ON THE STRUCTURE OF THE ATMOSPHERIC SURFACE LAYER

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ABSTRACT

With Prandtl's theory of the mixing length as the point of beginning, a theory concerning the structure of the atmospheric surface layer is proposed on similar assumptions as were put forward by Lettau. The novel feature in the present treatment lies in the fact that the acceleration due to the frictional part of the turbulence is considered to be dependent on stability, whereas Lettau assumed a constant value for this acceleration. Although this theory is not exact, it may promote a better understanding of atmospheric turbulence.

A dimensionless stability number is introduced; it enables one to obtain a simple survey of all states of the atmospheric surface layer. The theory is tested with observations of Rider (1954). The requirements for a further experimental program are established.

1. Introduction

When investigating atmospheric turbulence, one often encounters the fact that nearly every assumption which is made to develop a theory appears to be untenable during further analysis. Consequently one is prompted to try and work with ever more fundamental concepts and, as a result, ever greater difficulties are encountered. This picture of the present state of affairs comes to mind when reading the circumstantial and interesting survey on turbulence and transfer processes in the atmosphere by Priestley and Sheppard (1952). Of course, a fundamental analysis is essential to build up an entirely satisfactory theory. However, it may be profitable to develop in a less exact but more tentative manner a theory which is useful and capable of practical application. The exact basis of the theory may then be developed afterwards.

An example of such a course of action in the field of turbulence is provided by the theory of the mixing length as given by Prandtl (1932). Assuming the mixing length l proportional to the height z, Prandtl derived the well-known logarithmic wind profile; von Kármán (1930) produced a basis for Prandtl's assumption by means of his similarity hypothesis, while Hamel (1943) derived von Kármán's similarity hypothesis in an exact manner from the Navier-Stokes equations and so produced an exact theoretical basis for Prandtl's theory.

Conversely, the mixing length may now be defined by the relation:

$$l = k(z + z_0), \tag{1}$$

where k is von Kármán's constant, and z_0 is the roughness parameter. Moreover, from the experimental verification of the logarithmic wind profile, it may be inferred that the conditions postulated by Hamel are reasonably well satisfied when the atmosphere is in neutral equilibrium. These conditions are:

1. A steady turbulent flow exists;
2. The velocity components u, v and w may be split up into a mean component and a purely turbulent component:

$$u = \bar{u} + u', \textit{ etc.}, \text{ where } \bar{u}' = \bar{v}' = \bar{w}' = 0;$$

3. $u'w' \gg \nu\, \partial\bar{u}/\partial z$, or $K \gg \nu$, where ν = kinematic viscosity and K = coefficient of eddy transfer;
4. For the main flow, the relation $\mathrm{Re}_x = \bar{u}_m x \nu^{-1} \approx \infty$ must hold, where x is the distance in the direction of the main flow from the boundary of the surface, where it becomes homogeneous, to the point of measurement. For $\bar{u}_m$ one may, for instance, take the mean velocity through the atmospheric layer under consideration, such that $\bar{u}_m = h^{-1}\int_0^h \bar{u}\, dz$, where h is the height of the layer.

Hamel's work has been specially chosen by way of example, first because one has the impression that meteorologists are not aware of the exact foundation of the logarithmic wind profile, and secondly because the concept of the mixing length will be used in the following derivations.

As soon as a theory is developed in a less exact manner, with use of qualitative considerations, great care must be taken when interpreting the results. A typical example in which this is insufficiently done is given by Lettau's theory. Starting from some challengeable assumptions, Lettau (1949) constructs a theory which indeed possesses some interesting aspects, but which is certainly not founded on such a "sound physical basis" as the author claims. In the present article, it is shown that by slightly modifying his assumptions and by using a more cautious formulation a theory may be constructed which, it is true, will have fewer pretenses, but possibly more practical value.

2. The transfer equations

Using the above-listed conditions 1, 2 and 3, the equation of continuity, and the equation of state, Businger (1954) has shown that it is possible to derive the well-known equation of transfer of momentum and heat in a satisfactory manner from the Navier-Stokes equations and the Fourier equation:

$$\overline{w'u'} = -\tau/\rho = -u_*^2, \tag{2}$$

and

$$\overline{w'\theta'} = F/c_p\bar{\rho}, \tag{3}$$

where τ = shearing stress, ρ = density, u_* = friction velocity, θ = potential temperature, F = heat flux, and c_p = specific heat at constant pressure.

$\tau/\bar{\rho}$ and $F/c_p\bar{\rho}$ may be taken as constant in the bottom 25 m of the atmosphere. In accord with Lettau (1949), this layer will be called the "atmospheric surface layer." The upper limit of the "surface layer" is not sharply defined, but depends on the level where the term $(\partial p/\partial x)z$ can no longer be neglected and must be introduced in (2).

It is common practice to write the correlation coefficients $\overline{w'u'}$ and $\overline{w'\theta'}$ in the following form:

$$\overline{w'u'} = -K_m\,\partial\bar{u}/\partial z, \tag{4}$$

and

$$\overline{w'\theta'} = -K_h\,\partial\theta/\partial z, \tag{5}$$

where K_m = coefficient of eddy viscosity, and K_h = coefficient of eddy conductivity.

It should be kept in mind that, for adiabatic conditions, the relation

$$K_m = l^2\,\partial\bar{u}/\partial z \tag{6}$$

has obtained satisfactory theoretical foundation, so that on this point the theory is fairly complete. This, however, is not the case for K_h. It is not so easy to find a suitable and useful relation for this coefficient. The relation

$$K_h = K_m = K, \tag{7}$$

which formerly was generally accepted, has been strongly challenged on theoretical grounds by Ertel (1942; 1944) and by Priestley and Swinbank (1947); Swinbank (1951) and Pasquill (1949) claim to have proved experimentally that $K_h \neq K_m$. On the other hand, measurements by Rider and Robinson (1951) have shown that $K_h = K_m$ = const; moreover these writers argue that this constant can only be equal to 1.

Of an entirely different nature is a theory by van der Held (1947), which contains an argument in favor of the validity of (7). Starting from the model of turbulence as given by Prandtl, van der Held regards the elements of turbulence as molecules with an infinite number of degrees of freedom. He then follows up the analogy between molecular viscosity and the eddy viscosity K_m, and between the molecular thermal diffusivity a and the eddy conductivity K_h, making use of the well-known non-dimensional Prandtl number defined by $\mathrm{Pr} = \nu/a$. Now a relation exists, derived from the kinetic theory of gases, between Pr and the number of degrees of freedom of the molecules:

$$\mathrm{Pr} = (n+2)/(n+4, 5),$$

where n is the number of degrees of freedom. It follows that, for $n = \infty$, Pr = 1; or, if we may extend the analogy between molecular and turbulent motions,

$$\mathrm{Pr} = K_m/K_h = 1 \text{ or } K_m = K_h = K. \tag{7a}$$

From the above, it will be obvious that no unanimity exists on this point. In the following, we shall use (7), bearing specially in mind that the ratio K_m/K_h, while not necessarily equal to one, will be approximately constant for a given profile.

Equations (2) and (3) now become, with use of (4), (5) and (7),

$$K = \partial\bar{u}/\partial z = u_*^2, \tag{8}$$

and

$$K\,\partial\bar{\theta}/\partial z = -F/c_p\bar{\rho}. \tag{9}$$

These equations contain three unknowns: K, $\bar{u}$ and $\bar{\theta}$. Therefore, a third relation must be found to solve these equations. In the following sections, an attempt is made to find such a relation.

3. Frictional (mechanical) turbulence and convective turbulence

In an adiabatic atmosphere, the turbulence is entirely due to friction at the surface of the earth. As soon as a heat flux exists, turbulence will either increase or decrease through the liberation or absorption of convective energy. We may now make a formal distinction between turbulence caused by mechanical friction (the frictional or mechanical turbulence) and turbulence caused by convection (the convective turbulence). The problem is now to write down this formal distinction in an equation. It was Richardson (1920) who, for the first time, more or less succeeded in achieving this by means of energy considerations. The chief result of Richardson's analysis is the definition of the Richardson number, Ri, as a measure of the stability of the atmosphere.

$$\mathrm{Ri} = \frac{g\,\partial\bar{\theta}/\partial z}{\bar{T}(\partial\bar{u}/\partial z)^2}, \tag{10}$$

where g = gravity and T = absolute temperature. Measurements have shown that this number can be used as a stability parameter. A disadvantage, however, is that Ri varies with height. Moreover, in this form the description of the components of turbulence is still incomplete.

Attempts at a more complete description of both components of turbulence have been made by Rossby and Montgomery (1935) and by Lettau (1949). Lettau has given the more consistent definition, which nevertheless is not yet entirely satisfactory. He assumes a mean convective acceleration A, which may be written

$$A = (gl\, \partial\bar{\theta}/\partial z)/\bar{T},$$

where l is again the mixing length, which here has been used in the same sense as in deriving (6). Further, Lettau distinguishes between a total turbulent acceleration and a turbulent acceleration which would obtain if there were no flux of heat, *i.e.*, if the atmosphere were adiabatic. He writes

$$\frac{K^2}{l^3} = \frac{K_a^2}{l_a^3} - g\frac{l\, \partial\bar{\theta}/\partial z}{\bar{T}}, \tag{11}$$

(Lettau's so-called mixing velocity has been replaced by K/l) where the index a denotes quantities related to adiabatic conditions. The first term of the right-hand side indicates the contribution of friction to the turbulence. The further development of Lettau's theory makes it clear that this term is independent of stability, a fact which in our opinion is an objection to this theory; for the frictional turbulence will be determined by the shearing stress, which is certainly not independent of the stability. Since K_a and l_a refer to adiabatic conditions, the following relations hold:

$$K_a = u_{*a} l_a, \tag{12}$$

and

$$l_a = k(z + z_0). \tag{13}$$

So we may write, for (11),

$$\frac{K^2}{l^3} = \frac{u^2_{*a}}{k(z+z_0)} - \frac{gl\, \partial\bar{\theta}/\partial z}{\bar{T}}. \tag{14}$$

The difficulty now is that it is impossible to determine u_{*a} as soon as the atmospheric conditions are no longer adiabatic. It is necessary then to wait till the heat flux vanishes, while the further synoptic situation remains unchanged. For when the heat flux increases under otherwise similar circumstances, u_* will also increase as a result of the increasing instability of the atmosphere, a phenomenon which is often observed in the daily course of the wind velocity in case of clear weather. It is possible now to meet the above-mentioned objection by writing instead of (12),

$$K_f = u_* k(z + z_0), \tag{15}$$

in which the index f denotes that the quantity relates to the frictional part of the turbulence. This equation expresses the assumption that friction contributes to the turbulence in the same way whether the atmosphere is adiabatic or not. From (13) it is furthermore clear that l_a is not dependent on stability. So we may write

$$l_a = l_f = k(z + z_0). \tag{16}$$

With the aid of (15) it is now possible to formulate an equation analogous to (14), namely

$$\frac{K^2}{l^3} = \frac{u^2_*}{k(z+z_0)} - \frac{gl\, \partial\bar{\theta}/\partial z}{\bar{T}}. \tag{17}$$

Now we must take for u_* that value which holds for the moment under consideration. Besides the fact that qualitatively (17) is more acceptable than (14), the theory appears to develop further, in a simpler way than that due to Lettau. (We shall return to this in section 5, below.) However, it is fully realized that (17) requires further justification and proof; but, for the time being, this equation will be postulated.

Combining (15), (16) and (13) with (17), we obtain an equation analogous to (11),

$$\frac{K^2}{l^3} = \frac{K_f^2}{l_f^3} - \frac{gl\, \partial\bar{\theta}/\partial z}{\bar{T}}. \tag{18}$$

Although we have now come a step nearer to a solution of the problem, a new variable, l, has been introduced and therefore a fourth relation is required. Before derivation of such a relation, considerations of similarity are introduced to reduce the number of parameters and to define a suitable stability parameter.

4. Similarity and stability

From the well-known logarithmic law, $\bar{u} = u_* \times k^{-1} \ln[(z+z_0)/z_0]$, a non-dimensional velocity and a non-dimensional height may at once be derived, namely, $U = \bar{u}/u_*$ and $\zeta = (z+z_0)/z_0$. Substituting these expressions in (8) and introducing another non-dimensional parameter, $R = K/u_* z_0$, one obtains

$$\partial U/\partial\zeta = 1/R. \tag{19}$$

The similarity between (9) and (8) is brought out by writing (9) as

$$\partial\vartheta/\partial\zeta = 1/R, \tag{20}$$

where

$$\vartheta = (\bar{\theta} - \theta_0)/\theta_*, \quad \theta_* = -F/(u_* c_p \bar{\rho}),$$

and θ_0 = potential temperature at the surface. The meaning of θ_* may be clarified by identifying the velocity profile and the temperature profile. This can be achieved by using u_* as the unit of velocity and θ_* as the unit of potential temperature. The ratio of the scales of both profiles is given by $u_*/\theta_* = \sigma$. From this ratio and u_*, θ_* can be at once determined:

$$\theta_* = u_*/\sigma. \tag{21}$$

The problem is now to determine R as a function of height and stability; so, for instance,

$$R = R(\zeta, \mathrm{Ri}). \tag{22}$$

It has been shown experimentally that Ri can be used as a stability parameter. A disadvantage, however, is that Ri is an unknown function of height. Therefore, it is desirable to find another stability parameter which is independent of height. That such a quantity must exist may be deduced from the fact that, if Ri is known at one height, while u_* and θ_* are also known, the velocity and temperature profiles and consequently the rate of change of Ri with height are entirely determined.

One may now try to split up Ri in two parts, one of which is independent of height. The easiest way to achieve this is to express Ri in the quantities defined above. We then find

$$\mathrm{Ri} = \frac{g\,\partial\bar{\theta}/\partial z}{\bar{T}(\partial\bar{u}/\partial z)^2} = \frac{g\theta_* z_0}{\bar{T}u_*^2}R. \qquad (23)$$

This is really what we looked for: a term $(g\theta_* z_0)/\bar{T}u_*^2$ which is constant with height, and a known quantity R which varies with height. Writing

$$-(g\theta_* z_0)/\bar{T}u_*^2 = S_n, \qquad (24)$$

it follows that, instead of (22), one may write

$$R = R(\zeta, S_n). \qquad (25)$$

S_n is now used as stability parameter instead of Ri. The practical significance of this equation is that, if the profiles have once been determined for one given set of conditions involving a given value of S_n, they are also determined for any other set of conditions which yields the same value of S_n. Furthermore, the number S_n enables us to draw up a survey of all states of the atmospheric surface layer which satisfy Hamel's four conditions. However, the stability parameter thus defined can only assume its full importance when the above reasoning has been confirmed experimentally.

5. The relation $R(\zeta, S_n)$

For an adiabatic atmosphere $S_n = 0$, for $\theta_* = 0$. In unstable conditions, one finds $S_n > 0$, and in stable conditions $S_n < 0$.

The structure of the flow in neutral conditions, $S_n = 0$, is known and can be expressed in the non-dimensional units U and ζ:

$$R = k\zeta, \qquad (26)$$

so

$$U = (\ln \zeta)/k. \qquad (27)$$

As indicated earlier, for the calculation of $R(\zeta, S_n)$ in other than adiabatic conditions another relation between l and K, $\bar{\theta}$ and $\bar{u}$ is required in addition to (8), (9) and (18). Therefore, another assumption must be made. An obvious assumption, already made by Rossby and Montgomery (1935), is that (6) remains valid also for a non-adiabatic atmosphere. By use of (8), (15) and (16), this assumption reduces to

$$K/K_f = l/l_f. \qquad (28)$$

This relation, however, does not meet our requirements, because according to (18) K and l would decrease with increasing instability, which is contrary to experience. This also explains the unsatisfactory relation between the theory of Rossby and Montgomery and the observations.

However, by modification of (28), a qualitatively acceptable result can be obtained. For if we make the assumption that

$$K_f = l^2\,\partial\bar{u}/\partial z, \qquad (29)$$

we now find, using (8), (15) and (16), that

$$K/K_f = (l/l_f)^2. \qquad (30)$$

This assumption is very similar to Lettau's assumption,

$$K/K_a = (l/l_a)^2. \qquad (31)$$

The assumptions (30) and (31) are not equal, because $K_f \neq K_a$, which appears from (28) and (15); so l and K in (30) are not the same as l and K in (31). The problem is now fully determined by (8), (9), (18) and (30), so that a relation $R(\zeta, S_n)$ may be found.

On the contrary, the theory of Lettau is not yet so complete, because (8) and (9) combined with (11) and (31) produce still another parameter, u_*/u_{*a}, which is yet unknown. To make his theory complete, Lettau gives (in the present writer's opinion) a challengeable discussion about the daily course of the various quantities. He uses a certain level where $\partial\bar{u}/\partial z$ is independent of stability. However, we shall not reproduce this part of Lettau's theory; suffice it to refer to his work.

As has already been mentioned, it is possible to find a relation $R(\zeta, S_n)$ with the aid of (8), (9), (18) and (30). It is necessary, in doing so, to use the dimensionless parameters from section 4, above, and (15) and (16). After some calculations, we find

$$R = (k\zeta)^2 S_n + \tfrac{1}{2}k\zeta[1 + (1 + 4k\zeta S_n)^{\frac{1}{2}}]. \qquad (32)$$

Substituting (32) in (19) and integrating, one finds

$$U = \frac{1}{k}\left\{\ln\frac{[(1+4k\zeta S_n)^{\frac{1}{2}}-1][(1+4kS_n)^{\frac{1}{2}}+1]}{[(1+4k\zeta S_n)^{\frac{1}{2}}+1][(1+4kS_n)^{\frac{1}{2}}-1]} + \frac{2(1+4k\zeta S_n)^{\frac{1}{2}}}{[1+(1+4k\zeta S_n)^{\frac{1}{2}}]^2} - \frac{2(1+4kS_n)^{\frac{1}{2}}}{[1+(1+4kS_n)^{\frac{1}{2}}]^2}\right\}. \qquad (33)$$

This is a relation of the type required. It is, however, difficult to decide whether this equation agrees with the observations. The observations which so far have been taken are either incomplete or fail to satisfy the conditions required for the turbulence to be fully developed.

Although (32) and (33) are by no means sufficiently founded, and therefore may not be expected to give an adequate description of the structure of the atmospheric surface layer, they are represented graphically in figs. 1 and 2, respectively. The (ζ, S_n) diagram

TABLE 1. Comparison of various assumptions leading to a relation $R(\zeta, S_n)$.

Writer	1st assumption	2nd assumption	$R(\zeta, S_n)$
Rossby and Montgomery	$\frac{K^2}{l^3} = \frac{K_f^2}{l^3} - \beta g \frac{l\,\partial\bar{\theta}/\partial z}{\bar{T}}$	$\frac{K}{K_f} = \frac{l}{l_f}$	$k\zeta = R(1 - \beta S_n R)^{\frac{1}{2}}$
Lettau	$\frac{K^2}{l^3} = \frac{K_a^2}{l_a^3} - g \frac{l\,\partial\bar{\theta}/\partial z}{\bar{T}}$	$\frac{K}{K_f} = \left(\frac{l}{l_f}\right)^2$	$R = \left(\frac{u_*}{u_{*a}}\right)^2 k^2\zeta^2 S_n + \frac{1}{2}k\zeta \frac{u_{*a}}{u_*}\left\{1 + \left[1 + 4\left(\frac{u_*}{u_{*a}}\right)^3 k\zeta S_n\right]^{\frac{1}{2}}\right\}$
Businger	$\frac{K^2}{l^3} = \frac{K_f^2}{l_f^3} - g \frac{l\,\partial\bar{\theta}/\partial z}{\bar{T}}$	$\frac{K}{K_f} = \left(\frac{l}{l_f}\right)^2$	$R = k^2\zeta^2 S_n + \frac{k\zeta}{2}[1 + (1 + 4k\zeta S_n)^{\frac{1}{2}}]$
General	total turb. = frictional turb. + convective turb.	$\frac{K}{K_f} = f\left(\frac{l}{l_f}\right)$	$R = R(\zeta, S_n)$

(fig. 1) in particular is illuminating, since it shows how ultimately the entire structure of the surface layer can be represented by means of such a diagram. We shall return to this point below.

As long as $k\zeta S_n \ll 1$, (33) reduces by approximation to

$$U = k^{-1} \ln \zeta - 2S_n(\zeta - 1). \qquad (34)$$

Panofsky (1952) derived an equation equivalent to (34) by giving for K an expansion in two terms of a Taylor series,

$$U = k^{-1} \ln \zeta - (mz_0/k)(\zeta - 1),$$

where the parameter m obviously can be evaluated by the proposed theory as follows:

$$m = 2kS_n/z_0.$$

A synopsis of the various theories mentioned above, and the corresponding relations $R(\zeta, S_n)$ is given in table 1. Although The Rossby-Montgomery approach is quite different from Lettau's approach, the respective assumptions made by these investigators can be written in comparable form. This has been done, the various assumptions made being written in a form analogous to (18) and (28). β is a constant which must be determined experimentally. The ratio u_*/u_{*a} in Lettau's theory is a new parameter arising from the

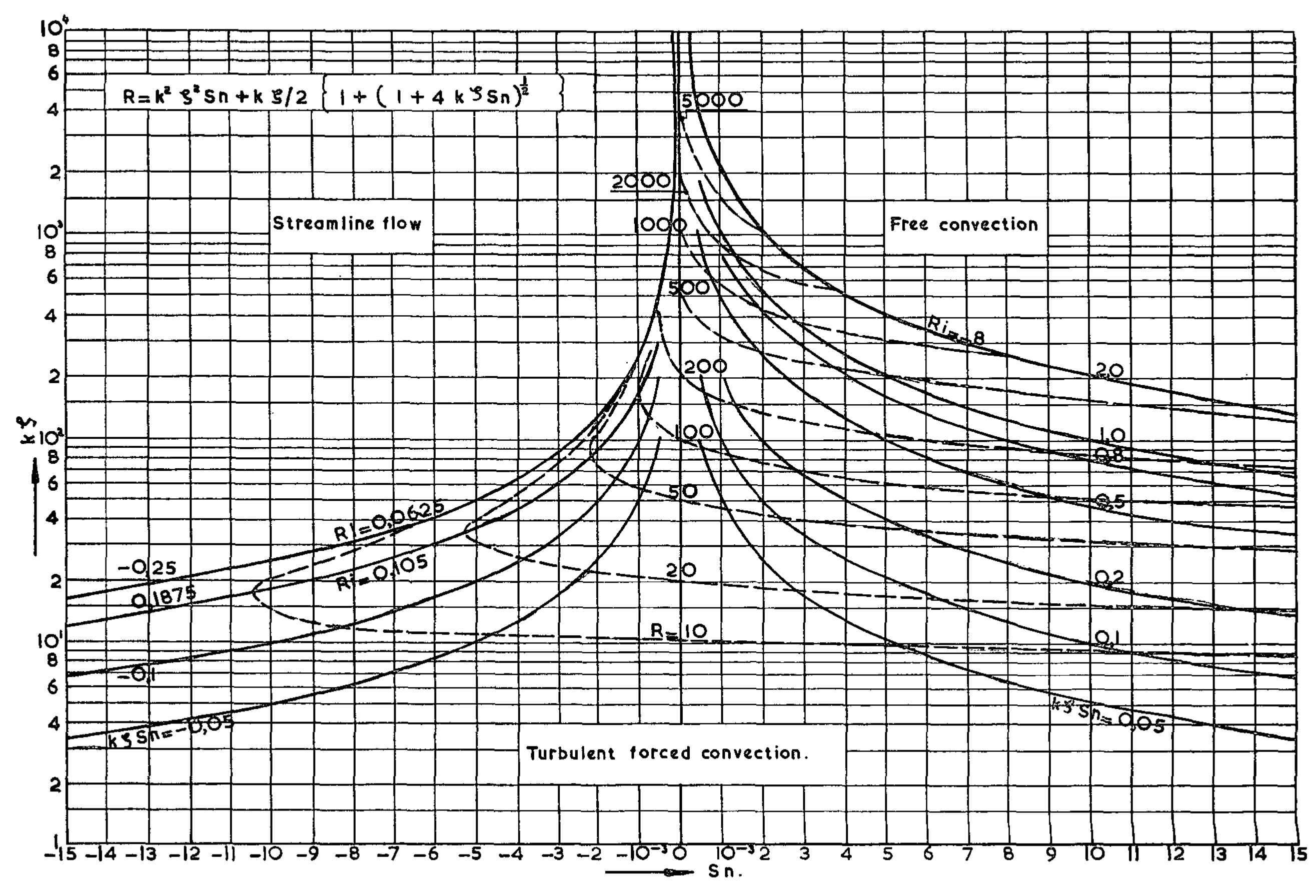

FIG. 1. (ζ, S_n) diagram.

fact that he relates his assumption to the adiabatic state. The table clearly shows that a large number of possibilities exists from which a choice can be made. Only a thorough theoretical investigation can settle the question as to which choice is the right one, although a well arranged experiment may serve to give some first indications.

6. The (ζ, S_n) diagram

It was already pointed out that fig. 1, in which ζ and S_n have been chosen as rectangular coordinates, is highly illuminating as regards the structure of the surface layer. The straight line $S_n = 0$ passes through the middle of the diagram and divides it into two parts. The left-hand side corresponds to a stable atmosphere, and the right-hand side to an unstable atmosphere. However, another division can be made which is more characteristic for this diagram. By means of (32) and (23), curves of R = const and Ri = const have been drawn.

Now, (32) determines a certain maximum value of Ri in the stable region, namely, Ri = 0.105. It is conceivable that this value corresponds to the transition from turbulent to laminar flow, so that a laminar region and a turbulent region may be distinguished.

A similar transition exists in the unstable region from forced convection to free convection.[1] Here also a critical value of Ri can be found at which the transition occurs. By means of theoretical consideration due to Taylor (1931), and measurements by Johnson and Heywood (1938), the following relation for the profiles in conditions of free convection may be derived:

$$R = \text{const} \times \zeta^{1.75}. \tag{35}$$

By fitting this relation continuously to (32), a critical value of Ri = −8 is found. A more comprehensive treatment of free convection has been given by Businger (1954). The conditions under which (32) is valid are therefore limited on two sides by a critical value of Ri. Under these conditions, a regime of forced convection is set up, since frictional turbulence predominates over convective turbulence throughout.

[1] By forced convection is understood the turbulent heat flux in regions where frictional turbulence predominates, while free convection occurs in regions of predominantly convective turbulence. In meteorology, the term convection is generally used instead of free convection.

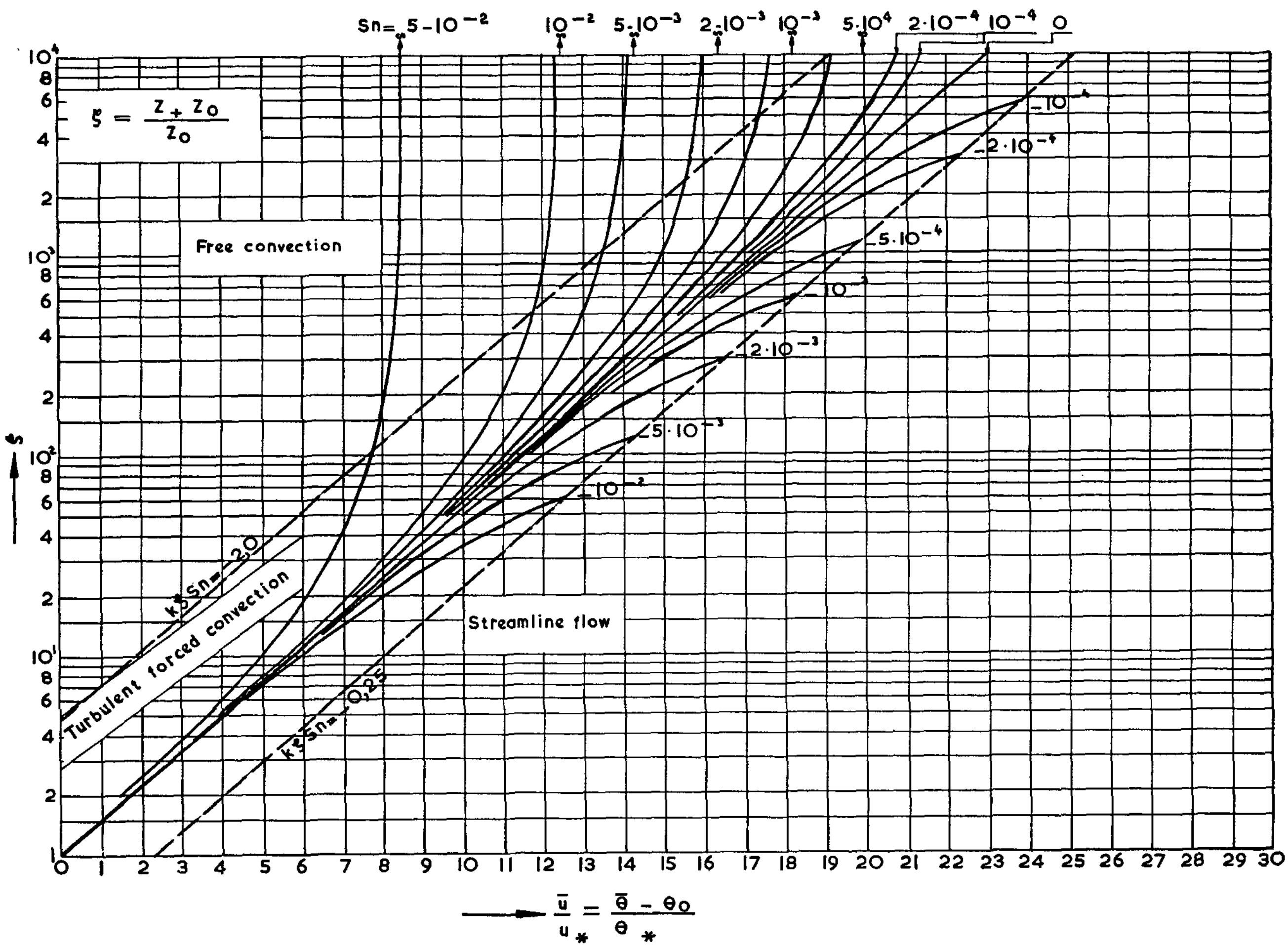

FIG. 2. Wind profile as function of S_n.

Through the above analysis, the whole classification has been given in essence.

By use of (32), it is possible to draw curves $U =$ const in fig. 1. Instead, a separate diagram has been constructed (fig. 2) with ζ and U as rectangular coordinates, so that a picture is obtained of the shape of the profiles.

7. Experimental verification

There is a great need for observations which are so complete that the theories can be tested. The experimental confirmation which Lettau claims to have found for his theory cannot in fact be accepted as such, since not all the independent theoretical parameters were determined experimentally.

The only set of observations which makes possible a suitable test has been published recently by Rider (1954). He has given direct measurements of the shearing stress, while the heat flux has been evaluated independently from a heat balance. Although the accuracy of either the shearing stress or the heat balance is small, these measurements are so complete as to prevent unlimited adjustment of the quantities u_*, z_0 and F to the observed wind profiles.

Although a test is possible with Rider's observations, it is difficult to determine the manner in which the test can best be carried out. The most convenient way appeared to be a comparison of S_n obtained by theory with S_n from observations. The theoretical S_n were obtained by comparison of the observed wind profiles with the theoretical ones. This was only possible after choice of a fixed value of z_0 for all observations. Careful considerations made it obvious that $0.34 \text{ cm} < z_0 < 0.44 \text{ cm}$; therefore, $z_0 = 0.4$ cm has been taken for the test. It being assumed further that the deviation of the wind profile from the logarithmic profile was negligible at 15 cm, S_n could be evaluated from the wind profile by interpolation in fig. 2. This value of S_n was considered the "theoretical" value. It was also possible to calculate S_n more directly from the observations in various other ways. The following ways were chosen:

1. $S_n = -\text{Ri}\,(\partial \bar{u}/\partial z)z_0/u_*$;
2. $S_n = -gF/(u_*^3 \bar{T} c_p \rho)$;

and

3. $S_n = -(\text{Ri}\,u_* z_0)/K_m$.

The quantities on the right-hand sides of these equations could be obtained from Rider's tables, except u_*, which was evaluated from the wind speed at 15 cm, and z_0, it again being assumed that the logarithmic profile was valid to this height.

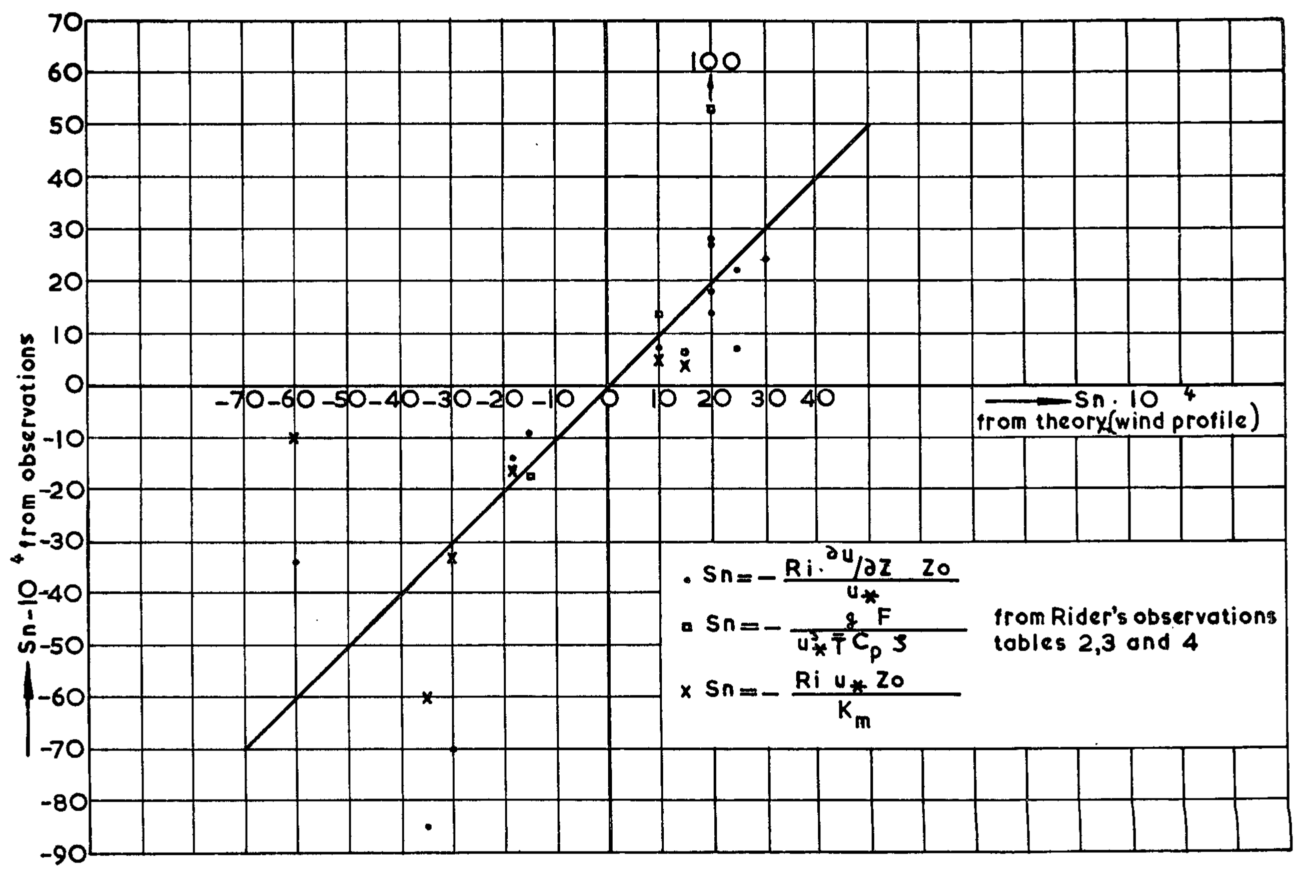

FIG. 3. Plot of S_n evaluated from profiles *versus* S_n from observations.

In table 2 and fig. 3, the results of this test are represented for the observations with $|S_n|$theor. $> 10^{-3}$. Although there is a rather large scattering of the points, there is a certain agreement between theory and observation. The observations in the stable region show particularly large spreading, which probably can be ascribed to non-stationary conditions of the surface layer. For good estimation of S_n from the profiles, it appeared to be desirable also to have measurements from heights above 2 m, especially in unstable conditions.

The results of this test cannot be considered as quite satisfactory, and the increased need for more complete observations is once more stressed.

It is seen that, to confirm a theory, it is necessary that *all* independent quantities are *measured* and that no unknown parameters are inferred from the theory.

On the basis of the preceding considerations, it is possible to formulate an observational program which is sufficiently complete. Starting from the principle that S_n and the shape of the velocity and temperature profiles must be measured independently, and requiring Hamel's four conditions to be realized as fully as possible, one notes that the theory can be tested if the following points are satisfied:

1. It is important to have available two very large fields with different z_0, the surfaces of which must be as uniform as possible, to ensure a constant z_0. Moreover, these fields should change as little as possible in the course of time, so that the observations can be reproduced. The minimum extent of the fields is determined by Re_x. There are reasons to suppose that Re_x must be $\geqslant 10^9$ for the profiles to be fully developed. This value of Re_x corresponds to a distance of 2 to 5 km. There are not many places where uniform terrains of such extent are found, which furthermore do not change with time. However, it is possible that the salt deserts in the United States satisfy these requirements.
2. The surface drag must be measured directly and in such a manner that the surface is not disturbed, for an accurate determination of u_* is of fundamental importance for testing the theory. No doubt, considerable experimental difficulties will be encountered; but in the writer's opinion, these should not be insuperable.
3. Complete measurements of heat- and water-budgets are required for an independent determination of F, a quantity which likewise is of fundamental importance for testing the theory. The experimental difficulties associated with these measurements have been largely overcome.
4. Finally, it is necessary that measurements of wind speed, temperature and humidity are made at different heights, *e.g.*, at 0.10, 0.20, 0.50, 1.00, 2.00, 5.00, 10.00 and 20.00 m. Here, also, the experimental difficulties have been largely overcome.

8. Remarks

The analysis presented above is by no means claimed to be a complete description of the atmospheric surface layer; but it contains a first fundamental step from which further progress can be made. There are many problems bordering on the problem discussed, which are just as fundamental and which may have even more practical value once they have been solved. Some of these problems are:

1. In which manner do the velocity- and temperature-profiles change during transitions from one field to another with a different value of z_0?
2. What is the influence of surface roughness on the flux of heat?
3. What are the critical values of Ri?
4. In which manner do the profiles vary as a function of the heat flux, other factors being equal?
5. What is the structure of the atmosphere throughout the boundary layer?

Although many workers have studied these and similar problems, no really satisfactory results have been obtained so far.

Finally, various investigators (Batchelor, 1950; Inoue, 1952; and others) have raised grave objections against the concept of the mixing length and have shown the inadequacy of this concept in certain cases. In its place, they have put forward the so-called statistical theory of turbulence. However, at present this statistical theory is only applicable to "homogeneous" media such as an adiabatic atmosphere, not to inhomogeneous media in which convective forces occur. It therefore seems that the rejection of the concept of the mixing length is at least premature, in particular because this concept has shown its usefulness in adiabatic conditions.

TABLE 2. Wind profiles for various stabilities observed by Rider. From these profiles, theoretical S_n is derived. Table shows dimensionless wind speed $U = \bar{u}/u_*$ at dimensionless height $\zeta = z/z_0$, $z_0 = 0.4$ cm.

ζ	Neutral profile	Observation number 7	16	17	23	24	25	29	31	33	35	40	41	42	43
500	15.5	13.8	14.1	14.2	13.7	14.1	13.7	17.9	14.3	14.6	14.0	19.4	22.0	24.0	20.0
375	14.8	13.3	13.4	13.7	13.3	13.8	13.2	16.3	13.8	14.0	13.6	17.5	20.0	21.4	18.8
250	13.8	12.7	13.0	12.9	12.9	13.3	12.9	14.6	13.1	13.2	12.8	15.6	16.0	18.0	16.4
188	13.1	12.4	12.5	13.0	12.4	12.7	12.3	13.7	12.5	12.6	12.0	15.3	15.6	16.5	14.9
125	12.1	11.5	11.7	11.9	11.8	12.3	11.4	12.3	11.8	11.8	11.6	12.5	12.8	13.6	13.3
94	11.3	10.7	11.2	11.4	11.3	11.4	11.2	11.6	11.2	11.3	11.2	12.0	11.5	12.5	12.4
62.5	10.4	10.0	10.0	10.3	10.1	10.5	10.1	10.5	10.2^{5}	10.1	10.4^{5}	10.9	10.3	11.1	10.6
37.5	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1*
u_*		17.5	12.9	10.7	15.6	13.1	10.8	13.3	26.1	14.7	11.2	6.5	3.2	4.5	3.3
10^4S_n		25	20	20	30	25	20	−15	15	10	20	−18	−30	−60	−35

* Value 9.1 follows from $U = k^{-1} \ln \zeta$, with $\zeta = 37.5$ and $k = 0.40$.

Lettau (1952), too, holds the view that the mixing length must not be used. However, it can easily be shown that Lettau's theory nevertheless uses this concept, which also follows from the fact that it is possible to formulate his assumptions in the form given in table 1.

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REFERENCES

Batchelor, G. K., 1950: The application of the similarity theory of turbulence to atmospheric diffusion. *Quart. J. r. meteor. Soc.*, **76**, 133–146.

Businger, J. A., 1954: Some aspects of the influence of the earth's surface on the atmosphere. *Meded. Verh. k. Ned. meteor. Inst.*, No. 61, 78 pp.

Ertel, H., 1942: Der vertikale Turbulenz-Wärmestrom in der Atmosphäre. *Meteor. Z.*, **59**, 250–253.

——, 1944: Über die Richtung der troposphärischen Turbulenz-Wärmestrom. *Meteor. Z.*, **61**, 8–11.

Hamel, G., 1943: Streifenmethode und Ähnlichkeitsbetrachtungen zur turbulente Bewegung. *Abhandl. Preus. Akad. Wiss., Math.-naturwiss. Klasse*, No. 8, 24 pp.

Held, E. F. M. van der, 1947: *Gelijkvormigheidsleer*. Utrecht, Standaard Dictaat, 50 pp.

Inoue, E., 1952: Some remarks on Lettau's new theories of the atmospheric surface layer. *J. Meteor.*, 9, 224–226.

Johnson, N. K., and G. S. P. Heywood, 1938: An investigation of the lapse rate of temperature in the lowest hundred metres of the atmosphere. *Geophys. Mem.*, No. 77, 5–49.

Kármán, Th. von, 1930: Mechanische Ähnlichkeit und Turbulenz. *Nachr. Ges. Wiss. Göttingen, Math.-phys. Klasse*, No. 58, 33 pp.

Lettau, H., 1949: Isotropic and non-isotropic turbulence in the atmospheric surface layer. *Geophys. Res. Pap.*, No. 1, 13–84.

——, 1952: Reply. *J. Meteor.*, 9, 225–226.

Panofsky, H. A., 1952: A comparison of different types of stress measurement. *Quart. J. r. meteor. Soc.*, **78**, 414–441.

Pasquill, F., 1949: Eddy diffusion of water vapour and heat near the ground. *Proc. roy. Soc., London*, A, **198**, 116–140.

Prandtl, L., 1932: Meteorologische Anwendung der Strömmigslehre. *Beitr. Physik. fr. Atmos.*, 19, 188–202.

Priestley, C. H. B., and W. S. Swinbank, 1947: Vertical transport of heat by turbulence in the atmosphere. *Proc. roy. Soc., London*, A, **189**, 543–561.

Priestley, C. H. B., and P. A. Sheppard, 1952: Turbulence and transfer processes in the atmosphere. *Quart. J. r. meteor. Soc.*, **78**, 488–529.

Richardson, L. F., 1920: The supply of energy from and to atmospheric eddies. *Proc. roy. Soc., London*, A, **97**, 354–373.

Rider, N. E., 1954: Eddy diffusion of momentum, water vapour and heat near the ground. *Phil. Trans. roy. Soc. London*, A, **246**, 481–501.

——, and G. D. Robinson, 1951: A study of the transfer of heat and water vapour above a surface of short grass. *Quart. J. r. meteor. Soc.*, **77**, 375–401.

Rossby, C.-G., and R. B. Montgomery, 1935: The layer of frictional influence in wind and ocean currents. *Pap. phys. Ocean. Meteor., Mass. Inst. Tech. and Woods Hole ocean. Instn.*, **3**, No. 3, 101 pp.

Swinbank, W. C., 1951: The measurement of vertical transfer of heat and water vapour by eddies in the lower atmosphere. *J. Meteor.*, **8**, 135–148.

Taylor, G. I., 1931: Effect of variation in density on the stability of superposed streams of fluid. *Proc. roy. Soc., London*, A, **132**, 354–373.