The Viscosity of Concentrated Suspensions and Solutions

H. C. Brinkman

Citation: The Journal of Chemical Physics **20**, 571 (1952); doi: 10.1063/1.1700493 View online: https://doi.org/10.1063/1.1700493 View Table of Contents: http://aip.scitation.org/toc/jcp/20/4 Published by the American Institute of Physics

Articles you may be interested in

A Mechanism for Non-Newtonian Flow in Suspensions of Rigid Spheres Transactions of the Society of Rheology **3**, 137 (1959); 10.1122/1.548848

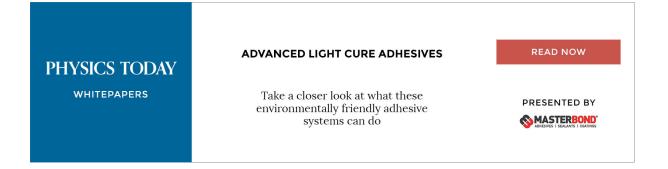
Anomalously increased effective thermal conductivities of ethylene glycol-based nanofluids containing copper nanoparticles Applied Physics Letters **78**, 718 (2001); 10.1063/1.1341218

A Treatment of the Viscosity of Concentrated Suspensions Journal of Applied Physics **23**, 1020 (1952); 10.1063/1.1702338

Empirical correlation finding the role of temperature and particle size for nanofluid (Al_2O_3) thermal conductivity enhancement Applied Physics Letters **87**, 153107 (2005); 10.1063/1.2093936

Role of Brownian motion in the enhanced thermal conductivity of nanofluids Applied Physics Letters **84**, 4316 (2004); 10.1063/1.1756684

A dynamic subgrid-scale eddy viscosity model Physics of Fluids A: Fluid Dynamics **3**, 1760 (1991); 10.1063/1.857955



The Viscosity of Concentrated Suspensions and Solutions*

H. C. BRINKMAN

Bosscha Physics Laboratory, University of Indonesia, Bandung, Indonesia (Received December 3, 1951)

An expression for the viscosity of solutions and suspensions of finite concentration is derived by considering the effect of the addition of one solute-molecule to an existing solution, which is considered as a continuous medium.

N recent years much attention has been given to the viscosity of concentrated solutions and suspensions. Guth and Simha¹ have derived the quadratic term in the concentration-dependence of the viscosity of suspensions of spherical particles by considering the hydronamic interaction of two spheres as derived by von Smoluchowski.² Their derivation, which leads to very involved calculations, seems to contain a numerical error, as was kindly pointed out to me by Dr. Simha.

Riseman and Ullman³ have derived formulas for the quadratic term in the concentration-dependence of the viscosity for dumbbell, rigid rod, and flexible chain macromolecules.

Burgers and, independently, Saito⁴ have developed a theory for spherical solute-molecules in which a single solute-molecule is placed in a field of flow, obtained by averaging over all possible positions of a second solutemolecule. This averaging process, however, leads to diverging integrals which make the result questionable.

In this paper we wish to point out that there is a very simple method of estimating the concentration-dependence of any quantity at higher concentrations if the result for infinite dilution is known. An analogous treatment was given by Onsager in his theory of the dielectric constant.⁵

Let us consider a suspension containing n soluteparticles in a total volume V. Its viscosity is a function of the concentration n/V and will be indicated by $\eta(n/V).$

Let us now consider the effect of adding one more solute-particle. If such a particle were added to a pure solvent of volume V we could immediately give an expression for the viscosity-increase. If the soluteparticle has a volume v_0 and a spherical shape, the viscosity would be multiplied by Einstein's well-known factor $(1+2.5v_0/(V+v_0))$.

Let us now assume this factor to be approximately valid for the result of the addition of one solute-molecule to an already existing solution containing n solutemolecules. This means that this solution is represented by a pure solvent of the same viscosity. The assumption would be exact if the *n* solute particles already present were much smaller than the one added. In our case, however, the substitution of a continuum for the solution is an approximation. In this way we obtain the difference-equation,

$$\eta\left(\frac{n+1}{V+v_0}\right) = \eta\left(\frac{n}{V}\right) \cdot \left(1 + \frac{5}{2} \frac{v_0}{V+v_0}\right),$$

which leads to

$$d\eta/dc_v = 5\eta/2(1-c_v),$$

where c_v is the concentration by volume nv_0/V and therefore

$$\eta = \eta_0 / (1 - c_v)^{5/2}$$

By expansion we find

$$\eta_{sp}/c_w = [\eta] + 0.7[\eta]^2 c_w + 0.42[\eta]^3 c_w^2 + \cdots,$$

where η_{sp} is the specific viscosity, $[\eta]$ the intrinsic viscosity, and c_w the concentration by weight. For spheres $[\eta] = 2.5v_0/M$, where M is the mass of a sphere.

This equation agrees quite well up to the term indicated with very careful measurements on asphaltemulsions by H. Eilers.⁶ Eilers represents his measurements by a formula

$$\eta = \eta_0 \left\{ 1 + \frac{1.25c_v}{1 - (c_v/0.78)} \right\}^2.$$

This is equivalent to

$$\eta_{sp}/c_w = [\eta] + 0.76[\eta]^2 c_w + 0.64[\eta]^3 c_w^2 + \cdots$$

The method discussed here for spherical molecules may of course be applied to any kind of molecules.

^{*} Paper presented at the XIIth Congress of Pure and Applied Chemistry, New York, 1951. ¹ E. Guth and R. Simha, Kolloid Z. 74, 266 (1936).

² M. v. Smoluchowski, Bull. Acad. de Sci. de Cracovie No. 1A (1911).

⁸ J. Riseman and R. Ullman, J. Chem. Phys. **19**, 578 (1951). ⁴ Private communication by Dr. Simha.

⁸ L. Onsager, J. Am. Chem. Soc. 58, 1986 (1936).

⁶ H. Eilers, Kolloid Z. 97, 313 (1941).