Wave 'Set-Down' and Set-Up

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'Set-down' and set-up, the negative and positive changes in mean water level due to the presence of a train of surface waves, was measured in a wave channel. Well outside the break point the experimental results are in good agreement with the theoretical relationship determined by Longuet-Higgins and Stewart. Near the break point, where the wave height is greater than predicted by first-order wave theory, the measured 'set-down' was consistently less than theory would predict from the deep water wave height. Inside the break point the bore height was found to be a linear function of the mean water depth. In this region, the gradient of the set-up, $d\bar{\eta}/dx$, was related to the beach slope $\tan\beta$ and the mean ratio of wave height to water depth $\bar{\gamma}$ by the equation $d\bar{\eta}/dx = -[1 + (8/3\bar{\gamma}^2)]^{-1} \tan\beta$.

INTRODUCTION

Although it has been known for some time that the configuration of the mean water level near the shore must be a function of the incoming wave field, relatively few measurements of this effect have been made. Isolated observations have shown that set-up does occur in the surf zone, but these observations are not sufficient to provide quantitative trends [*Savage*, 1957; *Fairchild*, 1958; *Dorrestein*, 1961; *Galvin, and Eagleson*, 1965].

A comprehensive series of laboratory observations by *Saville* [1961] has produced the best data available. Unfortunately, Saville was mainly interested in the conditions under which the waves overtop a sea wall; therefore, only a few of the data describe the simple case where the motion is confined to the beach face. In Saville's experiments the changes of mean pressure in the bottom boundary layer were measured at a few points, from just outside the breakers to the still water line. The pressures were converted to elevations, and an approximate profile of the mean sea level was plotted for various values of deep water wave height, wave period, and beach slope.

Recently, the changes in mean sea level near the shore have been the object of considerable theoretical interest. By considering the conservation of momentum flux in a normally incident wave train, *Longuet-Higgins and Stewart* [1960, 1962, 1963, 1964], *Whitham* [1962], and *Lundgren* [1963] derived theoretical expressions for the changes in sea level. These theoretical models predicted a lowering of the water level, 'set-down,' as the waves approach the break point and a steady rise in sea level, set-up, shoreward of the breakers, in general agreement with the observations.

Although Saville's measurements tend to confirm the applicability of the theoretical analysis to the description of the real physical situation in the nearshore region, they are not sufficiently extensive to check the quantitative agreement with the theory; for example, all the measurements of 'set-down' were made near the break point where the theory is least likely to give accurate predictions. Consequently, a series of experiments were conducted in the hydraulics laboratory of the Scripps Institution of Oceanography to examine in detail the suggestions made by *Longuet-Higgins and Stewart* [1962, 1963, 1964]. In particular, measurements of both 'set-down' and wave height were made (1) well outside the surf zone where the assumptions of the theory should be most applicable, (2) near the breaker point, where systematic departures from the theory might reasonably be expected, and (3) in the surf zone, to examine the linearity of the set-up and the relationship between the wave amplitude and the mean depth.

This study was a necessary first step in an investigation of the cause and spacing of rip currents. A subsequent three-dimensional investigation showed that the position of rip currents was determined by the longshore gradient in set-up on the beach. The gradient is caused by alternating zones of high and low breakers associated with the interaction of waves from deep water and of edge waves traveling along the shore [*Bowen*, 1967].

Theoretical Considerations

Longuet-Higgins and Stewart [1964] described some of the theoretical second-order effects of surface gravity waves in terms of a radiation stress, where the radiation stress is defined as the excess flow of momentum due to the presence of the waves. One of the phenomena discussed was the change in mean sea level η that occurs when water waves encounter a sloping beach.

If the velocity potential of the waves is given locally by

$$\phi = \frac{H\sigma}{2k} \cdot \frac{\cosh k(z + h)}{\sinh kh} \cdot \cos (kx + \sigma t) \qquad (1)$$

where H is the wave height, h is the still water depth, $k = 2\pi/L$ is the wave number, L is the wavelength, $\sigma = 2\pi/T$ is the radial frequency, and x and z are the horizontal and vertical coordinates. Then the x component of the radiation stress is given by

$$S_{xx} = E\left(\frac{2kh}{\sinh 2kh} + \frac{1}{2}\right) \qquad E = \tfrac{1}{8}\rho g H^2 \qquad (2)$$

where E is the wave energy per unit surface area. In shallow water this equation becomes

$$S_{xx} = \tfrac{3}{2} \cdot E = \tfrac{3}{16}\rho g H^2 \qquad (3)$$

In the steady state, the shoreward flux of momentum must be independent of x, the perpendicular to the shore. Momentum balance then gives

$$\frac{dS_{xx}}{dx} + \rho g(\bar{\eta} + h)\frac{d\bar{\eta}}{dx} = 0 \qquad (4)$$

If the beach slope is sufficiently small and the wave reflection is negligible, two distinct regions can be considered, one seaward and one shoreward of the break point.

Seaward of the breakers, wave energy is approximately conserved:

$$E \cdot Cn = \text{constant} \qquad (5)$$

where Cn is the group velocity of the wave train. *Longuet-Higgins and Stewart* [1962] showed that, using (5), equation 4 could be integrated to give

$$\bar{\eta} = -\frac{1}{8}\frac{H^2 k}{\sinh 2kh} \qquad (6)$$

where $\bar{\eta}$, the difference between the still water level and the mean sea level in the presence of waves, is always negative seaward of the break point.

Using (6), we can express $\bar{\eta}$ as a function of the local depth h and the deep water wave height H_0 and wave number k_0:

$$\bar{\eta} = -\tfrac{1}{8}H_0^2 k_0 \frac{\coth^2 kh}{2kh + \sinh 2kh} \qquad (7)$$

Since

$$kh \tanh kh = \sigma^2 h/g = k_0 h \qquad (8)$$

then

$$\bar{\eta} = -\tfrac{1}{4}H_0^2 k_0 f(k_0 h) \qquad (9)$$

where f, a function only of the nondimensional depth $k_0 h$, is shown in Figure 1. As the depth decreases, the mean water level is lowered by the presence of unbroken waves; there is a 'set-down' because the radiation stress increases steadily if no energy is dissipated.

Inside the break point the wave energy decreases shoreward, leading to a decrease in the radiation stress. Using similarity arguments, we

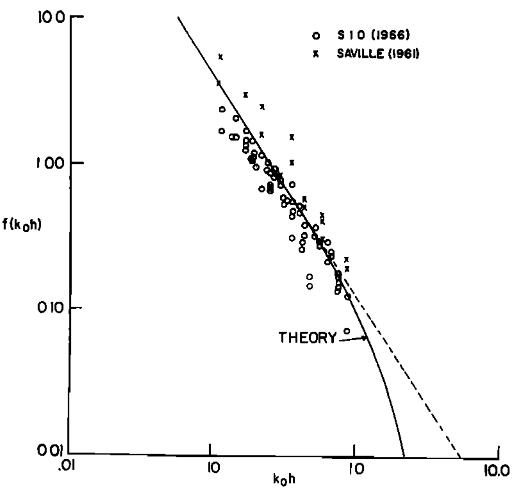

Fig. 1. 'Set-down' outside the break point expressed as a function of the nondimensional depth $k_0 h$. Solid line is from equation 9.

can assume that the height of the broken wave, or bore, remains an approximately constant proportion of the mean water depth

$$H = \gamma(\bar{\eta} + h) \qquad (10)$$

Although the waves are now far too steep for the second-order theory to remain valid, it is convenient to assume that $S_{xx} = (3/2)E$ [*Longuet-Higgins and Stewart,* 1964]. This gives

$$S_{xx} = \tfrac{3}{16}\rho g \gamma^2(\bar{\eta} + h)^2 \qquad (11)$$

Then from (4) and (11)

$$\frac{d\bar{\eta}}{dx} = -K\frac{dh}{dx} \qquad K = \frac{1}{1 + (8/3\gamma^2)} \qquad (12)$$

Thus, for a plane beach where $h = x \tan \beta$, the slope of the mean sea level should be constant and proportional to the beach slope.

Experimental Procedure

The measurements were made in the wave flume in the hydraulics laboratory. The flume is 40 meters long, 0.5 meter wide, and 0.75 meter deep and has glass walls. All the experiments were made on a smooth wooden beach with a 1:12 slope ($\tan \beta = 0.082$). Measurements of wave height were made visually using a Neyrpic-pointer gage with a maximum resolution of 0.01 cm. Observations were found to be reproducible to within ±0.05 cm outside the surf zone. Inside the break point the measurement of wave height is difficult and rather subjective; two people made separate measurements close to the break point, the results were accepted if agreement was within ±0.10 cm. Further inshore the bore was well developed and observations could again be made to ±0.05 cm.

The 'set-down' and set-up were measured by using a 3-mm ID glass tube connected by tygon tubing to a sensitive manometer. The glass tube was held normal to, and flush with, the beach, so that the opening might be confined to the bottom boundary layer. This tube was sufficiently narrow in relation to the manometer tube for no further damping to be necessary. In effect, the system measured the mean static pressure at the bottom of the flow. The assumptions involved in translating this measurement into surface elevation have been considered in detail by *Longuet-Higgins and Stewart* [1962]. The maximum resolution of the manometer was 0.005 cm. Measurements were reproducible to ±0.01 cm outside the surf zone and to ±0.02 cm between the break point and the mean run-up. It was found that the glass tube had to be kept very close to the bottom, or the opening would be partly above the boundary layer and a dynamic pressure rather than the static pressure would be recorded. If the opening was partly above the boundary layer, a larger pressure loss would occur, that is, an apparently greater 'set-down.' This may be the reason that Saville's experiments gave a larger 'set-down' than is predicted by the theory.

A summary of the experiments run in the hydraulics laboratory is given in Table 1, which shows the input wave conditions and some of the results. Profiles of wave height and 'set-down' were made for a region extending from well outside the surf zone to the point of mean run up on the beach for all the experiments except 24/17 and 24/20, for which the set-up was measured only inside the surf zone.

Experimental Results

The results of a typical experiment are shown in Figure 2. In the region well outside the break point the agreement between the measured and calculated values of the 'set-down' is excellent. This general agreement can also be seen in Figure 1, where the measured values of $f(k_0h)$, given by (9) as $f(k_0h) = -4\bar{\eta}/H_0^2k_0$ are plotted against k_0h for all the experiments except 24/17 and 24/20.

The theoretical curve from equation 9 is also plotted in Figure 1 with the data from Table 1 of Saville, which were analyzed by *Longuet-Higgins and Stewart* [1963]. Only the results from outside the break point are shown.

The results from the experiments in the hydraulics laboratory are all in good agreement with the theory. Some of the scatter is due to the fact that the 'set-down' associated with the reflected wave was not always negligible.

As the wave approaches the beach, the observed wave height becomes larger than that predicted by first-order wave theory and the wave profile is no longer sinusoidal. Theoretically, the 'set-down' should increase rapidly in this region; in fact, the measured values all show a rather small increase and then a general flattening as the waves reach the break point.

TABLE 1. Laboratory Measurements of Wave Set-Up, 'Set-Down,' and Run-Up

Experiment	Wave Period T, sec	Wave Length L_0, cm	Wave Height H_0, cm	Breaker Height H_b, cm	Still Water Depth at Break h_b, cm	Width of Surf Zone x_b, cm	'Set-Down' at Break $\bar{\eta}_b$, cm	Maximum Set-Up $\bar{\eta}_{max}$, cm	K	$\bar{\gamma}$	Observed Run-Up R, cm	Theoretical Run-Up $H_0 \tan \beta \cdot (H_0/L_0)^{-1/2}$, cm
71/3	0.82	105	3.60	4.40	4.15	75	−0.17	1.48	0.265	0.90	1.70	1.61
71/4	0.82	105	5.15	5.90	5.5	85	−0.19	1.60	0.255	0.88	1.84	1.93
51/4	1.14	202	4.20	6.60	5.0	85	−0.19	2.07	0.320	1.11	2.32	2.42
51/6	1.14	202	6.45	8.55	6.8	115	−0.32	2.95	0.345	1.15	3.25	3.00
51/8	1.14	202	9.00	10.60	9.7	155	−0.47	3.30	0.295	1.00	3.70	3.54
35/7	1.65	424	4.25	7.75	5.9	110	−0.18	3.37	0.390	1.22	3.66	3.53
35/10	1.65	424	5.85	9.65	6.8	125	−0.25	3.70	0.380	1.19	4.23	4.11
35/12	1.65	424	7.10	11.45	9.5	160	−0.26	4.15	0.310	1.17	4.75	4.56
35/15	1.65	424	8.90	13.00	9.7	165	−0.43	4.65	0.370	1.17	5.20	5.10
24/17	2.37	876	6.20	11.80	8.8	160	−0.30	4.50	0.365	1.24	6.17	6.10
24/20	2.37	876	7.50	12.70	9.2	170	−0.38	5.28	0.400	1.28	6.60	6.73

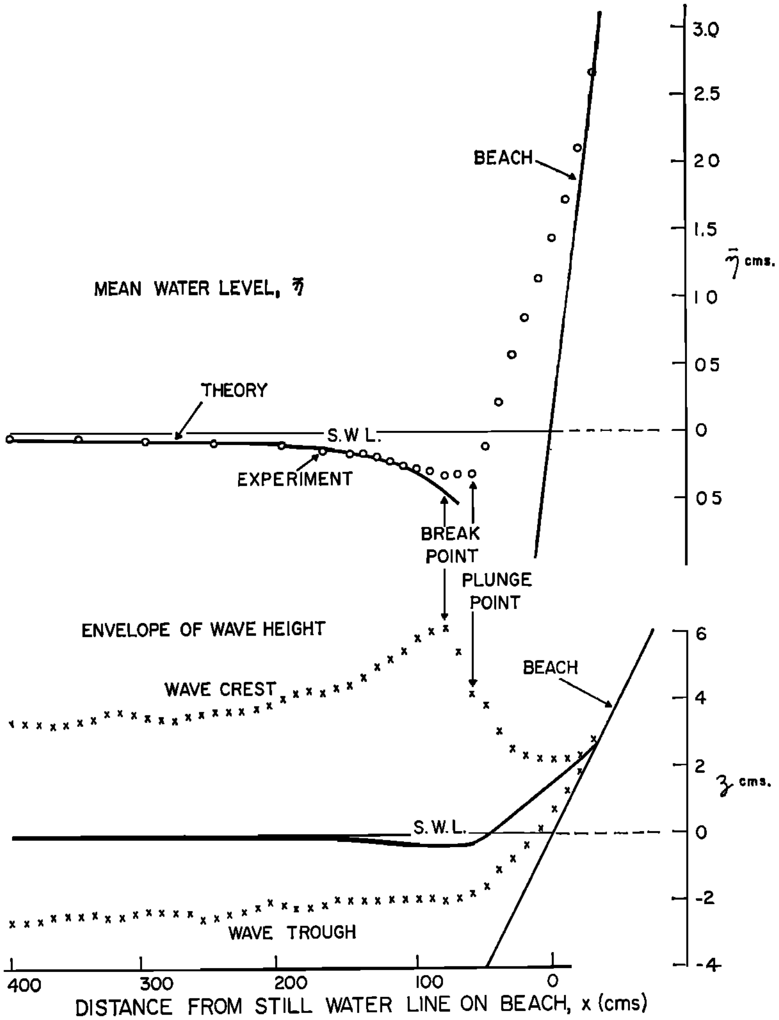

Fig. 2. Profile of the mean water level and the envelope of the wave height for a typical experiment. Theoretical plot is from equation 7. Wave period, 1.14 sec; $H_0 = 6.45$ cm; $H_b = 8.55$ cm; $\tan \beta = 0.082$.

As might be expected from (7), the difference between the observed and the theoretical 'set-down' is well correlated with the difference between the observed wave height and the wave height predicted by first-order theory.

Close to the break point the 'set-down' must be influenced by the fact that the solutions inside and outside the breakers must be patched together in a reasonable way. Experimentally, it was found that the 'set-down' was rather constant between the point where the crest of the wave begins to curl over and the point where the whole wave form collapses. These two points are defined as the break point and the plunge point (Figure 2). Inshore from the plunge point a region of rebound was observed where the broken wave reformed and then moved up the beach as a regular bore. The rebound was associated with a rather rapid rise in the mean water level.

Further inshore, where the bore was well formed, the set-up increased steadily, the gradient of the set-up being approximately constant as the theoretical results suggest. The measurements showed that in this region the wave height tends to be a linear function of the mean water depth,

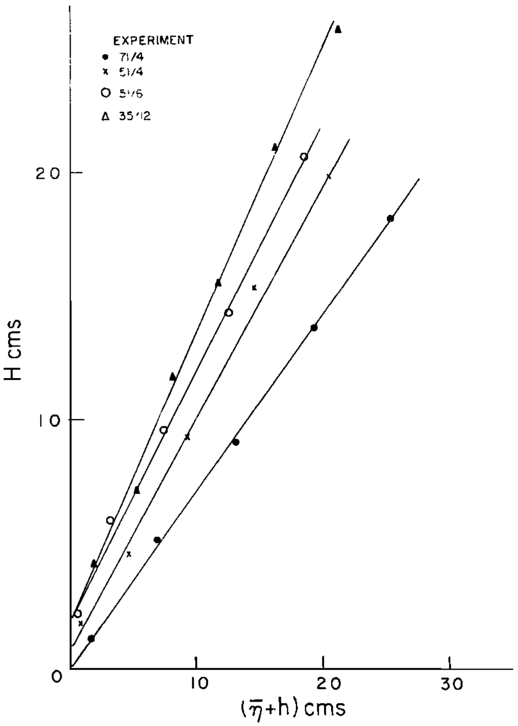

Fig. 3. Wave height inside the break point, near maximum set-up, as a function of the mean depth, showing linearity of the relation and the residual wave height at $\bar{\eta} + h = 0$.

$\bar{\eta} + h$ (Figure 3) as is assumed in equation 11. There was, however, always some residual wave height H_r at the beach, indicating that equation 10 should read $H = H_r + \gamma(\bar{\eta} + h)$. Then equation 12 becomes

$$\frac{d\bar{\eta}}{dx} = -\frac{\gamma^2(\bar{\eta} + h) + \gamma H_r}{(8/3 + \gamma^2)(\bar{\eta} + h) + \gamma H_r} \cdot \frac{dh}{dx}$$

so that, as $\bar{\eta} + h \rightarrow 0$, $d\bar{\eta}/dx \rightarrow - dh/dx$.

This suggests that very close to the shore the set-up should steepen, becoming tangential to the beach as $\bar{\eta} + h \rightarrow 0$. Perhaps surprisingly, this tendency can be seen quite clearly in several of the experiments (Figure 4). H_r is the vertical distance between the beach and the crest of the bore 'swashing' across the beach face that has just been 'dry.' Surface tension will tend to inhibit wave break in this region, but the long wavelength suggests that the dominant effects are viscous.

Equation 12, or its modified form (equation 13), seems to provide an excellent qualitative

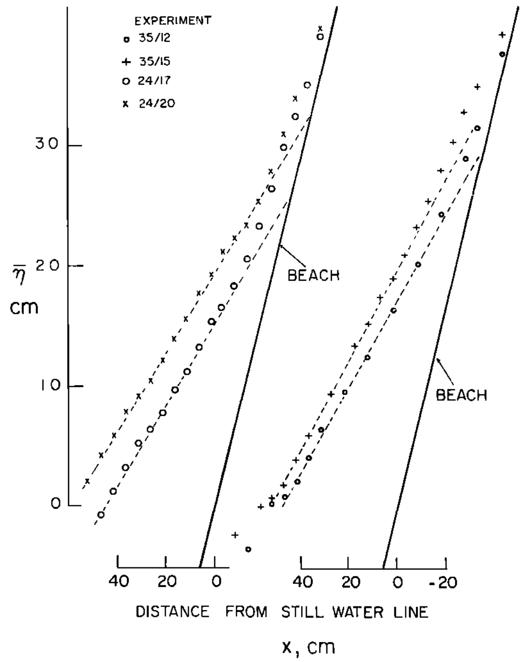

Fig. 4. Set-up on the experimental beach, showing the linearity of the set-up and the slope of the set-up becoming tangential to the beach slope as $\bar{\eta} + h \rightarrow 0$.

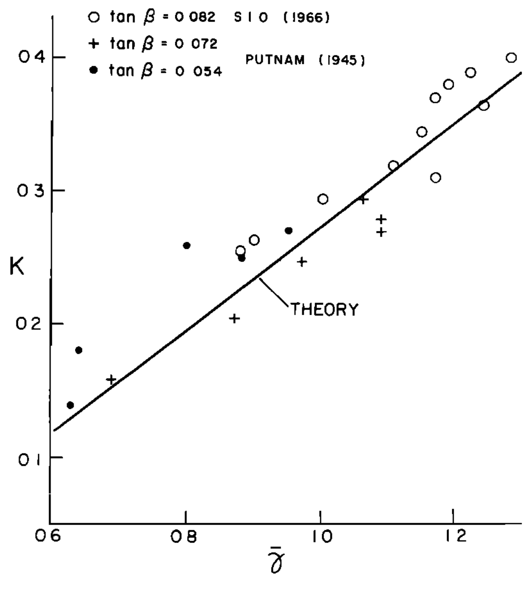

Fig. 5. The ratio K (set-up slope to beach slope) as a function of $\bar{\gamma}$, showing good agreement between experiment and theory as expressed by equation 12.

prediction of the set-up. The quantitative nature of the agreement was examined by plotting K, the ratio of the set-up slope to the beach slope, against $\bar{\gamma}$, the mean of the observed values of γ across the surf zone. From (12)

$$K = -\frac{d\bar{\eta}}{dx}\left[\frac{dh}{dx}\right]^{-1} = -\frac{1}{\tan\beta}\frac{d\bar{\eta}}{dx} \quad (13)$$

The value of K was taken as

$$\frac{1}{\tan\beta}(\bar{\eta}_{\max} - \bar{\eta}_b)/x_b$$

where $\bar{\eta}_b$ is the 'set-down' at the breakers, $\bar{\eta}_{\max}$ is the maximum set-up on the beach, and x_b is the width of the surf zone from break point to mean run-up (Table 1). Using this definition of K, we

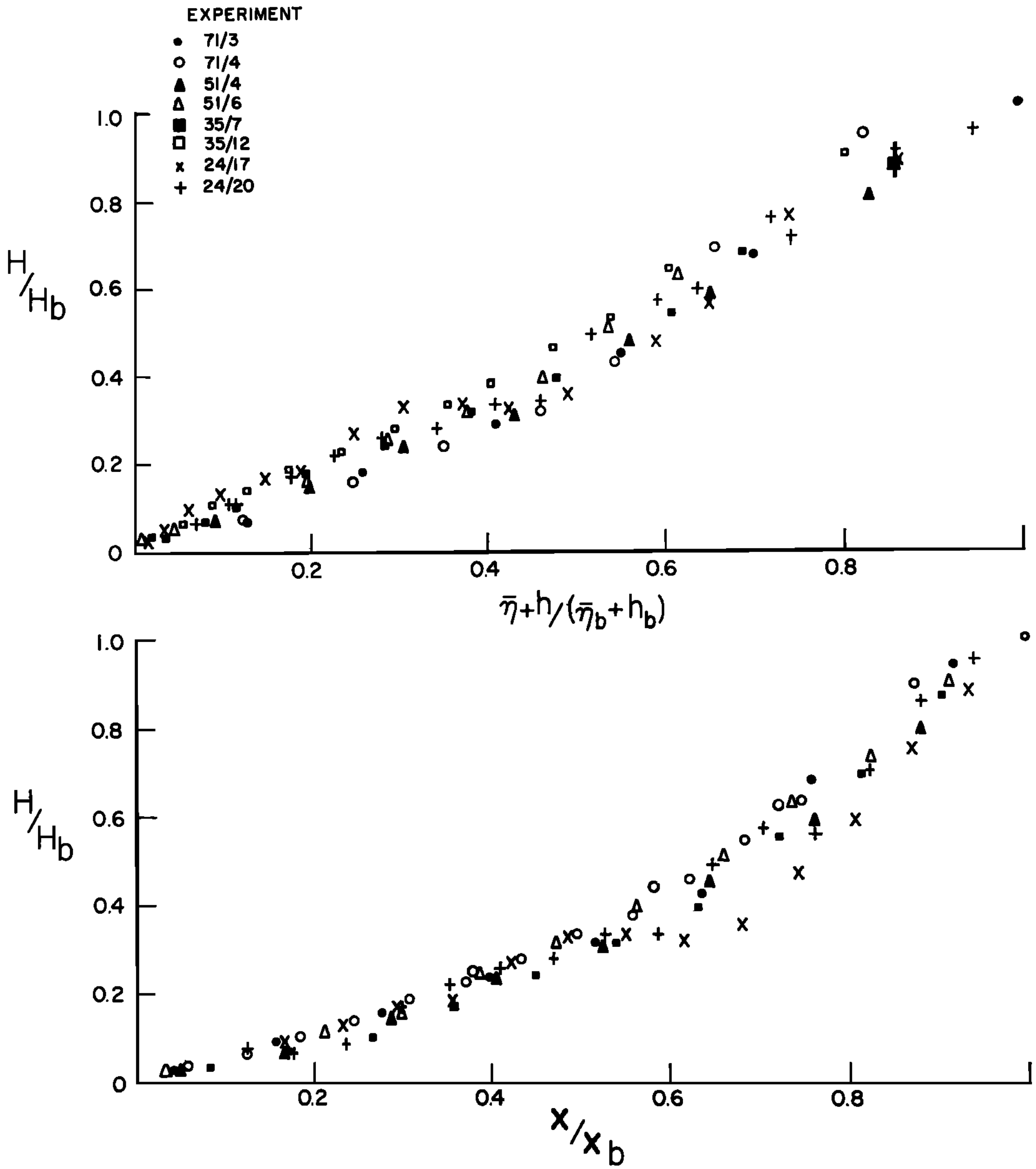

Fig. 6. Nondimensional plots of wave height H/H_b as a function of the depth and horizontal position, suggesting the geometrical similarity assumed in equation 10.

were able to analyze some of *Putnam's* [1945] data that had been taken on different beach slopes, and therefore we were able to extend the generality of the results (Figure 5). The theoretical curve is given from equation 12 as

$$K = \frac{1}{1 + (8/3\bar{\gamma}^2)}$$

In view of the fact that this analysis includes the regions of plunge and rebound, where $H/\bar{\eta} + h$ tends to increase and conditions are generally complex, the agreement between the measured and computed values is quite good.

The basis for this analysis of set-up is the similarity argument used to obtain equation 10; the measurements show that once the bore is well established this is indeed a reasonable assumption (Figure 3). To examine the similarity of the surf zone as a whole, a nondimensional wave height H/H_b, obtained by dividing the local wave height by the breaker height, was plotted against the nondimensional depth $(\bar{\eta} + h)/(\bar{\eta} + h)_b$ and the nondimensional horizontal position x/x_b for each experiment. The results are shown in Figure 6. The experimental points all lie along approximately the same line, suggesting the assumption that the flows are geometrically similar is certainly reasonable.

It is interesting to note that, although the gradient of the set-up is small (being less than the beach slope, which is assumed small), the maximum set-up that occurs when the gradient is integrated over a considerable distance need not be small. In fact, in all the experiments the maximum set-up at the beach was of the order of the wave amplitude (Table 1). Close to the shore line where the depth below the still water level h tends to zero, the set-up becomes the dominant contributor to the mean depth and the important depth parameter in this region is $\bar{\eta} + h$. This means that each wave runs upon the set-up due to the presence of the wave train.

Although the maximum set up on the beach forms a major part of the total run-up R (Table 1), the observed run-up is in reasonably good agreement with the empirical theory of *Hunt* [1959], who suggested that

$$R = C_p \cdot H_0 \tan \beta \cdot (H_0/L_0)^{-1/2}$$

where C_p, a porosity factor, is approximately 1 for solid beaches.

Discussion

Well outside the break point the agreement between the measured values and the values computed from equation 9 is very good. The 'set-down' can be accurately described knowing only the deep water wave height, the wave period, and the local depth.

Just outside the break point the assumptions of small-amplitude Airy wave theory are no longer justified, and here the measured values of 'set-down' are consistently less than the theory would predict. This is in contrast to the data of *Saville* [1961], which suggest that the 'set-down' should be rather larger than the theoretical value. (Figure 1).

In the surf zone the theory was based on the assumptions that led to equations 10 and 11. The measurements show that an assumption of similarity in the surf is certainly reasonable (Figure 6) and that, inside the plunge point, equation 10 describes the wave height rather well. It is surprising, however, that the set-up inside the surf zone should be described so well by equation 12 and 13, because it is not obvious why the assumption that $S_{xx} = (3/16)\rho g H^2$ in this region should

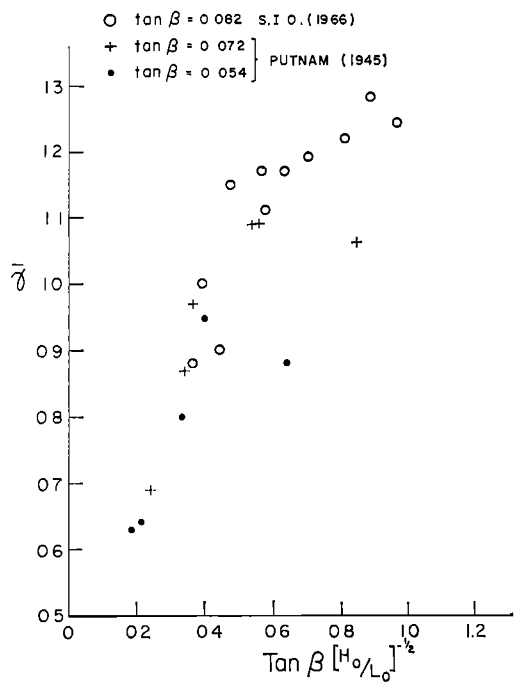

Fig. 7. The mean of the ratio of wave height to water depth in the surf zone $\bar{\gamma}$ as a function of β, H_0, and L_0.

be a good approximation. The waves in the surf zone are actually too steep to be described by the Airy wave theory from which this formula is derived. Unfortunately, the quantitative predictions of the theory depend on the value of γ or $\bar{\gamma}$. This value was measured directly in the experiments, but it is not accurately predictable in terms of the deep water conditions. The trend of $\bar{\gamma}$ as a $f(H_0, L_0, \tan \beta)$ is shown in Figure 7. The parameter $\tan \beta (H_0/L_0)^{-1/2}$, which occurs in the criterion for the breaking of a wave on the beach and also in the empirical relation for the vertical run up R, seems to be particularly relevant to the description of conditions in the surf zone.

Acknowledgments. This study was supported by the Coastal Engineering Research Center, U. S. Army Corps of Engineers, and by the Office of Naval Research under contract with the University of California.

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(Received October 24, 1967.)