An Extension to the Wide Swath Ocean Altimeter Concept

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***Abstract*—This paper presents a modification to the Wide Swath Ocean Altimeter allowing it not only to measure sea-surface currents directly, but also providing the possibility to correct for both baseline length and attitude errors.**

Keywords – Mesoscale Ocean Altimetry; Baseline Correction

I. INTRODUCTION

Conventional spaceborne radar altimeters have a relatively small (pulse limited) footprint of typically only a few kilometres. This means that they sample only a fraction of the sea surface each orbit and that there are always considerable gaps between adjacent passes (already at temperate latitudes the gaps between passes are already significantly greater than the footprint). As a result many assumptions and approximations must be made in order to derive ocean flow models and to resolve eddies.

The Wide Swath Ocean Altimeter concept was first proposed by the John Hopkins University and is an attempt to cover these shortcomings of the conventional altimeter by providing a swath of ~200km using two interferometric near-nadir looking antennas. These are separated by a fixed baseline in the across track direction. Each antenna is capable of covering a 100km wide, swath close to, but on either side of nadir, as pictured in figure 2. Through across-track interferometry local sea-surface slope can be determined which provides useful information on ocean currents allowing better modelling of global thermo-haline circulation as well as local phenomena leading to better inputs to climatological study.

However, neither conventional altimeters nor even WSOA are useful for studying coastal regions due to the high spatial and temporal variability of forcing factors. Wavemill was first conceived in an attempt to resolve this problem and is discussed in the following sections.

II. WAVEMILL CONCEPT

A simple modification to the WSOA concept might be to separate the two antennas not only in the across track direction, but also along track. In this way it would be possible to perform along-track interferometry potentially allowing the radial velocity vector of surface ocean currents to be determined.

Unfortunately, due to the extremely low incidence angle (of the order of 1-2º) the major component of the surface velocity vector thus derived will be in the vertical direction. While it is possible that this might have some application in terms of understanding wave dynamics the real goal of ocean ATI is to determine surface currents. From the literature e.g. [4], it seems that an incidence angle of 30-45º is optimal for this application. However, it is desirable to keep the swath(s) of the instrument close to nadir in order that the resulting wide-swath altimetry measurements can be reliably 'tied' to those of the conventional pulse-limited altimeter, which is nadir-looking. So pointing the antenna beam out to 30º (perpendicularly to the along-track direction) is not an option. Instead, one way to solve this problem might be to squint the beam in azimuth to obtain a 30º incidence angle at the near edge of the swath.

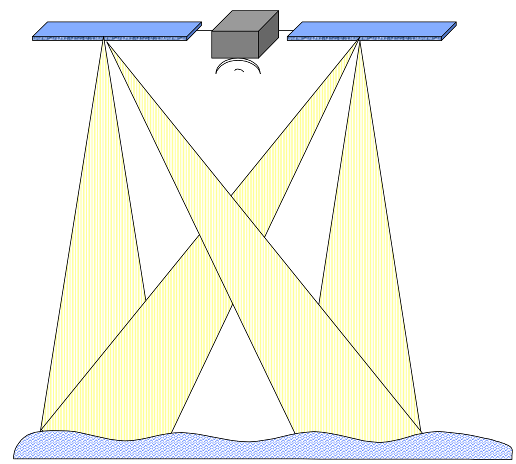

Figure 1. Wide Swath Ocean Altimeter - Standard Configuration

Assuming a satellite at 500km altitude, then this could be achieved by squinting the beam forward. If the near edge of the swath were taken to be 20km from nadir then this would put the imaged patch of ocean 265km ahead of the satellite.

Without squinting the beam, ATI results in a radial velocity measurement for surface currents i.e. perpendicular to the flight direction. However, since ATI measures the velocity of ocean currents in the direction of the line of sight of the radar this means that such a squinted beam system would actually provide the velocity vector ($\sin^{-1}(20/265)$) = 4.3º off azimuth. This is interesting because if a second similarly squinted beam were also pointed aft of the satellite then the two interferometric measures of surface currents would have a separation angle of 171.4º between them meaning that there would be the possibility of deriving a 2-D surface current image from the measurements.

However, what is clear is that matching the 100km broadside swath width, this configuration results in an impractical 1334km beam width. Also, it will be appreciated that the (nearly) broadside current vector is being poorly sampled and therefore susceptible to error.

In order to resolve these problems it is necessary to abandon the WSOA configuration of beams in favour of the full "Wavemill" solution. In this arrangement the two antennas (still mounted with both an along and across track baseline) produce four beams each to allow for both along and across track interferometry forward and aft of the satellite. These beams are squinted by 45º fore and aft so that they sample the surface currents orthogonally allowing generation of two 2-D surface current images (either side of the satellite) while keeping the errors to a minimum. The near edge of each swath is 30º for optimal ATI [4]. The full *Wavemill* arrangement is shown in figure 2.

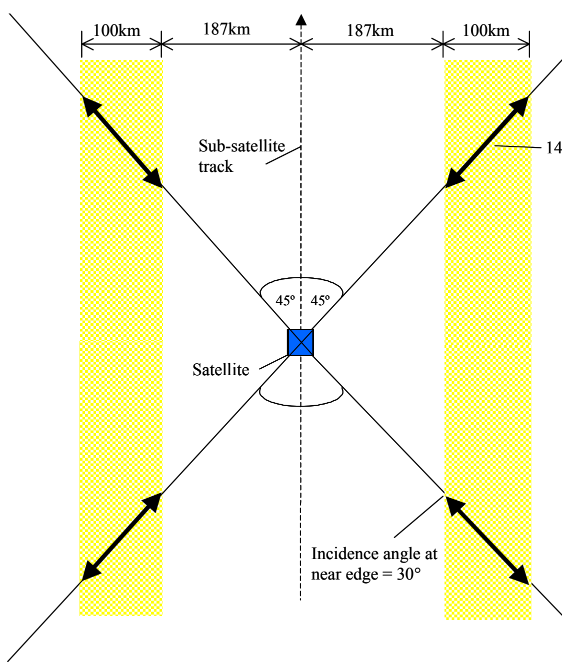

Figure 2. Full "*Wavemill*" swath configuration

The result of this is that instead of just having the surface topography available and using it to make assumptions based on models about what ocean currents will look like under such circumstances, there will also be a *direct* measurement of ocean current to assist in solving the problem. This will provide valuable additional information for scientists in determining the dynamics of the world's oceans and, in particular, the coastal regions. In addition, the offset antenna configuration may be useful in providing an extra input into the attitude calibration of the instrument.

III. Ocean Current Measurement

A. Along-Track Interferometry (ATI)

A simple ATI system has two side-looking coherent radars separated in the along-track direction by a baseline, *B*. After processing and performing spatial co-registration, the image produced by the fore antenna $A_a(t+\tau)$ lags that of the aft antenna $A_f(t)$ by the time taken for the spacecraft to move the distance B such that $\tau = B/v_s$ where v_s is the velocity of the spacecraft. It is clear that the two images of the same scene taken at slightly different times but from the same point in space will contain differences in the phase observed due to the mean Doppler velocity, v_0, resulting from any surface scatterers which are moving radially with respect to the spacecraft. This Doppler velocity can be determined from the cross-correlation:

$$v_0 = \frac{\lambda}{2}\frac{\arg\langle A_1(t)A_2^*(t+\tau)\rangle}{2\pi\tau} \tag{1}$$

where λ is the wavelength of the radar. The unambiguous range of values for the phase is $[-\pi, \pi]$ corresponding to an unambiguous Doppler velocity range of $[-\lambda/4\tau, \lambda/4\tau]$. Here, it is assumed that one of the antennas both transmits and receives while the other receives only. For a system in which both antennas alternatively transmit and receive the baseline separation is doubled.

B. Along-Track Baseline

The length of the along-track baseline determines to a large extent the ability of the interferometer to discriminate surface velocities. Too small a baseline results in low temporal sampling and hence noisy Doppler estimates. Too large a baseline and the time between observations is also too long resulting in temporal decorrelation. In [1] it is suggested that for low to moderate resolutions (around 30m) and wind speeds, the coherence time, τ_s, of scattering from the ocean surface can be approximated by $3\lambda/u$. This is consistent with a reported coherence time over the ocean of 7ms at 14GHz. The effect of degrading the resolution is for the coherence to reach its asymptotic value sooner.

In order to determine a suitable along-track baseline, it is useful to look at the results produced by the hybrid along- and across-track airborne SAR described by [3] since they claim to have recorded ocean currents in the order of tens of centimetres per second. This was achieved with a pair of SAR antennas separated in the along-track direction by only 3.4cm. Scaling this for a Ku-band *Wavemill* instrument at 500km altitude (theirs was X-band at 3200m) would suggest that reasonable results could be achievable with an along-track baseline of 3.7m.

C. Sea Surface Height Measurement

The primary purpose of a WSOA-type mission is to measure sea-surface height by means of across-track interferometry. *Wavemill* does the same but with a modified geometry.

1) Across-Track Interferometry (XTI)

The principle of XTI for determining topography is well covered by [3] and so is not repeated here, fig. 4 shows the relevant viewing geometry.

2) Elevation Accuracy

The elevation accuracy required for most applications of ocean altimetry data is of the order of 2cm hence it is very important to keep the sources of error to an absolute minimum. From the equations above five sources of error can be deduced due to uncertainties in the:

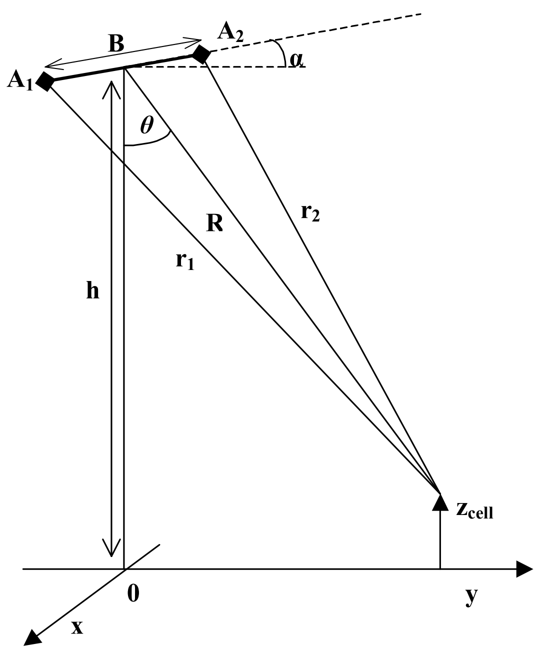

Figure 3. Geometry of Across-Track Interferometry

1. Satellite altitude h => $\delta z = \delta h$
2. Baseline roll angle, α => $\delta z = R\sin\theta.\delta\alpha$
3. Baseline length, B => $\delta z = -R\sin\theta\tan(\theta-\alpha)\dfrac{\delta B}{B}$
4. Random and systematic errors in the measurement of the interferometric phase, φ => $$\delta z = R\frac{\lambda\sin\theta}{2\pi B\cos(\theta-\alpha)}\delta\varphi$$
5. Translation of radar timing measurement to geometric range, R => $\delta z = -\cos\theta\delta R$

[4] goes on to simplify these errors assuming angles θ and φ are small. In the case of WSOA, this is true, however for *Wavemill* while φ is still small, θ is around 30º and the small angle approximation does not hold so most terms become more significant than in the WSOA case. The relevant importance of each source of error is discussed below:

Platform height errors are dominated by orbital error. Dedicated earth observation missions currently achieve platform height accuracies of a few centimetres.

Range errors result principally from timing errors on board i.e. local oscillator jitter. Modern spaceborne LOs have jitters better than a few picoseconds and hence this error is negligible.

Precise knowledge of the baseline roll is critical for a WSOA-type instrument; from the equation given it can be seen that just 1 arc second of baseline roll results in an error of 48cm at an across-track position of 100km. For comparison, a typical value of roll error obtained by the AOCS for ENVISAT is a remarkably good 54 arc seconds.

The elevation error resulting from a change in baseline length is relatively small for WSOA. Even so, 0.1mm error in baseline length knowledge results in 7.8cm elevation error.

Non-random differential phase must be known to an accuracy of better than 0.1º in order to achieve accuracy in the order of centimetres.

Points 1 and 5 hold the same for *Wavemill* but for points 2-4 there is a significant difference:

Due to the fact that the look angle, θ, is much larger in this configuration, *Wavemill* is more than three times as sensitive to baseline roll error than WSOA. Just one arc second of error corresponds to 1.73m of height error at far swath.

Baseline length error is much more of a problem for *Wavemill*. An error of only 0.1mm corresponds to 2.35m of error at far swath.

Wavemill is also about 4.5 times as sensitive as WSOA to non-random differential phase errors requiring knowledge of better than 0.02º in order to achieve accuracy in the order of centimetres. This is still a very much smaller error than 2 and 3. A possible solution to these errors is elaborated in the next section.

D. Baseline Attitude Determination

Roll – With its four beams for each of the two antennas, it is possible to produce two phase-difference diagrams (interferograms) on both the left- and the right-hand sides of the spacecraft (i.e. four interferograms in total).

Assuming that the average height of the ocean (averaged over a region of, say, 1x1km) does not change in the time between imaging with the fore beams and the aft beams, then when the difference is made between the fore and aft interferograms (differential interferometry), any resulting phase error (corresponds to height error) will be due to spacecraft attitude error or roll (note that this only works for asymmetrically arranged antennas i.e. with both an across- and along-track baseline component).

In addition, an error found between, say, the two left-hand interferograms will be equal and opposite in sign to the error found between the two right-hand interferograms and as such constitutes additional confirmation that the error is due to roll (see figure 4). This process can be expressed mathematically as so. If:

$\phi_{FLL}(x_i, y_j)$ and $\phi_{FLR}(x_i, y_j)$ express the phase of the pixel (x_i, y_j) seen by the fore beams of the left- and right-hand antennas respectively for the swath on the *left-hand side* of the sub-satellite track and $\phi_{FRL}(x_i^/, y_j^/)$ and $\phi_{FRR}(x_i^/, y_j^/)$ are the equivalent for the *right-hand side*. The equivalents for the aft beams are therefore $\phi_{ALL}(x_i, y_j)$, $\phi_{ALR}(x_i, y_j)$, $\phi_{ARL}(x_i^/, y_j^/)$ and $\phi_{ARR}(x_i^/, y_j^/)$.

The interferogram generated between the left and right fore beams for the left-hand swath is therefore given by:

$$\delta\phi_{FL}(x_i, y_j) = \left|\phi_{FLL}(x_i, y_j) - \phi_{FLR}(x_i, y_j)\right|$$

and is principally determined by the parallel baseline and the local sea surface height.

For the other side (right) and aft beams, three more similar interferograms can be derived:

$$\delta\phi_{FR}(x_i^/, y_j^/) = \left|\phi_{FRL}(x_i^/, y_j^/) - \phi_{FRR}(x_i^/, y_j^/)\right|$$

$$\delta\phi_{AL}(x_i, y_j) = \left|\phi_{ALL}(x_i, y_j) - \phi_{ALR}(x_i, y_j)\right|$$

$$\delta\phi_{AR}(x_i^/, y_j^/) = \left|\phi_{ARL}(x_i^/, y_j^/) - \phi_{ARR}(x_i^/, y_j^/)\right|$$

Taking now the differential interferogram of the two same side interferograms we get:

$$\delta\delta\phi_L(x_i, y_j) = \left|\delta\phi_{FL}(x_i, y_j) - \delta\phi_{AL}(x_i, y_j)\right| \text{ and}$$

$$\delta\delta\phi_R(x_i^/, y_j^/) = \left|\delta\phi_{FR}(x_i^/, y_j^/) - \delta\phi_{AR}(x_i^/, y_j^/)\right|$$

Always assuming that the sea surface height averaged over the pixel has not changed from the time of the forward acquisition to the time of the aft acquisition (differential interferograms are usually used for surface change detection) then these differential interferograms hold only information about any geometric (or parallel baseline) difference between the left and right antennas. To extract this information it is necessary first to determine the equivalent differential interferograms for a reference system with no attitude errors – $\delta\delta\phi_{L\_ref}(x_i, y_j)$ and $\delta\delta\phi_{R\_ref}(x_i^/, y_j^/)$ – and then calculate the differential-differential interferogram between these and the actual measured ones:

$$\delta\delta\delta\phi_{L\_geom.}(x_i, y_j) = \left|\delta\delta\phi_L(x_i, y_j) - \delta\delta\phi_{L\_ref}(x_i, y_j)\right|$$

$$\delta\delta\delta\phi_{R\_geom}(x_i^/, y_j^/) = \left|\delta\delta\phi_R(x_i^/, y_j^/) - \delta\delta\phi_{R\_ref}(x_i^/, y_j^/)\right|$$

Examples of the results of this process are given for both baseline roll and baseline length error in figures 4 and 4.

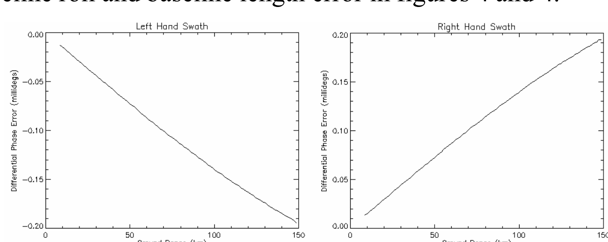

Figure 4. Impact of 0.1 degree of baseline roll

This shows that it is possible to use the fore and aft differential interferograms in order to determine baseline roll error.

Baseline Error – Not only can the Wavemill configuration allow for roll estimation but it can also provide an estimation of the baseline length by comparing the interferograms produced by the fore and aft beams. Figure 5 shows the residual error

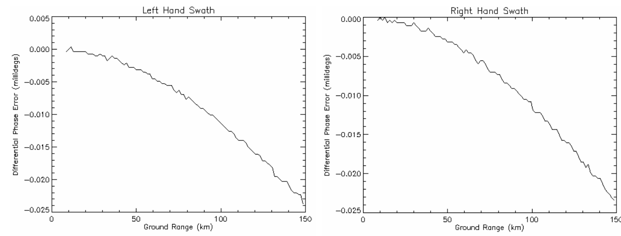

Figure 5. Impact of baseline error of 1cm

The important point to notice here is that this measurement gives the actual error in baseline length from *phase centre to phase centre*, something which is impossible using instrument mounted measuring apparatus (e.g. lasers) or indeed in any other way.

Yaw – The error due to yaw is not considered here since according to [2], the significance of the height error due to yaw is four order of magnitude less than the error due to roll for a WSOA system. To put that into perspective, consider the ERS AMI-SAR's Doppler bandwidth of 1332Hz. The Doppler beamwidth for the same instrument was 0.28^o and the achieved Doppler tracking error was less than 30Hz, which would suggest a yaw pointing knowledge of better than 0.006^o.

IV. Summary

A novel earth observation instrument has been presented which expands on the Wide-Swath Ocean Altimetry concept in order to provide not only accurate altimetry across wide swaths for mesoscale ocean applications, but also surface current measurements for coastal waters. This constitutes a combination of features not previously been considered for a single remote sensing instrument. Moreover, this innovative concept allows for direct knowledge, from the data, of baseline attitude and length (between phase centres) by means of differential interferometry. While these baseline parameters are of paramount significance for interferometric applications, deriving this knowledge is not possible with other current configurations.

As such the Wavemill concept represents an excellent instrument for oceanographers and the general science community involved in climate modelling, pollution monitoring, coastal erosion monitoring etc. and there is also a wider application for instance to shipping and weather forecasting/modelling.

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