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Momentum exchange at the sea surface by wind stress and understress

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SUMMARY

Momentum exchange at the interface between two fluids is considered using general reasoning on energy dissipation in the coupled planetary boundary layers.

It is shown that the surface shearing stress can be expressed as a function of the relative surface geostrophic velocity between the two fluids, and that the surface velocity is a linear combination of the two surface geostrophic velocities. This latter result is in agreement with a result previously derived by E. B. Kraus directly from the momentum equations.

The analysis enables the ratio of the dissipation rates between the two fluids in the interfacial boundary layer to be determined. For the coupled ocean–atmosphere, we find this ratio to be approximately $(\rho_1/\rho_2)^{1/2}$ where ρ_1 and ρ_2 are the densities of air and water, and we have made use of observations of the surface drift velocity and a dimensional argument which both suggest that the drag coefficients in the two fluids are approximately equal.

From the form of the expression for the surface shearing stress, it is clear that *both* fluids lose energy in the coupled boundary layer, and also that they either transfer energy to each other or extract energy from each other by interaction at the interface.

These energy fluxes are carried by two elemental stresses due to the general circulations of the atmosphere and the ocean respectively and which we call 'wind stress' and 'understress'.

The mutual loss of energy implies that the interface exerts a drag on *both* the atmosphere and the ocean, and the interfacial energy exchanges show that the ocean does work on the atmosphere analogous to the work done by the atmosphere on the ocean.

1. INTRODUCTION

An understanding of the physics of the air–sea interface is very important in general circulation studies of the ocean and the atmosphere. We present some general results on the geostrophic interactions between the two fluids which are independent of the details of the frictional processes within the planetary boundary layers.

A feature of the analysis is a discussion of the frame of reference in which the results are to be presented. We seek to show that this is by no means a trivial consideration.

In particular, on expressing the dissipation rate for the coupled ocean–atmosphere in a frame relative to the earth it is shown that the physics is described by the interaction of two stresses due respectively to the surface geostrophic velocities of the two fluids. These two stresses, which we call 'wind stress' and 'understress', are essentially independent vectors owing to the difference in scales between the horizontal turbulence of the atmosphere and the ocean.

2. EKMAN LAYER ENERGETICS

(*a*) *Single Ekman layer*

The steady state momentum equations for the Ekman layer are

$$\left.\begin{aligned} -f\rho V &= -\partial p/\partial x + \partial \tau_{xz}/\partial z \\ f\rho U &= -\partial p/\partial y + \partial \tau_{yz}/\partial z \end{aligned}\right\} \tag{1}$$

in which Ox, Oy and Oz are respectively towards the east, and north and vertically upwards, ρ is density, p is pressure, (U, V) and (τ_{xz}, τ_{yz}) are respectively the horizontal components of velocity and vertical shearing stress, and $f = 2\Omega \sin\theta$ is the Coriolis parameter, in which Ω is the angular speed of rotation of the earth and θ is latitude.

On defining the geostrophic velocity components,

$$V_g = (1/f\rho)\partial p/\partial x \qquad U_g = -(1/f\rho)\partial p/\partial y, \tag{2}$$

these equations may be written in the form

$$-f\rho\hat{V} = \partial\tau_{xz}/\partial z \qquad f\rho\hat{U} = \partial\tau_{yz}/\partial z \tag{3}$$

where $\hat{U} = U - U_g$ and $\hat{V} = V - V_g$ are the components of the frictional velocity. The rate of dissipation per unit area in the Ekman layer by a fluid of constant density is

$$D = \int_0^\infty (\boldsymbol{\tau}_z \cdot \partial\hat{\mathbf{U}}/\partial z)\, dz \tag{4}$$

where $\hat{\mathbf{U}} = (\hat{U}, \hat{V})$ and $\boldsymbol{\tau}_z = (\tau_{zx}, \tau_{zy})$, and $z = 0$ is the surface coordinate, which may be integrated by parts to yield

$$D = [\boldsymbol{\tau}_z \cdot \hat{\mathbf{U}}]_0^\infty - \int_0^\infty (\partial\boldsymbol{\tau}_z/\partial z \cdot \hat{\mathbf{U}})\, dz \tag{5}$$

in which the frictional velocity and the vertical gradient of shearing stress are orthogonal such that the integral is identically zero. On evaluating the first term, assuming the surface is at rest and using the definition of the frictional velocity, we obtain

$$D = \boldsymbol{\tau}_s \cdot \mathbf{U}_{go} \tag{6}$$

where $\mathbf{U}_{go}$ is the surface geostrophic velocity and $\boldsymbol{\tau}_s$ is the surface shearing stress. Hence the rate of dissipation is equal to the rate of working of the surface shearing stress on the surface geostrophic velocity.

(*b*) *Coupled Ekman layers*

The preceding analysis has several important consequences when applied to a coupled fluid system which consists of immiscible upper and lower fluids. In the discussion we consider in particular the coupled ocean–atmosphere and we will refer throughout to the upper fluid as air and lower fluid as water (Fig. 1). The results, however, may be applied to any stable two-layer fluid system.

For the coupled Ekman layers we have the pairs of equations

$$-f\rho_1\hat{V}_1 = (\partial\tau_{xz}/\partial z)_1 \qquad f\rho_1\hat{U}_1 = (\partial\tau_{yz}/\partial z)_1 \tag{7}$$

and

$$-f\rho_2\hat{V}_2 = (\partial\tau_{xz}/\partial z)_2 \qquad f\rho_2\hat{U}_2 = (\partial\tau_{yz}/\partial z)_2 \tag{8}$$

where the subscripts 1 and 2 refer respectively to the atmosphere and the ocean. On applying the preceding reasoning we obtain, for the dissipation rate per unit area in the atmosphere and ocean respectively,

$$D_1 = \boldsymbol{\tau}_s \cdot (\mathbf{U}_{g1} - \mathbf{U}_s) \tag{9}$$

and

$$D_2 = \boldsymbol{\tau}_s \cdot (\mathbf{U}_s - \mathbf{U}_{g2}) \tag{10}$$

where $\mathbf{U}_{g1}$ and $\mathbf{U}_{g2}$ are the surface geostrophic velocities in the atmosphere and the ocean, and $\mathbf{U}_s$ is the surface drift velocity; and for the coupled system, the dissipation rate per

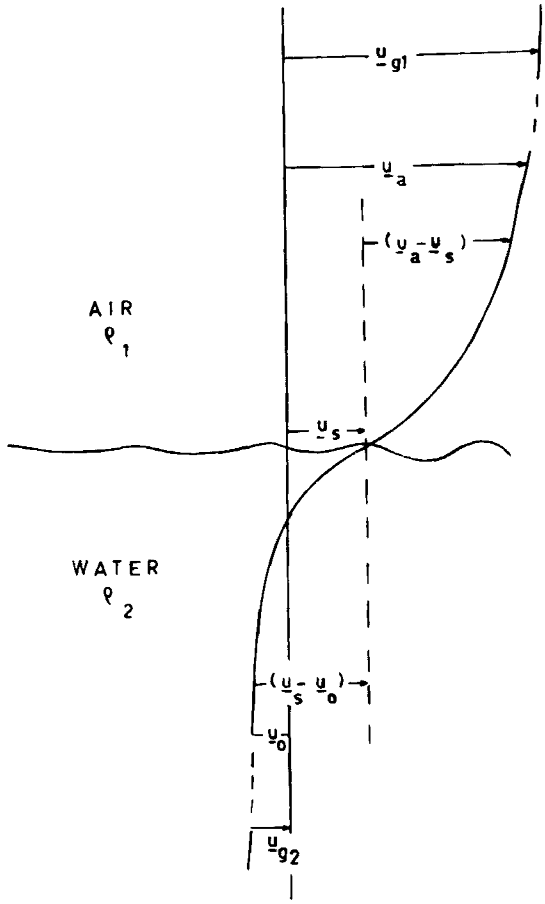

Figure 1. The coupled ocean–atmosphere planetary boundary layers.

unit area is given by

$$D = \boldsymbol{\tau}_s \cdot (\mathbf{U}_{g1} - \mathbf{U}_{g2}). \tag{11}$$

Now since $D_1 \geqslant 0$ and $D_2 \geqslant 0$, the surface shearing stress must be expressible in the forms

$$\boldsymbol{\tau}_s = A(\mathbf{U}_{g1} - \mathbf{U}_s) + \boldsymbol{\phi}_1 \tag{12}$$

$$\boldsymbol{\tau}_s = B(\mathbf{U}_s - \mathbf{U}_{g2}) + \boldsymbol{\phi}_2 \tag{13}$$

in which $\boldsymbol{\phi}_1$ and $\boldsymbol{\phi}_2$ are orthogonal to $(\mathbf{U}_{g1} - \mathbf{U}_s)$ and $(\mathbf{U}_s - \mathbf{U}_{g2})$ respectively, and A and B are positive constants, the values of which depend on the dynamics and structure of the Ekman layers.

Hence on solving Eqs. (12) and (13) for $\mathbf{U}_s$ and $\boldsymbol{\tau}_s$ we obtain

$$\mathbf{U}_s = (A\mathbf{U}_{g1} + B\mathbf{U}_{g2})/(A + B) + (\boldsymbol{\phi}_1 - \boldsymbol{\phi}_2)/(A + B) \tag{14}$$

and

$$\boldsymbol{\tau}_s = \{AB/(A + B)\}(\mathbf{U}_{g1} - \mathbf{U}_{g2}) + (B\boldsymbol{\phi}_1 + A\boldsymbol{\phi}_2)/(A + B). \tag{15}$$

By applying similar reasoning to the total dissipation rate (Eq. (11)), since $D \geqslant 0$, we also require that $\{(B\boldsymbol{\phi}_1 + A\boldsymbol{\phi}_2)/(A+B)\}$ should be orthogonal to $(\mathbf{U}_{g1} - \mathbf{U}_{g2})$. The three orthogonality conditions yield the three equations

$$\boldsymbol{\phi}_1 \cdot \{B(\mathbf{U}_{g1} - \mathbf{U}_{g2}) - \boldsymbol{\phi}_1 + \boldsymbol{\phi}_2\} = 0$$

$$\boldsymbol{\phi}_2 \cdot \{A(\mathbf{U}_{g1} - \mathbf{U}_{g2}) + \boldsymbol{\phi}_1 - \boldsymbol{\phi}_2\} = 0$$

$$(B\boldsymbol{\phi}_1 + A\boldsymbol{\phi}_2) \cdot (\mathbf{U}_{g1} - \mathbf{U}_{g2}) = 0$$

the solution of which is

$$\boldsymbol{\phi}_1 = \boldsymbol{\phi}_2 = \boldsymbol{\phi}. \tag{16}$$

Hence we obtain the general relations

$$\mathbf{U}_s = (A\mathbf{U}_{g1} + B\mathbf{U}_{g2})/(A+B) \tag{17}$$

$$\boldsymbol{\tau}_s = \{AB/(A+B)\}(\mathbf{U}_{g1} - \mathbf{U}_{g2}) + \boldsymbol{\phi} \tag{18}$$

together with the condition that the vectors $(\mathbf{U}_{g1} - \mathbf{U}_s)$, $(\mathbf{U}_s - \mathbf{U}_{g2})$ and $(\mathbf{U}_{g1} - \mathbf{U}_{g2})$ are collinear regardless of the dynamics or structure of the Ekman layers.

On evaluating $\hat{\mathbf{U}}_1$ and $\hat{\mathbf{U}}_2$ at the interface ($z = 0$) in Eqs. (7) and (8) and substituting Eq. (17), we find also that

$$-\left.\frac{(\partial \boldsymbol{\tau}_z/\partial z)_2}{(\partial \boldsymbol{\tau}_z/\partial z)_1}\right|_{z=0} = r \tag{19}$$

in which $r = (\rho_2/\rho_1)A/B$.

The expression for the surface drift velocity (Eq. (17)) in terms of r, and the conditions on collinearity have been derived previously by Kraus (1977) directly from the momentum equations.

In this discussion we focus our attention on the expression for the shearing stress (Eq. (18)) which is the complementary relation to Eq. (17).

On substituting for $\boldsymbol{\tau}_s$, the expressions for the dissipation rates are

$$D_1 = \{AB^2/(A+B)^2\}(\mathbf{U}_{g1} - \mathbf{U}_{g2})^2 \tag{20}$$

$$D_2 = \{A^2B/(A+B)^2\}(\mathbf{U}_{g1} - \mathbf{U}_{g2})^2 \tag{21}$$

$$D = \{AB/(A+B)\}(\mathbf{U}_{g1} - \mathbf{U}_{g2})^2 \tag{22}$$

and for the ratio of the dissipation rates in the two fluids

$$D_1/D_2 = B/A. \tag{23}$$

The above results have been obtained in the frame of reference of the relative motion between the two fluids.

We consider next the implications of the analysis for a frame of reference relative to the earth, which is that of the momentum equations.

Firstly, on expressing $\boldsymbol{\phi}$ by the general relation

$$\boldsymbol{\phi} = \{AB/(A+B)\}(\mathbf{U}^*_{g1} - \mathbf{U}^*_{g2}) \tan \alpha \tag{24}$$

where $|\mathbf{U}^*_{g1}| = |\mathbf{U}_{g1}|$ and $|\mathbf{U}^*_{g2}| = |\mathbf{U}_{g2}|$, and $\mathbf{U}^*_{g1}$ and $\mathbf{U}^*_{g2}$ are rotated 90° to the left-hand side of $\mathbf{U}_{g1}$ and $\mathbf{U}_{g2}$ respectively such that α is the angle of rotation of $\boldsymbol{\tau}_s$ to the left-hand side of $(\mathbf{U}_{g1} - \mathbf{U}_{g2})$, and substituting in Eq. (18) we obtain

$$\boldsymbol{\tau}_s = \{AB/(A+B)\}(\mathbf{U}_{g1} + \mathbf{U}^*_{g1} \tan \alpha - \mathbf{U}_{g2} - \mathbf{U}^*_{g2} \tan \alpha)$$

which may be expressed in the form

$$\boldsymbol{\tau}_s = \boldsymbol{\tau}_1 - \boldsymbol{\tau}_2 \tag{25}$$

where

$$\boldsymbol{\tau}_1 = \{AB/(A+B)\}(\mathbf{U}_{g1} + \mathbf{U}^*_{g1} \tan \alpha)$$

and

$$\boldsymbol{\tau}_2 = \{AB/(A+B)\}(\mathbf{U}_{g2} + \mathbf{U}^*_{g2} \tan \alpha).$$

In Eq. (25), $\boldsymbol{\tau}_s$ is partitioned into two elemental stresses, $\boldsymbol{\tau}_1$ due to the motion of the atmosphere and $\boldsymbol{\tau}_2$ due to the motion of the ocean.

Secondly, *relative to the earth*, we have the rate of energy extraction per unit area from the atmosphere, $D'_1 = \boldsymbol{\tau}_s \cdot \mathbf{U}_{g1}$, and the rate of energy extraction per unit area from the ocean, $D'_2 = -(\boldsymbol{\tau}_s \cdot \mathbf{U}_{g2})$, are such that

$$D = D'_1 + D'_2. \tag{26}$$

D'_1 and D'_2 differ respectively from D_1 and D_2 because an energy flux $(\boldsymbol{\tau}_s \cdot \mathbf{U}_s)$ crosses the interface (positive from air to water) during the dissipation process.

On using Eq. (25) we obtain the expressions

$$D'_1 = \boldsymbol{\tau}_1 \cdot \mathbf{U}_{g1} - \boldsymbol{\tau}_2 \cdot \mathbf{U}_{g1} \tag{27}$$

$$D'_2 = \boldsymbol{\tau}_2 \cdot \mathbf{U}_{g2} - \boldsymbol{\tau}_1 \cdot \mathbf{U}_{g2} \tag{28}$$

which have the following interpretation.

For the atmosphere (Eq. (27)) the first term represents the rate of loss of energy per unit area and the second term represents the rate of working of the ocean per unit area on the atmosphere, and for the ocean (Eq. (28)) the corresponding terms represent the rate of energy loss per unit area, and the rate of working per unit area of the atmosphere on the ocean. Note also that the ratio D'_2/D'_1 in general depends on $\mathbf{U}_{g1}$ and $\mathbf{U}_{g2}$, in contrast to the ratio D_2/D_1 which we have shown to be a local function independent of $\mathbf{U}_{g1}$ and $\mathbf{U}_{g2}$.

Two important limits illustrate the general results.

(i) $\mathbf{U}_{g2} = 0$. For a zero surface geostrophic current Eq. (17) reduces to the expression

$$\mathbf{U}_s = \{A/(A+B)\}\mathbf{U}_{g1}$$

and hence the surface drift velocity lies exactly along the direction of the surface *geostrophic* wind (cf. Kraus 1977). The rate of energy dissipation per unit area from Eq. (26) is equal to the rate of energy loss per unit area by the atmosphere, $D = \boldsymbol{\tau}_1 \cdot \mathbf{U}_{g1}$. Note, however, that the ratio of the dissipation rates in the ocean and the atmosphere remains $D_1/D_2 = B/A$ such that part of the dissipation due to the surface geostrophic wind occurs in the ocean (in an Ekman layer which reduces the surface drift velocity to zero at depth).

(ii) $B/A \gg 1$. Suppose that $\mathbf{U}_{g1}$ is given, then from Eq. (17) $\mathbf{U}_s \rightarrow \mathbf{U}_{g2}$ and from Eq. (23) all the dissipation occurs in the atmosphere. This limit approximates the situation for flow over a solid surface which is moving at a velocity $\mathbf{U}_{g2}$. If $\mathbf{U}_{g2} = 0$, the problem reduces to the single Ekman layer over a surface at rest, as in the introductory discussion (*a*), for which we have the relation

$$\boldsymbol{\tau}_s = \boldsymbol{\tau}_1 \tag{29}$$

3. Nomenclature for the stresses

We wish to present a simple nomenclature for the stresses which carry the energy transfers between the two fluids.

Firstly consider the single Ekman layer. Over a surface at rest the surface shearing stress is due solely to the action of the wind, and hence there is no corruption in using the terms 'wind stress' and 'surface shearing stress' interchangeably. This equivalence is expressed formally in Eq. (29) which is an identity for $\boldsymbol{\tau}_s$ and $\boldsymbol{\tau}_1$.

Similarly for the ocean there is an equivalence between the bottom shearing stress and bottom stress as universally applied to the stress exerted by the current on the ocean bottom.

In the coupled ocean–atmosphere $\boldsymbol{\tau}_1$ is also the wind stress, i.e. it is the stress due to the motion of the atmosphere relative to the earth. We propose that $\boldsymbol{\tau}_2$, which is the corresponding stress due to the motion of the ocean, cf. Eq. (25), should be called the 'understress'. This name is preferable to 'current stress' which is the exact analogue of 'wind stress' because shearing stresses act both at the surface and on the bottom of the ocean.

Two main arguments are presented for the use of the two-stress notation. (1) It is a formal expression that for the coupled ocean–atmosphere the terms 'surface shearing stress' and 'wind stress' are not synonymous. The shearing stress arises from the interaction of two independent general circulations, one in the ocean and the other in the atmosphere. (2) It is an essential vehicle for conveying the physics of the coupled boundary layer. The small change in the frame of reference for $\boldsymbol{\tau}_s$, between that of the relative motion between the two fluids (Eqs. (12) and (13)), and that of the earth (Eq. (25)) may at first glance appear to be trivial, since the current speed is usually much less than the wind speed. This inference, however, overlooks the fact that the change in datum is of the same order as the ocean current.

Relative to the earth (our dynamical reference) the surface geostrophic wind and current velocities are the only fundamental velocities, and the wind stress and understress play distinct roles of comparable significance, see section 5 and Bye (1985).

4. Models of coupled Ekman layers

We will illustrate the previous discussion with two models for which we may obtain expressions for A and B.

(a) *Laminar coupled Ekman layers*

The classical Ekman model (Ekman 1905) assumes that the density and also the viscosity are constant in each fluid. Thus

$$\partial \tau_{xz}/\partial z = \eta \partial^2 U/\partial z^2 \qquad \partial \tau_{yz}/\partial z = \eta \partial^2 V/\partial z^2 \tag{30}$$

where η is the viscosity. The solution for the coupled Ekman layers has been obtained by Kraus (1972), and it is found that the surface shearing stresses in the atmosphere and ocean respectively are of the form

$$\left.\begin{aligned} \tau_{sx} &= q_1(U_{g1} - U_s - V_{g1} + V_s) \\ \tau_{sy} &= q_1(U_{g1} - U_s + V_{g1} - V_s) \end{aligned}\right\} \tag{31}$$

$$\left.\begin{aligned} \tau_{sx} &= q_2(U_s - U_{g2} - V_s + V_{g2}) \\ \tau_{sy} &= q_2(U_s - U_{g2} + V_s - V_{g2}) \end{aligned}\right\} \tag{32}$$

where $q_1 = \rho_1(\frac{1}{2}f\nu_1)^{1/2}$ and $q_2 = \rho_2(\frac{1}{2}f\nu_2)^{1/2}$, $f > 0$, in which $\nu_1 = \eta_1/\rho_1$ and $\nu_2 = \eta_2/\rho_2$ where ν_1 and ν_2 are respectively the kinematic viscosities forthe atmosphere and the ocean.

On expressing these results in the form of Eqs. (12) and (13) we have,

$$A = q_1 \qquad B = q_2$$

$$\boldsymbol{\phi}_1 = q_1(-V_{g1}+V_s, U_{g1}-U_s)$$

$$\boldsymbol{\phi}_2 = q_2(-V_s+V_{g2}, U_s-U_{g2})$$

and hence the surface drift velocity is given by the relation

$$\mathbf{U}_s = (q_1\mathbf{U}_{g1} + q_2\mathbf{U}_{g2})/(q_1 + q_2) \tag{33}$$

cf. Kraus (1977). On substituting for $\mathbf{U}_s$ we obtain

$$\boldsymbol{\phi} = \{q_1q_2/(q_1 + q_2)\}(-V_{g1} + V_{g2}, U_{g1}-U_{g2}) \tag{34}$$

from which follows the classical result that $\boldsymbol{\tau}_s$ lies at an angle of 45° to the left-hand side of $(\mathbf{U}_{g1} - \mathbf{U}_{g2})$ in the northern hemisphere ($\alpha = 45°$). The dissipation rate is given by

$$D = \{q_1q_2/(q_1+q_2)\}(\mathbf{U}_{g1}-\mathbf{U}_{g2})^2. \tag{35}$$

(*b*) *Turbulent coupled Ekman layers*

The simulation of the frictional processes for turbulent flow gives rise to the quadratic expressions

$$\boldsymbol{\tau}_s = \rho_1K_1|\mathbf{U}_{g1} - \mathbf{U}_s|(\mathbf{U}_{g1} - \mathbf{U}_s) + \boldsymbol{\phi}_1 \tag{36}$$

$$\boldsymbol{\tau}_s = \rho_2K_2|\mathbf{U}_s - \mathbf{U}_{g2}|(\mathbf{U}_s - \mathbf{U}_{g2}) + \boldsymbol{\phi}_2 \tag{37}$$

where K_1 and K_2 are drag coefficients in each fluid for the component of stress in the direction of the relative motion.

These relations are of the form of Eqs. (12) and (13) with

$$A = \rho_1K_1|\mathbf{U}_{g1} - \mathbf{U}_s| \tag{38}$$

$$B = \rho_2K_2|\mathbf{U}_s - \mathbf{U}_{g2}| \tag{39}$$

and on applying the general results we obtain

$$\boldsymbol{\phi} = \boldsymbol{\phi}_1 = \boldsymbol{\phi}_2 \quad \text{and} \quad \boldsymbol{\tau}_s = G(\mathbf{U}_{g1} - \mathbf{U}_{g2}) + \boldsymbol{\phi} \tag{40}$$

in which

$$G = \frac{AB}{A+B} = \frac{\rho_1K_1|\mathbf{U}_{g1} - \mathbf{U}_{g2}|}{(1+\delta)^2} = \frac{\rho_2K_2|\mathbf{U}_{g1} - \mathbf{U}_{g2}|}{(1+\delta)^2}\,\delta^2$$

and

$$\mathbf{U}_s = (\mathbf{U}_{g2} + \delta\mathbf{U}_{g1})/(1+\delta) \tag{41}$$

where

$$\delta = A/B = (\rho_1 K_1/\rho_2 K_2)^{1/2}. \tag{42}$$

These results completely specify the energetics of the coupled Ekman layers provided that K_1 and K_2 are known. Note that on substituting for $\boldsymbol{\phi}$ from Eq. (24) and using Eq. (41) we have

$$\boldsymbol{\phi}_1 = \rho_1 K_1 |\mathbf{U}_{g1}^* - \mathbf{U}_s^*|(\mathbf{U}_{g1}^* - \mathbf{U}_s^*) \tan\alpha$$

where $|\mathbf{U}_s^*| = |\mathbf{U}_s|$ and $\mathbf{U}_s^*$ is rotated 90° to the left-hand side of $\mathbf{U}_s$. Thus from Eq. (36)

$$|\boldsymbol{\tau}_s| = \rho_1 K_1 \sec\alpha |\mathbf{U}_{g1} - \mathbf{U}_s|^2$$

and similarly from Eq. (37)

$$|\boldsymbol{\tau}_s| = \rho_2 K_2 \sec\alpha |\mathbf{U}_s - \mathbf{U}_{g2}|^2$$

and hence $K_1 = C_{f1}\cos\alpha$ and $K_2 = C_{f2}\cos\alpha$, where C_{f1} and C_{f2} are the geostrophic drag coefficients for air and water respectively, see Arya (1975).

Observations of the wind drift velocity, $\{\delta/(1+\delta)\}\mathbf{U}_{g1}$ (e.g. Kraus 1972), usually show it to be about 3% of the surface geostrophic wind and suggest that $\delta \sim 0{\cdot}03$.

By Eq. (23) δ is also the ratio of the dissipation rates in the ocean and the atmosphere, and hence its determination is also of importance for the thermodynamics of the coupled ocean–atmosphere system.

On assuming that $\rho_1 = 1{\cdot}2\,\mathrm{kg\,m^{-3}}$ and $\rho_2 = 10^3\,\mathrm{kg\,m^{-3}}$ we obtain $(\rho_1/\rho_2)^{1/2} \sim 0{\cdot}034$ and hence on comparing this result with the observational estimate for δ it appears that $K_1/K_2 \sim 1$ so that the geostrophic drag coefficients in the air and water should have similar magnitudes. Kraus (1977) has given an independent dimensional argument which supports this result, namely that the stress gradients should scale according to the Ekman depths of the respective fluids. Thus,

$$-\left.\frac{(\partial\boldsymbol{\tau}_z/\partial z)_2}{(\partial\boldsymbol{\tau}_z/\partial z)_1}\right|_{z=0} \sim \frac{|\boldsymbol{\tau}_s|(w_*/f)^{-1}}{|\boldsymbol{\tau}_s|(u_*/f)^{-1}} = (\rho_2/\rho_1)^{1/2}$$

where $u_* = (|\boldsymbol{\tau}_s|/\rho_1)^{1/2}$ and $w_* = (|\boldsymbol{\tau}_{s1}|/\rho_2)^{1/2}$ are respectively the friction velocities in air and water, and on substituting this result in Eq. (19) we obtain $A/B = (\rho_1/\rho_2)^{1/2}$ and hence also from Eq. (42) that $K_1/K_2 \sim 1$.

Further experimental studies on this important ratio, however, are clearly necessary. Observations also indicate that the angle of rotation (α) is usually small, and adjacent to the interface ($z = 0$) in both fluids we have approximate constant stress layers in which

$$\boldsymbol{\tau}_s \sim \rho_1 K|\mathbf{U}_a - \mathbf{U}_s|(\mathbf{U}_a - \mathbf{U}_s) \qquad |z_1 f|/u_* \ll 1 \tag{43}$$

$$\boldsymbol{\tau}_s \sim \rho_2 K'|\mathbf{U}_s - \mathbf{U}_o|(\mathbf{U}_s - \mathbf{U}_o) \qquad |z_2 f|/w_* \ll 1 \tag{44}$$

where $\mathbf{U}_a$ refers to the wind velocity at a height $|z_1|$ with the drag coefficient K, and $\mathbf{U}_o$ refers to the current velocity at a depth $|z_2|$ with the drag coefficient K'.

By arguments similar to the above we obtain

$$\boldsymbol{\tau}_s \sim \boldsymbol{\tau}_W - \boldsymbol{\tau}_F \tag{45}$$

in which $\boldsymbol{\tau}_W = C\mathbf{U}_a$ and $\boldsymbol{\tau}_F = C\mathbf{U}_o$ where

$$C = \rho_1 K|\mathbf{U}_a - \mathbf{U}_o|/(1+\varepsilon)^2 \quad \text{and} \quad \varepsilon = (K\rho_1/K'\rho_2)^{1/2}.$$

$\boldsymbol{\tau}_W$ and $\boldsymbol{\tau}_F$ are the wind stress and the understress referred *jointly* to the height $|z_1|$ in the atmosphere and the depth $|z_2|$ in the ocean. These formulae, and the corresponding expression for the surface drift velocity,

$$\mathbf{U}_s \sim (\varepsilon \mathbf{U}_a + \mathbf{U}_o)/(1 + \varepsilon) \tag{46}$$

are useful approximate relations which have been used to illustrate the principles of the coupled analysis (Bye 1985).

5. Discussion

The preceding analysis has sought to examine the dissipation processes in the coupled ocean–atmosphere boundary layer.

We can discuss the results in two frames of reference, firstly that of the relative geostrophic motion between the two fluids, and secondly with respect to the motion of the earth. Both approaches are valid and useful.

The major conclusion from the arguments on the relative motion is that the dissipation rates in the two fluids are partitioned according to Eq. (23). This ratio can be evaluated for laminar flow, and approximately for turbulent flow (Eq. (42)) and may be expected to be important for the thermodynamics of the coupled ocean–atmosphere system, see Bye (1985).

Arguments from an absolute frame of reference yield other information. In particular an answer can be obtained to the question where does the energy come from that is to be dissipated in the coupled boundary layer? This question cannot be addressed in the relative frame as in general energy crosses the interface between the two fluids in the dissipation process. We seek the answer through Eqs. (27) and (28) in which the rate of dissipation is partitioned into two terms representing loss of energy by the respective fluids, and two interactive or rate-of-working terms.

The energy loss terms show that the fluids lose energy in proportion to the squares of their geostrophic velocities, and imply that the interface exerts a drag on *both* the atmosphere (by wind stress) and the ocean (by understress). The importance of this drag for the oceanic general circulation has been argued in Bye (1985).

The rates-of-working terms may take either sign depending on the relative alignment of the geostrophic flows. If $\boldsymbol{\tau}_2 \cdot \mathbf{U}_{g1} > 0$ the ocean transfers energy to the atmosphere, and if $\boldsymbol{\tau}_1 \cdot \mathbf{U}_{g2} > 0$ the atmosphere transfers energy to the ocean; and vice versa. The intrinsic interest of the rate-of-working terms lies in the differing horizontal scales of the geostrophic motions of the two fluids—the rate of working of the atmosphere on the ocean (by wind stress) has been extensively studied, but the rate of working of the ocean on the atmosphere (by understress) which emerges as a complementary process would appear also worthy of study.

The difference between the rates of energy loss and the rates of interfacial energy transfer is the rate of dissipation, see Eq. (26). These considerations provide a simple framework in which momentum exchange at the sea surface can be discussed.

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