The Development of an Inversion Technique to Extract Vertical Current Profiles from X-Band Radar Observations

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ABSTRACT

The influence of wave-current interactions on time series of marine X-band radar backscatter maps at the mouth of the Columbia River (MCR) near Astoria, Oregon, is examined. The energetic wave environment at the MCR, coupled with the strong tidally forced currents, provides a unique test environment to explore the limitations in accurately determining the magnitude and vertical structure of upper-ocean currents from wavefield measurements. Direct observation in time and space of the wave-induced radar backscatter and supporting acoustic Doppler current profiler (ADCP) current measurements provide a rich dataset for investigating how currents shift the observed wave dispersion relationship. First, current extraction techniques that assume a specific current-depth profile are tested against ADCP measurements. These constrained solutions prove to have inaccuracies because the models do not properly account for vertical shear. A forward solution using measured current profiles to predict the wavenumber-Doppler shift relationship for the range of ocean waves sensed by the radar is introduced. This approach confirms the ocean wavefield is affected by underlying vertical current shear. Finally, a new inversion method is developed to extract current profiles from the wavenumber-dependent Doppler shift observations. The success of the inversion model is shown to be sensitive to the range of wavenumbers spanned by observed Doppler shifts, with skill exceeding 0.8 when wavenumbers span more than 0.1 rad m^{-1} . This agreement when observations successfully capture the broadband wavefield suggests the X-band backscatter is a viable means of remotely estimating current shear.

1. Introduction

Upper-ocean currents and current shear at the air-sea boundary play important roles in the vertical mixing of entrained atmospheric gasses; advection of pollution plumes, including spilled oil, and wave dynamics; and influencing other navigational and scientific concerns (Phillips 1966; Halpern 1977; Davis et al. 1981). Despite their importance, accurate in situ measurements of nearsurface currents remain difficult using traditional oceanographic tools due to platform motion complications and wave contamination of the current signal. Traditional point measurement current meters also lack spatial coverage (Paduan and Rosenfeld 1996). Remote sensing techniques have recently received considerable attention with the use of high-frequency (HF) radar, which provides maps of ocean currents with spatial resolutions of

O(1-6) km out to distances of tens of kilometers and is used operationally around the United States (e.g., Terrill et al. 2006; Harlan et al. 2009, 2011). Shifting to higherfrequency electromagnetic (EM) signals, such as incoherent marine X-band radar, offers the advantage of providing sea surface backscatter maps from a relatively large range (out to approximately 3-8 km, depending on user-defined parameters and environmental conditions). The large spatial scales of remote sensing techniques, however, makes comparison to in situ current measurements in the open ocean difficult because of potentially weak currents and the presence of horizontal current shear from eddies or current fronts (Kohut et al. 2006; Ohlmann et al. 2007). The strong periodic currents within the tidally forced mouth of the Columbia River (MCR) make the region an ideal test bed for the development and validation of X-band current estimation techniques. Furthermore, the dynamics of tidally forced river inlets are a concern for navigation, civil, and scientific communities, which will directly benefit from the ability to retrieve accurate current and current shear information remotely.

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The use of radar as a tool to study wave-current interaction began with Crombie's (1955) observations of Doppler shifts using high-frequency radar. Since then, many studies have taken advantage of the sensitivity of different types of radar to ocean wave celerity to estimate both surface current velocity and bathymetry (e.g., Barrick 1972; Shuchman 1979; Alpers et al. 1981b; Young et al. 1985; Bell 1999). Marine X-band provides an advantage over other radar types by offering near-realtime imaging of a broad range of ocean wavelengths, ranging between approximately 20 and 200 m. The simultaneous measurement of time-space properties of the backscatter allow for exploration in the frequencywavenumber domain. Where HF radar techniques involve direct observations of radio wave Doppler shifts, X-band backscatter images are processed to supply a broadband wavenumber-Doppler shift relationship from which currents can be estimated.

Incoherent radars, such as the one used in this study, differ from coherent radars in that incoherent systems supply only backscatter magnitude and not the phase of the returned EM signal. Thus, a single scan from an incoherent radar supplies information about the locations but not velocities of individual scatterers. The modulation of incoherent marine X-band backscatter is a result of Bragg scatter with centimeter-scale ocean roughness that is modified by the underlying surface gravity waves via three mechanisms: 1) hydrodynamic modulation, which, due to orbital velocities and wave-wave interactions, leads to the collection of capillary waves near the crests of longer waves; 2) shadowing by wave crests in low-grazing-angle geometries (typical of marine radar deployment); and 3) geometric scatter that occurs from the forward face of the ocean waves (Alpers et al. 1981a; Nieto Borge et al. 2004). A series of sequential backscatter images therefore captures the spatial and temporal evolution of the surface wavefield, which can then be transformed into the wavenumber-frequency domain using a Fourier transform (Young et al. 1985). Surface current information is present in the observed Doppler shift-wavenumber relationship and can be extracted using various techniques (e.g., Stewart and Joy 1974; Ha 1979; Young et al. 1985).

In previous studies, currents are extracted from wavenumber-dependent Doppler shift measurements under strict assumptions of the underlying current– depth profile shape (e.g., that currents are either depth uniform or vary linearly with depth) (Young et al. 1985; Senet et al. 2008; Teague et al. 2001; Trizna and Xu 2006; Hessner and Bell 2009). One aim of this work is to carry out a rigorous comparison of X-band-derived currents under both uniform and linear current profile assumptions to in situ current profile measurements. These



FIG. 1. A map of the Columbia River mouth, where gray is land and white is water. Light gray contours show bathymetry collected by USGS surveys. The dashed circle denotes the approximate maximum range of the X-band radar, and the triangle indicates the location of a bottom-mounted ADCP. The variance ellipse of the currents during RIVET2 is shown in black.

comparisons are made over multiple tidal cycles to evaluate the success of each method in the presence of varying degrees of current shear.

There has been limited success in estimating depthvarying currents from wavefield measurements. Qualitative observations of current shear have been made using broadband Doppler shift-wavenumber measurements (Ha 1979; Fernandez et al. 1996; Dugan et al. 2008) by noting fluctuations in the Doppler shift-wavenumber relationships, suggesting the presence of vertical current shear. Ha (1979) attempted to use the wavenumber dependence of Doppler shift measurements to solve for the arbitrary current-depth profile (i.e., without an assumption of its shape) using a four-frequency HF system. This small number of discrete wavenumbers, however, did not adequately constrain the inversion, yielding noisy results. In this study the concept of extracting currents from the observed Doppler shifted dispersion relationship is extended. The ability of the X-band radar to image a broad range of ocean wavelengths is used in combination with a constrained least squares technique of the wavenumber-dense data to invert for current profiles.

2. Data collection

As a part of the Office of Naval Research (ONR)funded Riverine and Estuarine Transport Experiment 2 (RIVET2) field campaign, X-band backscatter was collected at the MCR near Astoria, Oregon, from 24 May 2013 to 4 June 2013. The land-based radar system was located inside the mouth at the south jetty (Fig. 1).

Supporting current and water depth measurements were collected using a bottom-mounted acoustic Doppler



FIG. 2. ADCP measurements of the tidally forced currents over a 3-day period. Warm colors indicate upriver flow and cool colors indicate downriver flow.

current profiler [ADCP; 1000-kHz Nortek acoustic wave and current (AWAC) sensor] within the radar field of view (Fig. 1), collecting vertical profiles of three-dimensional currents at 15-min intervals in 0.5-m bins. The overlaid current variance ellipse, representing the depth-average variances of channel and crosschannel currents, shows the currents were strongly polarized due to the channel geometry. This led to the adoption of the convention that current directions were limited to positive (up channel) and negative (down channel). The vertical structure of the streamwise current (Fig. 2) shows strong tidally forced currents, ranging between -3.5 and $1.3 \,\mathrm{m \, s^{-1}}$. Stronger currents occurred during ebb tides when the tidal and river forcing were aligned. The vertical current shear structure was also tidally dependent (Fig. 3). During slack conditions, tidal forcing became small and the flow was dominated by the river forcing, resulting in slightly negative depth-uniform currents. Flood and ebb conditions, however, were dominated by strong tidal forcing and complicated saltwater-freshwater flow, resulting in strong current shear.

An example of a single radar scan (0800 UTC 3 June 2013) shows strong reflection from land features that illuminate the coastline, as well as the north and south jetties bounding the inlet (Fig. 4). Each radar scan was georeferenced using known stationary points in the field of view. Backscatter from incoming waves can be seen within the inlet. Overlaid on this scan is a $750 \text{ m} \times 750 \text{ m}$ box centered over the position of the ADCP, which represents the region over which data were processed to study wavecurrent interactions. Bathymetry contours under the inspection square (Fig. 5) collected by the U.S. Geological Survey (USGS) show waves within the inspection square were propagating in water depths ranging between -10and -15 m. To capture the large spatial scale of the MCR, X-band operating parameters (Table 1) were tuned to maximize the usable radar range by collecting a large number of samples (1024 samples per beam) with a coarse range resolution (approximately 7.5 m).

Measurements of the relevant environmental conditions during RIVET2 include the wind conditions measured at the radar site and incident wave conditions collected approximately 8km offshore by a California Data Information Program buoy (Figs. 6a and 6b).

The signal strength of radar backscatter is influenced by a combination of wave height and wind conditions. A signal-to-noise ratio (SNR) was calculated by defining noise as the mean energy return in a high-wavenumber, low-frequency region of the backscatter spectrum, far removed from expected ocean wave information (Young et al. 1985). The evolution of the SNR for data recorded within the inspection box region demonstrates the radar signal's complicated relationship to wind and incident wave conditions approaching the MCR (Fig. 6c). Sections of sustained high SNR correspond to times of large magnitude and near-incident (northwest) wind and waves.



FIG. 3. Ebb, flood, and slack current profiles measured by the ADCP on 31 May 2013.



FIG. 4. A section of a single X-band scan of the MCR, where light and dark shading represent high and low signal return, respectively. The radar is located in the middle of the white disk in the lower right, which denotes the 500-m blanking range. The study region (white square) is centered over the bottom-mounted ADCP (triangle).

The study of wave–current interaction is dependent on the relative angle between the current and overlying wave propagation directions. By estimating the local wave direction as the direction of the maximum SNR, the relative wave–current direction $\Delta\theta_{W-C}$ was calculated. The cosine of this angle was used as a metric of the degree to which the waves and currents are collinear (Fig. 6d).

The 3-day time period of sustained SNR and collinear waves and currents $[\cos(\Delta\theta_{W-C} > 0.98)]$ in Fig. 6 (1200 UTC 29 May 2013 to 1200 UTC 1 June 2013) defines the time period of X-band backscatter used for this current extraction study.

3. Methods

a. Doppler shift extraction

For each radar scan, backscatter within the inspection square was selected to create square subsections. A total of 128 successive subsections (representing 3 min of data collection) were stacked to create a cube of data in (x,y,t), which was transformed into directional wavenumberfrequency (k_x, k_y, ω) space via a 3D fast Fourier transform (FFT). To increase SNR, four subsequent FFT results were averaged with 50% overlap. The result, therefore, represented 6 min of data with 8 degrees of freedom. A dispersion mask was applied to remove data separated from the zero-current dispersion line by a maximum expected current magnitude. Two examples



FIG. 5. Bathymetry [North American Vertical Datum of 1988 (NAVD88)] from USGS surveys shows that water depth under the inspection square varies between -10 and -15 m. The ADCP (asterisk) was mounted at a depth of -11 m.

of the azimuthally integrated results of the FFT, representative of ebb and flood current conditions (Figs. 7a and 7b, respectively), display the location of X-band backscatter energy in the wavenumber magnitude– frequency domain. In the absence of currents, linear wave theory suggests waves behave according to the dispersion relationship

$$\omega^2 = \sqrt{gk} \tanh(kh), \tag{1}$$

where ω is the wave frequency, k is the wavenumber magnitude, and h is the local water depth that was measured by the ADCP's pressure sensor.

The dominant energy during the ebb tide (Fig. 7a) lies below the zero-current line, representing the Doppler shift effect of a strong current adverse to the wave

TABLE 1. Operational parameters for the land-based X-band radar during the RIVET2 campaign that collected data from 24 May 2013 to 4 Jun 2013.

RIVET2 X-band collection parameters	
Radar type	Furuno 2117bb
Peak output power	12 kW
Antenna length	8 ft
Beamwidth	0.95° (horizontal)
	20° (vertical)
Radar scan rate	42 rpm
Range resolution	7.5 m
Maximum range	$\sim 8 \mathrm{km}$
Radio wave frequency	$9410 \pm 30 \mathrm{MHz}$
Bragg scatterer wavelength	3.2 cm
Antenna height	16–20 m
Analog-to-digital converter resolution	12 bits



FIG. 6. Environmental conditions data collected at MCR near Astoria, including (a) offshore wave conditions and (b) wind conditions at the radar site. (c) The X-band SNR (solid), resulting from the wave and wind conditions. The SNR threshold (dotted) used to separate low-quality noise. (d) Relative wave–current direction displayed as the cosine of the absolute difference between wave and current directions, $\Delta\theta_{W-C}$. This study focuses on the time period of sustained SNR and small $\Delta\theta_{W-C}$ from 30 May to 1 Jun.

direction, whereas during the flood (Fig. 7b), much of the energy lies just above the zero-current line, suggesting waves and currents traveling in the same direction. However, there is also a portion of energy that appears high above the dispersion line in the higher wavenumbers $(0.1-0.14 \text{ rad m}^{-1})$ during the flood tide. This energy was attributed to the first harmonic of the peak energy, which is located on the line described by the harmonic dispersion relationship for water of finite depth,

$$\omega_p = (p+1)\sqrt{\frac{gk}{p+1}} \tanh\left(\frac{kh}{p+1}\right),$$

where p = 0 represents the fundamental mode and p = 1represents the first harmonic (Senet et al. 2001). Harmonics in the backscatter data originate from nonlinearities in the observed wavefield, as well as from nonlinearities introduced from the near-grazing imaging process of the wavefield, which results in shadowing. If the observed harmonic signal were due to wave nonlinearities, then the signal would be enhanced at times of maximum ebb, when oncoming currents steepen waves. However, because the observed harmonic signal appeared at flood tide, during which the grazing angle between the antenna and ocean surface decreased by an average of 15%, the harmonics were most likely caused by the nonlinearities in the imaging process when waves shadowed the radar signal. This



FIG. 7. Examples of the location of returned backscatter energy in wavenumber–frequency space during (a) ebb and (b) flood current conditions. Darker colors indicate higher energy return than lighter colors. The depth-corrected dispersion relationship (solid blue) and first harmonic relationship (dashed blue) indicate expected energy locations in the zero-current condition. The solid red line denotes selected $\Delta\omega(k)$ profiles for current estimation.

signal, therefore, contained no additional exploitable Doppler shift information and was removed. To eliminate this signal from the wavenumber–frequency domain, energy that introduced large discontinuities $(>0.1 \text{ rad s}^{-1})$ in the frequency–wavenumber profile was isolated and deleted.

The remaining ridge of energy in the wavenumberfrequency domain was isolated, and wavenumberdependent Doppler shifts were calculated by subtracting the dispersive frequency [(1)] from the observed frequency.

Young et al. (1985) developed a method of current extraction from X-band radar using the Doppler shift expression

$$\Delta \omega(k) = \mathbf{k} \cdot \mathbf{u}_{\text{eff}}(k) = k u_{\text{eff}}(k) \cos(\Delta \theta_{\text{C-W}}), \qquad (2)$$

where $\Delta \omega$ is the Doppler shift and **k** is the wavenumber vector. The term $\mathbf{u}_{eff}(k)$ is the 2D *effective current velocity* vector that is wavenumber dependent, which represents a weighted depth-average effect currents have on the wavefield (Stewart and Joy 1974). The scalar product in (2) implies that waves are Doppler shifted only by current components collinear with the wave propagation (Phillips 1966). For this study, because of polarized currents (Fig. 1) and waves (Fig. 6d) within the inspection square, the vector quantities **k** and \mathbf{u}_{eff} were replaced with the channel direction magnitudes k and u_{eff} . Because a time period was selected such that $\cos(\Delta\theta_{C-W}) > 0.98$, the error introduced by this simplification into the Doppler shift calculation for this study was less than 2%.

The relationship between the wavenumber-dependent effective velocity $u_{\text{eff}}(k)$ and the depth-varying current u(z) was first derived by Stewart and Joy (1974) in deep water (i.e., $kh \rightarrow \infty$) and then expanded to account for finite depth effects by Kirby and Chen (1989). The finite depth relationship is

$$u_{\text{eff}}(k) = \frac{\Delta\omega}{k} = \frac{2k}{\sinh(2kh)} \int_{h}^{0} u(z) \cosh[2k(h+z)] dz.$$
(3)

The weighting function within the integral indicates that Doppler shifts of all wavenumbers are heavily influenced by near-surface currents.

b. Depth-uniform and linear current estimation

If the currents affecting the wavefield are depth uniform, or $u(z) = u_0$, then the evaluation of (3) shows that the effective velocity is independent of wavenumber, and $u_{\text{eff}} = u_0$. This simplification reduces (2) into the linear expression

$$\Delta \omega = k u_0, \tag{4}$$

which implies that depth-uniform currents can be estimated with a linear fit of observed Doppler shiftwavenumber profiles. Because u_0 is wavenumber independent, it describes the bulk effect underlying currents have on the wavefield. The term u_0 is therefore referred to as the velocity of encounter or bulk current velocity (e.g., Dankert and Rosenthal 2004). In an example from data collected on 0520 UTC 31 May 2013 (Fig. 8), a linear regression of the Doppler shiftwavenumber observations during the maximum ebb current had a slope of $-1.9 \,\mathrm{m \, s^{-1}}$, which defines an estimate of the bulk current magnitude for this time period. This linear regression technique to calculate bulk currents was repeated throughout the 3-day time period, and results were compared with depth-averaged ADCP current velocities.

In the presence of depth-varying currents, the effective velocity remains wavenumber dependent. Previous studies carried out in deep water have estimated depthvarying currents from Doppler shift measurements by assuming that the currents vary linearly with depth (e.g., Stewart and Joy 1974; Ha 1979; Young et al. 1985). Evaluating (3) with a linear current profile yields



FIG. 8. An example of a linear fit to Doppler shift observations, where the -1.9 m s^{-1} slope represents an estimate of a depthuniform current. These data were selected during an ebb tide at 0520 UTC 31 May 2013.

$$u_{\rm eff}(k) = u \left[z = -\frac{1}{2k} \tanh(kh) \right], \tag{5}$$

which implies that the observed effective velocity at wavenumber k is equal to the geophysical current velocity at a depth of $z = -(2k)^{-1} \tanh(kh)$. The expression in (5) can therefore be used to map effective velocities derived from radar observations to estimate current–depth profiles.

The strong, depth-dependent current shear measured by the ADCP (Fig. 3) suggests that the accuracy of radarderived current estimates would be enhanced by removing assumptions of the current-depth structure. To account for the effects of arbitrary current profiles on the wavefield, the integral in (3) can be used in either the "forward" or the "inverse" problem. The forward problem involves transforming measured current profiles (e.g., from an ADCP) into Doppler shift–wavenumber profiles to be compared to X-band measurements. The inverse problem is the extraction of current profiles from X-band measurements of the wavefield.

c. The forward problem

To transform the discrete ADCP current-depth profiles into Doppler shift-wavenumber profiles, (3) was discretized to match the ADCP measurement interval. The resulting finite sum is

$$u_{\text{eff}}(k_j) = \frac{\Delta\omega_j}{k_j} \approx \frac{2k_j}{\sinh(2k_jh)} \sum_{i=1}^n u_i \cosh[2k_j(h+z_i)]\Delta z,$$
(6)

where there are *n* depth bins of size Δz and *h* is the measured water depth. The sum was evaluated using ADCP current measurements for each k_j measured by the X band to build a Doppler shift–wavenumber profile for each time to be directly compared to those observed by the radar.

d. The inverse problem

The inverse problem involves extracting current profiles u(z) from the Doppler shift-wavenumber observations using (3). The inversions of Laplacian-type expressions, such as this, are plagued by the amplification of inherent measurement noise and truncation error (Ha 1979). Two methods were used in this study to stabilize the inversion: 1) the Gauss-Legendre method was applied to reduce the integral to quadrature (Weeks 1966; Cohen 2007) and 2) a constrained least squares approach was used to invert the resulting matrix expression (Twomey 1977; Wunsch 1996).

Gauss–Legendre quadrature suggests that an integral can be approximated by the finite sum

$$\int_{-1}^{1} f(x) \, dx \approx \sum_{i=1}^{n} f(x_i) w_i, \tag{7}$$

where x_i and w_i are the quadrature points and weights, respectively, which are chosen by requiring that (7) be satisfied exactly for any polynomial with order m < n - 1(Golub and Welsch 1969). The orthogonal polynomials for which this condition holds are the Legendre polynomials, where x_i are their zeros and w_i are weights, both of which are well tabulated (e.g., Cohen 2007). To reduce (3) to the quadrature form in (7), first the substitution $x = 1 + 2zh^{-1}$ is made. The integral expression becomes

$$\frac{u_{\rm eff}(k)\sinh(2kh)}{kh} = \int_{-1}^{1}\cosh[2k(h+z(x))]u[z(x)]\,dx\,,$$

where

$$z(x) = \frac{h}{2}(x-1).$$

Using (7), the quadrature form becomes

$$\frac{u_{\text{eff}}(k_j)\sinh(2k_jh)}{k_jh} \approx \sum_{i=1}^n \cosh[2k_j(h+z_i)]u_iw_i,$$

where $z_i = (h/2)(x_i - 1)$ and *n* is Legendre polynomial order. This sum can be rewritten as the matrix equation

$$d = \mathbf{G}m$$

where

$$d_j = \frac{u_{\text{eff}}(k_j)\sinh(2k_jh)}{k_jh},$$

$$G_{ij} = \cosh[2k_j(h+z_i)],$$

$$m_i = u(z_i)w_i.$$

In this way, the model is now in a form that can be solved using least squares techniques, which minimize the model misfit $\mathbf{G}m-d$ with respect to m. The number of discrete solutions is defined by the order of the Legendre polynomial, which is adjustable to minimize noise amplification. To further stabilize the inversion, the curvature of the resulting current profiles was constrained using a model-weighting matrix. Using the Taylor series expansion of $\mathbf{u}(z)$, the second derivative can be approximated as

$$\frac{\partial^2 u_i}{\partial z^2} \approx \frac{u_{i-1} - 2u_i + u_{i+1}}{\Delta z^2}$$

or in matrix form as

$$\frac{\partial^2 u_i}{\partial z^2} \approx \frac{1}{\Delta z^2} \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$
$$u = \frac{1}{\Delta z^2} \begin{bmatrix} 1/w_1 & -1/w_2 & 0 & 0 \\ 1/w_1 & -2/w_2 & 1/w_3 & 0 \\ 0 & 1/w_2 & -2/w_3 & 1/w_4 \\ 0 & 0 & 1/w_3 & -2/w_4 \end{bmatrix} m \equiv \mathbf{C}m.$$

Minimizing both the misfit and the new model weighting matrix $\mathbf{C}^{T}\mathbf{C}$ with respect to *m* results in the constrained least squares solution

$$m = \left[\mathbf{G}^{\mathrm{T}}\mathbf{G} + \lambda(\mathbf{C}^{\mathrm{T}}\mathbf{C})\right]^{-1}\mathbf{G}^{\mathrm{T}}d$$
(8)

(Twomey 1977), where λ is a tunable parameter defining the extent to which the result is constrained by **C**. The Legendre order *n* and the adjustable parameter λ were tuned by inverting the results of the forward problem and comparing with the original ADCP current profile measurements. The tuned model was then applied to invert current profiles from X-band wavefield measurements.

Model skill was used as the error metric to quantify the comparison between estimated currents from the X-band inversion and those measured by the ADCP. The definition of model skill (Bogden et al. 1996; Hetland 2006) is

skill =
$$1 - \frac{\langle (u_{adcp} - u_{XBand})^2 \rangle}{\langle u_{adcp}^2 \rangle}$$
, (9)



FIG. 9. (a) ADCP depth-averaged currents (dashed) and X-band bulk current estimates (dotted) vs time for the 3-day period. (b) Scatterplot of X-band bulk currents vs ADCP depth-averaged currents. Solid line indicates a 1-to-1 relationship and dashed line denotes linear fit.

where angle brackets denote an average (in time or depth). Zero misfit between ADCP and inverted currents results in a maximum skill of 1, whereas large inverted currents that disagree with ADCP observation can yield a negative skill. One advantage of model skill is the normalization of error during times of low signal. Another advantage of the skill metric is that it is a single number evaluating either depth or temporally averaged inversion results. For this study, model skill was used to evaluate the success of current inversions in both time and depth.

4. Results

a. Depth-uniform and linear current estimation

Currents were estimated under the depth-uniform current assumption using the linear regression in (4) over the selected 3-day time period. These results were compared to depth-averaged ADCP currents (Fig. 9). Time periods of missing X-band data correspond to low SNR (less than 5 dB), for example, due to low wind speed (below 3 m s^{-1}) (Fig. 6c). The time series shows the agreement was good during slack tides but reduced as the currents reached flood and ebb conditions (Fig. 9a). The scatterplot shows the best agreement occurred when the depth-averaged current was between -0.25 and -0.5 m s^{-1} , corresponding to times when the current was dominated by the river outflow (Fig. 9b).

Current profiles were estimated under the linear current–depth assumption using (5) and the results were compared with ADCP measurements (Fig. 10). The skill of the current estimates was calculated using (9) (Fig. 10c). The skill ranges between 0.7 and 0.9 during ebb tides, whereas during flood conditions the skill ranges between -1 and 0.5. This behavior is consistent with the truncated wavenumber range during flood tides due to the harmonic contamination (e.g., Fig. 7b). Because solutions from this method exist only at depths defined by the effective depth $z = -(2k)^{-1} \tanh(kh)$, the current profile estimates were constrained to a narrow range of depths, between -2.5 and -4.75 m. Therefore, although model skill suggests that currents estimated under the linear current assumption show reasonable agreement with ADCP measurements during ebb conditions, these current estimates are limited to a small range of depths.

b. The forward problem

To compute the forward problem, the weighted sum in (6) was used to transform ADCP current measurements to Doppler shift–wavenumber profiles. The results were then directly comparable to X-band Doppler shift observations (Fig. 11). Missing X-band data in the higher wavenumbers during flood tides were due to the presence of the harmonics, which overwhelm the fundamental signal (e.g., Fig. 7). Results from the forward problem qualitatively



FIG. 10. (a) X-band-derived current profiles assuming a linear current-depth relationship, or $u_{eff}[\tanh(kh)/(2k)] = u(z)$. (b) ADCP depth-current profile measurements in the along-channel direction. Black lines bound the depth range of radar-derived current estimates made under the linear current-depth assumption. (c) Skill of the current estimate made under the linear current-depth assumption calculated from (9).

agree with X-band profiles over the 3-day time period, exhibiting similar structure in the wavenumber dependence of the observed Doppler shifts.

To quantitatively assess the results of the forward problem (Fig. 11b), the average slope of each $\Delta\omega(k)$ profile was calculated (Fig. 12). If the observed currents were depth uniform, then the slopes of the forward problem results would be the same as the depth-averaged currents [i.e., as shown in (9)]. Discrepancy between the observed depth-averaged currents and the mean slopes, therefore, represents the effect of current shear on the Doppler shift– wavenumber relationship (Fig. 12). Furthermore, agreement between X band and forward problem average slopes suggests that the effect of current shear on the Doppler shift–wavenumber relationship is accurately represented by the model developed by Kirby and Chen (1989).

c. The inverse problem

1) MODEL TUNING

The Doppler shift–wavenumber profiles from the forward problem were used as input into (8) to tune the inversion method. The Legendre polynomial order and the tunable parameter λ were varied, until the best match with the measured ADCP current profiles was found (Fig. 13), which occurred with n = 7 and $\lambda = 0.04$. Residual discrepancy between these inversion results and measured current profiles were caused by 1) the truncation of the current profiles near the surface to avoid surface contamination and 2) the finite range of wavenumbers used in the forward problem calculation, defined by the resolution and box size imaged by the X-band radar.

2) INVERSION OF X-BAND BACKSCATTER

The tuned inversion model [(8)] was used to estimate the current profiles from the X-band $\Delta\omega(k)$ profiles (Fig. 14a). Current profiles range from the river bed to the surface, whereas ADCP current measurements (Fig. 14b) are limited to a smaller depth range (indicated by black lines in Fig. 14a) due to instrument mounting height and surface contamination. The vertical structure of the inverted currents compares well with ADCP structure qualitatively, showing the complicated exchange between tidal forcing and river outflow through tidal evolutions. Equation (9) was used to calculate the skill of the inversion result as a function of time (Fig. 14c). With skill exceeding 0.7, the inversion model shows good results during ebb and slack current conditions. However, the skill



FIG. 11. (a) ADCP depth-current profile measurements. (b) The Doppler shift-wavenumber profiles resulting from transforming the ADCP current profiles via the forward problem. (c) Observed X-band Doppler shift-wavenumber profiles.

drops below zero during flood currents, when $\Delta\omega(k)$ are truncated due to harmonic contamination. The depth dependence of skill was calculated during ebb and slack conditions (Fig. 15), showing that inverted currents agree well with ADCP currents from the surface (skill > 0.8) to 9 m, where skill falls below 0.7.

5. Discussion

Results of the depth-uniform analysis show that error current estimates under the depth-uniform assumption exceeded 20% within one hour of slack currents (Fig. 9). This confirmed that the presence of current shear plays an important role in estimating currents from wavefield measurements. The simplest way to include the effects of current shear was to assume a linear depth–current relationship, which resulted in a model to directly map effective velocities of individual wavenumbers to currents of unique effective depths (Fig. 10). Although current shear, solutions were limited to a small depth range of approximately 2.5 m. Furthermore, because of the direct mapping between individual wavenumbers to individual water depths, missing data and truncated Doppler shift data led to discontinuous and truncated current–depth profiles. Because of the small depth range and inconsistency of the current estimates, a more rigorous solution was sought after that involved the entire depth–current profile.

The forward problem investigated the sensitivity of the wavefield to the arbitrary current profile shapes



FIG. 12. Time series of the slopes of the $\Delta\omega(k)$ profiles from X-band observations (dotted) and the forward problem (solid). Dashed line indicates ADCP depth-averaged currents.



FIG. 13. The outcome of inverting the forward problem result (solid) is compared to measured current profiles (dashed) to tune the inversion method. In this example, Legendre order n = 7 and $\lambda = 0.01$.

measured by the ADCP. The Doppler shift–wavenumber profiles resulting from the forward problem compared well with those observed by the X band (Figs. 11 and 12), implying that the model developed by Kirby and Chen (1989) appropriately describes the observed wave–current relationship in the MCR. Furthermore, the result of the forward problem shows that the observed current shear significantly altered the Doppler shift–wavenumber relationship, suggesting that the retrieval of current shear information from Doppler shift measurements is possible. A method was therefore developed to calculate the inversion of the model used in the forward problem. Free parameters were tuned by forcing the inversion of the forward problem results to match ADCP current profiles (Fig. 13). Residual error between the two was partially due to error in the forward problem, which arose from the truncation of ADCP profiles near the riverbed (due to instrument mounting and blanking range) as well as below the surface (to avoid surface contamination).

The inversion method resulted in current estimates that successfully captured the evolution of the current profile throughout multiple tidal cycles in the MCR, with model skill exceeding 0.8 during ebb and slack current conditions (Fig. 14). The drop in model skill during flood tides (Fig. 14c), when harmonic contamination resulted in a narrower usable wavenumber range, shows the success of the inversion was dependent on broadband Doppler shift observations. The depth dependence of skill (Fig. 15) shows agreement between inverted and measured currents were best in the top 9 m of the ADCP range. The skill of current estimates below



FIG. 14. (a) The result of the inversion of current profiles from X-band backscatter. Black lines bound the region measured by the ADCP. (b) ADCP current measurements in the along-channel direction. (c) Time series of the skill of the X-band inversion method.



FIG. 15. The depth dependence of the skill of the X-band inversion method.

-10 m declined rapidly, as waves within the observed wavenumber range were less sensitive to deeper currents as indicated by (3).

One source of error in the results of the inversion was the bathymetric variation under the inspection square, which varied over a range of $\pm 2 \text{ m}$ (Fig. 5). The range of bathymetry-induced Doppler shifts caused the "smearing" of energy in the wavenumber–frequency space (e.g., Fig. 7), which led to uncertainty in the Doppler shift estimation.

Water depth uncertainty contributed to error in the calculation of Doppler shifts and in the process of inverting the Doppler shifts to estimate currents. To examine the effects of error in water level on each of these parts of the inversion method separately, the water level was varied by $\pm 2m$ first before and then after Doppler shifts were estimated (Fig. 16). By varying the water level before Doppler shifts were estimated, model skill was affected by the error in the wavefield measurements [i.e., the dispersion relationship (1)] and the inversion process (8). By varying the water level after Doppler shifts were calculated, the wavefield measurements were assumed to be accurate and variations in model skill only reflected the sensitivity of the inversion model to water level. Water-level error in the dispersion relationship calculations resulted in much higher sensitivity to water level, with model skill falling to 0 when water depth was altered by +1.5 m. Depth error in the inversion process alone, however, led to skill variations of less than 0.1 over depth variations between ± 2 m. The maximum skill at a depth offset of 0m confirms that there was no bias in the water level used for current inversions in this study.

Both the time and depth dependence of skill suggest that the results of this study could be improved by



FIG. 16. Sensitivity of skill to error in water-level estimate. Solid line describes water-level sensitivity of the inversion process assuming perfect Doppler shift estimates. Dotted line includes water depth sensitivity of the process of estimating Doppler shifts.

expanding the wavenumber range used in the inversion. Raising the antenna height, for example, would minimize wave shadowing and therefore avoid the harmonic contamination during flood tides. Increasing the range resolution of the radar backscatter measurements would extend the Nyquist wavenumber cutoff by sampling smaller waves. Increasing the size of the inspection square would result in a smaller low wavenumber cutoff, potentially increasing the inversion skill below -10-m depth. However, a larger inspection area would be sensitive to a larger range of water depths, resulting in more uncertainty in Doppler shift estimation.

The accuracy of Doppler shift estimates depends on the time window of the FFT to exceed the wavefield's decorrelation time scale. At a minimum one should ensure that time windows capture multiple wave periods of the longest measurable wave. However, increasing the FFT time window to sample multiple wave groups would ensure the maximum energy in each frequency band is observed, increasing the SNR. A potential improvement to the inversion example shown in this study, therefore, is to explore the effect of much longer time windows to sample across several time scales across which wave groups pass.

The success of future applications of the current inversion method will rely on both the presence and the ability to observe a broadbanded wavefield. Wavefields comprised of only short waves are largely unaffected by deeper currents, limiting the depth range of inversion results. Conversely, the inversion of wavefield measurements that contain only low wavenumber information will limit the ability to resolve near-surface currents.

6. Conclusions

In this work, X-band backscatter was collected in the mouth of the Columbia River, Oregon, to investigate the validity of current and current shear estimations from Doppler shift measurements. Although the wellestablished method that assumes uniform depth-current profiles provided reasonable current estimates during times of low shear, discrepancies exceed 20% within one hour of slack tide. The estimation of current profiles under a linear depth-current profile assumption was shown to be limited to a depth range of approximately 2.5 m and to be sensitive to observation noise. Through the use of the integral transform (Kirby and Chen 1989) that takes current shear into account, ADCP current profiles were transformed into Doppler shift-wavenumber profiles for comparison with X-band observations. The agreement between ADCP and X-band observations was improved by the inclusion of the observed current shear information. This result indicated the X-band observations of the wavefield contained information about the underlying current shear, which should be taken into account when estimating currents from Doppler shift observations.

A new inversion model to estimate current-depth profiles from X-band observations of the wavefield's Doppler shift-wavenumber relationship was introduced. Applying the model to estimate current profiles in the MCR showed good agreement to concurrent ADCP measurements, with a model skill exceeding 0.8 during ebb and slack current conditions. The X-band-derived results were shown to accurately depict depth and time variable phenomena such as the tidally forced countercurrent at depth during tidal transition. The ability to remotely sense such phenomena suggest this new model can be used to address many types of nearshore scientific, navigational, and civil concerns. To support studies of open ocean currents on the submesoscale, future work can expand this inversion method to be applicable for use in deep-water environments. Expansion to the open ocean introduces geometric complications, with relative directions of waves of various wavenumbers and currents of various depths becoming important. Furthermore, smaller current magnitudes in the open ocean make it more difficult to extract Doppler shift-wavenumber profiles from wavefield observations.

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