Surface Stresses Produced by Rainfall [1,2]

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ABSTRACT

The previous solution to the problem of determining the effect of rainfall on the sea-surface stress is found to be inadequate. A correct treatment confirms that rainfall may contribute significantly to the surface stress and may under some conditions produce stresses comparable in value to the wind stress. Also the drag of the drops in the lower air layer may induce measurable distortions from the logarithmic wind profile.

1. Introduction

Whether raindrops, by virtue of their horizontal momentum, transmit an appreciable amount of stress to the sea surface was considered by Van Dorn (1953). His conclusion "It thus appears as if heavy rainfall can considerably augment the stress due to the wind" has been widely quoted (see Deacon and Webb, 1962; Malkus, 1962; Roll, 1965). However, since Van Dorn's analysis was incorrect, the whole problem should be re-examined.

The stress τ_R produced by rainfall can be written as

$$\tau_R=\rho_d U_0 R, \tag{1}$$

where ρ_d is the density of rainwater, U_0 the horizontal speed of the drop at impact, and R the rainfall rate. If U_0 is a substantial fraction of the wind speed at anemometer level, then a rain of several centimeters per hour with a strong wind could produce stresses of significant magnitude, and even moderate rains in light winds might produce noticeable stresses. The problem is to determine what fraction of the wind speed is retained by the raindrops as they pass through the wind-shear layer near the surface.

In his analysis Van Dorn assumed a drop of 0.6 cm diameter, which is much larger than most raindrops. Typical raindrop-size distributions show median volume diameters between 0.1–0.2 cm (Cataneo and Stout, 1968). In addition, Van Dorn used an incorrect value of the drag coefficient for his raindrop. A more serious error, however, was taking the horizontal force on the drop as the result of the horizontal wind speed rather than as the horizontal component of the total force. Our final result turns out to be substantially the same as Van Dorn's; this agreement apparently comes about because of several compensating errors in Van Dorn's calculation.

2. The model

We consider a drop of mass m, density ρ_d, and cross-sectional area A, falling in the negative z direction with speed w. The drop is considered spherical and its deformation neglected. We consider the air to be moving in the positive x direction with speed v and the drop's

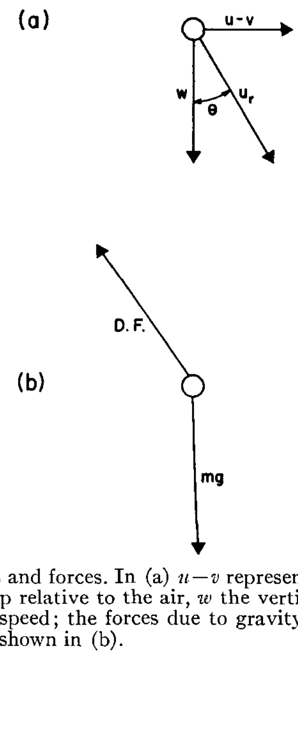

FIG. 1. Speeds and forces. In (a) $u-v$ represents the horizontal speed of the drop relative to the air, w the vertical velocity, and u_r the total air speed; the forces due to gravity and drag of air on the drop are shown in (b).

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[2] *Editor's note.* Although this paper is concerned essentially with a micrometeorological problem, its impact is perhaps most significant to the oceanographer since the surface wind stress is the governing factor for a major fraction of oceanic motions. For this reason it is felt that the work of Caldwell and Elliott is appropriate to this Journal. Moreover, it was indeed one of the intentions in establishing this Journal, as one of the publications of the American Meteorological Society, that the interplay between the oceanic and atmospheric sciences be enhanced through papers having an interdisciplinary appeal such as this one.

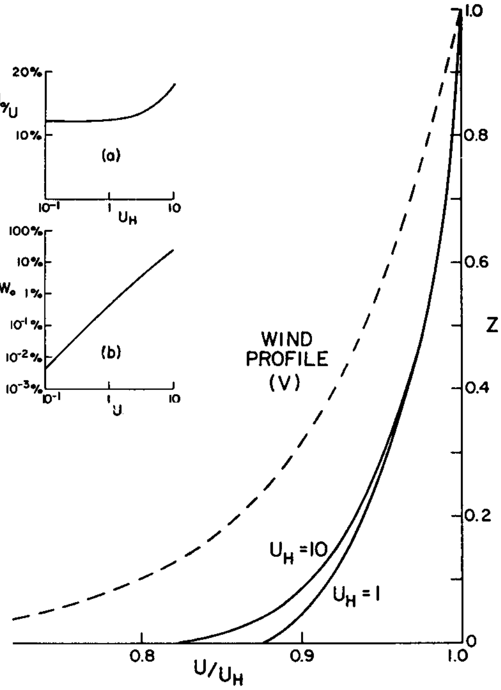

FIG. 2. Slowing down of the drop as it falls through the boundary layer, for several values of U_H, given that $w_t^2/(gh)=0.43$ and $Z_0=10^{-5}$. Insert (a) shows the variation of relative total change of horizontal speed, $\Delta U_0/U_H$, with U_H, while (b) shows the variation of total change in vertical speed ΔW_0 with U_H. (Note that the subscript H has been inadvertantly omitted from the abscissa.)

speed in the x direction to be u. Thus, the horizontal speed of the drop through the air is $u-v$ and the total speed u_r through the air is

$$u_r=[(u-v)^2+w^2]^{\frac{1}{2}}. \tag{2}$$

The forces on the drop consist of the force of gravity mg and the drag force given by

$$D.F.=\tfrac{1}{2}\rho_a C_D A u_r^2, \tag{3}$$

where ρ_a is the air density and C_D the drag coefficient. Fig. 1 shows these speeds and forces.

The equations of motion for the drops are thus:

$$m\,du/dt=-\tfrac{1}{2}\rho_a C_D A u_r^2 \sin\theta, \tag{4a}$$

$$m\,dw/dt=\tfrac{1}{2}\rho_a C_D A u_r^2 \cos\theta - mg, \tag{4b}$$

where $\sin\theta$ and $\cos\theta$ are $(u-v)/u_r$ and w/u_r, respectively. Eqs. (4a) and (4b) can be made nondimensional by dividing both by mg. Noting that

$$mg=\tfrac{1}{2}\rho_a C_D A w_t^2, \tag{5}$$

where w_t is the terminal velocity of the drop, we can write Eqs. (4) in nondimensional form with velocities scaled by w_t and time scaled by w_t/g; thus,

$$dU/dT=-[(U-V)^2+W^2]^{\frac{1}{2}}(U-V), \tag{6a}$$

$$dW/dT=[(U-V)^2+W^2]^{\frac{1}{2}}W-1, \tag{6b}$$

where $U=u/w_t$, $V=v/w_t$, $W=w/w_t$ and $T=tg/w_t$. This assumes C_D is not a function of Reynolds number, a condition which holds with sufficient accuracy for drops above 0.1 cm diameter.

The wind speed v is a function of height and we assume that at some height h the wind and the drop are moving together at the speed v_h. Below this level the wind is presumed to be given by a logarithmic wind profile

$$V=\frac{V_h \ln(Z/Z_0)}{\ln(1/Z_0)}, \tag{7}$$

where $Z_0=z_0/h$, $Z=z/h$ and $V_h=v_h/w_t$.

If (7) is substituted into (6a) and (6b), the velocity of the drop can be assessed at any point subject to the initial conditions

$$\left.\begin{array}{l} V=U=V_h \\ Z=1 \\ W=1 \end{array}\right\} \text{ at } T=0.$$

The quantity w_t/g is a measure of the time required for a drop, initially at rest in still air, to attain its terminal velocity and h/w_t is the time required for a drop, falling at w_t, to traverse the depth h. Their ratio, $w_t^2/(gh)$, represents the reciprocal of the number of "time constants" a drop takes to fall through the layer. If $w_t^2/(gh)$ is large, the drop spends only a small amount of time in the layer and will be relatively unaffected by

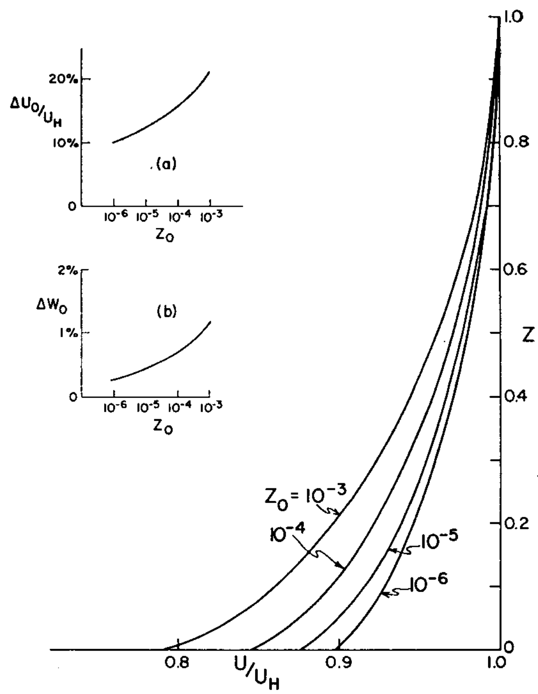

FIG. 3. Slowing down of the drop for several values of Z_0 given that $w_t^2/(gh)=0.43$ and $U_H=1$. Insert (a) shows the variation of relative total change of horizontal speed, $\Delta U_0/U_H$, with Z_0, while (b) shows the variation of total change in vertical speed with Z_0.

velocity changes there; conversely, a small value of $w_t^2/(gh)$ means that the drop will have enough time to respond with the result that the drop will follow the wind velocity fairly closely. Since all necessary information about the drop—its size, drag coefficient, etc.—is contained in the value of terminal velocity, $w_t^2/(gh)$ is a convenient nondimensional measure of the drop size.

3. Results

We mentioned above that Van Dorn used the wrong expression for the forces, and assumed a linear wind profile rather than a logarithmic one. Since he found only a 12% reduction in drop speed as the drop fell from 10 m to the surface with his linear profile, he naturally considered the logarithmic profile would show even less reduction. However, had he used the correct expressions for a linear profile, he would have found a reduction of over 40%, and had he used a drop diameter of 2 mm he would have found a reduction of drop speed of almost two-thirds.

Eqs. (6) were therefore solved by numerical integration for a logarithmic wind profile with various values of the parameters. Figs. 2–5 display the results. In the first instance, we chose $w_t^2/gh=0.43$ which, for a height h of 10 m, corresponds to a drop diameter of 0.2 cm. The parameter Z_0 was taken as 10^{-5} which, for these conditions, corresponds to a roughness length of 10^{-2} cm, a representative value for the ocean under moderate winds (Roll, 1965, p. 139). Fig. 2 shows the results of integrating (6) with these values for several selected values of V_h. The figure gives the ratio of U/U_h as a function of Z as well as the variations of horizontal and vertical speed with U_h [the wind speed profile $V(Z)$ is shown for comparison]. It is evident that the actual value of the wind speed makes very little difference to the drop speed, save for large values of U_h (curves for values of $U_h<1$ are indistinguishable from the curve $U_h=1$). We would conclude that the drop hits the surface with between 80 and 90% of its speed at anemometer height, regardless of that speed, for the values of Z_0 and $w_t^2/(gh)$ chosen here. It remains to be seen if these parameters have much effect. It is also worth noting that the fallspeed can differ considerably from w_t in cases of large U_h.

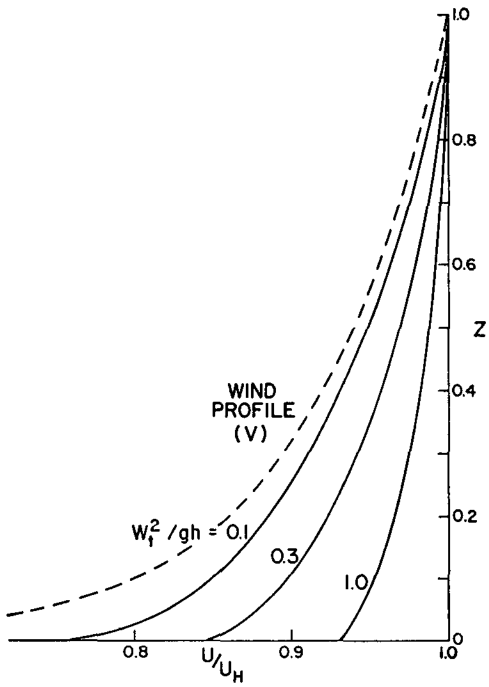

FIG. 4. Slowing down of the drop for several values of $w_t^2/(gh)$ given that $U_H=1$ and $Z_0=10^{-5}$.

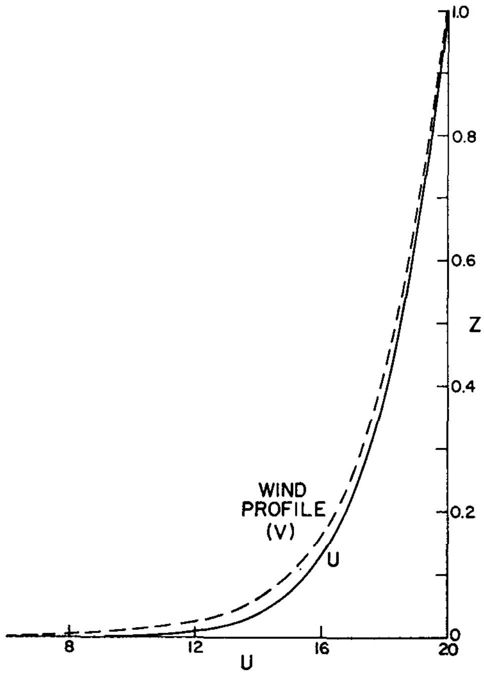

FIG. 5. Extreme case of slowing down of the drop: $w_t^2/(gh)=0.04$, $U_H=20$ and $Z_0=10^{-4}$.

Fig. 3 shows the results of integrating (6) with $w_t^2/(gh)=0.43$ and $U_h=1$ for various values of Z_0. The lowest values of Z_0 (10^{-6}) would be typical of fairly calm seas whereas the highest value (10^{-3}) would be likely found only over land (corresponding to a roughness length of 1 cm for an anemometer height of 10 m). Again, save for very rough surfaces, the reduction in drop speed is only 10–20%.

Fig. 4 gives U/U_h for various values of $w_t^2/(gh)$ taking $Z_0=10^{-5}$ and $V_h=1$. Again only the very smallest drops are reduced in speed by more than 20%.

Fig. 5 shows a fairly extreme case: $w_t^2/gh=0.4$, $V_h=20$ and $Z_0=10^{-4}$. For a 10 m anemometer height this would be a drop of about 0.05 cm diameter in a 40 m sec^{-1} wind falling over a surface of 0.1 cm roughness. Even this drop retains roughly one-half of its speed at the surface.

a. The stress produced by rainfall

Cataneo and Stout (1968) have presented raindrop-size spectra, for rainfall rates of 2.3, 10.8 and 51.4 mm hr^{-1}, which we will assume are fairly representative. Based on their data the median-momentum-drop diameter was 0.13 cm for the lightest rainfall, about 0.16 cm for the intermediate and 0.19 cm for the heaviest. Furthermore, less than 10% of the total momentum was carried by drops <0.11 cm in the intermediate and heavy rains and about 18% in the light rains. From these figures we can see that most rains will carry the bulk of their horizontal momentum with them to the surface and can therefore contribute a measurable amount to the stress. Thus, using (1) with $U_0 = 10$ m sec^{-1}, a rainfall rate of 25 mm hr^{-1} would produce a stress of about ~0.6 dyn cm^{-2}, a not inconsiderable value.

If the wind stress τ_W is written as $\tau_W = \rho_a C_{10} U_{10}^2$, where C_{10} is the drag coefficient appropriate to a 10 m anemometer height, then the ratio of τ_R to τ_W can be expressed in the form

$$\tau_R/\tau_W = (\rho_d/\rho_a)(0.85R)/(C_{10}U_{10}), \quad (8)$$

taking the average reduction in drop speed to be about 15%. If C_{10} is chosen[3] as 1.2×10^{-3}, then τ_R/τ_W can be written conveniently as $1.6R/U_{10}$ if R is in cm hr^{-1} and U_{10} in m sec^{-1}.

Thus, a rain of several centimeters per hour in winds of several meters per second could produce stresses comparable in magnitude to the wind stress; even in winds as high as 10–20 m sec^{-1} it could produce a noticeable increase.

Van Dorn's theoretical analysis was undertaken to explain his observation of a significant increase in stress during rainfall. His experiments can be taken as confirmation of this effect.

b. The stress on the air

Although the drops do not lose much momentum, they do lose some and this in turn must go toward accelerating the air. It remains to determine whether this would produce noticeable increases in wind speed in the lower layers.[4]

The acceleration of the air is given by

$$du_a/dt = (m_w/m_a)(du_d/dt), \quad (9)$$

where m_w is the mass of water, m_a the mass of air to which the momentum is transmitted, and du_d/dt the deceleration of the drops.

The mass of air can be written as

$$m_a = \rho_a A \Delta Z, \quad (10)$$

where A is the surface area over which the rain falls and $\Delta Z (= w_t \Delta t)$ is the height through which the drop falls in time Δt.

The mass of water involved is

$$m_w = \rho_d A R \Delta t. \quad (11)$$

Substituting (10) and (11) into (9) and writing du_d/dt as $w_t du_d/dz$ gives finally

$$du_a/dt = 0.23 R du_d/dz, \quad (12)$$

when R is expressed in cm hr^{-1} and du_d/dz can be evaluated from one of the appropriate Figs. 2–4.

Consider, as examples, that $h = 10$ m and $v_h = 10$ m sec^{-1}. For fairly large drops Fig. 3 will suffice. The largest values of dU/dz are found in the lowest meter where the average values range from 3–7% per meter. The largest value would produce an acceleration of about 10 cm sec^{-1} min^{-1} and the smaller one about half that in the presumed condition in a 1 cm hr^{-1} rain. Therefore, the air can be accelerated by the rainfall by a substantial amount. Since 10–20% of the drops' momentum is lost to the air, and since in some cases the total momentum transferred to the surfaces by the drop can be comparable to that transferred by the air, the stresses introduced into the air by the drops can be 10–20% of the wind stress. Thus, measurable changes in the wind speed profile are quite possible in heavy rainstorms, particularly over "rough" terrain. Clearly, this increase in wind speed in the lowest layer would result in a greater transfer of momentum to the surface so that some or most[5] of the momentum lost by the drops to the air would find its way into the surface by this route. We do find interesting the possibility that the logarithmic profile could be distorted in the lowest meter or so by moderate to heavy rains, but we know of no observations to confirm this. Perhaps this is because micrometeorological measurements are rarely made in this kind of weather.

[3] Prof. H. Charnock, in a private communication to the authors, has pointed out that over the sea the effective roughness (hence C_{10}) can be altered by rainfall, thus producing an indirect influence on τ_W.

[4] Note, however, that we have assumed a fixed wind profile in the foregoing analysis. A more exact analysis would treat the dynamics of the raindrops and of the air simultaneously.

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[5] For sustained rainfall, in which a quasi-steady state has been attained, Prof. Charnock points out that, indeed, the total stress, $\tau_W + \tau_R$, should be nearly independent of elevation in the lowest 10–20 m.