The radiation of infrasound into the atmosphere by surface waves in the ocean

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Abstract The radiation of infrasound into the atmosphere and ocean due to nonlinear interaction of surface gravity waves is examined. It is shown that the radiation of infrasound into the atmosphere depends on the sound wave propagating into the ocean. The results of numerical calculations of certains characteristics of the radiated sound for various surface-wave spectra are presented.

Introduction

It is known that storm regions in the ocean are sources of atmospheric infrasound, which can then propagate over very long distance [1, 2]. It appears that surface waves can make a substancial contribution to this radiation. By radiating infrasound into the water, for example, these waves are almost entirely responsible for the observed microseism level [3]. Radiation of sound into the water as a result of nonlinear surface-wave interaction was investigated in [3, 4]. A similar mechanism of radiation of sound into the atmosphere was analyzed in [5], and this analysis was used in [6], with consideration of sound propagation conditions in the atmosphere, as a basis for certain estimates.

However, it was found that the analysis of the phenomenon in [5] was not quite correctly formulated and too greatly simplified. A result of this study was that the radiation of sound into the water and into the atmosphere was perfectly symmetrical. In reality, however, this is not the case. The ocean surface is acoustically absolutely compliant for radiation of sound into the water. As a result, bipolar sound sources produced, as it were, by forces applied to the ocean surface appear on that surface. One such force in the case being considered here will be the nonlinear force $F = \rho\zeta\partial^2\zeta/\partial t^2$ which arises on displacement of the free surface $\zeta(x, y, t)$, where $\rho\zeta$ is the mass of the displaced liquid element per unit area and $\partial^2\zeta/\partial t^2$ is acceleration. For the radiation of sound into the atmosphere, on the other hand,the ocean surface is a surface of volume-velocity sources (monopoles) because of its low acoustic compliance. The displacement of the interface due to nonlinear interactions in the ocean (normal velocity $w = \partial\zeta/\partial t$) may serve as an example of such a source. But force-driven acoustic sources do not usually form on a surface with low compliance. Coincidentally, the generation of sound into the atmosphere will be of monopolar nature, and not dipolar as was concluded in [5].

Below we shall set forth the results of complete analysis of the radiation of sound into both the water and the atmosphere with consideration of the nonlinear interaction of the waves at the interface between these media. With regard to the radiation of sound into the water, this will produce nothing new compared with [4]. For the radiation into the air, however, the estimates of [6] will be improved upon greatly. Thus, although the order of magnitude of the radiated sound intensity will not change, its directional characteristic will be substantially different. It is found, for example, that the radiation at small angles to the horizontal is much stronger than was found in [6].

Moreover, it will be seen below that the radiation of sound into the water and the atmosphere is also asymmetrical because the radiation into the atmosphere depends strongly on the interface reaction of the sound wave propagating into water. Thus the acoustic field in the atmosphere cannot be considered in isolation from the acoustic field in the water. On the other hand, the radiation into the water can be treated by substituting a vacuum for the atmosphere, as was done in [4].

1 Statement of problem and basic equations

We shall consider the two half-spaces $z > 0$ and $z < 0$, which are filled with air and water, respectively, and are characterized by equilibrium density profiles $\rho_{j0}(z)$ and sonic velocities c_j ($j = 1$ for air and $j = 2$ for water). We shall denote the displacement of the boundary by $\zeta(x, y, t)$. We shall describe the motions in the air and water with the hydrodynamic equations of inviscid compressible fluids with consideration of the force of gravity. In Euler coordinates, these equations take the form

$$\begin{gathered} \rho_j \left(\frac{\partial \mathbf{v}_j}{\partial t} + (\mathbf{v}_j \nabla)\, \mathbf{v}_j \right) = -\nabla p_j - g\rho_j \nabla z \\ \frac{\partial \rho_j}{\partial t} + \nabla(\rho_j \mathbf{v}_j) = 0 \end{gathered} \tag{1}$$

Where g is the acceleration of gravity and p_j, ρ_j, and v_j are the pressure, density, and velocity of parcels of the medium. Assuming that the equilibrium entropy is constant for each of the media and regarding oscillatory processes are adiabatic, we write the equations of state in the linear approximation in the form (note de bas: In our case, consideration of nonlinearity in the equations of state yields negligibly small terms.)

$$p_j - p_{j0}(0) = c_j^2 \left[\rho_j - \rho_{j0}(0)\right] \tag{2}$$

Dynamic and kinematic boundary conditions must be satisfied at the interface $z = \zeta(x, y, t)$:

$$p_1 = p_2, \quad \mathbf{v}_j \nabla z = \partial\zeta/\partial t \qquad \text{at} \qquad z = \zeta(x, y, t). \tag{3}$$

Putting $v_j = 0$, we find the equilibrium density and pressure profiles for the ocean and the atmosphere from the first equation of (1), (3) with consideration of (2):

$$\begin{gathered} \rho_{j0}(z) = \rho_{j0}(0) \exp\left\{-gz/c_j^2\right\}, \\ p_{j0}(z) = p_{j0} + \left[\rho_{j0}(z) - \rho_{j0}(0)\right] c_j^2, \\ p_{j0} = p_{10}(0) = p_{20}(0). \end{gathered} \tag{4}$$

At $v_j \neq 0$, we represent all quantities that appear in (1)-(3) in the form of expansions in a certain small parameter s:

$$\begin{gathered} \rho_j = \rho_{j0}(z) + s\rho_{j1} + s^2\rho_{j2} + ... \\ p_j = p_{j0}(z) + sp_{j1} + s^2 p_{j2} + ... \\ \mathbf{v}_j = s\mathbf{v}_{j1} + s^2\mathbf{v}_{j2} + ... \\ \zeta = s\zeta_1 + s^2\zeta_2 + ... \end{gathered} \tag{5}$$

We substitute (5) into (1)-(3) after first transferring (3) to the plane $z = 0$ by series expansion in powers of ζ and equate terms with equal powers of s. It is easily seen that (2) will then give a simple relation between ρ_{ji} and p_{ji} $(i = 1, 2, ...)$:

$$p_{ji} = c_j^2 \rho_{ji}$$

It will subsequently be convenient to use the quantity $\mathcal{P}_{ji}$ which is related to p_{ji} by

$$p_{ji} = \rho_{j0}(z)\mathcal{P}_{ji} \tag{6}$$

Now the first equation of (1) is written as follows for the i-th approximation:

$$\partial v_{ji}/\partial t + \nabla\mathcal{P}_{ji} = \mathbf{f}_{ji} \tag{7}$$

where $f_{j1} = 0$, $\mathbf{f}_{j2} = \nabla\{(\mathcal{P}_{j1}^2/2c_j^2)\text{-}(\mathbf{v}_{j1}^2/2)\} + \mathbf{v}_{j1} \times \mathbf{curl}\ \mathbf{v}_{j1}$.

It follows from (7) that when $i = 1 : (\partial/\partial t)(\mathbf{curl}\ \mathbf{v}_{j1}) = 0$, and we may put $\mathbf{curl}\ \mathbf{v}_{j1} = 0$ for the oscillatory processes under consideration. Consequently, $\mathbf{f}_{j2} = \nabla\{(\mathcal{P}_{j1}^2/2c_j^2)$ - $(\mathbf{v}_{j1}^2/2)\}$ and, similarly, $\mathbf{curl}\ \mathbf{v}_{j2} = 0$ for the second approximation. Thus,there exists a velocity potential (note de bas : The possibility of introduction of the velocity potential arises out the assumption of constant entropy in the media and the linearity of the equation of state (2)) $\phi_{j,i}$, i.e.,

$$\mathbf{v}_{ji} = -\nabla\phi_{ji} \tag{8}$$

As a result, we obtain the following system of equations and boundary conditions for the i-th approximation from (1)-(3) :

$$\Delta\phi_{ji} - \frac{g}{c_j^2}\frac{\partial\phi_{ji}}{\partial z} - \frac{1}{c_j^2}\frac{\partial^2\phi_{ji}}{\partial t^2} = S_{ji}, \qquad \mathcal{P}_{ji} = \tfrac{\partial\phi_{ji}}{\partial t} + F_{ji},$$
$$\left.\frac{\partial\phi_{ji}}{\partial z}\right|_{z=0} + \frac{\partial\zeta_i}{\partial t} = Q_{ji}, \qquad (\mathcal{P}_{2i} - m\mathcal{P}_{1i})_{z=0} - g(1-m)\zeta_i = R_i, \tag{9}$$

where

$$F_{j1} = S_{j1} = Q_{j1} = R_1 = 0,$$
$$F_{j2} = \frac{\mathcal{P}_{j1}^2}{2c_j^2} - \frac{(\nabla\phi_{j1})^2}{2}, \qquad S_{j2} = -\frac{1}{c_j^2}\frac{\partial}{\partial t}(\nabla\phi_{j1})^2,$$
$$Q_{j2} = \left.\frac{-\partial^2\phi_{j1}}{\partial z^2}\right|_0 \zeta_1 + \nabla\phi_{ji}|_0\nabla\zeta_1,$$
$$R_2 = -\left(\frac{\partial\mathcal{P}_{21}}{\partial z} - m\frac{\partial\mathcal{P}_{11}}{\partial z}\right)_0 \zeta_1 + \frac{g}{c_1^2}\left(n^2\mathcal{P}_{21} - m\mathcal{P}_{11}\right)_0\zeta_1 - \frac{1}{2}\frac{g^2}{c_1^2}(n^2 - m)\zeta_1^2,$$
$$m = \rho_{10}(0)/\rho_{20}(0), \quad n = c_1/c_2.$$

Uniqueness of the solution of system (9) requires, in addition, that the solution decrease or be a receding sound wave as $z \to \pm\infty$.

2 Acoustic field in the ocean and the atmosphere

The small parameter $m \simeq 1/800$ appears in (9). Analysis of the first-approximation solution of the equations leads to another small parameter (κ is the wave number of the surface wave):

$$\delta_j = \left(\frac{g}{\kappa c_j^2}\right)^{1/2} = \frac{\omega}{\kappa c_j} \simeq \frac{\lambda_{surface}}{\lambda_{sound}} \tag{10}$$

the wavelength ratio of the surface and sound waves of equal frequency. Thus, for example, we have $\delta \simeq 5 \cdot 10^{-2}$ for air at a frequency of $0.1Hz$. The parameter δ_1 decreases with increasing frequency.

Let us assume that there is a continuous surface-wave spectrum in the first approximation. The solution of system (9) for $i = 1$ will be

$$\begin{aligned} \phi_{j1} &= \mathrm{i}\int_{-\infty}^{\infty} \frac{\omega(\kappa)}{\alpha_j(\kappa)} a(\vec{\kappa}) \exp(\alpha_j(\vec{\kappa})z + \mathrm{i}\psi) d\vec{\kappa} \\ \mathcal{P}_{j1} &= \int_{-\infty}^{\infty} \frac{\omega^2(\kappa)}{\alpha_j(\kappa)} a(\vec{\kappa}) \exp(\alpha_j(\vec{\kappa})z + \mathrm{i}\psi) d\vec{\kappa} \\ \zeta_1 &= \int_{-\infty}^{\infty} a(\vec{\kappa}) \exp(\mathrm{i}\psi) d\vec{\kappa} \end{aligned} \tag{11}$$

where $\vec{\kappa} = \{\kappa_x, \kappa_y\}$, $\kappa = |\vec{\kappa}|, \psi = \vec{\kappa}\vec{r} - \omega(\kappa)t + \epsilon(\vec{\kappa})$ $(\epsilon(\vec{\kappa})$ is a certain phase angle), and, accurate terms of order δ_j^2 inclusive:

$$\begin{gathered} (1+m)\omega^2(\kappa) = (1-m)g\vec{\kappa}, \\ \alpha_j(\kappa) = (-1)^j \kappa\{1 - [1 - (-1)^j]\delta_j^2/2\} \end{gathered} \tag{12}$$

Substitution of (11) into the right-hand sides of the second-approximation equations (9) endows the later with terms dependent on r and t : $\exp\{\mathrm{i}(\vec{\kappa} \pm \vec{\kappa}')\vec{r} - \mathrm{i}[\omega(\kappa) \pm \omega(\kappa')t\}$ where $\vec{\kappa}$ and $\vec{\kappa}'$ are two arbitrary vectors within the limits of the specified wave spectrum. It was shown in [4] that only components with the pus sign in the exponent correspond to sound waves receding from the boundary, and that $\vec{\kappa}$ and $\vec{\kappa}'$ must satisfy the condition

$$c_1|\vec{\kappa} + \vec{\kappa}'| \leqslant [\omega(\kappa) + \omega(\kappa')] \tag{13}$$

To calculate the field at finite distances z from the interface, it is also necessary to consider inhomogeneous sound waves, for which $c_1|\vec{\kappa} + \vec{\kappa}'| > [\omega(\kappa) + \omega(\kappa')]$. This was done for the ocean [7].

Confining ourselves to waves that satisfy (13) (we shall call this region Γ), we obtain:

$$\begin{aligned} F_{j2} &= \int_\Gamma A_j(\vec{\kappa}, \vec{\kappa}') \exp(\alpha_{j+}(\vec{\kappa})z + \mathrm{i}\psi_+) d\vec{\kappa} d\vec{\kappa}' \\ S_{j2} &= \int_\Gamma B_j(\vec{\kappa}, \vec{\kappa}') \exp(\alpha_{j+}(\vec{\kappa})z + \mathrm{i}\psi_+) d\vec{\kappa} d\vec{\kappa}' \\ Q_{j2} &= \int_\Gamma C_j(\vec{\kappa}, \vec{\kappa}') \exp(\alpha_{j+}(\vec{\kappa})z + \mathrm{i}\psi_+) d\vec{\kappa} d\vec{\kappa}' \\ R_2 &= \int_\Gamma D(\vec{\kappa}, \vec{\kappa}') \exp(\mathrm{i}\psi_+) d\vec{\kappa} d\vec{\kappa}' \end{aligned} \tag{14}$$

where

$$\psi_+ = \vec{q}\vec{r} - \Omega t + \epsilon, \qquad \vec{q} = \vec{\kappa} + \vec{\kappa}', \qquad \Omega = \omega(\kappa) + \omega(\kappa'), \qquad \epsilon = \epsilon(\vec{\kappa}) + \epsilon(\vec{\kappa})$$

$$A_j = \frac{1}{4}\omega(\kappa)\omega(\kappa')\left[\gamma_j + \frac{\omega(\kappa)\omega(\kappa')}{c_j^2\alpha_j(\kappa)\alpha_j(\kappa')}\right]a(\vec{\kappa})a(\vec{\kappa}')$$

$$B_j = \frac{-\mathrm{i}\Omega}{2c_j^2}\omega(\kappa)\omega(\kappa')\gamma_j a(\vec{\kappa})a(\vec{\kappa}')$$

$$C_j = \frac{-\mathrm{i}}{4}\left\{\Omega\alpha_{j+} - \left[\omega(\kappa)\alpha_j(\kappa') + \omega(\kappa')\alpha_j(\kappa)\right]\gamma_j\right\}a(\vec{\kappa})a(\vec{\kappa}')$$

$$D = -\frac{1}{4}\left\{(1-m)\left[\omega^2(\kappa) + \omega^2(\kappa')\right] - \frac{g}{c_j^2}\left[n^2\beta_2 - m\beta_1 - g(n^2-m)\right]\right\}a(\vec{\kappa})a(\vec{\kappa}'),$$

$$\alpha_{j+} = \alpha_j(\kappa) + \alpha_j(\kappa'), \qquad \gamma_j = 1 - \frac{\vec{\kappa}\vec{\kappa}}{\alpha_j(\kappa)\alpha_j(\kappa')}, \qquad \beta_j = \frac{\omega^2(\kappa)}{\alpha_j(\kappa)} + \frac{\omega^2(\kappa')}{\alpha_j(\kappa')}.$$

It is natural to seek the second approximation solution of system (9) in the form:

$$\mathcal{P}_{j2} = \int_\Gamma \Phi_j(z)\mathrm{e}^{\mathrm{i}\psi_+}d\vec{\kappa}d\vec{\kappa}' \tag{15}$$

Substituting (15) into (9), we obtain a system of ordinary differential equations in Φ_j, whose general solution satisfying the conditions as $z \to \pm\infty$ takes the form

$$\Phi_j = \left[A_j - \frac{\mathrm{i}\Omega B_j}{\alpha_{j+}^2\left(1 - \frac{q}{c_j^2\alpha_{j+}} + \frac{k_j^2 - q^2}{\alpha_{j+}^2}\right)}\right]\mathrm{e}^{\alpha_{j+}z} + a(\vec{\kappa})a(\vec{\kappa}')\mu_j\mathrm{e}^{\mathrm{i}\lambda_j\kappa} \tag{16}$$

where $k_j = \Omega/c_j$ and

$$\lambda_j = (-1)^{j+1}k_j\left[\left(1 - \frac{q^2}{k_j^2} - \frac{g^2}{4c_j^2k_j^2}\right)^{1/2} + \mathrm{i}(-1)^j\frac{g}{2c_j^2k_j}\right]. \tag{17}$$

The acoustic-pressure spectral amplitude μ_j of the waves receding from the boundary are determined by substitution of (16) into the conditions for $z = 0$ (see (9)).

We note that the condition (13) is satisfied only if $\vec{\kappa} \simeq -\vec{\kappa}'$ and then $q/\kappa - \delta_1$ is the small parameter introduced previously. This enables us to expand the μ_j in powers of δ_j. If we confine ourselves to the principal terms of the expansion and neglect the parameter m by comparison with unity, the ocean pressure $\mathcal{P}_{22}$ will not depend on the atmosphere pressure $\mathcal{P}_{12}$ in (9),i.e.,

$$\begin{aligned} L_2\phi_{22} = S_{22}, \quad \mathcal{P}_{22} = \frac{\partial\phi_{22}}{\partial t} + F_{22}, \\ \mathcal{P}_{22}\big|_0 = R_2, \quad \frac{\partial\zeta_2}{\partial t} = -\left.\frac{\partial\phi_{22}}{\partial z}\right|_0 + Q_{22}, \end{aligned} \tag{18}$$

where $L_j \equiv \Delta - c_j^{-2}\partial^2/\partial t^2$. The first three equations of system (18), together with the condition for $z \to +\infty$, fully determine ϕ_{22} and $\mathcal{P}_{22}$. The last equation of (18) determines the second approximation for the interface displacement ζ_2. It is also found to be independent of the pressure in the atmosphere. This result is quite natural because of the large density difference between water and air. Specifying a pressure distribution on the plane $z = 0$ in the system (18) corresponds to dipolar sound sources. Thus, the radiation of sound into the ocean can be described by the distribution of the dipoles on the undisturbed interface.

Calculation of the spectral amplitude μ_2 on the basis of system (18) gives

$$\mu_2(\kappa) = -\omega^2(\kappa), \tag{19}$$

which agrees with the analogous expression obtained in [4].

Now, regarding $\partial\zeta_2/\partial t$ as known, we obtain from the solution of the problem for the acoustic field in the ocean a closed equation system for the atmospheric fluctuations in the form

$$\begin{aligned} L_1\phi_{12} = S_{12}, \quad \mathcal{P}_{12} = \partial\phi_{12}/\partial t + F_{12}, \\ \left.\frac{\partial\phi_{12}}{\partial z}\right|_{z=0} = -\frac{\partial\zeta_2}{\partial t} + Q_{12}. \end{aligned} \tag{20}$$

This time the sound sources are monopoles (the normal velocity distribution is specified on the plane $z = 0$). An estimate of orders of magnitude indicates that the term Q_{12} in the right-hand side of the last equation in (20) stands in relation to the term $\partial\zeta_2/\partial t$ as δ_1 stands to unity. Hence it is clear that the acoustic field in the water cannot be left out of account in calculating the acoustic field in the atmosphere. The field in the water is largely responsible for determining the term $\partial\zeta_2/\partial t$. Solution of system (20) with neglect of Q_{12} by comparison with $\partial\zeta_2/\partial t$ results in the following expression for the spectral amplitude of the acoustic pressure in the atmosphere :

$$\mu_1(\kappa, q) = \omega^2(\kappa)(n^2 - \sin^2\theta_1)^{1/2}/\cos\theta_1, \tag{21}$$

where $\theta_1 = \arcsin(qc_1/2\omega(\kappa))$ is the angle between the propagation direction of the sonic wave and the z-axis.

The latter expression vanishes at θ_1 equal to the angle of total internal reflection ($\theta_1 = \theta_i = \arcsin n$). This is explained by the fact that at $\theta_1 = \theta_i$ the acoustic wave in the water propagates along the plane $z = 0$ without causing displacements along the z-axis ($\zeta_2 = 0$).

At $\theta_1 = \pi/2$ the expression for μ_1 increases without limit because of the in-phase action of the surface monopole sound sources in these directions. In the neighborhood of the angles θ_i and $\pi/2$, it is necessary to include higher terms in expanding in the small parameters δ_j and m. We present expressions obtained for μ_1 with consideration of terms of the first order in δ_j and m:

$$\mu_1 = -\mathrm{i}\omega^2(\kappa)\frac{(\sin^2\theta_1 - n^2)^{1/2} - 2\delta_1\left[1 - \sin^2\theta_1\left(1 - \frac{1}{2}(\vec{q}\vec{\kappa}/q\kappa)^2\right) + 5n^2/8\right]}{\cos\theta_1 - \mathrm{i}\left[(\delta_1/4) - m(\sin^2\theta_1 - n^2)^{1/2}\right]} \tag{22}$$

At $\theta_1 \simeq \theta_i$ the improved relation (22) tells us little that is new, because μ_1 again vanishes at a certain θ_1 near θ_i ($\theta_1 \simeq \theta_i + (2/n)\delta_1^2$).

3 Spectral characteristics of the radiated sound

The acoustic pressure in a wave radiated on interaction of surface waves will be given by relation (15) with Φ_j substituted from (16), where it is necessary to consider only the second term (the first term corresponds to inhomogeneous waves):

$$\mathcal{P}_j = \int_{-\infty}^{\infty}\int_{q<2\omega(\kappa)/c_1} \mu_j(\kappa, q)a(\vec{\kappa})a(\vec{q}-\vec{\kappa})\exp(\mathrm{i}\lambda_j z + \mathrm{i}\psi_+)d\vec{\kappa}d\vec{q}. \tag{23}$$

Here we have introduced the new variables of integration $\vec{\kappa}$ and $\vec{q} = \vec{\kappa} + \vec{\kappa}'$. The correlation function of the acoustic pressure at the two point r_1 and r_2 at level z and at times t_1 and t_2, with consideration of the δ correlation of the random variables $a(\vec{\kappa})$ and $a(\vec{q}-\vec{\kappa})$ (see, for example, [4]), is

$$\left\langle \mathcal{P}_j(\vec{r_1}, z, t_1)\mathcal{P}_j'(\vec{r_2}, z, t_2)\right\rangle = \int_{-\infty}^{\infty}\int_{q<2\omega(\kappa)/c_j} |\mu_j|^3 a^2(\vec{\kappa})a^2(\vec{q}-\vec{\kappa})\exp\mathrm{i}[(\lambda_j - \lambda_j*)z + \vec{q}\vec{r} - 2\omega(\kappa)\tau]d\vec{\kappa}d\vec{q}, \tag{24}$$

where $\vec{r} = \vec{r_1} - \vec{r_2}$, $\tau = t_1 - t_2$, $a^2(\kappa)$ is the spatial two-dimensional wave spectrum. Assuming that it is sufficiently smooth and considering the smallness of q compared to κ, we cant put $a^2(\vec{q}-\vec{\kappa}) \simeq a^2(-\vec{\kappa})$. Unfortunately, very little is known concerning $a^2(\vec{\kappa})$. Introducing the polar coordinates κ, ϕ, $d\vec{\kappa} = \kappa d\kappa d\phi$ we shall assume for simplicity that $a^2(\vec{\kappa}) = A(\kappa)S(\phi)$ then

$a^2(-\vec{\kappa}) = A(\kappa)S(\pi+\phi)$. The Fourier transform of (24) with respect to $\vec{r}$ and τ will give the space-time spectrum of the square of the radiated acoustic pressure:

$$P_j^2(\vec{q},\omega,z) = \frac{1}{2}\int_0^{2\pi} S(\phi)S(\pi+\phi)\left|\mu_j(\vec{q},\frac{\omega}{2})\right|^2 \cdot A^2\left(\kappa\left(\frac{\omega}{2}\right)\right) \mathrm{e}^{\mathrm{i}(\lambda_j-\lambda_j*)z}\left(\kappa\frac{d\kappa}{d\omega}\right)_{\omega/2} d\phi. \quad (25)$$

It is also convenient to introduce the wave frequency spectrum $\Phi(\omega)$ which is related to $A(\kappa)$ by

$$\Phi(\omega) = A[\kappa(\omega)]\left(\kappa\frac{d\kappa}{d\omega}\right)_\omega \int_0^{2\pi} S(\phi)d\phi.$$

As a result, with consideration of the approximate dispersion equation $\omega^2 = g\kappa$, we obtain for $P_j^2(\vec{q},\omega,z)$

$$P_j^2(\vec{q},\omega,z) = 2\frac{S_1}{S_0^2}\exp[\mathrm{i}(\lambda_j-\lambda_j*)z]g^2\left|\mu_j\left(q,\frac{\omega}{2}\right)\right|^2\omega^{-2}\Phi^2\left(\frac{\omega}{2}\right), \quad (26)$$

where

$$S_0 = \int_0^{2\pi} S(\phi)d\phi, \qquad S_1 = \int_0^{2\pi} S(\phi)S(\pi+\phi)d\phi.$$

We shall characterize the propagation direction of the sound wave by the angle θ_j between the normal to its front and the z-axis and by the azimuth ϕ_a ($\cos\phi_a = \vec{q}\nabla x/q$). Then $q = (\omega/c_j)\sin\theta_j$; and $d\vec{q} = qdad\phi_a = (\omega^2/c_j^2)\cos\theta_j\sin\theta_j d\theta_j d\phi_a$. The acoustic anerdy flux into the solid angle $d\Omega_j = \sin\theta_j d\theta_j d\phi_a$ in the frequency banc $d\omega$ is determined by the expression $I_j d\Omega_j d\omega = \rho_j P_j^2(d\vec{q}d\omega/2c_j)$. With (4), (17), (21), and (26), we obtain

$$I_j = \frac{S_1}{16S_0^2}\frac{\rho_{j0}(0)g^2}{c_j^3}\psi_j\omega^3\Phi^2\left(\frac{\omega}{2}\right), \quad (27)$$

where

$$\psi_1(\theta_1) = |\sin^2\theta_1 - \sin^2\theta_i|/\cos\theta_1; \qquad \psi_2(\theta_2) = \cos\theta_2. \quad (28)$$

Following [6], we take the wave frequency spectrum in the following form for numerical calculations of the intensity of the radiated sound:

$$\Phi(\omega) = (\pi M/2\omega^6)\exp(-2g^2/\omega^2V^2), \quad (29)$$

where V is the wind velocity and $M = 3.05 \cdot 10^4 cm^2/sec^5$. Substitution of (29) into (27) gives

$$I_j = 2^{-6}\pi^{-7}M^2\frac{S_1}{S_0^2}\rho_{j0}(0)\frac{g^2}{c_j^3}\psi_j f^{-9}\exp\left(-\frac{4g^2}{\pi^2 f^2 V^2}\right). \quad (30)$$

The maximum value of I_j is reached at the frequency $f_m = (2\sqrt{2}/3\pi)(g/V)$:

$$I_{j\,max} = 2.2 \cdot 10^7 (S_1/S_0^2)\rho_{j0}(0)(g^2\psi_j/c_j^3)(V/g)^9. \quad (31)$$

We note that if we calculated I_j by the scheme described in [6], we would obtain an expression analogous to (30) with $\psi_1 = \cos\theta_1$. In our case, ψ_1 becomes larger than $\cos\theta_1$, beginning at angles $\theta_1 \simeq 46^\circ$, and increases rapidly with increasing θ_1. This results in a substantial increase in the intensity of the sound radiated into the atmosphere for glancing angles and, consequently, increases the possible importance of this mechanism for explanation of the observed (see, for example, [8]) atmospheric infrasonic noise due to storm regions in the ocean.

Let us also note that somewhat different frequency characteristics are obtained for the radiated sound if we work from another representation of the wave frequency spectrum. Thus, for example, Phillips' spectrum for the equilibrium frequency range is widely known in oceanology [9]. We shall use a modification of the Phillips' spectrum proposed in [10], which is valid for lower frequencies:

$$\Phi(\omega) = 8.1 \cdot 10^{-3} g^2 \omega^{-5} \exp[-0.74(\omega_0/\omega)^4], \quad (32)$$

where ω_0 is the characteristic frequency of the surface waves. Selecting ω_0 such that the maximum values of (29) and (32) will coincide, we obtain $\omega_0 \simeq 0.93g/V$. Substitution of (32) into (27) yields

$$I_j = 3.3 \cdot 10^{-5}\pi^{-7}\frac{S_1}{S_0^2}\rho_{j0}(0)\frac{g^6}{c_j^3}\psi_j f^{-7}\exp\left[-1.48\left(\frac{\omega_0}{\pi f}\right)^4\right]. \quad (33)$$

Here the maximum of I_j corresponds to the frequency $f_m = 0.28g/V$ and equals

$$I_{j\,max} = 1.5 \cdot 10^{-6}(S_1/S_0^2)\rho_{j0}(0)(g^6\psi_j/c_j^3)(V/g)^7. \quad (34)$$

Comparison of (33)-(34) with (30)-(31), at a wind velocity $V = 10m/sec$, for example, indicates that a low frequencies (on the order of a fraction of a Hertz), surface waves with a spectrum in the form of (29) lead to much larger values of the radiated acoustic energy, so that we have the ratio

$$\frac{(I_{j\,max})_{(31)}}{(I_{j\,max})_{(34)}} \simeq 15.$$

At high frequencies, e.g., $f = 10Hz$, the opposite is observed:

$$\frac{(I_j)_{(30)}}{(I_j)_{(33)}} \simeq 27.$$

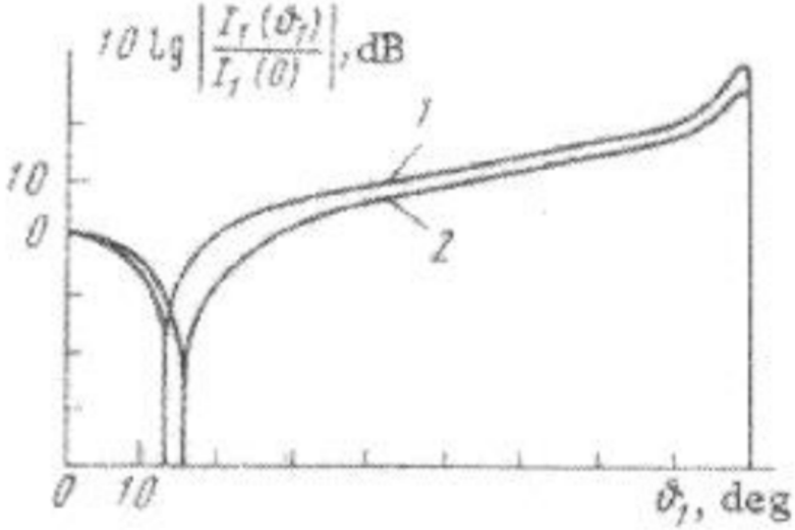

Fig. 1: Flux of acoustic energy radiated into the atmosphere vs. direction of radiation : 1) calculation by approximate relation (30); 2) computer calculation for $f = 0.125Hz$.

To supplement the above analytical analysis of equation system (9), the latter was solved exactly on a BESM-6 computer, so that the range of validity of the approximate relations (27)-(31) can be estimated. As we have already noted, the error of the approximate relationships decreases with increasing frequency of the radiated sound. Curve 1 on the figure was plotted from (27), using (28) as an expression for ψ_1. The curve for $f = 1Hz$ that was obtained by the exact calculation is indistinguishable from curve 1. Curve 2 was plotted from the exact calculations for $f = 0.125Hz$. This time the deviation from curve 1 is noticeable, especially in the angle range around $\theta = \theta_i$. However, the disagreement in this range could be reduced substantially if curve 1 were plotted with ψ_1 calculated with the series expansion in powers of $\theta_1 - \theta_i$ (see the first relation in (22)).

We note that the frequency dependence of the radiated sound intensity changes as $\theta_1 \to \theta_i$. In fact, when $\theta_1 = \theta_i$ we have from (22) $|\mu_1| \sim \omega^2 \delta_1 \sim \omega$.

We then obtain $\psi_1 \sim \omega^{-2}$ in (30), so that $I_1 \sim f^{-11}$ instead of f^{-9} at θ_1, far from θ_i.

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