Wave Dependent Momentum Fluxes in Operational Oceanography

Göran Broström[1]

Kai Christensen[1], Johannes Röhrs[2], Øyvind Sætra[1], and Jan Erik Weber[3]

[1]*Norwegian Meteorological Office, Oslo, Norway*
[2] *Norwegian Meteorological Office, Bergen, Norway*
[3]*Dep. Of Geophysics, University of Oslo, Norway*

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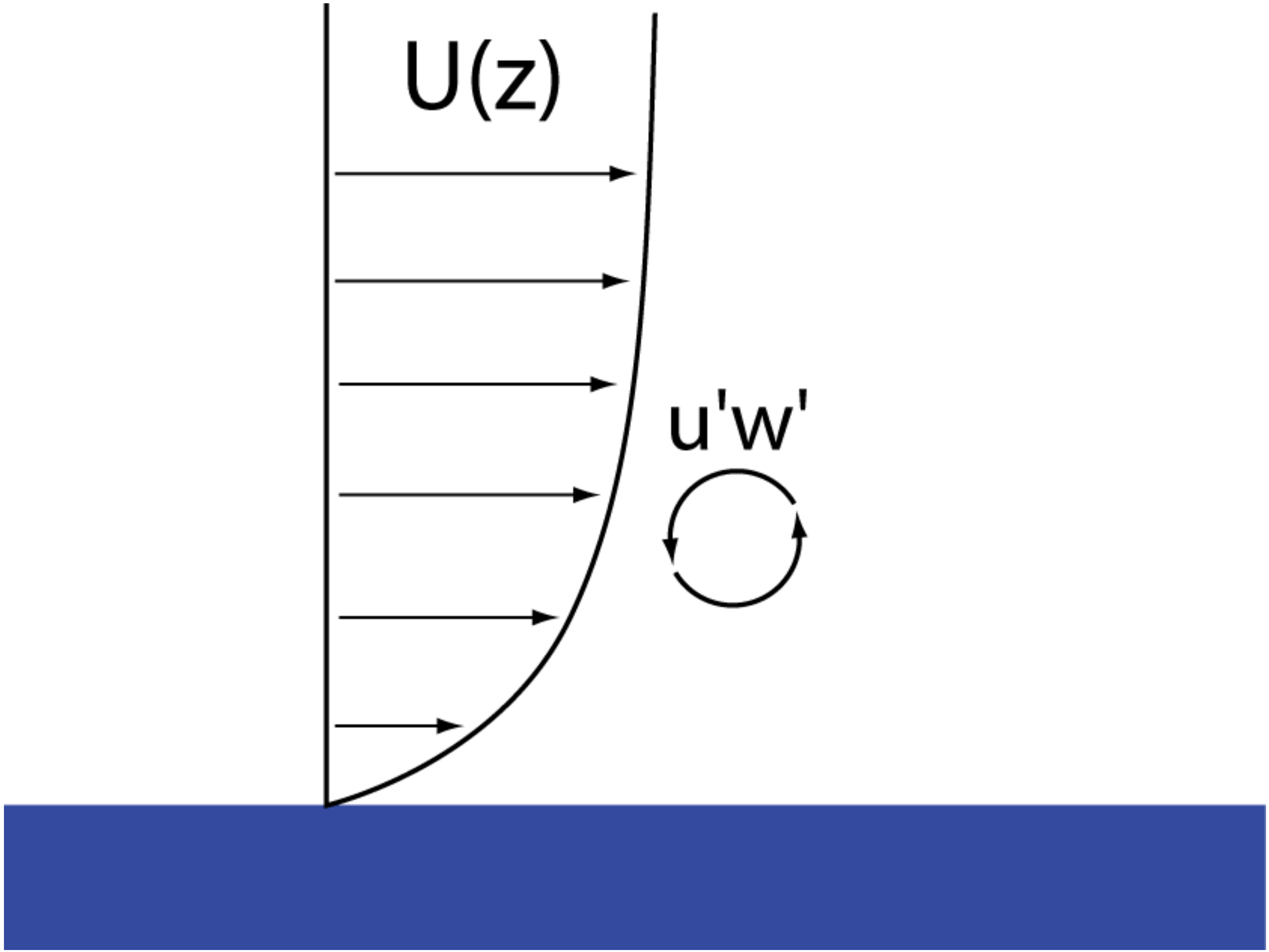
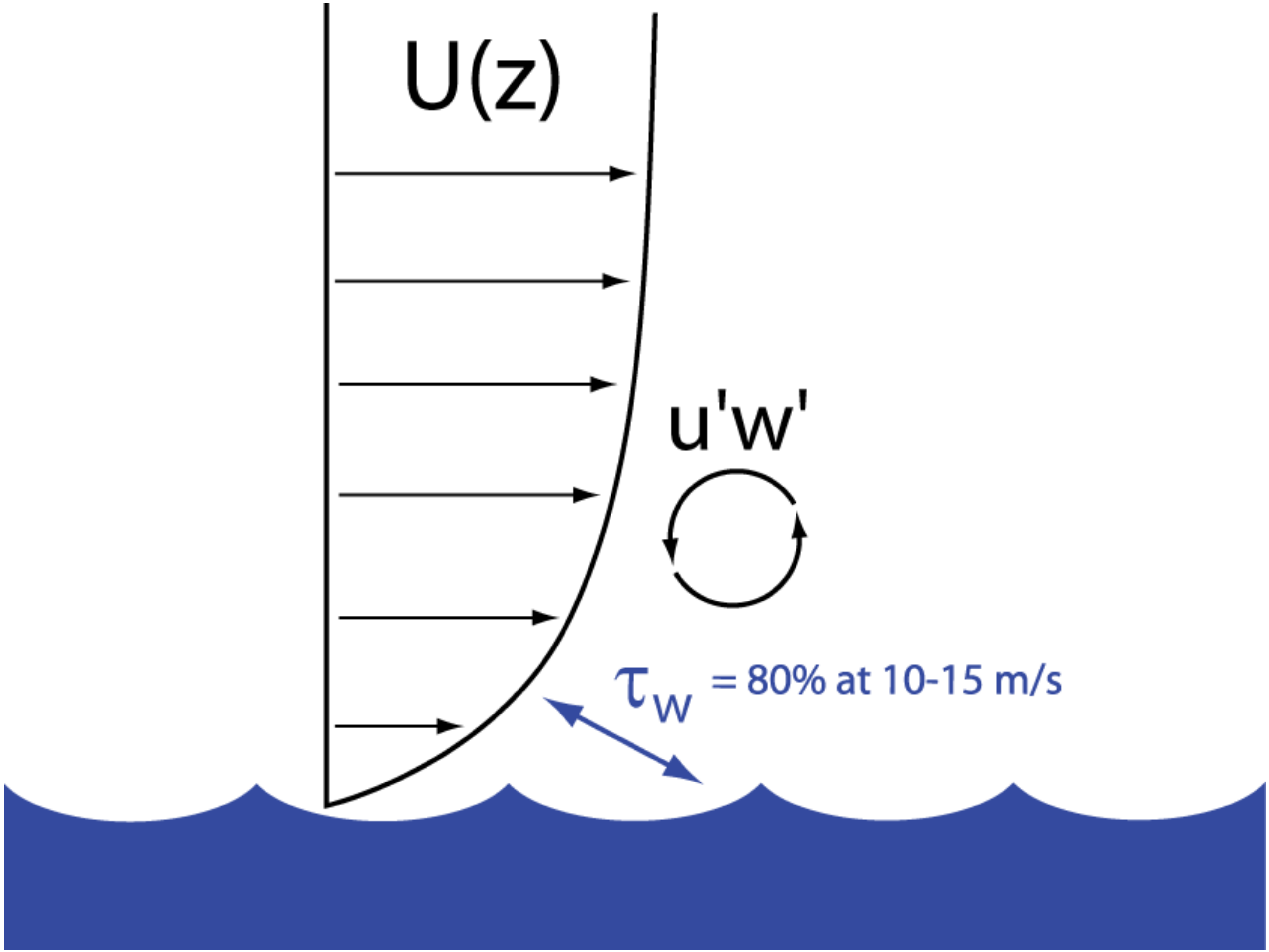
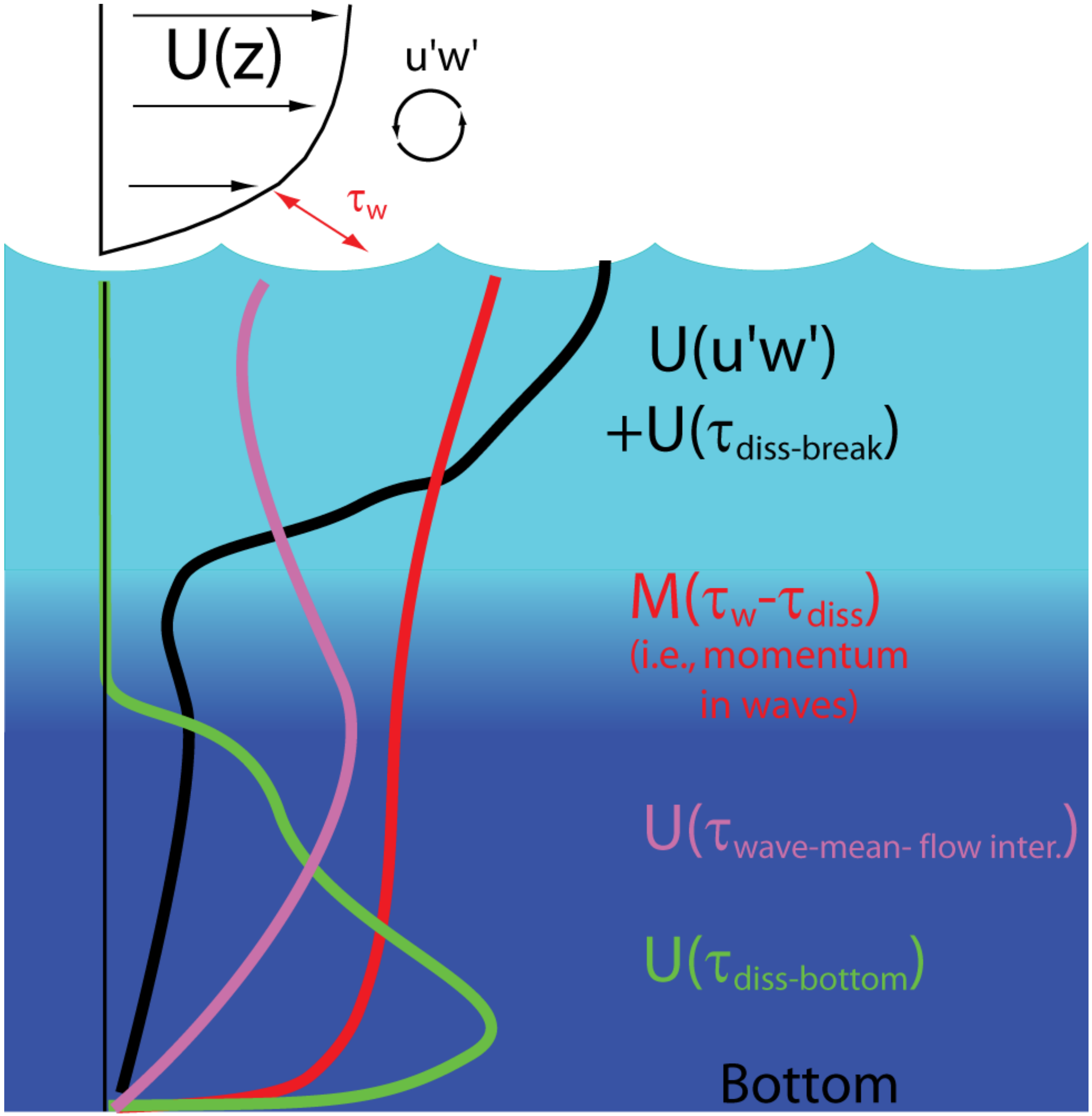
A MyOcean Research and Development Project

- Motivation
- How to include wave forcing in ocean models
- An mid-latitude storm example
- Products that will be made available by the project

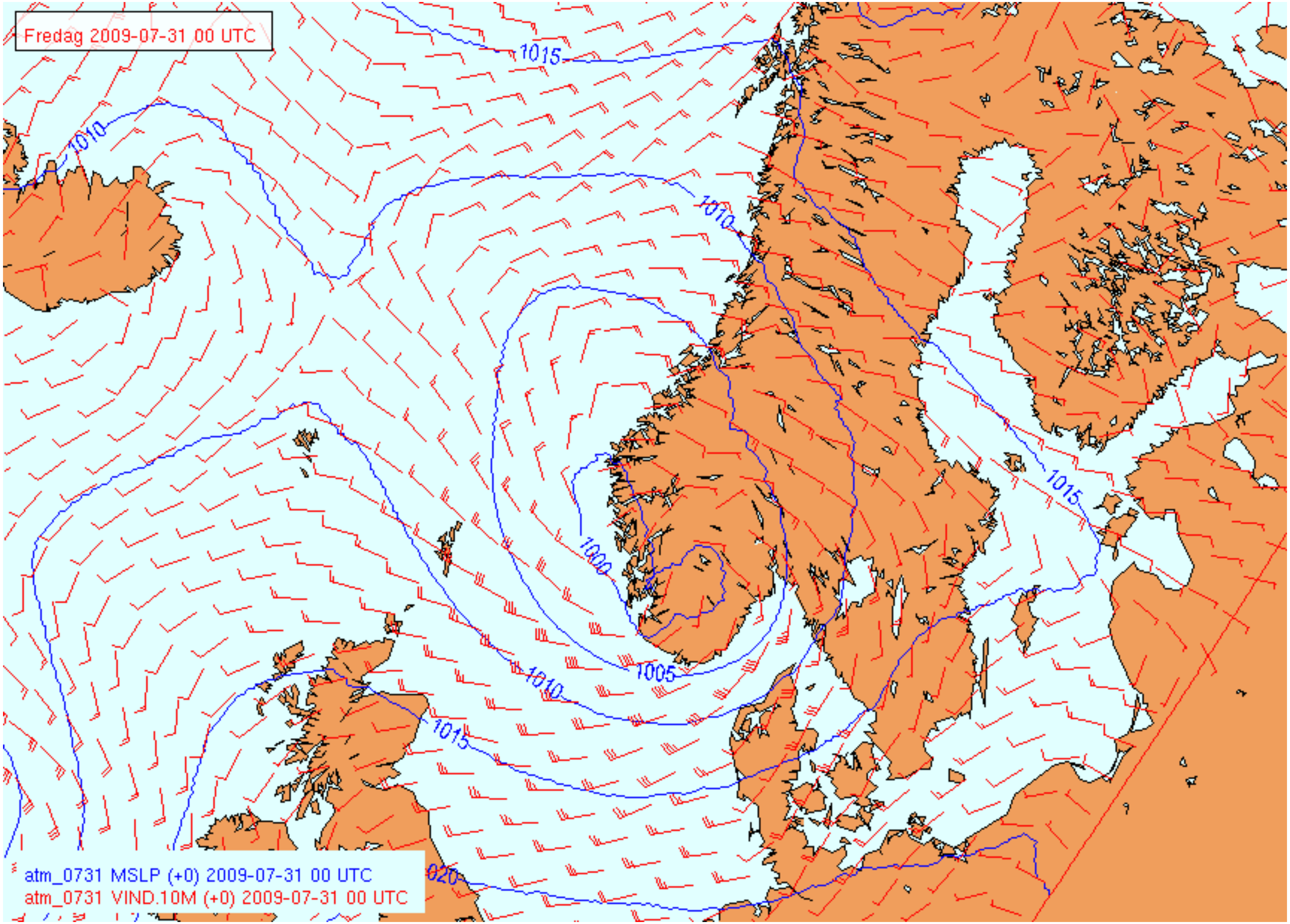

U(z)
u'w'

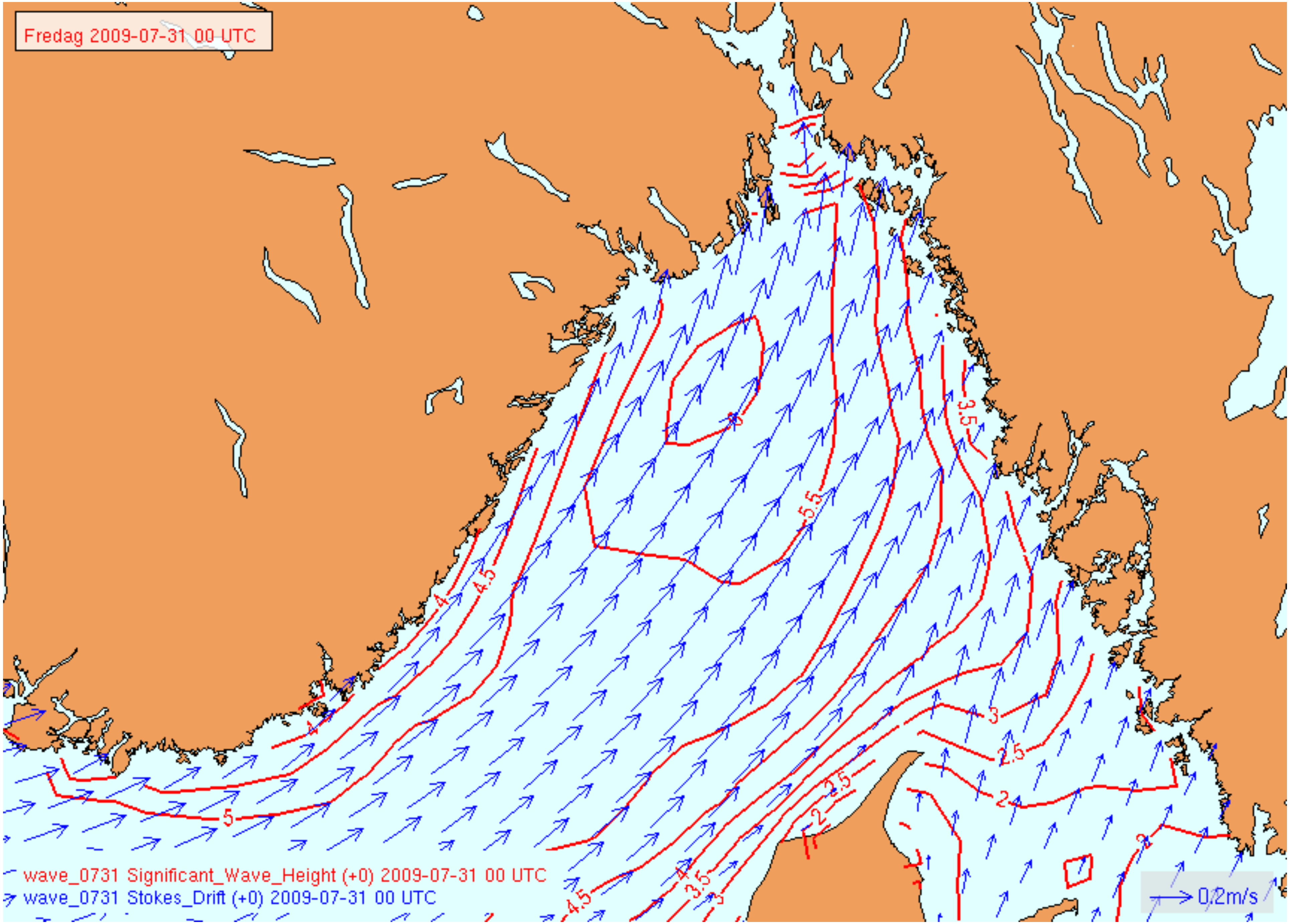

U(z)
u'w'
τ_W = 80% at 10-15 m/s

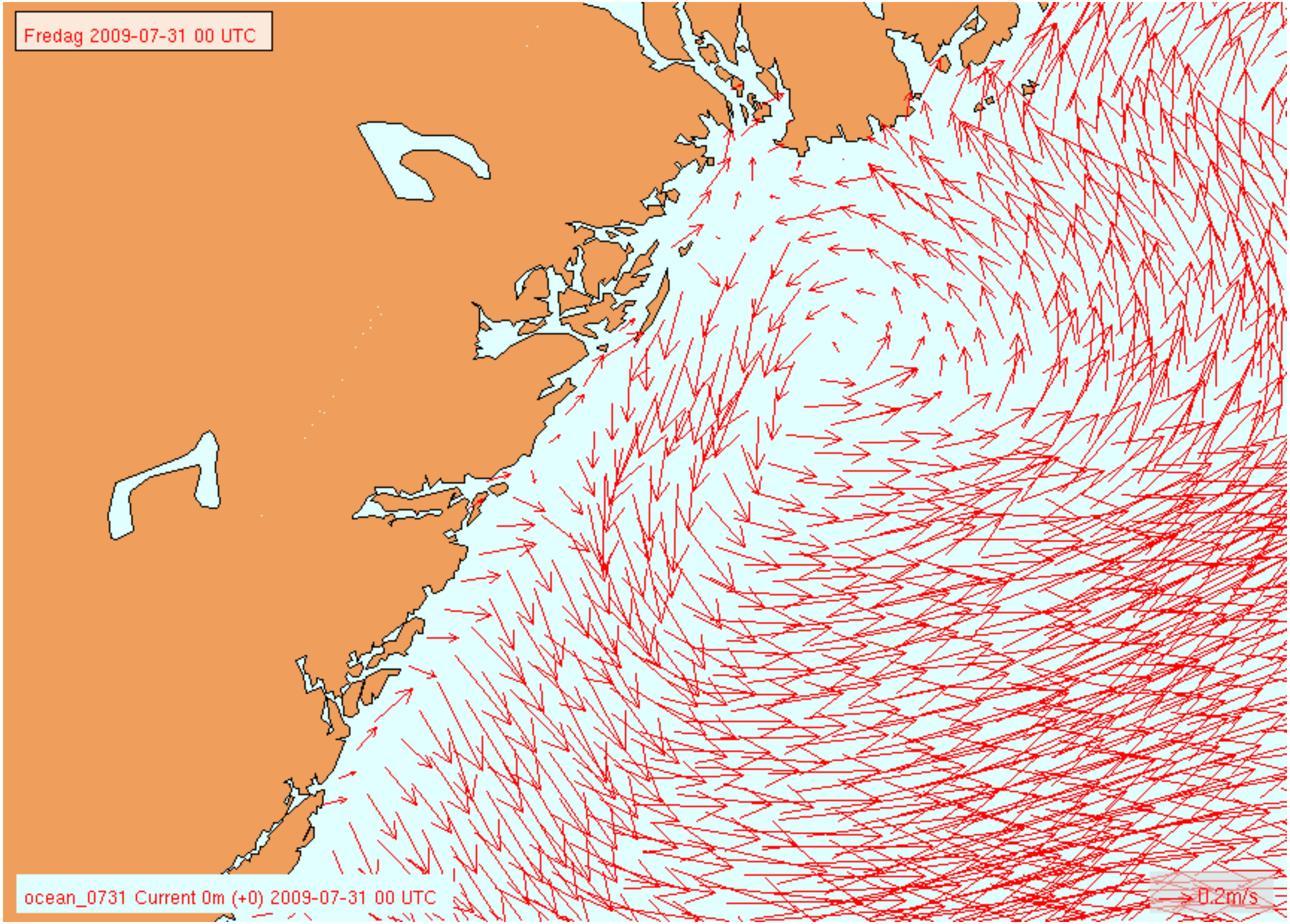


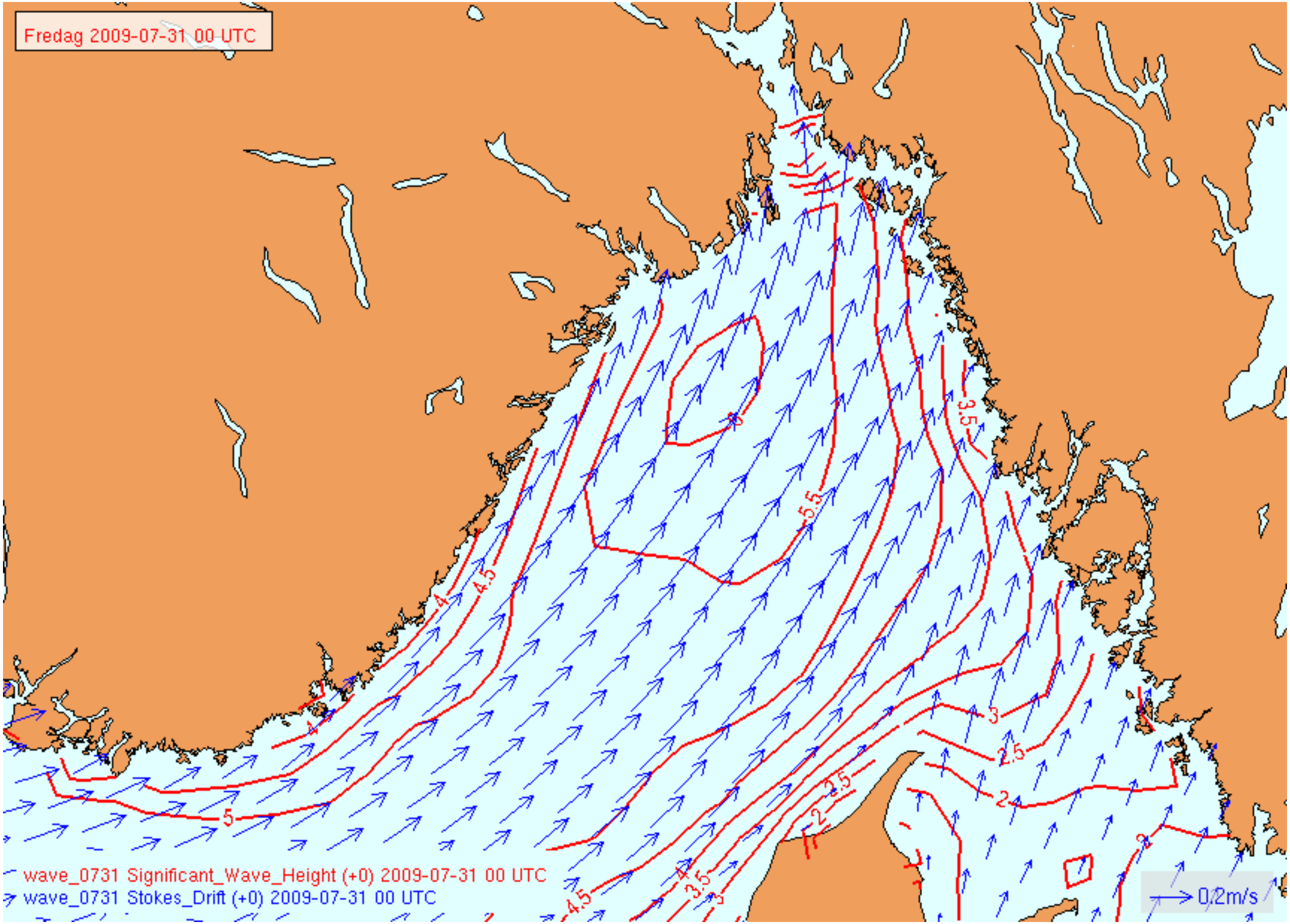
U(z)
u'w'
τw
U(u'w')
+U(τdiss-break)
M(τw-τdiss)
(i.e., momentum
in waves)
U(τwave-mean- flow inter.)
U(τdiss-bottom)
Bottom

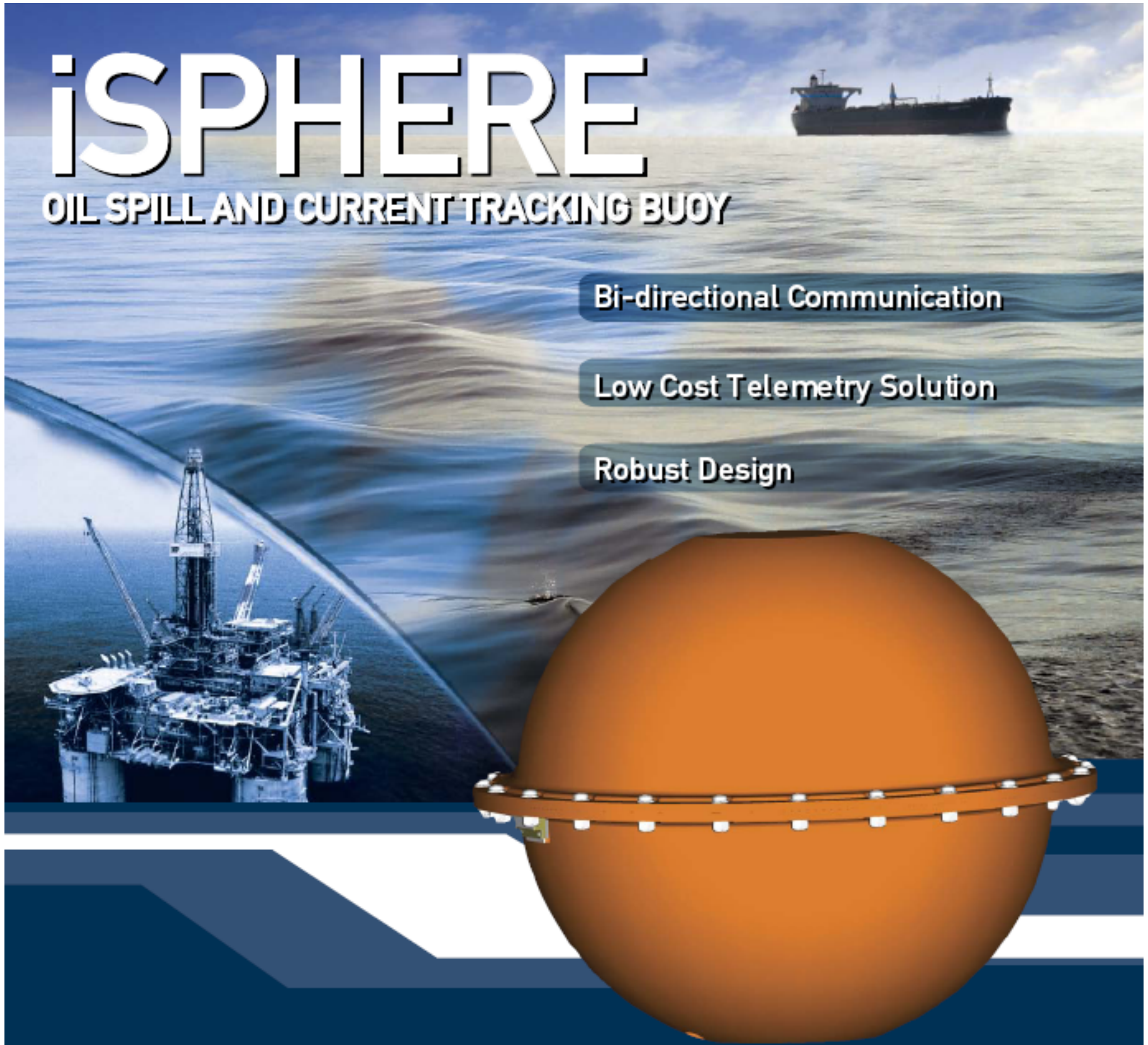

02.08.2009 05:13:41 (+0,0 hrs) Dir=49 Lat=N58 57' 49,53" Lon=E009 43' 12,05" Alt=807ft MSL WGS 1984


Fredag 2009-07-31 00 UTC
1015
1010
1010
1000
1005
1010
1015
1015
020
atm_0731 MSLP (+0) 2009-07-31 00 UTC
atm_0731 VIND.10M (+0) 2009-07-31 00 UTC


Fredag 2009-07-31 00 UTC
wave_0731 Significant_Wave_Height (+0) 2009-07-31 00 UTC
wave_0731 Stokes_Drift (+0) 2009-07-31 00 UTC
0.2m/s


Fredag 2009-07-31 00 UTC
ocean_0731 Current 0m (+0) 2009-07-31 00 UTC
0.2m/s

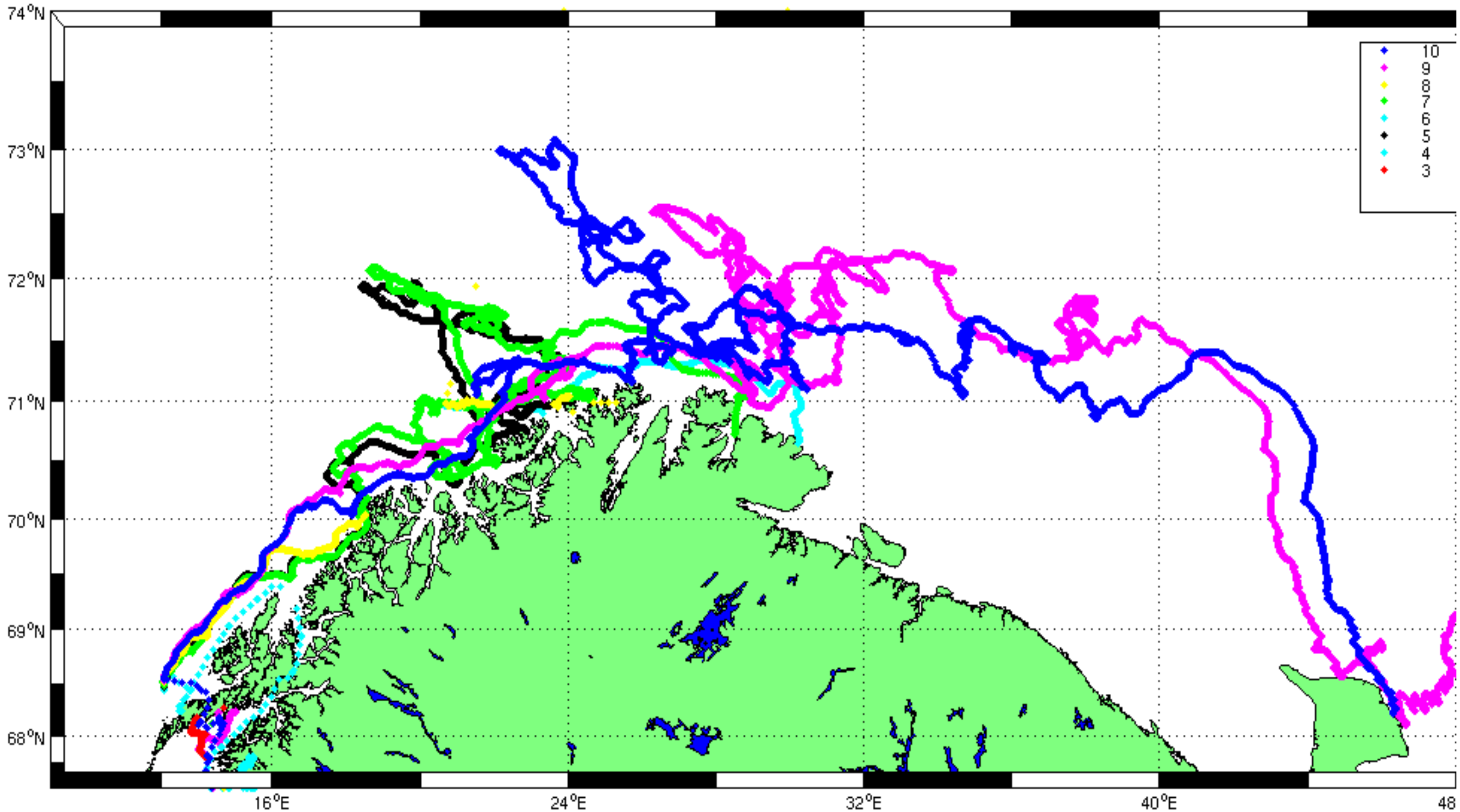

Fredag 2009-07-31 00 UTC
wave_0731 Significant_Wave_Height (+0) 2009-07-31 00 UTC
wave_0731 Stokes_Drift (+0) 2009-07-31 00 UTC
0.2m/s
6
5.5
5
4.5
4
3.5
3
2.5
2



The iSPHERE is an expendable, low cost, bi-directional spherical drifting buoy. The drifter was developed to meet the demanding needs of the offshore oil industry, ocean freight industry and the oceanographic scientific community. The buoy was designed specifically to track and monitor oil spill incidences. The iSPHERE drifter also provides the user with essential real-time sea surface temperature data and GPS positional data.

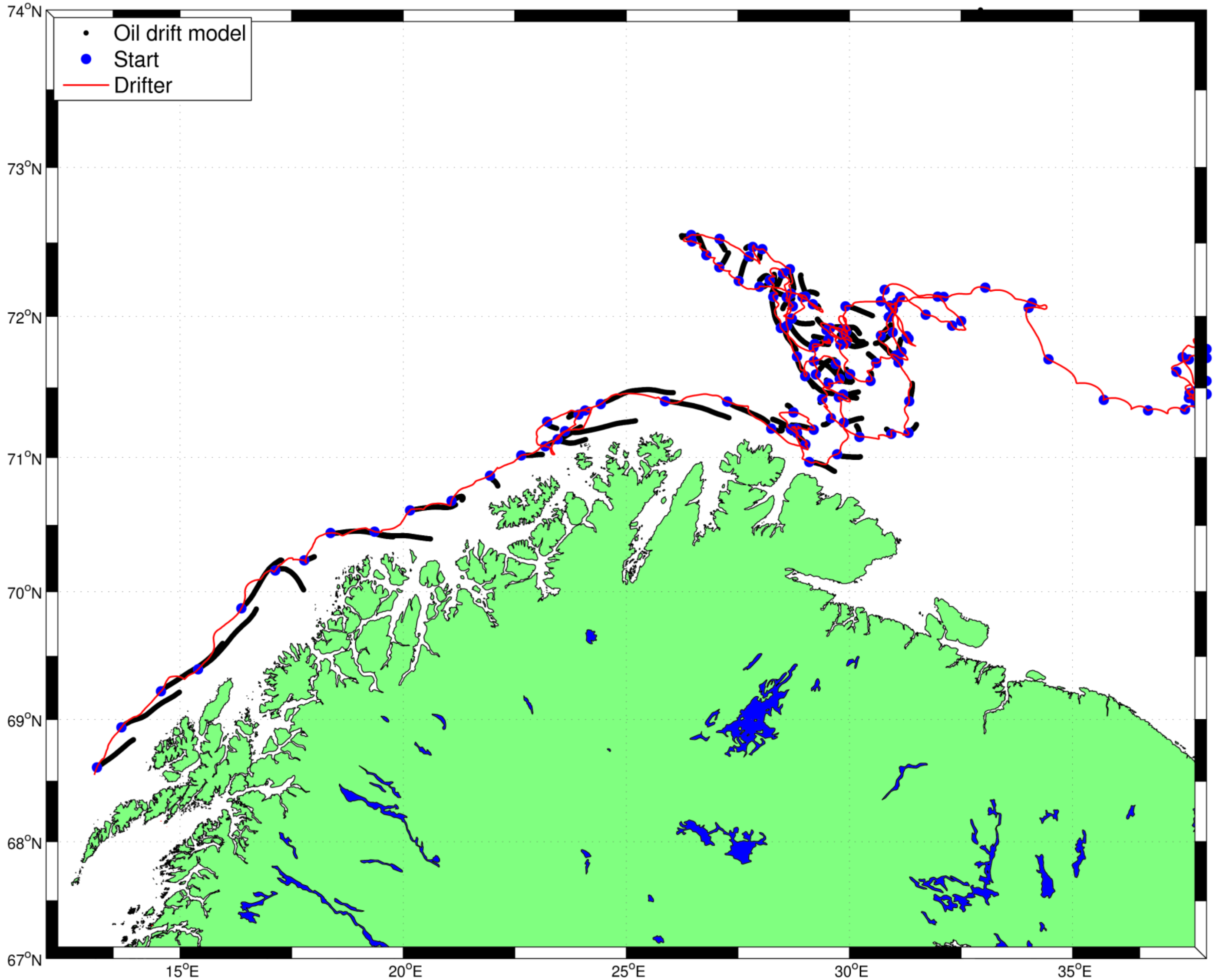
The robust design of the iSPHERE allows the buoy to be deployed effortlessly from a vessel or an oil platform. The standard operating life of the buoy is approximately 180-365* days.

MetOcean
21 Thornhill Drive
Dartmouth, Nova Scotia B3B 1R9
CANADA
Tel: +1 902 468-2505
Fax: +1 902 468-4442
www.metocean.com
sales@metocean.com



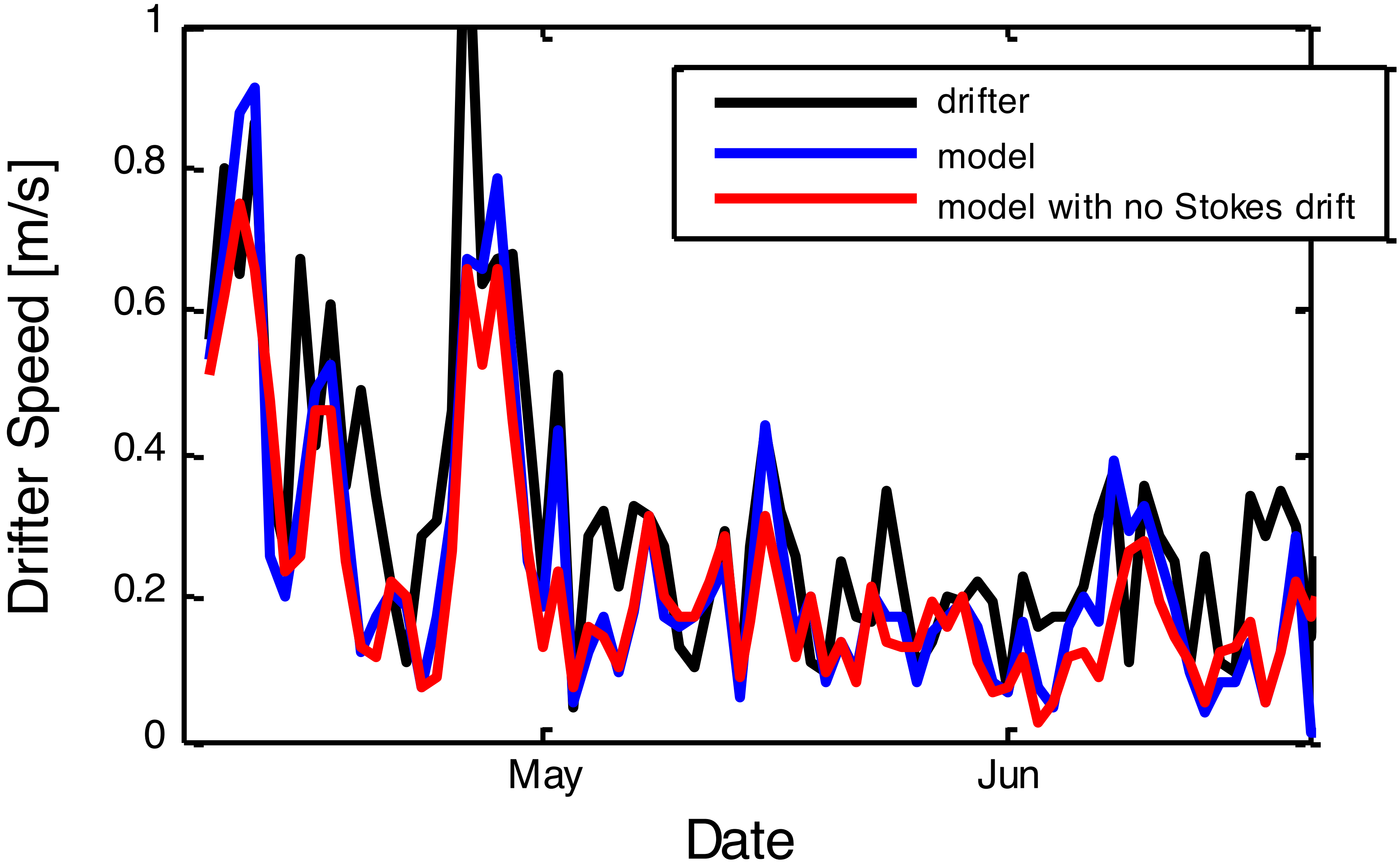
74°N
73°N
72°N
71°N
70°N
69°N
68°N
16°E
24°E
32°E
40°E
48
10
9
8
7
6
5
4
3

Comparing oil drift simulations with drifter trajectories



No Stokes drift

- 11 % reduction in average speed



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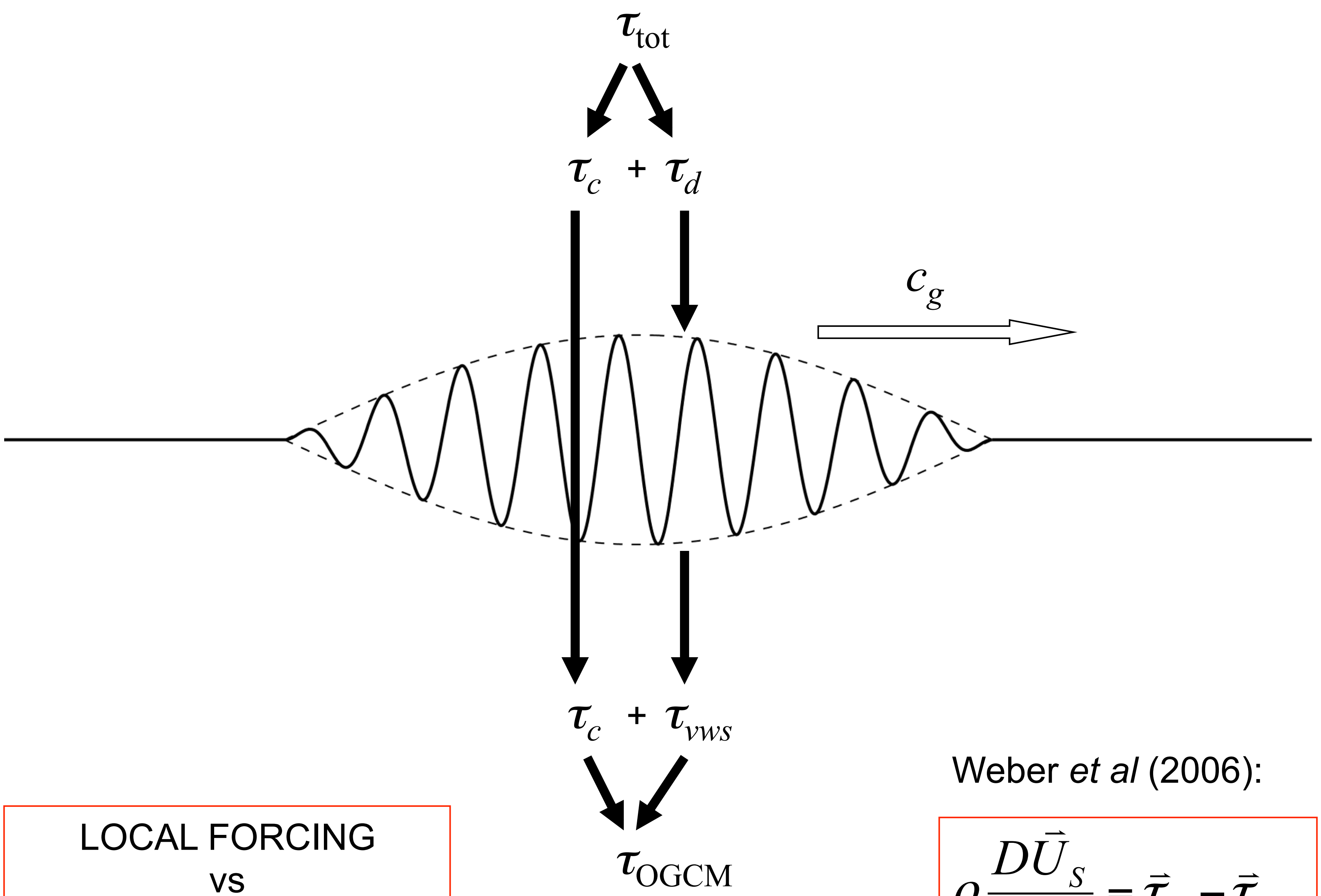
Wave models

$$\frac{\partial}{\partial t}\left(\frac{\mathbf{E}}{\omega}\right)+\nabla\cdot\left\{\left(\mathbf{U}+\mathbf{c}_g\right)\frac{\mathbf{E}}{\omega}\right\}=\mathbf{S}_{in}+\mathbf{S}_{nl}+\mathbf{S}_{diss}$$

S_{in}: input from winds
S_{nl}: non-linear wave wave ineraction
S_{diss}: dissipation by wave breaking or (bottom) friction

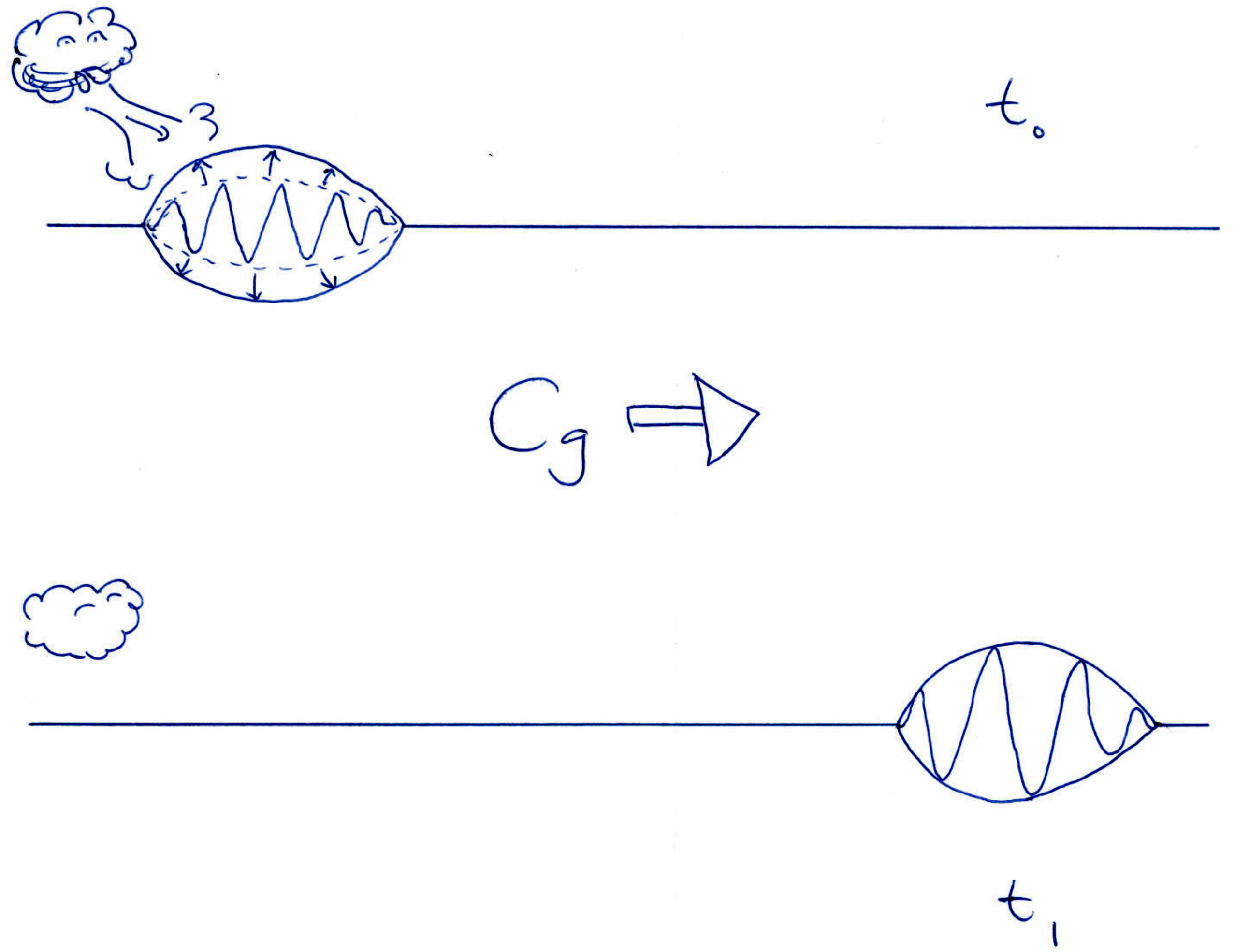
Wave momentum is given by E/c:
The equation for wave momentum is similar to the above equation



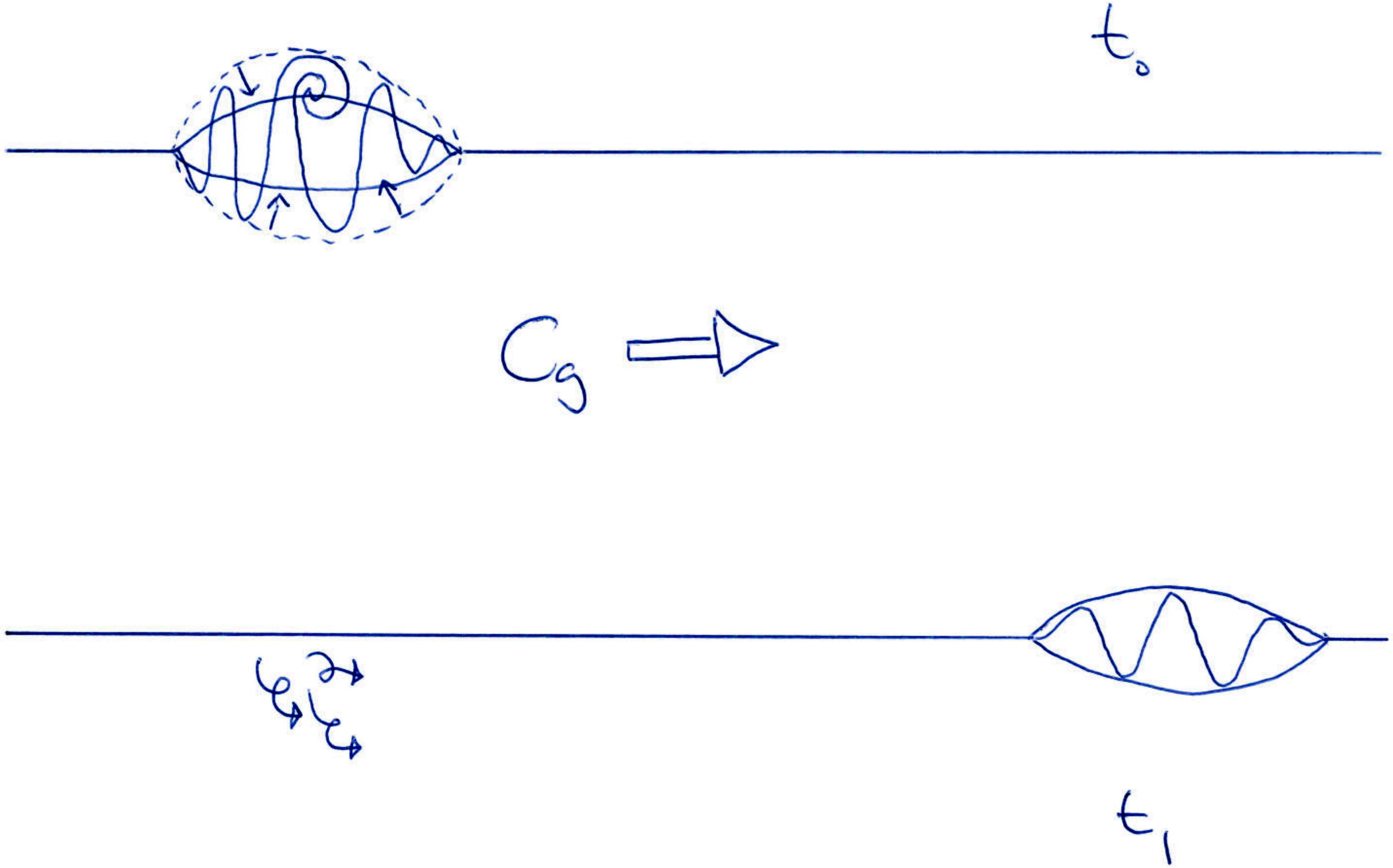
LOCAL FORCING
vs
ADVECTION BY GROUP

Weber *et al* (2006):

$$\rho \frac{D\vec{U}_S}{dt} = \vec{\tau}_d - \vec{\tau}_{vws}.$$



t₀
Cg
t₁



t_0
C_g
t_1

How to include wave in an ocean model?

Divide all fields into a mean field and an oscillating part

$$\mathbf{u} = \mathbf{U} + \tilde{\mathbf{u}}$$

$$p = P + \tilde{p}$$

U: mean field characterized by a long time scale T

$\tilde{u}$: Oscillating wave field characterized by a short time scale $\tilde{t}$

$$\tilde{\eta} = a\cos(kx - \omega t)$$

$$\tilde{u} = a\omega \frac{\cosh(k(H+z))}{\sinh(kH)}\cos(kx - \omega t)$$

$$\tilde{w} = a\omega \frac{\sinh(k(H+z))}{\sinh(kH)}\sin(kx - \omega t)$$

$$\tilde{p} = \frac{a\rho\omega^2}{k}\frac{\cosh(k(H+z))}{\sinh(kH)}\cos(kx - \omega t)$$

Start with Navier Stokes
and continuity eqs.

$$\frac{d\mathbf{u}}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial \mathbf{x}}$$

$$\nabla \cdot \mathbf{u} = 0$$

$$\mathbf{u} = \mathbf{U} + \tilde{\mathbf{u}}$$

$$p = P + \tilde{p}$$

$$\frac{d\left(\mathbf{U} + \tilde{\mathbf{u}}\right)}{dt} = -\frac{1}{\rho}\frac{\partial\left(P + \tilde{p}\right)}{\partial \mathbf{x}}$$

$$\nabla \cdot \left(\mathbf{U} + \tilde{\mathbf{u}}\right) = 0$$

Apply a ”time averaging
operator” { }

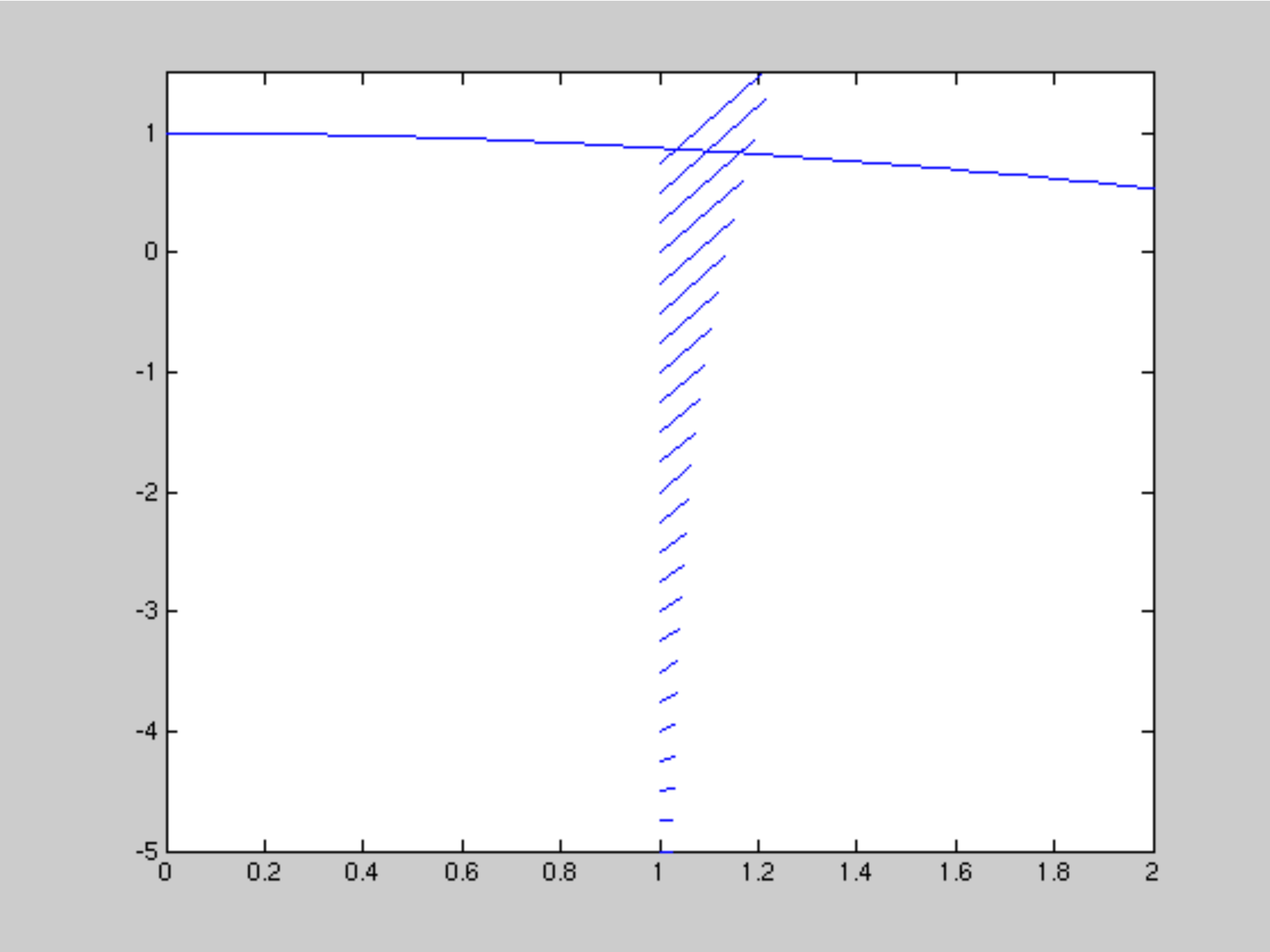
$$\left\{\frac{d\left(\mathbf{U} + \tilde{\mathbf{u}}\right)}{dt}\right\} = -\frac{1}{\rho}\left\{\frac{\partial\left(P + \tilde{p}\right)}{\partial \mathbf{x}}\right\}$$

$$\left\{\nabla \cdot \left(\mathbf{U} + \tilde{\mathbf{u}}\right)\right\} = 0$$

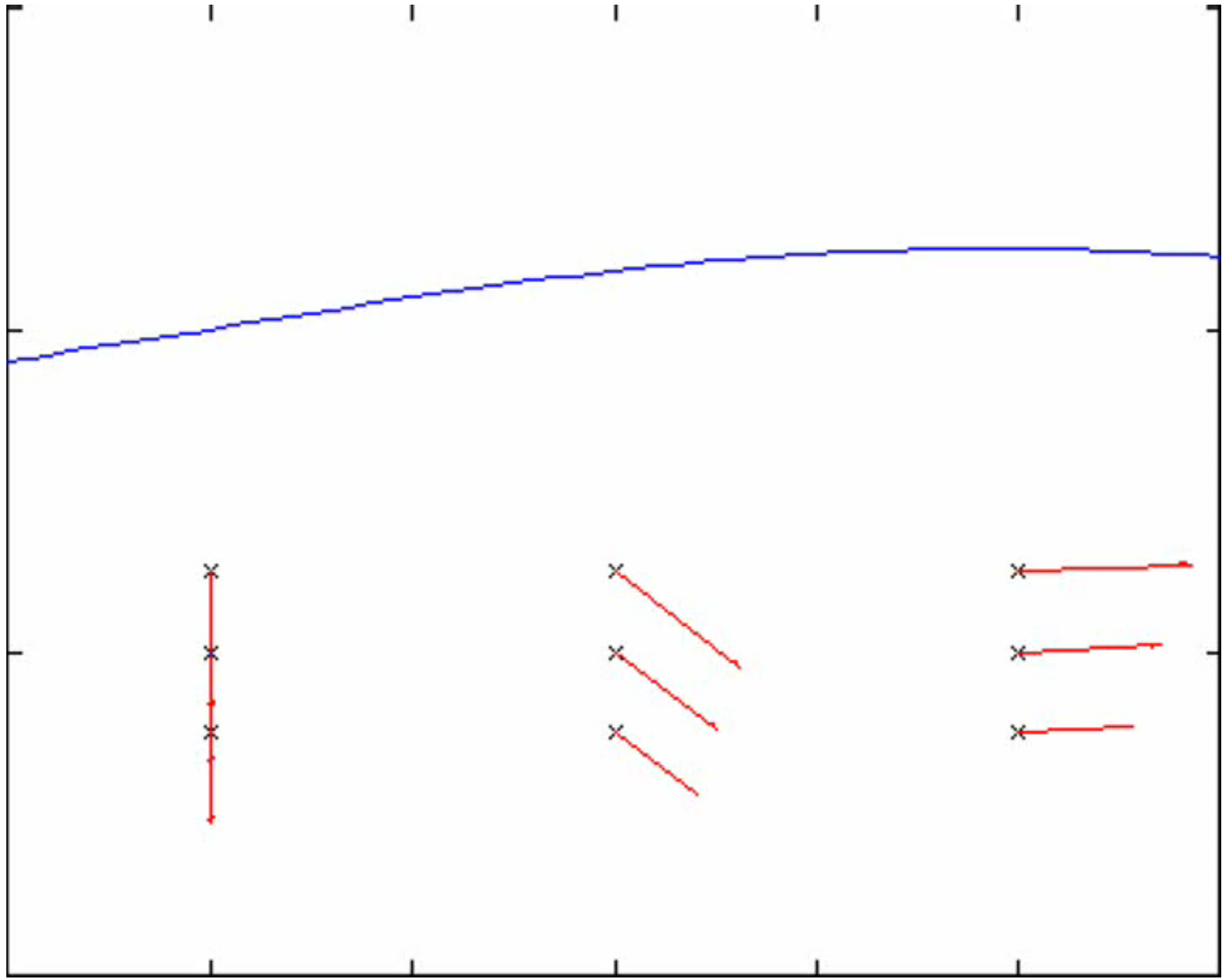
Some algebra gives …

However, let us take a few steps back and see what the problem is.

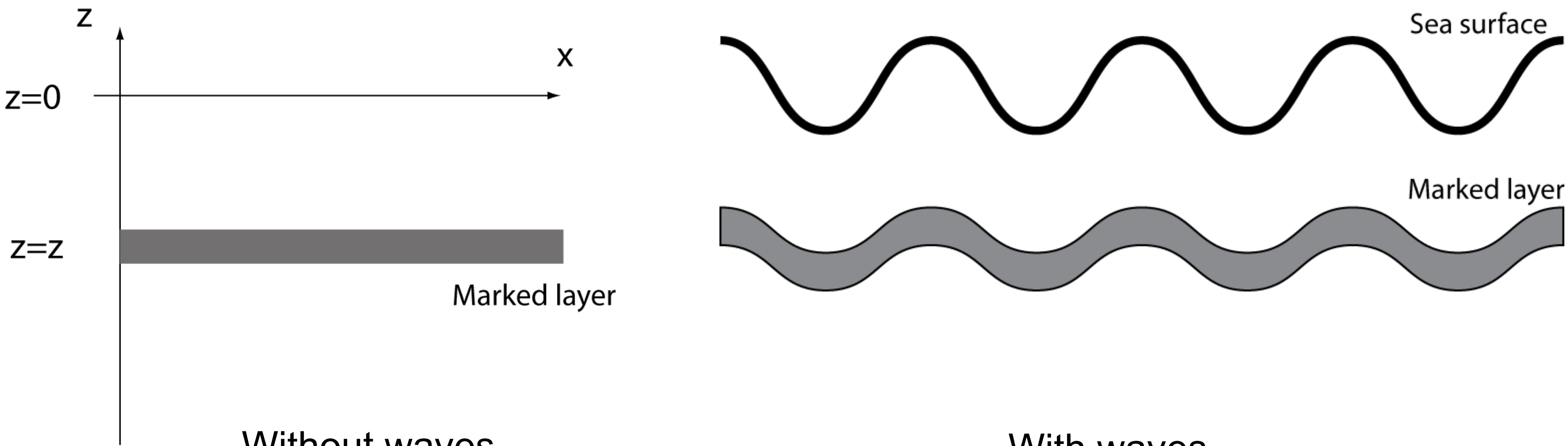
Velocity vectors in a wave field



Particle drift (Lagrangian Stokes drift)



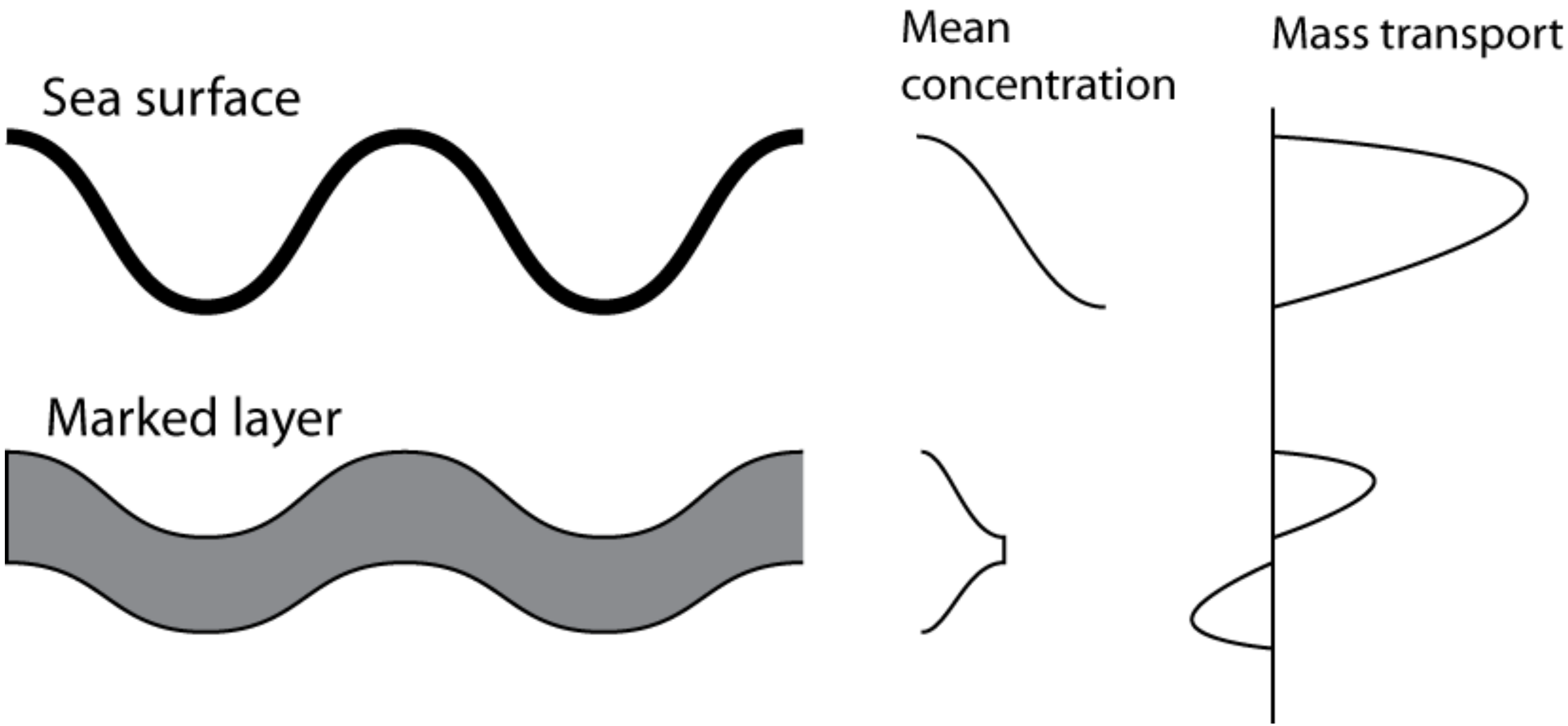
Let us consider a tracer field with concentration C=1 between ζ_1 and ζ_2 and is zero otherwise



Without waves

With waves

Taking an Eulerian mean

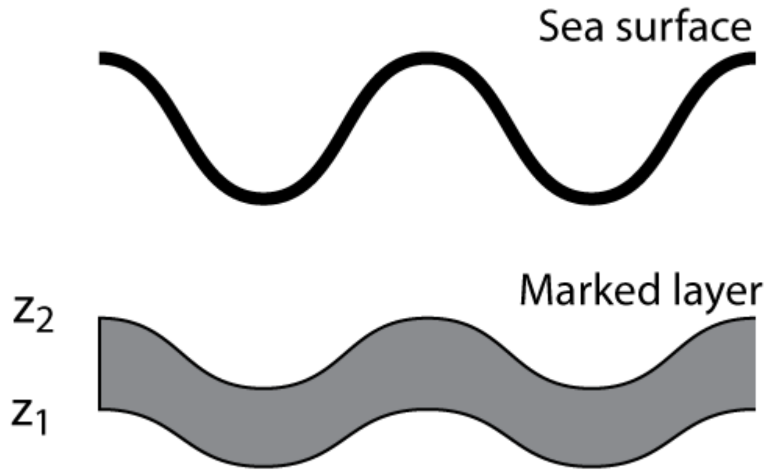


Note that the sea surface is not well defined using Eulerian averaging. Furthermore, the concentration *C* has a wider vertical distribution and a ”complicated” horizontal transport when waves are present. However, the total transports are the same in Eulerian and Lagrangian coordinates!

An alternative time mean operator to calculate drift.

The horizontal mean velocity of the marked fluid at a vertical section can be calculated as

$$\{u\} = \frac{1}{\delta z}\overline{\int_{\zeta_1}^{\zeta_2} u(z)\,dz}$$



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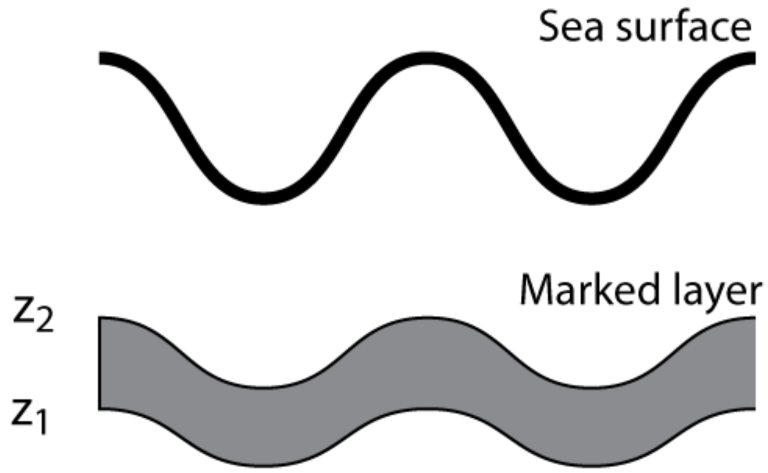
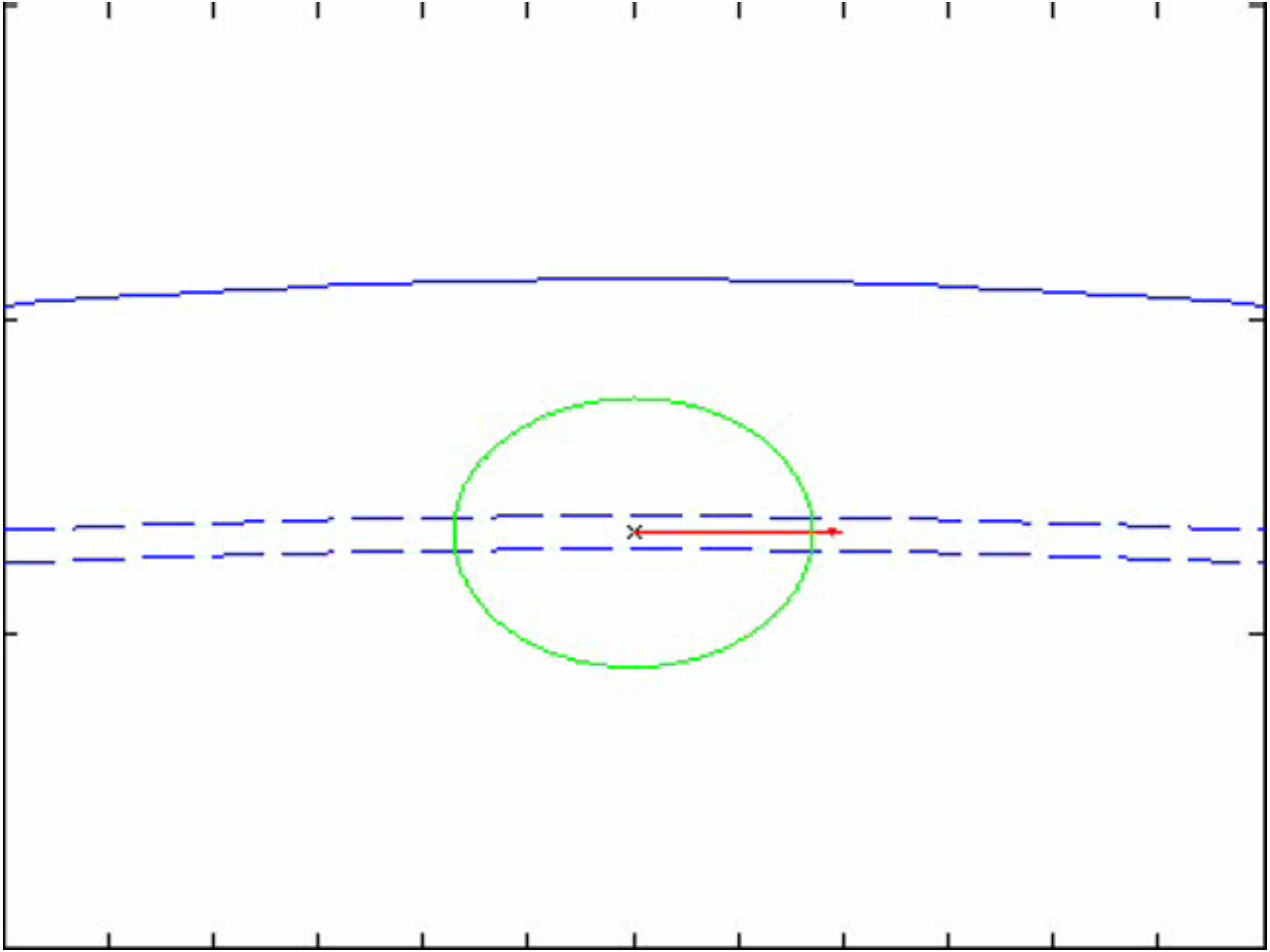


$$\{u(Z)\} = \frac{\partial}{\partial Z}\overline{\int_{-H}^{Z+\varsigma(t,Z)} u(z)dz}$$

Alternative, easier to use formulation.
Broström et al, JPO, 2008.

Illustration of the velocity of the marked layer



Start with Navier Stokes and continuity eqs.

$$\frac{d\mathbf{u}}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial \mathbf{x}},$$
$$\nabla \cdot \mathbf{u} = 0,$$

$$\mathbf{u} = \mathbf{U} + \tilde{\mathbf{u}},$$
$$p = P + \tilde{p},$$

$$\frac{d\left(\mathbf{U} + \tilde{\mathbf{u}}\right)}{dt} = -\frac{1}{\rho}\frac{\partial\left(P + \tilde{p}\right)}{\partial \mathbf{x}},$$
$$\nabla \cdot \left(\mathbf{U} + \tilde{\mathbf{u}}\right) = 0.$$

Apply a "time averaging operator" {}

$$\left\{\frac{d\left(\mathbf{U} + \tilde{\mathbf{u}}\right)}{dt}\right\} = -\frac{1}{\rho}\left\{\frac{\partial\left(P + \tilde{p}\right)}{\partial \mathbf{x}},\right\}$$
$$\left\{\nabla \cdot \left(\mathbf{U} + \tilde{\mathbf{u}}\right)\right\} = 0.$$

Some algebra gives …

We need to use the following averaging operator $\{u\} = \frac{1}{\delta z}\overline{\int_{\zeta_1}^{\zeta_2} u(z)\,dz}$

$$\frac{d\left\{\mathbf{U}+\widetilde{\mathbf{u}}\right\}}{dt} = -\frac{1}{\rho}\frac{\partial\left\{P+\widetilde{p}\right\}}{\partial\mathbf{x}}$$

$$\nabla\cdot\left\{\mathbf{U}+\widetilde{\mathbf{u}}\right\} = 0.$$

$$\left\{a\right\} = \frac{1}{\overline{\delta z}}\overline{\int_{\zeta_1(t,z)}^{\zeta_2(t,z)} a\,dz}$$

Some algebra gives …

$$\left\{\mathbf{U}\right\} = \mathbf{U},$$

$$\left\{\widetilde{u}\right\} = U_{St},\ \left\{\widetilde{w}\right\} = 0,$$

$$\left\{\widetilde{u}\widetilde{u}\right\} \neq 0,\ \left\{\widetilde{w}^2\right\} \neq 0,$$

$$\left\{P+\widetilde{p}\right\} \neq P,$$

$$\left\{U\widetilde{u}\right\} \approx UU_{St}.$$

$\left\{\widetilde{w}\widetilde{u}\right\} = 0$ for linear waves

$\left\{\widetilde{w}\widetilde{u}\right\} \neq 0$ near boundaries,

or cases with sloping bottoms etc

Averaging over fluctuations

Turbulence:

$$\langle \mathbf{U} \rangle = \mathbf{U}, \langle \mathbf{u}' \rangle = 0, \langle \, \mathrm{p}' \rangle = 0$$

$$\langle \mathbf{U}\mathbf{u}' \rangle = 0$$

$$\langle \mathbf{u}'\mathbf{u}' \rangle \neq 0$$

Simple but unknown

Waves:

$$\{\mathbf{U}\} = \mathbf{U},$$

$$\{\widetilde{u}\} = U_{St}, \ \{\widetilde{w}\} = 0,$$

$$\{\widetilde{u}\widetilde{u}\} \neq 0, \ \{\widetilde{w}^2\} \neq 0, \ \{\widetilde{w}\widetilde{u}\} \neq 0$$

$$\{P + \widetilde{p}\} \neq P,$$

$$\{U\widetilde{u}\} \approx UU_{St}.$$

Complicated but known

Averaging over fluctuations

Turbulence:

$$\frac{d\mathbf{U}}{dt} + \frac{\partial}{\partial \mathbf{x}}\left\langle \mathbf{u}'\mathbf{u}' \right\rangle = -\frac{1}{\rho}\frac{\partial P}{\partial \mathbf{x}}$$

$$\nabla \cdot \mathbf{U} = 0.$$

$\mathbf{u}'\mathbf{u}'$

unknown

Waves:

$$\frac{d\left(\mathbf{U} + \mathbf{U}_{St}\right)}{dt} + \mathbf{S} = -\frac{1}{\rho}\frac{\partial P}{\partial \mathbf{x}}$$

$$\nabla \cdot \left(\mathbf{U}\right) = 0.$$

Known
but complicated

S is the wave induced stress given by

$$S_{11} = \left\{ \frac{\partial}{\partial x}\widetilde{u}^2 + \frac{\partial}{\partial z}\widetilde{u}\widetilde{w} + \frac{1}{\rho}\frac{\partial}{\partial x}(P + \widetilde{p}) \right\} + p(z = \zeta_1)\frac{\partial \zeta_1}{\partial x} + p(z = \zeta_2)\frac{\partial \zeta_2}{\partial x}$$

$$-\frac{1}{\rho}\frac{\partial}{\partial x}P - U_{St}\frac{\partial U_{St}}{\partial x},$$

$$S_{22} = \left\{ \frac{1}{\rho}\frac{\partial}{\partial y}(P + \widetilde{p}) \right\} - \frac{1}{\rho}\frac{\partial}{\partial y}P,$$

Calculating the wave induced stresses

$$S_{11} = \left\{ \frac{\partial}{\partial x}\widetilde{u}^2 + \frac{\partial}{\partial z}\widetilde{u}\widetilde{w} + \frac{1}{\rho}\frac{\partial}{\partial x}(P + \widetilde{p}) \right\} + p(z = \zeta_1)\frac{\partial \zeta_1}{\partial x} + p(z = \zeta_2)\frac{\partial \zeta_2}{\partial x}$$

$$- \frac{1}{\rho}\frac{\partial}{\partial x}P - U_{St}\frac{\partial U_{St}}{\partial x},$$

$$S_{22} = \left\{ \frac{1}{\rho}\frac{\partial}{\partial y}(P + \widetilde{p}) \right\} - \frac{1}{\rho}\frac{\partial}{\partial y}P,$$

Correct to second order in wave steepness:

Needs second order estimate for pressure fluctuations.

We also need the boundary conditions for the integration that is correct to second order

See e.g., Broström et al. 2008, JPO, 38, 1122.

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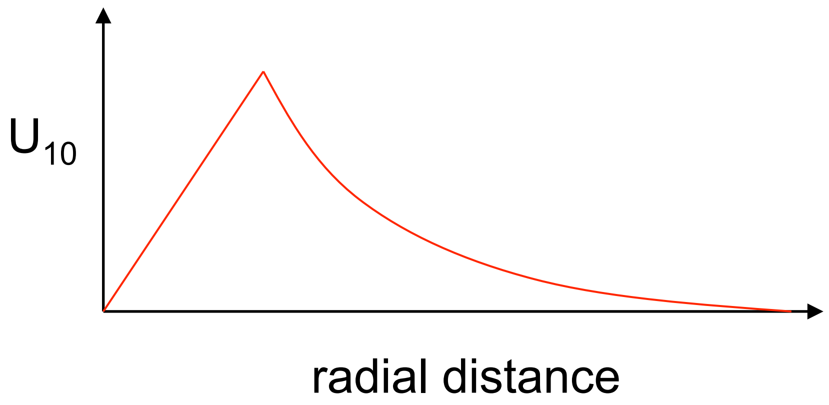
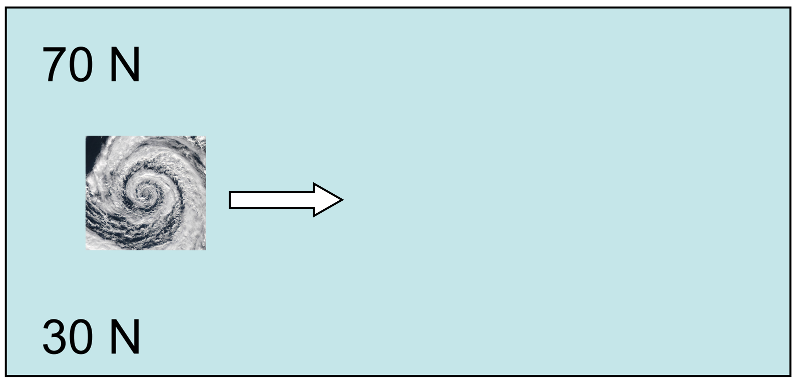
Numerical experiment

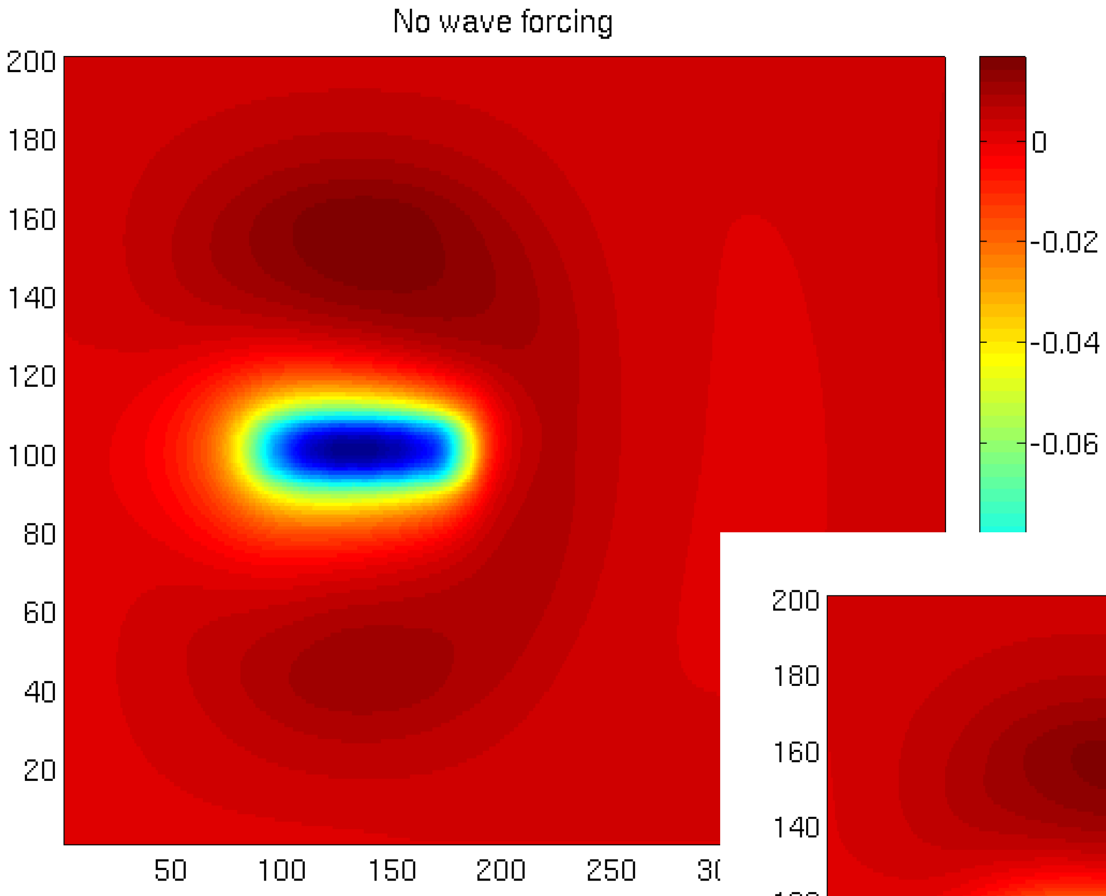


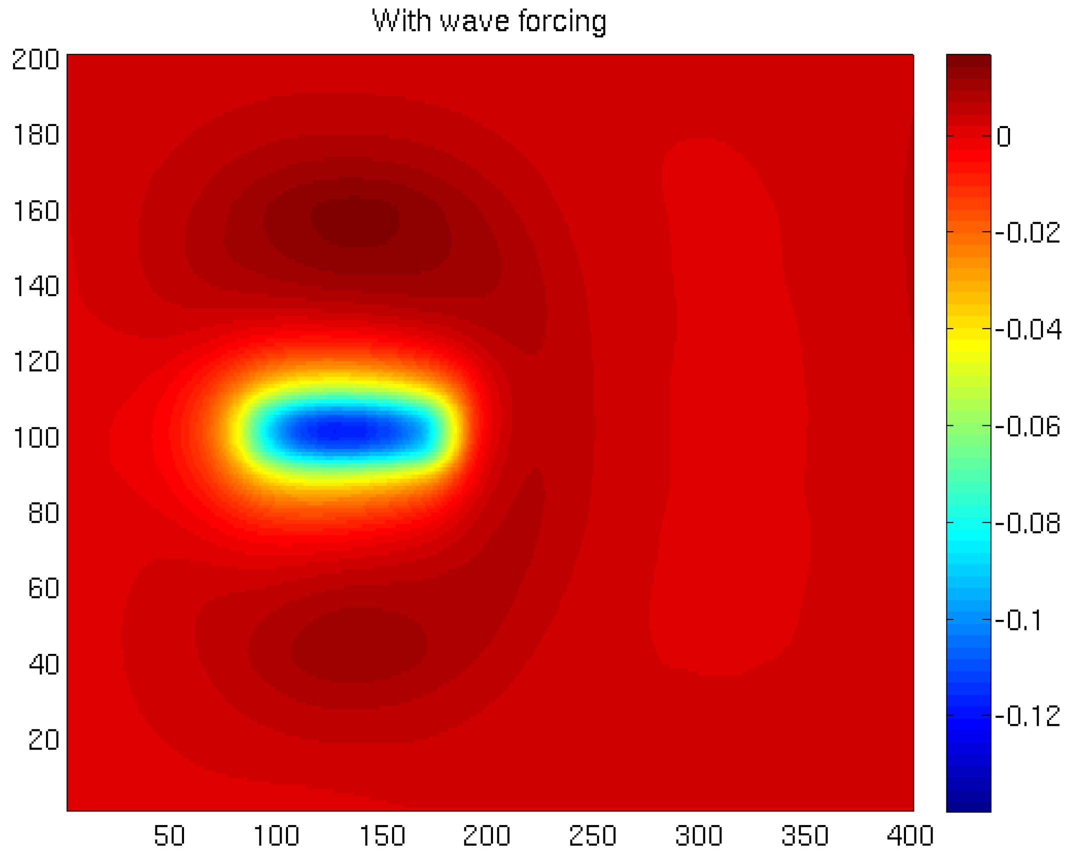


dx = 20 km, 401 x 201 x 30 gridpoints

- MITgcm/WAM
- Total atmospheric momentum flux kept constant
- Only wave-induced surface stress, body forces not implemented yet
- Low pressure system as Rankine vortex: max wind speed 25 m/s, advection speed 10 m/s
- No land, constant depth 500 m
- Flow relaxation zone at the basin boundaries (MITgcm)

Mean sea surface height





Differences up to 10-20%. Waves radiate out of the domain, reducing overall momentum flux to the ocean.

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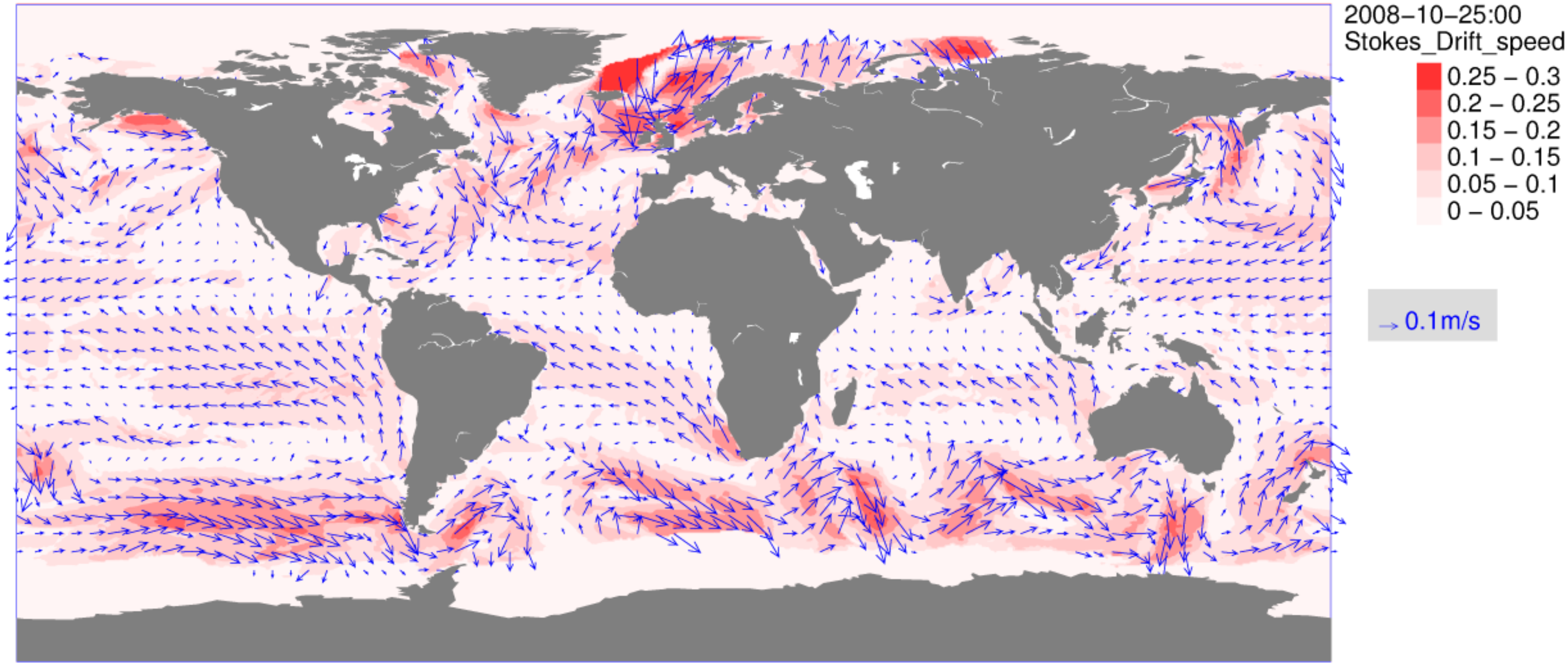
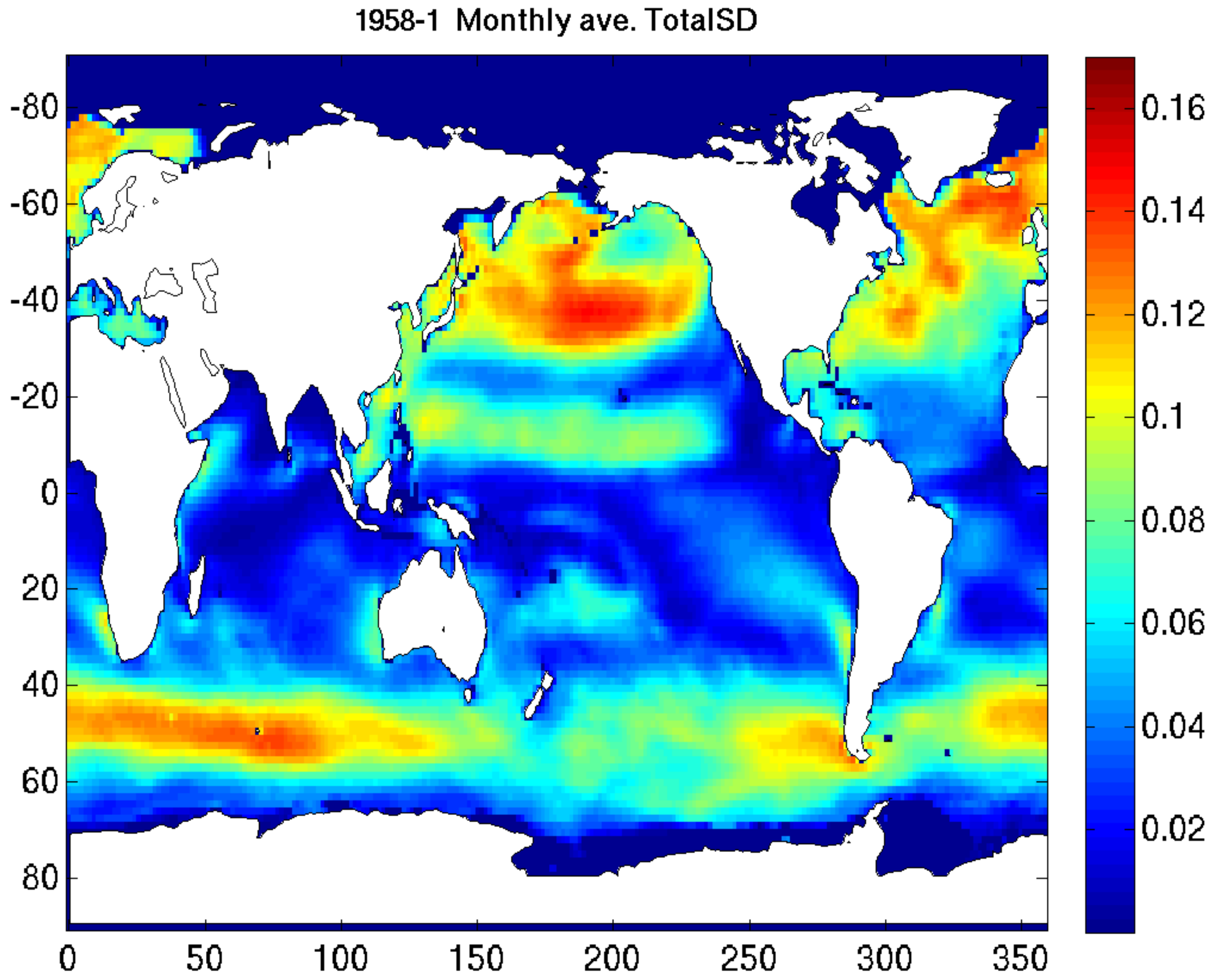
An example of the Stokes drift

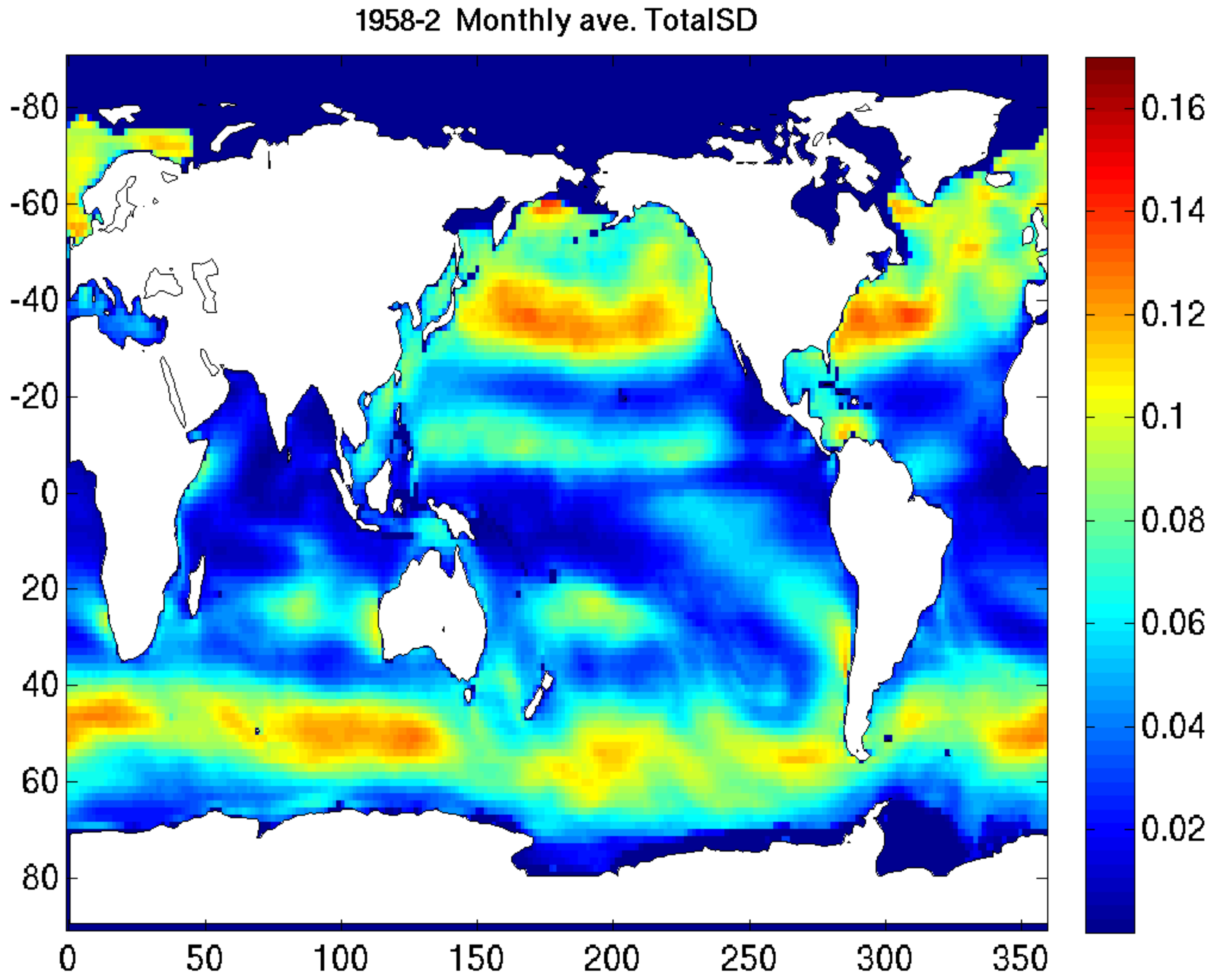


Calculated from ECMWF wave model (routine output)
Will be made available routinely in WamFlux

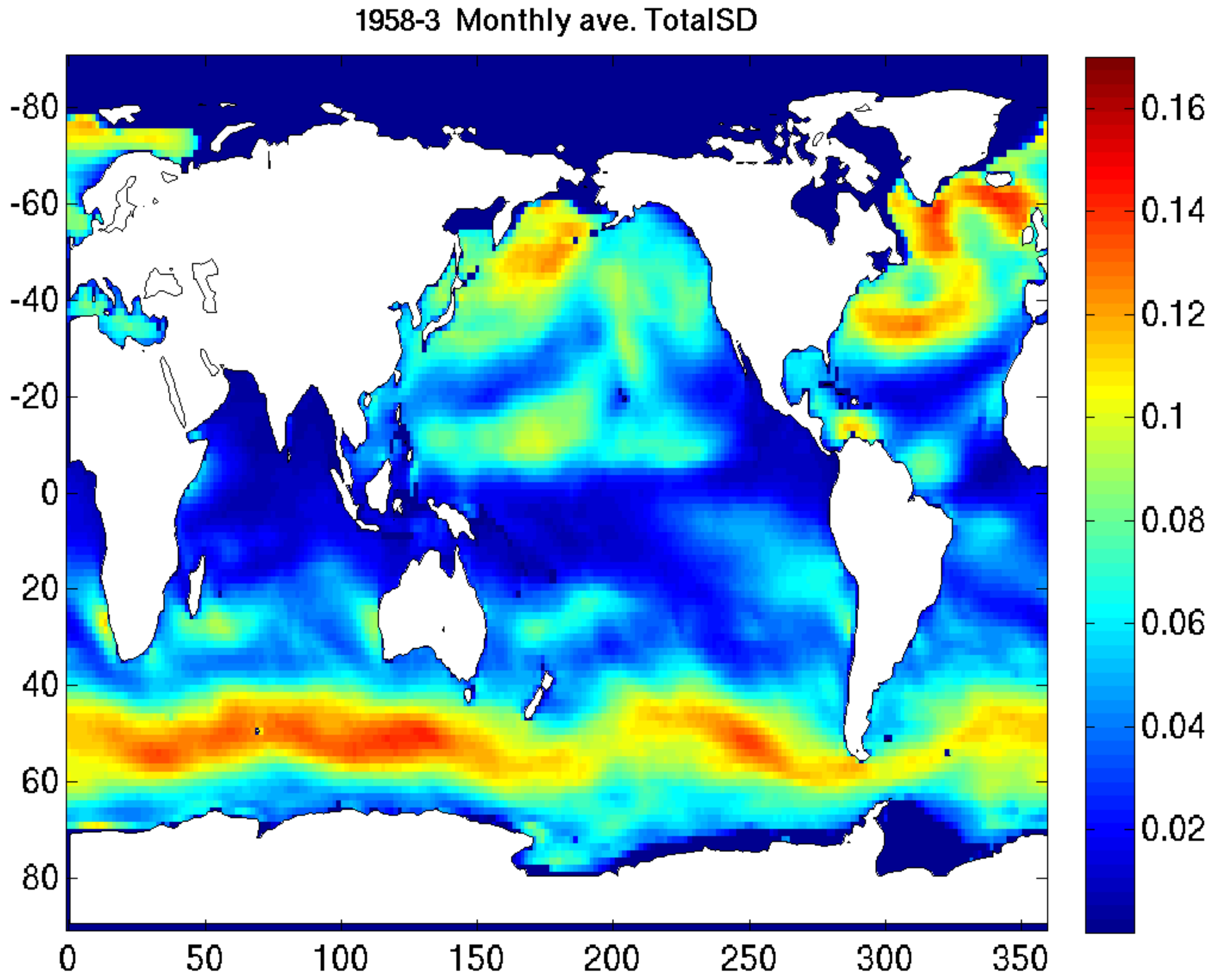
Can be used for drift calculations,
and for calculating Coriolis-Stokes force



1958-1 Monthly ave. TotalSD
-80
-60
-40
-20
0
20
40
60
80
0
50
100
150
200
250
300
350
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02



1958-2 Monthly ave. TotalSD
-80
-60
-40
-20
0
20
40
60
80
0
50
100
150
200
250
300
350
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02

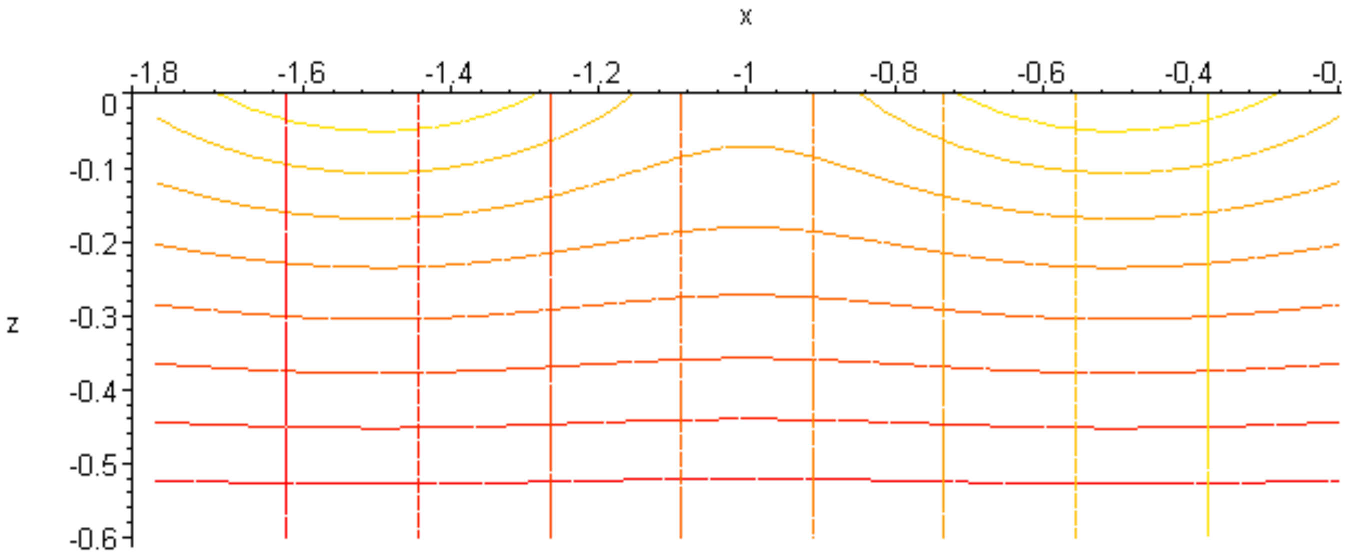

1958-3 Monthly ave. TotalSD
-80
-60
-40
-20
0
20
40
60
80
0
50
100
150
200
250
300
350
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02

Products in WamFlux

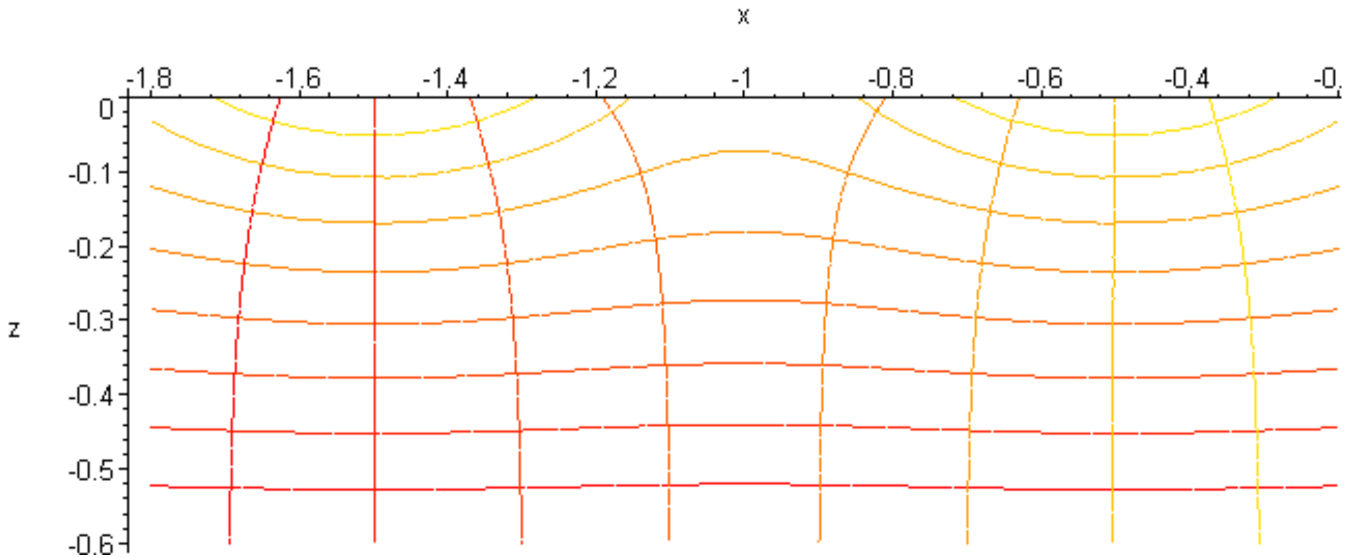
- Stresses calculated from wave model
  - Stress at sea surface
  - (stress at bottom)
- Stokes drift
  - Coriolis-Stokes force (routine to calculate 3D)
  - 2D Coriolis-Stokes force (estimated depth dependence)
  - (Evaluating importance of Stokes drift)
  - (Evaluating importance of wave-mean flow forcing)

Discussion

- Wave-current interactions are needed in OGCMs for important applications (SAR, oil spill mitigation, bio-models).
- Several overlapping/competing theories.
  - All of them tend to be quite complex, some rely heavily on certain assumptions concerning the wave field (e.g. stationarity).
  - The present work is essentially a reformulation of the classical well-known well-accepted description of barotropic wave forcing on mean fields by Longuett-Higgins and Stewart (1960, 1964).
- Stresses/forces estimates for ocean models can be improved using wave models that i) provide superior physics for the exchange formulation and ii) assimilates reliable satellite products for wave height.



Mellor transformation



Orthogonal curvilinear transformation

Calculating the radiation stresses

$$S_{11} = -\frac{\rho}{2}\frac{\partial}{\partial x}\left(cU_{St}\right) - 2\frac{\partial}{\partial x}\left(\frac{1}{H}\left[\frac{c_g}{c} - \frac{1}{2}\right]E\right) + \frac{1}{2k}\left\{\rho cU_{St} + \frac{2}{H}\left[\frac{c_g}{c} - \frac{1}{2}\right]E\right\}\frac{\partial k}{\partial x}$$

$$S_{22} = -\frac{\partial}{\partial x}\left(\frac{1}{H}\left[\frac{c_g}{c} - \frac{1}{2}\right]E\right)$$

Does not have the same vertical structure as the studies by Mellor (2003, 2007), partly due to a different treatment of the pressure term.

Mellor used a fixed coordinate transformation and does not allow waves to change in time or space.

Final model

$$\frac{d\left(\mathbf{U}+\mathbf{U}_{St}\right)}{dt}+\mathbf{S}=-\frac{1}{\rho}\frac{\partial P}{\partial \mathbf{x}}$$

$$\nabla\cdot\left(\mathbf{U}\right)=0.$$

$$S_{11}=\left\{\frac{\partial}{\partial x}\tilde{u}^2+\frac{\partial}{\partial z}\tilde{u}\tilde{w}+\frac{1}{\rho}\frac{\partial}{\partial x}(P+\tilde{p})\right\}+p(z=\zeta_1)\frac{\partial \zeta_1}{\partial x}+p(z=\zeta_2)\frac{\partial \zeta_2}{\partial x}$$

$$-\frac{1}{\rho}\frac{\partial}{\partial x}P-U_{St}\frac{\partial U_{St}}{\partial x},$$

$$S_{22}=\left\{\frac{1}{\rho}\frac{\partial}{\partial y}(P+\tilde{p})\right\}-\frac{1}{\rho}\frac{\partial}{\partial y}P,$$

$$\left\{a\right\}=\frac{\partial}{\partial z}\int_{-H}^{z+\varsigma(z)}\left\{a\right\}dz$$