical application

(a) *Choice of section*

Application to an actual levelling between England and France was limited by the simultaneous availability for a fairly long period of good tide-gauge records on either side of the channel, and of voltage records for estimation of mean flow. By courtesy of the General Post Office Engineering Department records of voltage across one of the cross-channel telegraph cables between St Margaret's Bay, near Dover, and Sangatte, near Calais, have been made with very few interruptions since 1953. Several tide gauges have been operating in south-east England during this period. The marégraphes of northern France were mostly destroyed at the end of the 1939–45 war and have only recently begun to be replaced. Some records taken at Shoreham and Dieppe, used by Bowden (1956), extended for only a few months at the end of 1953. They were used for a pilot analysis, whose results will be mentioned later, but they were considered too short and the section too remote from the cable section for the results to be very reliable. Finally, the section Ramsgate–Dunkerque (figure 1) was chosen for a full analysis, since good tide-gauge records were available at both ports during the whole of 1957 and 1958, and the area is fairly near the Dover Strait. However, the section is not ideal, because of the many sand bars (figure 1) and the proximity of the Goodwin Sands and other banks which are dry at Low Water Springs.

The records for Ramsgate were obtained from the local harbour authority, and those for Dunkerque through the kind co-operation of the Service Hydrographique de la Marine, who also supplied figures relating the gauge zero to the zero of the 'Nivellement Général'. The corresponding levelling data for the Ramsgate gauge were re-estimated specially for this work by the Ordnance Survey, as part of the 3rd Geodetic Levelling of Great Britain (Burnett & Carmody 1960). The Ordnance

Survey also undertook the arduous task of abstracting the hourly readings from both sets of tide gauge records, and evaluating the 25 h averages.

The tide-gauge at Ramsgate was established in 1843 by Messrs G. J. Rennie of London, and overhauled in 1956 by Lea Recorder Co. Apart from normal wear and tear, it has always worked well. The gauge at Dunkerque is a modern high-precision instrument of the type 'Marégraphe St Chamond–Granat' described by Imbert (1954). It was installed in June 1956, and apart from an interruption during September to October 1956, has performed well ever since.

The time span analysed was from noon 4 January 1957 to noon 28 December 1958. This period of 723 days (about 49 tidal fortnights) is nearly an exact number of cycles of most of the principal tidal harmonic components, and so possible errors due to unremoved tidal effects are minimal. 25 h averages every 6 h were taken in preference to the more elaborate 'X_0 tidal filter' (Doodson & Warburg 1945) in order to simplify the hand computations and checking. The tidal variation left in the differences of 25 h means at Ramsgate and Dunkerque is very small, and negligible when 6-hourly values are averaged over a complete number of days. The interval of 6 h eliminates S_2 components completely, and prevents other semi-diurnal components from appearing like the long-period components required for analysis of the second-order terms. (With a 12 h interval, M_2 appears like *MSf*.)

(*b*) *Pressure and wind*

The atmospheric pressure differences were computed from pressures at three stations read at 6 h intervals from the Meteorological Office Daily Weather Reports. Winds were at first computed from the pressure gradients, by a standard method, but it was found that this often gave values of W^2 very much in excess of those recorded directly at nearby stations. Accordingly, values of W and θ_w were read every 6 h from records of lightships stationed near the section. During 1957, records from the 'Dyck' (see figure 1), held by the National Institute of Oceanography, were used; during 1958, records from the 'Noord Hinder' (51° 39′ N., 02° 34′ E.), published by the Royal Netherlands Meteorological Institute. Values of W had to be converted from Beaufort numbers by means of a standard scale, but the possible inaccuracies so caused were too small to be of any consequence in the long run. Both pressure and wind terms were smoothed over 24 h by the operator

$$\bar{X}_0 = \tfrac{1}{4}(\tfrac{1}{2}X_{-12} + X_{-6} + X_0 + X_6 + \tfrac{1}{2}X_{12})$$

to accord with the 25 h tidal averages as far as possible.

(*c*) *Voltage measurements and calibration*

The electrical potential between the St Margaret's Bay–Sangatte cable and its earth is recorded continuously at the St Margaret's Bay G.P.O. Repeater Station by an ordinary pen milliammeter. The circuit is similar to that described in Bowden's figure 3 *b* (not 3 *a*), and consists essentially of a 0·03 F capacitance shunt to smooth out transient earth–current effects of the duration of a minute or so, and a 30 μF path for the a.c. telephone signals. We may represent the recorded e.m.f. E in the form

$$E = E_0 + E_1(t) + K\Phi(t), \tag{12}$$

where E_0 is a constant value relative to the true zero-voltage (closed circuit) reading, due to electro-chemical effects between the cable screen and the sea and possibly to d.c. earth currents. $E_1(t)$ is an oscillatory voltage due to variable earth currents which are known to be of fairly regular daily period with occasional irregular disturbances at times of magnetic storms (Chapman & Bartels 1940). The last term of equation (12), due to the water flux Φ is the dominant part of E, and the factor K varies only within about $\pm 10\%$ of a constant value due to seasonal changes in conductivity. Although K can in theory be calculated if the conductivity of the rocks below the sea bed is known (Longuet-Higgins 1949), in practice it has to be determined by direct measurements, which are also necessary for the determination of E_0.

A full-scale calibration of the cable e.m.f. was carried out during May 1960, when, with the aid of two Admiralty coastal survey ships and current meters supplied by Kelvin Hughes, Ltd, over 6000 readings of current speed and direction were made at six stations along the line between St Margaret's Bay and Sangatte. At each station, currents were recorded at all depths every 30 min for about 40 h. The analysis and results are described fully by Cartwright (1961), so only a brief outline need be given here. Values of $\overline{u}h(t)$ relative to the line of measurements were evaluated for each half-hourly set of readings, and the principal tidal constituents computed from them. For separating the closer tidal constituents an analysis of a 29-day series of hourly values of E was used as a reference. The factor K was evaluated by integrating the M_2 tidal component across the channel, since the semidiurnal component of $E_1(t)$ is certainly very small compared with that of $K\Phi$. Comparison of the diurnal components, using this value of K, gave an estimate of the amplitude of $E_1(t)$, which, though we do not require to know it in the present investigation, was shown to be of the same order of magnitude as the (small) diurnal components of $K\Phi$.

The residual d.c. flow at each recording station was calculated fairly accurately, but since they were not all simultaneous some of the measurements had to be adjusted for the fortnightly variation in mean flow during the recording period, indicated by the mean values of $E(t)$. Thus adjusted, the six values of d.c. flow could be integrated across the channel to obtain the d.c. value of Φ, from which E_0 could then be deduced by using the value of K. A value $E_0 = -86$ mV was found, equivalent to a uniform flow of 11·6 cm/s between St Margaret's Bay and Sangatte, which in turn would produce an increment of 3·9 cm in $\Delta\zeta$.

Unfortunately, little is known about 'permanent' earth currents; according to Chapman & Bartels (1950, § 13.11) they are indistinguishable from contact potentials at the electrodes in land measurements, but thought to be less than 1 mV/km. How far our measurement of $E_0 = -83$ mV is due to a permanent earth current or to contact potentials between the cable screen and the sea is not known, but in the absence of any further information, we have assumed it to be reasonably constant, and have applied it to the voltages measured during 1957 and 1958 as if this were so.

The seasonal variations in K were estimated by Bowden's method of comparing monthly mean tidal ranges of potential and of water level at Ramsgate. The ratios are plotted in figure 2 as proportions of the 2 yearly average, which was 134 mV/ft.

They are on the whole greatest in September, when sea temperature is greatest, and least around March, in accordance with Bowden's measurements, though the amplitude of the variation evidently varies from year to year. The corresponding ratio for May 1960, when the cable was calibrated, was 131 mV/ft., and so in application the ratios shown in figure 2 had also to be multiplied by 134/131, an increase of 2 %.

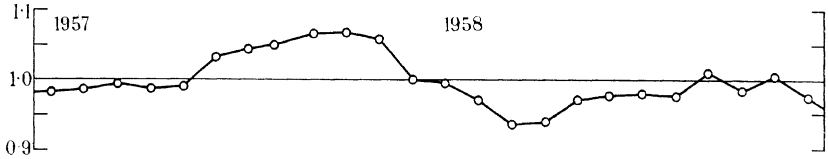

FIGURE 2. Seasonal variation of sensitivity of cable voltage per water transport, relative to mean value for 1957 and 1958.

(d) *Estimation of β*

At an early stage of the work, the running mean values at 6-hourly intervals,

$$\Delta'\zeta = \zeta_R - \zeta_D - \frac{1}{\rho g}\Delta p_a + \frac{L}{\rho g h'}\tau_a + \frac{f}{g}\int_0^L \bar{u}\,\mathrm{d}y, \tag{13}$$

where ζ_R and ζ_D are the 25 h mean water levels at Ramsgate and Dunkerque measured from their respective gauge-zeros, were computed on the assumption that the flow coefficient $\beta = 1$ in equation (10). In other words, in evaluating the Coriolis effect from the flow measured by the cable potentials, $\bar{u}$ was assumed to be uniform across the section. However, it was soon noticed that $\Delta'\zeta$ so estimated contained larger variations than expected, positively correlated with the variations of Ω. On the other hand values of $\zeta_R - \zeta_D$ are negatively correlated with Φ, as seen in figure 3. This indicated that the estimate $\Delta'\zeta$ was being over-compensated for the Coriolis set-up; that is, the assumed value $\beta = 1$ was too small.

It was evident that the effective value of β could only be estimated by an empirical method. Twelve periods of 25 days, each containing about 100 sets of measurements, were taken at roughly equal intervals over the 2-year span, and each was analyzed for a linear regression between Φ and $\Delta''\zeta$, where $\Delta''\zeta$ consisted of the expression on the right-hand side of (13) with the last term omitted. The coefficients of correlation r between $\Delta''\zeta$ and Φ, and corresponding values of β are tabulated below.

year	dates	r	β
1957	4 Jan. to 29 Jan.	−0·96	2·2
	20 Mar. to 14 April	−0·71	4·4
	3 June to 28 June	−0·75	1·8
	17 Aug. to 11 Sept.	+0·08	—
	6 Oct. to 31 Oct.	−0·10	—
	31 Oct. to 25 Nov.	−0·90	1·9
1958	14 Jan. to 8 Feb.	−0·45	4·2
	5 Mar. to 30 Mar.	−0·68	3·1
	30 Mar. to 24 April	−0·82	1·8
	8 July to 2 Aug.	−0·74	2·0
	21 Sept. to 16 Oct.	−0·61	2·8
	10 Nov. to 5 Dec.	−0·77	1·3

Values of r are seen to be usually large and negative, as should be expected, with two exceptions where the correlation was disturbed presumably either by unusual flow patterns in the water or by effects of magnetic storms on the cable voltages. The values of β vary rather irregularly, but are all significantly greater than 1. As a compromise, the nine sets of values for which r was between $-1\cdot0$ and $-0\cdot6$ were combined to give an estimate of $\beta = 2\cdot12$ with overall correlation coefficient $-0\cdot80$. Finally, this value of β was used for the whole period in the combination $(f\,\Phi/g\beta\bar{h})$ to replace the last term in (13).

It is interesting to note that if we calculate a value of β appropriate to the main semi-diurnal tidal flow by means of the amplitudes and phases of u along the y axis which are evaluated in the next section (see figure 5), we find it is only 1·09. This is no doubt partly because the flow distribution across the channel of the semi-diurnal tide is different from that of the residual flow, which must be concentrated in the deeper parts of the channel. Such differences are borne out by the direct measurements of Van Veen (1938) and Cartwright (1961), although these were made in a different part of the channel, and at times of slight wind. It has also been pointed out that the empirical estimate of β from non-tidal flows will contain an element due to bottom friction as in (8), which would reduce the set-up and hence increase the apparent value of β. The effect of bottom friction during a tidal cycle would of course be quite different in character, and would be difficult to estimate by direct analysis of measurements of $\Delta\zeta$ because of the effect of acceleration.

Apart from the small second-order corrections considered below, the sums $\Delta'\zeta$ given by (13) are the best estimates of the true difference of the reference levels at the two places which we can obtain by direct measurement. Figure 3 shows the daily values of the uncorrected differences $\zeta_R - \zeta_D$ (measured from the gauge zeros), the Coriolis correction $f\,\Phi/g\beta\bar{h}$, and the differences with fully correction, $\Delta'\zeta$. Only the averages centred at mid-day are shown, for convenience. Gaps in the upper two records represent missing data for either ζ_R, ζ_D or E; only the corresponding gaps in the bottom record $\Delta'\zeta$ were filled in, by linear interpolation, since the latter has least variation. It is clear that a good deal of the variation in $\zeta_R - \zeta_D$ is eliminated by the addition of the Coriolis term; indeed the method of estimating β ensured this. The effects of wind and pressure included in $\Delta'\zeta$ also help to smooth out some irregularities, but these are on the whole rather small and are therefore not shown individually.

Nevertheless, $\Delta'\zeta$ still has a standard deviation of 4·88 cm, compared with 7·35 cm for the uncorrected difference $\zeta_R - \zeta_D$. Since its more obvious irregularities appear to be independent of the Coriolis correction and are already present in $\zeta_R - \zeta_D$, they are unlikely to be due to earth-current anomalies in E, or to substantial changes in the flow parameter β. They are mostly of too long a time scale—of the order of a month—to be entirely accounted for by swell waves, or by errors in tide-gauge chart alignment. Further, the absence of any obvious correlation with spring tides, indicated by letters S, suggests that second-order hydrodynamic effects are not important, as will be confirmed below. There remain the possibilities of differential changes in water density due to freshwater discharge from the Rivers Thames or Rhine, or of irregular (non-tidal) earth movements caused perhaps by

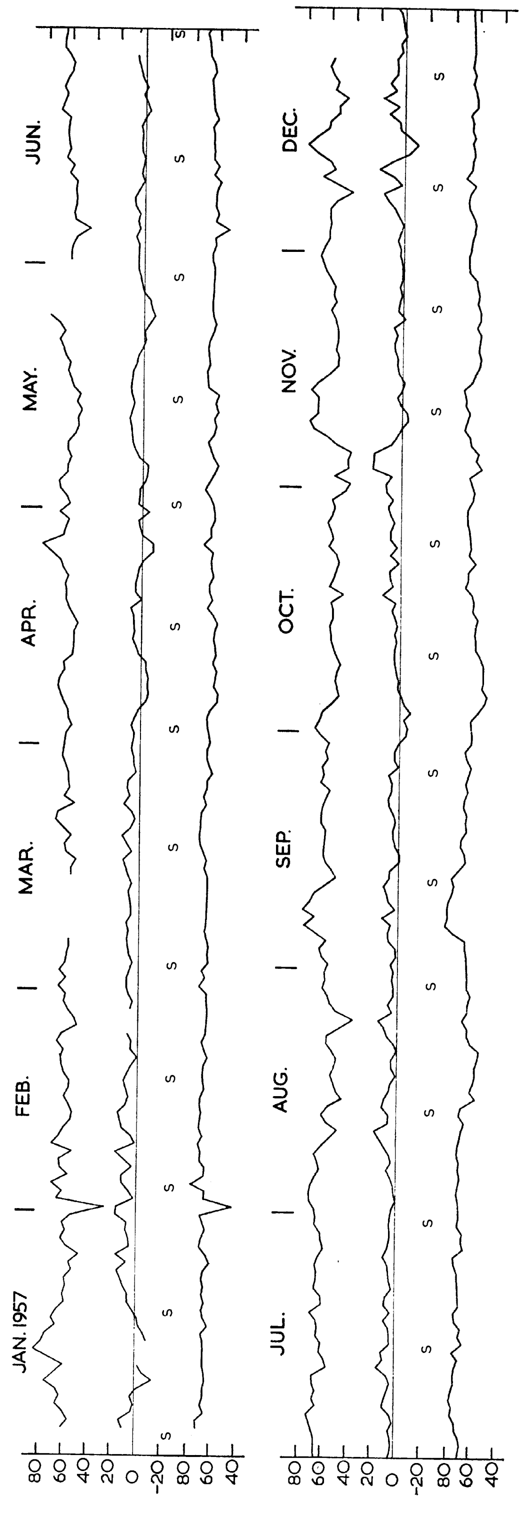

FIGURE 3. See facing page for legend.

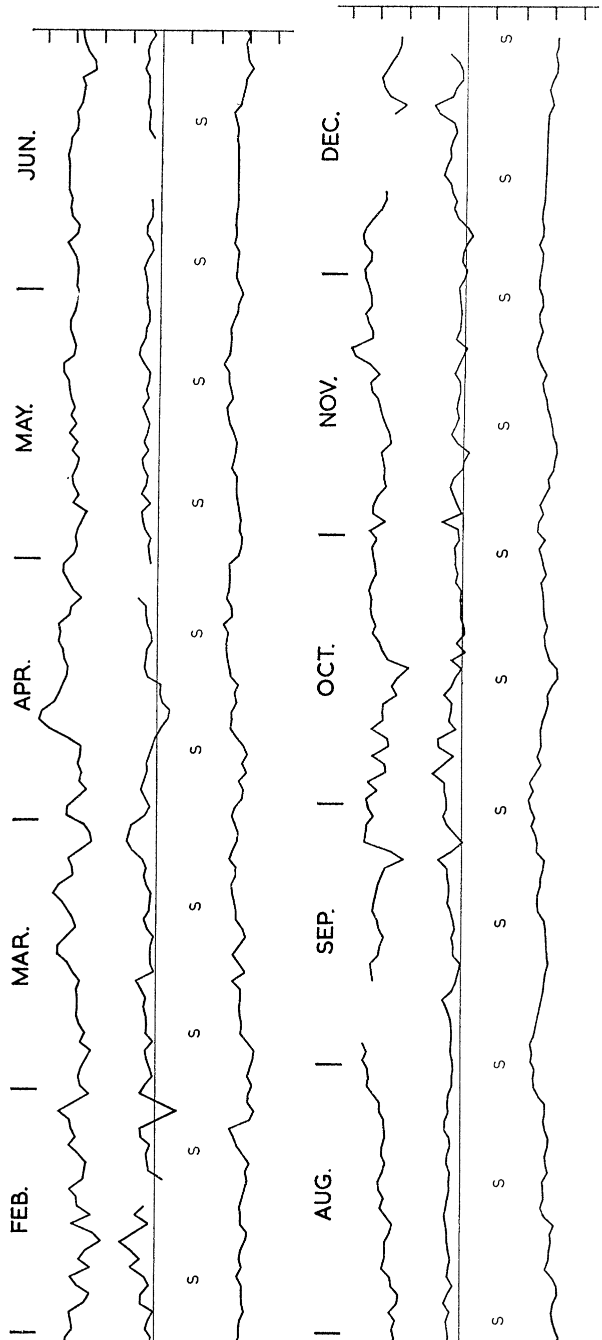

FIGURE 3. Top curves, daily values of 25 h means of $(\zeta_R - \zeta_D)$ (arbitrary zero); middle curves, Coriolis set up deduced from cable voltage; bottom curves, corrected differences $\Delta'\zeta$.

varying moisture content of the soil and affecting the zeros of the tide gauges. However, not enough is known about these factors to be able to assess them, and so we must merely hope that their average effects over two years are fairly small, or at least stable.

(e) *Second-order hydrodynamic effects*

As mentioned earlier, two methods were used to attempt to discover any effect of the second-order term involving $\overline{u\,\partial v/\partial x}$. The method of harmonic analysis for long tidal periods can be outlined as follows. Dropping the bars for convenience, we may represent the values of u and $\partial v/\partial x$ at any point $(0, y)$ by equations of the type

$$u = U_m \cos(\sigma_m t - \phi_m) + U_s \cos(\sigma_s t - \phi_s) + U_n \cos(\sigma_n t - \phi_n) + u_r(t)$$
$$\partial v/\partial x = V'_m \cos(\sigma_m t - \psi_m) + V'_s \cos(\sigma_s t - \psi_s) + V'_n \cos(\sigma_n t - \psi_n) + v'_r(t),$$

where the suffixes m, s and n refer to the dominant semi-diurnal tidal harmonic constituents M_2, S_2 and N_2, respectively, and u_r and v'_r include the remaining constituents and random variations, in general of smaller amplitude than the first three. We may safely assume constant amplitude ratios

$$\frac{U_s}{U_m} = \frac{V'_s}{V'_m} = k_1 = 0{\cdot}33, \quad \frac{U_n}{U_m} = \frac{V'_n}{V'_m} = k_2 = 0{\cdot}19,$$

the numerical values being derived from tidal analyses by Cartwright (1961) with a slight adjustment for the 19 yearly variation factor f (Doodson & Warburg 1945), appropriate to 1 January 1958. Further, since the tidal ellipses are very elongated in the region we may assume

$$\phi_m = \psi_m,\ \phi_s = \psi_s,\ \phi_n = \psi_n \quad \text{(or with 180° added)}$$

approximately. With these assumptions, the product $u\,\partial v/\partial x$ has the leading terms

$$\begin{aligned} u\,\partial v/\partial x = {} & \pm\tfrac{1}{2} U_m V'_m (1 + k_1^2 + k_2^2) \pm U_m V'_m [k_1 \cos(\sigma_{10} t - \phi_s + \phi_m) \\ & + k_2 \cos(\sigma_{02} t - \phi_m + \phi_n) + k_1 k_2 \cos(\sigma_{12} t - \phi_s + \phi_n)] + \dots, \end{aligned} \tag{14}$$

where
$$\sigma_{10} = \sigma_s - \sigma_m = 1{\cdot}0159\ \text{deg/h} \quad \text{(period 14·77 days)},$$
$$\sigma_{02} = \sigma_m - \sigma_n = 0{\cdot}5444\ \text{deg/h} \quad \text{(period 27·55 days)},$$
$$\sigma_{12} = \sigma_s - \sigma_n = 1{\cdot}5603\ \text{deg/h} \quad \text{(period 9·61 days)},$$

and the corresponding terms with frequencies such as $\sigma_m + \sigma_n$ are omitted (periods about $\frac{1}{4}$ day), as they are largely suppressed in the 25 h averages. The result of integrating with respect to y across the width of the channel is essentially similar to (14), and expresses the contribution of the term to $\Delta\zeta$. If, therefore, we make a harmonic analysis of $\Delta'\zeta$ (which consists essentially of the supposed constant difference in level datums less the correction we are investigating), and obtain, for example at frequency σ_{10}, an amplitude A_{10} and phase p_{10}, then the constant contribution of $-g^{-1}\int \overline{u\,\partial v/\partial x}\,.\mathrm{d}y$ to $\Delta\zeta$ is

$$(2k_1)^{-1}(1 + k_1^2 + k_2^2)\, A_{10} \cos(p_{10} + \phi_s - \phi_m). \tag{15}$$

The argument of the cosine in (15) should ideally be 0 or 180°, according to the differences between the phases ϕ and ψ.

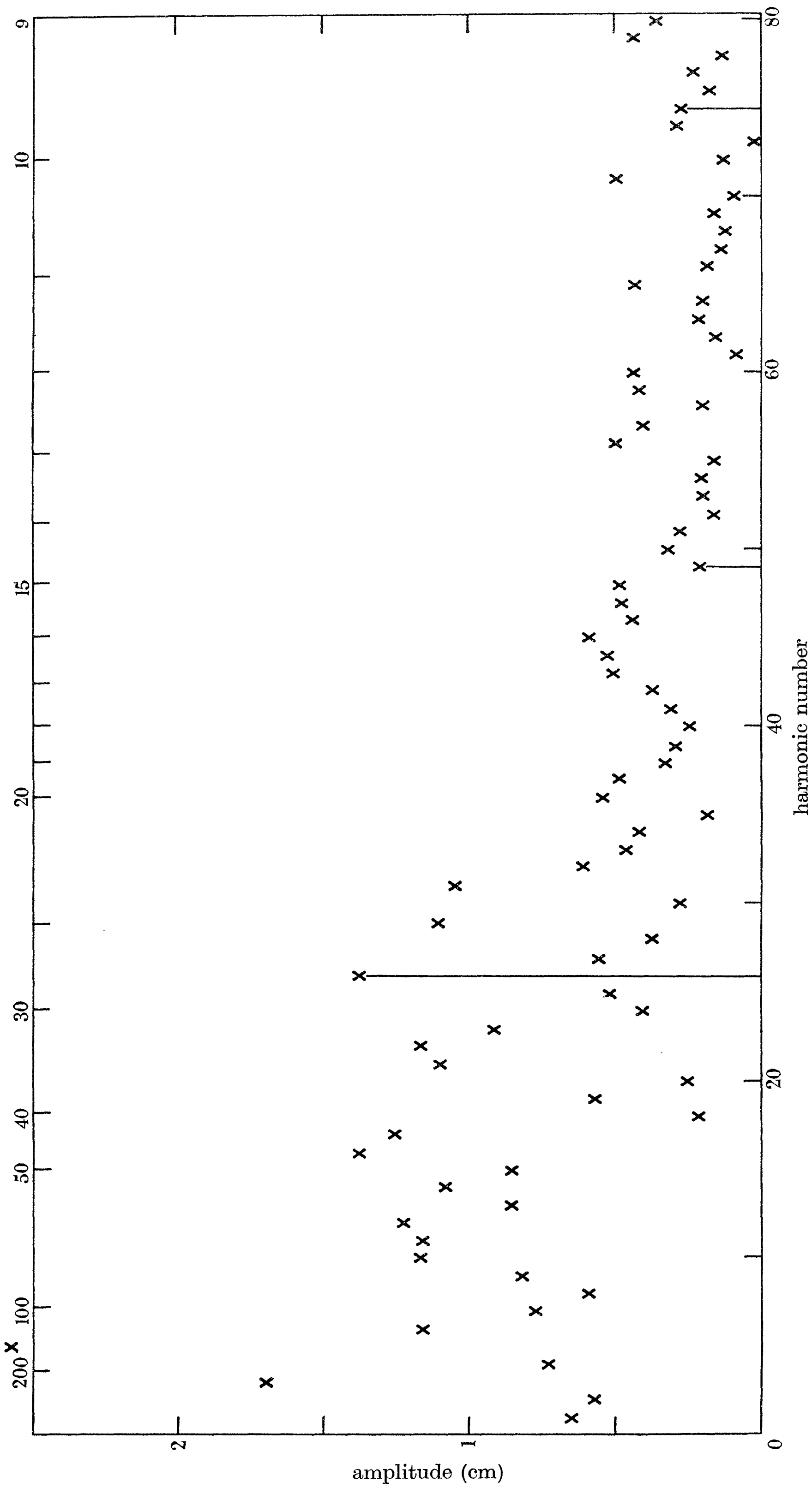

FIGURE 4. Harmonic amplitudes of 723 days' values of $\Delta'\zeta$. Those with special tidal significance marked by vertical lines.

Figure 4 shows the amplitudes of the first 80 Fourier harmonics of the 2892-long series of $\Delta'\zeta$, obtained by electronic computer. They are in fact values of $|R_k|$, where

$$R_k = \sum_{r=-\frac{1}{2}N}^{\frac{1}{2}N}{}'' \Delta'\zeta(rt') \exp(2\pi i kr/N) \quad (N = 2892, t' = 6h),$$

and Σ'' means a summation with the first and last terms halved. The frequencies σ_{02}, σ_{10}, σ_{12}, correspond fairly closely to the harmonics $k = 26$, 49 and 75, respectively, indicated by vertical lines. It is clear that the amplitudes at these three frequencies are of the same order of those at adjacent frequencies which can only be due to random variations uncorrelated with tides. In fact, harmonic 49, which, corresponding to $M_2:S_2$ interaction, should be the greatest of the three, is actually lower than the background 'noise'. It can also be shown to be of the same order of magnitude as the reduced amplitude due to the quarter-diurnal component MS_4, which also contributes to harmonic 49 in virtue of the 6 h time intervals. Harmonic 26 is a little larger than its neighbours, and if true would indicate a constant value (15) of $-3{\cdot}6$ cm (the argument of the cosine is actually 150°), meaning that $\Delta'\zeta$ should be increased by 3·6 cm. However, since this would only be consistent with an amplitude of 2·4 cm at harmonic 49, it cannot be relied upon, but must be largely due to random variation.

On the whole, then, the method of harmonic analysis fails to show up any second-order effect above the level of random noise, confirming the qualitative absence of variations correlated with spring tides in figure 3. The other method, using the known tidal currents in the region, seems to be more reliable in the present case, and does indicate a small second-order effect.

British Admiralty tidal stream data were used, for convenience as compiled in the German Hydrographic Institute's Atlas.† Data were taken from all fourteen stations within 20 km of the section line, namely numbers 38 to 44, and 175 to 182, excepting number 177. Their positions are shown as black circles in figure 1. At each station, hourly values of current speed and direction are given for spring tides from 6 h before to 6 h after high water at a standard port, in most of the present cases, Dover. These current vectors were all resolved into u and v components relative to our axes fixed on Ramsgate and Dunkerque, and from each set the mean semi-diurnal amplitude and phase were extracted by means of a simple numerical filter. This extraction was necessary because the currents as recorded rightly contained shallow water effects, whose inclusion, though unimportant, would make the work unnecessarily complicated. The phases were adjusted if necessary to refer to the time of high water at Dover.

By this process, the mean spring tide semi-diurnal current of frequency σ at each station, co-ordinates x, y, was specified by the four components u_1, u_2, v_1, v_2, where

$$u(x,y,t) = u_1(x,y)\cos\sigma t + u_2(x,y)\sin\sigma t,$$
$$v(x,y,t) = v_1(x,y)\cos\sigma t + v_2(x,y)\sin\sigma t.$$

These components varied somewhat irregularly with x and y, partly because of variations in depth (see profile in figure 1), and also probably because of errors in the

† *Atlas der Gezeitenströme fur die Nordsee, den Kanal, und die Britischen Gewasser*, Hamburg, 1956.

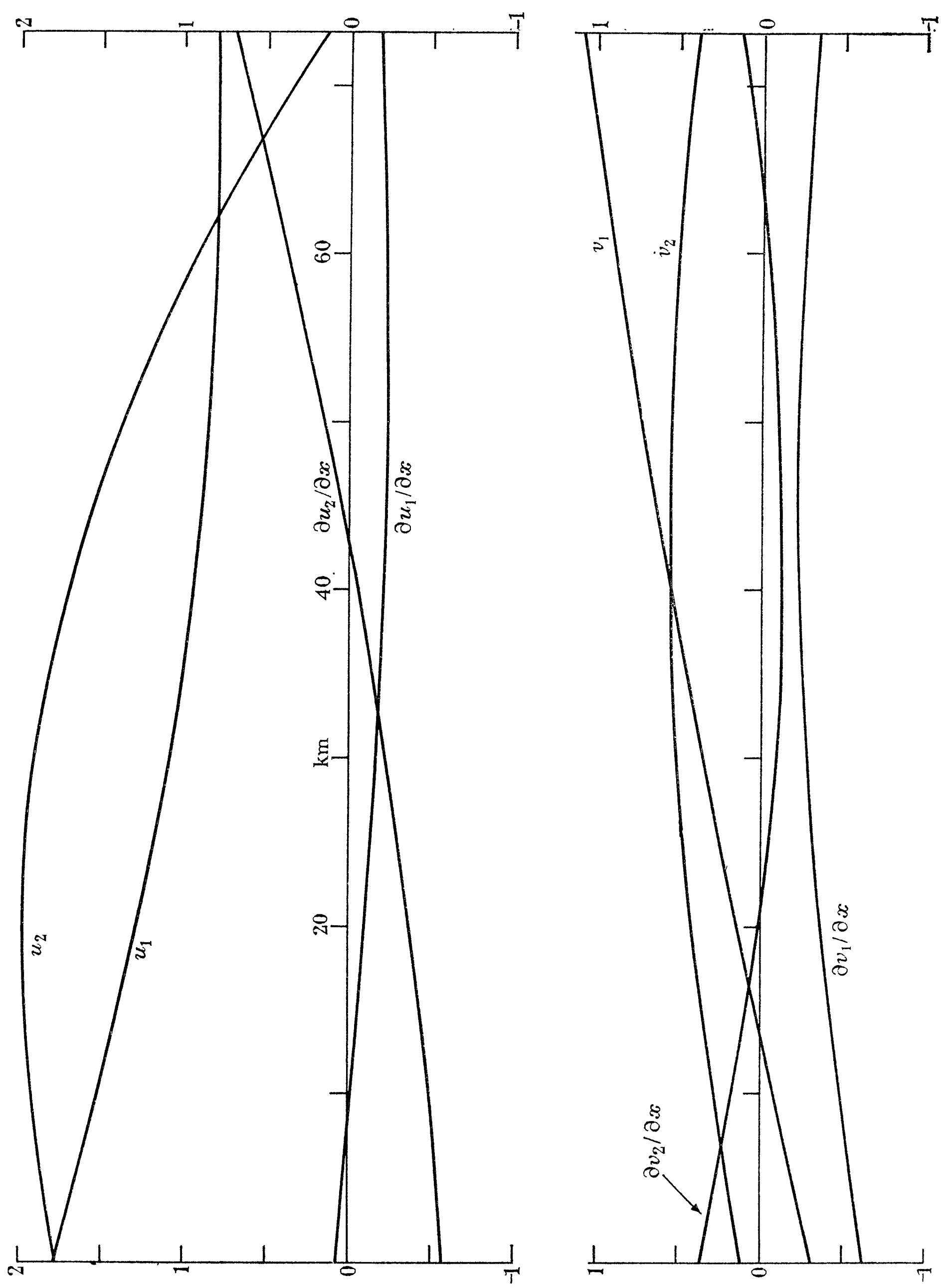

FIGURE 5. Phased amplitudes of mean semi-diurnal spring tidal velocities (knots) and velocity-gradients (knots/kilometre) computed for the Ramsgate–Dunkerque section.

original readings on which the data were based. In order to interpolate suitable values of u and v and their x derivatives along the line $x = 0$, a smooth function of the form

$$Ay^2 + By + C + x(Fy^2 + Gy + H) \qquad (16)$$

was calculated for least-square deviation from each set of values of u_1, u_2, v_1 and v_2. The least r.m.s. deviations from the functions (16) were respectively

0·24, 0·31, 0·30 and 0·37 knots,

not unduly large for values of order 1 to 2 knots. The resulting four velocity components and their x-derivatives along the 74 km of the cross-section $x = 0$ (all quadratic functions of y) are plotted in figure 5.

With the numerical constants in (16), it was then easy to compute the average second-order contribution to $\Delta\zeta$ over a spring tidal cycle, namely

$$\partial\zeta = -(2\times 1{\cdot}4g)^{-1}\int_0^L (u_1\,\partial v_1/\partial x + u_2\,\partial v_2/\partial x)_{x=0}\,\mathrm{d}y. \qquad (17)$$

The divisor 1·4 in (17) is an allowance for the average velocity product over the depth, in accordance with van Veen's (1938) empirical rule,

$$u(z) = u(0)\,(1-z/h)^{0{\cdot}2} \quad (v(z) \text{ similar}),$$

since all the basic current data were for the surface $z = 0$. The result is $\delta\zeta = +2{\cdot}7$ cm for mean spring tides, and on multiplying by the factor

$$(1+k_1^2)/(1+k_1)^2 = 0{\cdot}63,$$

we obtain a final average over all states of the tide,

$$\partial\zeta = +1{\cdot}7 \text{ cm},$$

which should be added as a constant increment to $\Delta'\zeta$.

5. Results on levelling

(a) *Ramsgate–Dunkerque*

Taking into account all the factors discussed above, the final average value of $\Delta'\zeta + \delta\zeta$ over the 723-day period is 19·6 cm, made up of the following contributions

Ramsgate—mean sea level above 'O.D.'	+3·7 cm
Dunkerque—mean sea level above 'N.G.'	−8·0
mean $\zeta_R - \zeta_D$	+11·7 cm
correction for mean wind stress	+1·0
air pressure	−0·3
Coriolis stress	+5·5
$(u\,\partial v/\partial x)$	+1·7
total	+19·6 cm

The sums are shown geometrically in figure 6 (*a*). The actual figure derived for the mean of ζ_D was 329·6 cm, referred to the zero of the Dunkerque tide gauge. The official figure supplied by the 'Service Central Hydrographique de la Marine' for the level of this gauge zero is 337·6 cm below the basic French level datum known as

'Zéro officiel du Nivellement Général de la France' (N.G.); hence the figure $-8\cdot0$ cm quoted above. The figure for ζ_R was similarly adjusted to the British 'Ordnance Datum' (O.D.) by the use of a figure for the level of the Ramsgate gauge zero supplied by the Ordnance Survey from special measurements carried out recently for scientific purposes, as part of the Third Geodetic Levelling of Great Britain (Burnett & Carmody 1960). (This figure is not at present available for publication.)

The mean sea levels quoted do not of course agree with the official figures for the ports, since the latter are usually based on average values for several previous years. The figures $\bar{\zeta}_R = -10$ cm.† (Admiralty Tide Tables) and $\bar{\zeta}_D = -15$ cm. (Annuaire des Marées) are both lower than the mean values shown above, but are not inconsistent with them in view of the fluctuations of several centimetres in successive yearly means, reported for example in the publications of A.I.O.P.‡

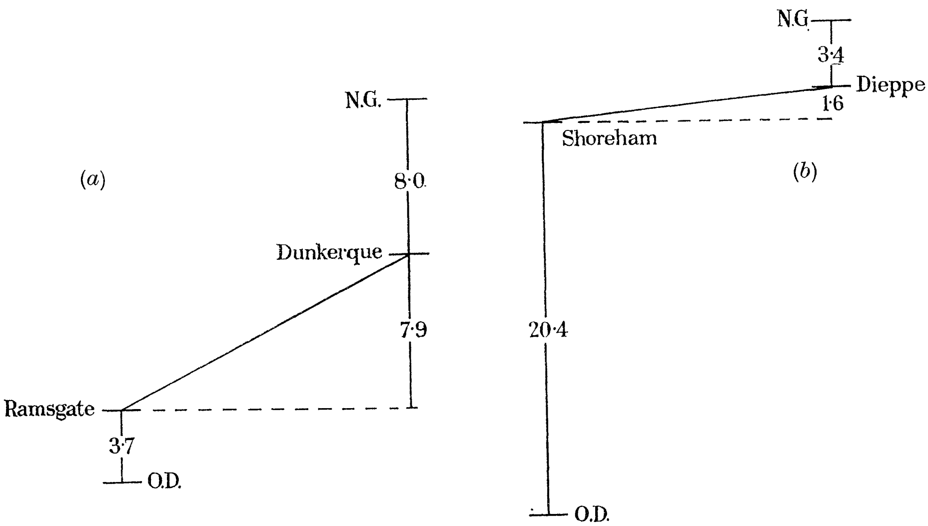

FIGURE 6. Diagram of mean sea level and set-up for (*a*) Ramsgate–Dunkerque, 1957–1958, (*b*) Shoreham–Dieppe, Sept.–Oct. 1953.

The standard deviation of the 2892 values of $\Delta'\zeta$ is $4\cdot9$ cm, and of their 30-day means, $1\cdot9$ cm, so that errors due to random variations should be reduced in the 2-year mean value of level difference to a standard error of less than $0\cdot5$ cm. However, this is obviously an underestimate, as it takes no account of errors in estimating or neglect of constants such as E_0, or variables with possible non-zero average such as the effects of wave set-up, or earth movements. A generous estimate of the standard error due to such factors is $1\cdot5$ cm. Errors in the geodetic data relating the gauge zeros to the respective datum levels are no doubt also possible, but estimation of these is outside the scope of this paper. We may finally state, then, that according

† The official figure is based on a year's readings in 1864. University of Liverpool Tidal Institute quote the figure $\bar{\zeta}_R = +13$ cm for 1961 (private communication).

‡ Association International d'Océanographique Physique, Permanent Service for Mean Sea Level. The figures for French ports given in these publications are obtained from instruments known as 'médimarémetres', and refer to a 'zéro rationnel du N.G.', which at Dunkerque is 15 cm lower than the 'zéro officiel du N.G.' referred to above.

to the present investigation, Ordnance Datum (Newlyn) was, during 1957–58, 19·6 cm below Zéro Officiel du Nivellement Général de la France, with a standard error of ± 1·5 cm.

(*b*) *Shoreham–Dieppe*

Previously, one of the authors (J. C.) had made a similar analysis of the water levels at Shoreham and Dieppe (about 90 miles south-west of the Ramsgate section) during the 61-day period September to October 1953. The results are certainly less reliable than the main analysis described above, because they are derived from a much shorter period and a wider stretch of water ($L = 138$ km), and the records from the Dieppe tide gauge are not so well authenticated as those from the recent installation at Dunkerque. However, they provide an interesting comparison.

By correlation of E and $\Delta'\zeta$, a flow coefficient $\beta = 1{\cdot}17$ was deduced with correlation coefficient $-0{\cdot}85$. The cable constant E_0 was estimated by comparison with continuous current measurements at the Varne lightship made between July 1953 and May 1954 (Carruthers 1928, 1935; records also used by Bowden 1956), and was found to be equivalent to a mean current of $-8{\cdot}8$ cm/s, or -65 mV. This compares favourably with the value $E_0 = -86$ mV obtained by more elaborate and accurate measurements in 1960. Using these figures, the following results were obtained (see figure 6*b*),

Shoreham—mean sea level above O.D.	$+20{\cdot}4$ cm
Dieppe—mean sea level above N.G.	$-3{\cdot}4$
mean $\zeta_S - \zeta_{Di}$	$+23{\cdot}8$ cm
correction for mean wind stress	$+0{\cdot}1$
air pressure	$-0{\cdot}4$
Coriolis stress	$+1{\cdot}9$
total	$+25{\cdot}4$ cm

It was not possible to estimate the contribution from $(u\,\partial v/\partial x)$, but it is probably less than 1 cm in magnitude, because the currents have less amplitude and lateral variation than along the Ramsgate section. The ζ value for Dieppe, $-3{\cdot}4$ cm, consisted of a mean sea level of 485·6 cm above a gauge zero which was stated to be 489 cm below 'Zéro officiel du Nivellement Général de la France'. As for Ramsgate, the ζ value for Shoreham was adjusted to Ordnance Datum (Newlyn) using a recent (1960) levelling specially carried out by the Ordnance Survey. The previous levelling, made about 40 years earlier, would have reduced ζ_S and the final total by some 5 cm,† making better agreement with the result derived from Ramsgate and Dunkerque, but it is probable that the later levelling is more accurate and appropriate, even though seven years later than the tide-gauge readings.

Because of the short duration and various uncertainties, a standard error for the final estimate of level difference, 25·4 cm, is about 3·0 cm. The standard error of the difference between this estimate and the value 19·6 cm derived from the Ramsgate section is $\sqrt{(1{\cdot}5^2 + 3{\cdot}0^2)} = 3{\cdot}4$ cm, so that the actual difference, 5·8 cm, between the

† The value 19 cm for the level difference quoted by Cartwright (1960, 1961) was based on the older levelling figure for Shoreham, and also included some minor errors which have since been corrected.

two estimates is not significantly large, on the assumption of the normal law of errors. A suitable weighted average is

$$1{\cdot}80\left(\frac{19{\cdot}6}{1{\cdot}5^2}+\frac{25{\cdot}4}{3{\cdot}0^2}\right) = 20{\cdot}8\,\text{cm},$$

but the authors would prefer to adhere to the result from Ramsgate–Dunkerque because of its greater reliability in every respect.

The apparent difference of some 17 cm between the mean sea levels at Shoreham and Ramsgate is of little significance, because of the different times and durations of the records. Only simultaneous records of sea level are strictly comparable, and such figures are not at present available for these ports. One might expect the mean level at Ramsgate to be a few centimetres lower than that at Shoreham, because of the greater Coriolis effect and the generally greater tidal velocities across the section of the channel at Ramsgate.

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References

Allard, P. 1952 *Bull. Inf. Com. Central Océanogr.* **4**, 8, 307–310.
Bowden, K. F. 1956 *Phil. Trans.* A, **248**, 517–551.
Bowden, K. F. & Fairbairn, L. A. 1956 *Proc. Roy. Soc.* A, **237**, 422–438.
Burnett, Col. & Carmody, Mjr. 1960 *Bull. Géod.* (*Paris*), **55**, 55–58.
Carruthers, J. N. 1928, 1935 *The flow of water through the Straits of Dover*, Fishery Investigations, (2), (Pt. 1, **11**, 1, 109 pp.) (Pt. 2, **14**, 4, 67 pp.). London: H.M.S.O.
Cartwright, D. E. 1960 *Bull. Géod.* (*Paris*), **55**, 97–99.
Cartwright, D. E. 1961 *J. Inst. Navig.* **14**, 2, 130–151.
Chapman, S. & Bartels, J. 1940 *Geomagnetism*, **1**, 542 pp.; **2**, 1049 pp. Oxford University Press.
Charnock, H. 1956 *Nature, Lond.* **177**, 62–63.
Charnock, H. 1959 *Geophys. J. R. Astr. Soc.* **2**, 3, 215–221.
Courtier, A. 1953 *Hydrographic Rev.* (*Monaco*), **10**, 2, 124–129.
Darbyshire, J. & Darbyshire, M. 1955 *Quart. J. R. Met. Soc.* **81**, 349, 333–339.
Doodson, A. T. 1960 *Bull. Géod.* (*Paris*), **55**, 69–88.
Doodson, A. T. & Warburg, H. D. 1941 *Admiralty manual of tides*, 270 pp. London: H.M.S.O.
Dorrestein, R. 1962 *Proc. 2nd Tech. Conf. on Hurricanes*, pp. 230–240. Washington: U.S. Dept. Commerce.
Imbert, B. 1954 *Bull. Inf. Com. Central Océanogr.* **6**, 1, 36–40.
Lacombe, H. 1949 *Ann. Hydrog.* (*Paris*), 1361, 1–39.

Longuet-Higgins, M. S. 1949 *Mon. Not. R. Astr. Soc. Geophys. Suppl.* **5**, 8, 285–307.
Longuet-Higgins, M. S. & Stewart, R. W. 1962 *J. Fluid Mech.* **13**, 4, 481–504.
Nørlund, N. E. 1945, 1946 *Geod. Inst. Skrifter (Copenhagen)*, **3**, 6 and **3**, 8.
Proudman, J. 1953 *Dynamical oceanography*. London: Methuen. 409 pp.
Proudman, J. 1960 *Bull. Géod. (Paris)*, **55**, 89–90.
Rossiter, J. R. 1960 *Bull. Géod. (Paris)*, **55**, 105–106.
Stewart, R. W. 1957 *J. Mar. Res.* **16**, 1, 34–39.
Vantroys, L. 1958 *Bull. Inf. Com. Central Océanogr.* **10**, 1, 14–18.
Van Veen, J. 1938 *J. Cons. int. Explor. Mer.* **13**, 1, 7–36.
Waalewijn, A. 1959 *Rijkswaterstaat Comm. (The Hague)*, **1**, 61–86.