On the Balance Between Growth and Dissipation in an Extreme Depth-Limited Wind-Sea in the Southern North Sea

E. BOUWS AND G. J. KOMEN

Royal Netherlands Meteorological Institute, De Bilt, The Netherlands

(Manuscript received 5 October 1982, in final form 28 February 1983)

ABSTRACT

Bouws has given a discussion of a severe storm in the southern North Sea, on 3 January 1976 near Texel, one of the Friesian islands. This storm was characterized by a remarkable steadiness of wind and wave parameters. The steadiness of the wave parameters was apparently the result of depth limitations, which prevented further wave evolution. Here the various terms in the energy balance equation for wave growth in shallow water are estimated. The relative importance of wind input, surface dissipation, bottom dissipation, advection and nonlinear transfer is discussed. For a certain choice of dissipation parameters, a good balance can be obtained. This is in agreement with the steadiness of the observed wave conditions.

1. Introduction

Recent interest in the modeling of wind waves on shallow water (Vincent, 1982; Sanders and Bruinsma, 1983) has shown that our knowledge of the various processes contributing to the evolution of the wave spectrum is still fragmentary. As in deep water, wind input, nonlinear transfer, dissipation and advection are important, but, in addition, bottom and tidal effects can come into play. Here we will study a unique set of wave measurements with the purpose of learning more about the relative importance of these different physical processes. Some of them are relatively well-known, such as the nonlinear transfer which can be calculated from first principles (Hasselmann and Hasselmann, 1981) and the wind input which has been carefully studied in the Bight of Abaco experiment (Snyder *et al.*, 1981). Much less is known about the dissipative terms. In fact, an important motivation for the present study was our desire to learn more about these dissipative terms.

We use results obtained by Bouws (1980) in his analysis of an extreme storm in the North Sea. The storm center was located between the Dogger Bank and the Southern Bight, where bottom depths vary around 35 m. During this storm, wind speed ($U_{10} \approx 25$ m s^{-1}) and direction ($\sim 300°$, on shore) were quite constant. The measurements were made with a waverider near the Lightship *Texel*, 25 km west of the Dutch coast. At this location the tidal range is 1.25 m and tidal current speeds remain below 1 m s^{-1}. Moreover, the tidal currents are alternating between 210 and 20°, almost normal to the mean wave direction. Tidal effects on our analysis are therefore negligible. The significant wave height H_s increased for some time, until it reached an average maximum level of 6.8 m (see Fig. 1). A wave spectrum developed that on the average was well described by the JONSWAP spectral shape (Hasselmann *et al.*, 1973) with peak frequency $f_m = 0.086$ Hz, Phillips constant $\alpha = 0.01$, and a peak enhancement factor $\gamma = 2$. The width of the enhanced peak was not very stable. Here we have taken an average JONSWAP width $\sigma_a = \sigma_b = 0.08$. The "*Texel* spectrum" so defined is shown in Fig. 2, together with the mean of 16 spectra, during the fully grown stationary situation. It should be noted that the JONSWAP spectrum is defined sufficiently well in the range $0.8f_m < f < 2f_m$ only. Therefore, we will restrict our considerations to this interval in the following. Kitaigorodskii *et al.* (1975) have suggested that in extreme shallow-water situations the wind-sea spectrum has an f^{-3} saturation range rather than the deep-water f^{-5} dependence. The effect is important when $kd < 1$, with k the wavenumber and d the water depth. In the present case at the peak frequency $kd = 1.24$, so the Kitaigorodskii effect is not very strong. For a JONSWAP spectrum with five parameters, it is difficult to detect deviations from a f^{-5} law. Therefore we continue to use a JONSWAP spectrum. The reduced value of γ (2 instead of the mean JONSWAP value 3.3) may well reflect the Kitaigorodskii reduction.

2. Analysis

The *Texel* spectrum consisted of pure wind sea, in the sense that at all times the waves propagated at a slower speed than the wind speed. The evolution of the spectrum can be described by the energy-balance equation for the rate of change of the two-dimensional frequency spectrum:

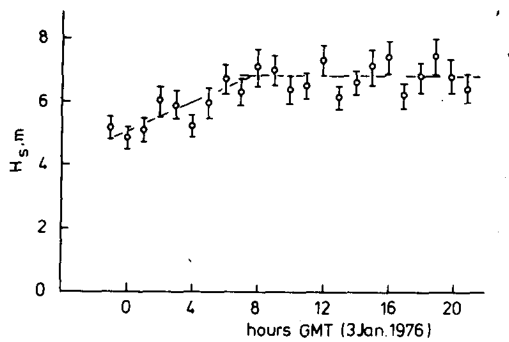

FIG. 1. Wave heights at Lightship *Texel*, with 80% confidence interval, calculated from the equivalent number of degrees of freedom of the spectra.

$$\frac{\partial F}{\partial t}(f, \theta) = S_{\text{in}} + S_{\text{nl}} + S_{\text{diss}} + S_{\text{bot}} + S_{\text{cur}} - \nabla \cdot (\mathbf{c}_g F). \tag{1}$$

Here f and θ denote wave frequency and direction, and $\mathbf{c}_g$ is the group velocity of waves with frequency f. During wave growth, the evolution was duration limited rather than fetch limited. Obviously in the fully grown, stationary situation, the six terms on the right (wind input, nonlinear transfer, dissipation, bottom effect, current interaction and advection) must cancel.

First let us summarize what is known about the individual source terms in (1). Most wave modelers take S_{in} of the form

$$S_{\text{in}} = \max\left\{0, B\omega\left[\frac{U_{10}\cos(\Phi - \theta)}{c} - 1\right]F(f, \theta)\right\}. \tag{2}$$

The constant B was measured by Snyder *et al.* (1981). They found $B = 3 \times 10^{-4}$. In (2) ω is the angular frequency $2\pi f$, U_{10} the wind speed (direction Φ) at 10 m height, and c the phase velocity of waves with frequency f in water of depth d. In deep water, in the conventional picture of wave growth, waves grow until saturation is reached, the short ones first, longer ones later. Finally, wave growth stops altogether when the wave speed exceeds the wind speed. In shallow water this stage is not reached because the influence of the bottom manifests itself. An exception are waves traveling at a sufficiently large angle to the wind direction. To estimate the total wind input at a given frequency (i.e., integrated over angles) we have to make an assumption about the angular dependence of F. Here, we assumed the empirical spreading function found for JONSWAP wind-sea spectra (D. E. Hasselmann *et al.*, 1980):

$$F(f, \theta) = I(p)^{-1}\cos^{2p}[(\theta - \bar{\theta})/2]F(f),$$

$$p = 9.77(f/f_m)^{\beta}, \quad \beta = \begin{cases} 4.06, & f < f_m \\ -2.34, & f \geq f_m \end{cases},$$

$$I(p) = 2^{1-2p}\pi(2p)!/(p!)^2,$$

where $\bar{\theta}$ is the mean direction of wave propagation. The resulting wind input for the *Texel* spectrum has been plotted in Fig. 3a.

The nonlinear transfer in shallow water has been calculated from first principles by Hasselmann and Hasselmann (1981). For the *Texel* spectrum, we obtained the transfer given in Fig. 3b. The main effect is a transfer of wave energy from the right of the spectral peak to low frequencies on the left of the peak. As a result, the effective growth on the forward face gets enhanced.

In the past, several attempts have been made to parametrize the deep water dissipation. Recently, Hasselmann and Hasselmann (1983) tried

$$S_{\text{diss}} = -c_0\left(\frac{E\omega_m^4}{g^2}\right)^6 \frac{\omega^4}{\omega_m^3}F(f, \theta), \tag{3}$$

with $c_0 = 1.2 \times 10^9$. Here E is the total variance, ω_m the peak frequency and g the gravitational acceleration. This expression was obtained in an attempt to simulate fetch-limited wave growth in deep water. However, it should be noted that the correct growth laws remain poorly known, as became evident from the work of the SWAMP group (1983). A few examples of duration-limited growth curves are given in Fig. 4. Drawn in are the deep- and shallow-water growth curves of the KNMI operational-wave-prediction model GONO, a deep-water growth curve from Kahma (1983) and a shallow-water growth curve based on the *CERC Shore Protection Manual* but modified so as to get a continuous transition to Kahma's curve (see Appendix), the deep-water growth curve of the numerical HYPA model, and the results

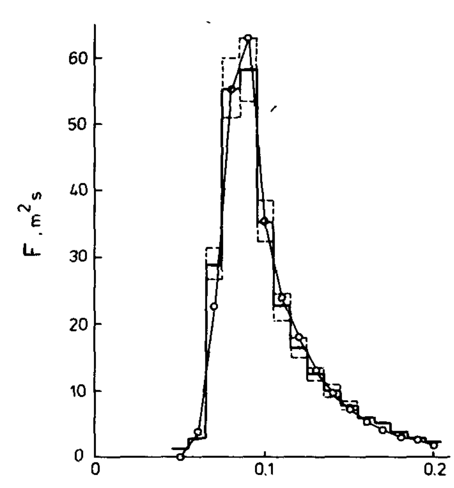

FIG. 2. The *Texel* spectrum, as composed of 16 spectra during the interval, with average maximum level of H_s = 6.8 m. Also shown are 80% confidence intervals and the approximation of the *Texel* spectrum by the JONSWAP spectrum with α = 0.01, f_m = 0.086 Hz, γ = 2 and $\sigma_a = \sigma_b$ = 0.08, for comparison with measured spectra, taking into account the frequency bin size of 0.01 Hz.

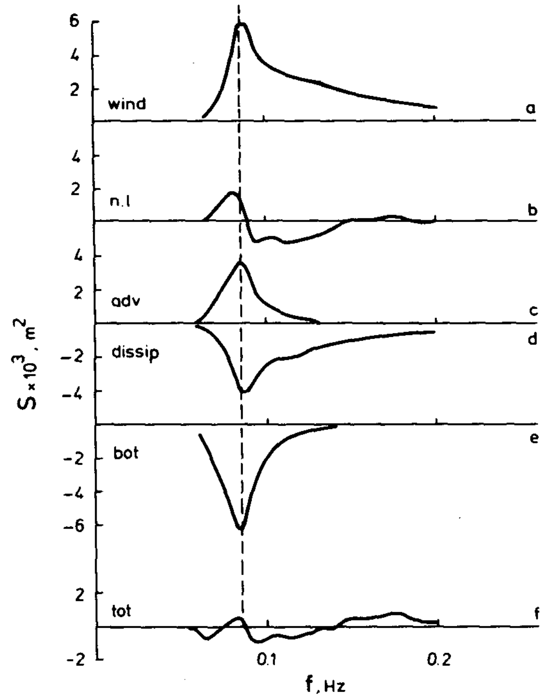

FIG. 3. Various source terms contributing to the energy balance in the fully grown depth-limited wind sea on 3 January 1976 near *Texel:* (a) wind input; (b) nonlinear energy transfer; (c) advection; (d) deep water dissipation; (e) bottom dissipation; (f) addition of source terms. Depth is 35 m and wind speed 25 m s^{-1}.

of the exact calculations of Hasselmann and Hasselmann (1983). The discrepancies are not well understood. It has been conjectured that they are related to the wind-field variability. Anyway, this type of approach to the dissipation has not been very rewarding. Therefore, we have chosen a different approach. This approach is based on an earlier paper of Hasselmann (1974). In this paper, he presented a theory of dissipation due to whitecapping, in which he found

$$S_{\rm diss} = -\eta\omega^2 F(f, \theta). \qquad (4)$$

The parameter η could not be calculated from first principles, but it was estimated from the assumption that for high frequencies a balance exists between wind input, whitecapping and nonlinear transfer. As a result he found

$$\eta = \omega_m^{-1}\{2.2 \times 10^{-4}[1 - 0.3g(\omega_m U)^{-1} + 2\alpha^2\lambda]\}, \qquad (5)$$

with α the Phillips constant, ω_m the peak frequency, and λ a factor between 0.12 and 0.14 (depending on the spectral shape). In the following, we will use (4), rather than (3), as it is based on a definite assumption about the underlying physics. Strictly speaking, Eq. (4) cannot be applied to the *Texel* spectrum, as it is derived for deep water. However, we have estimated the necessary corrections and we found that they are relatively small, of the order of $[1 - c_g(\omega, \infty)/c_g(\omega, d)]$ which is about 15%. We will neglect them. In Fig. 3d we have plotted $S_{\rm diss}$ for $\eta = 1.9 \times 10^{-4}$ s.

Next we have to consider the bottom influence. We do not attempt to interpret bottom dissipation in terms of a physical mechanism such as percolation, friction or bottom motion. Instead we represent the bottom influence on empirical grounds by

$$S_{\rm bot} = \frac{c_2}{g^2}\frac{\omega^2}{\sinh^2 kd}F(f, \theta). \qquad (6)$$

This is in agreement with the JONSWAP result for swell dissipation, as given by Hasselmann *et al.* (1973). They obtained $c_2 = 0.038$ m^2 s^{-3}. The frequency dependence of (6) is illustrated in Fig. 3e. Because of the hyperbolic sine in (6) the minimum is just below the peak frequency. Theoretically, Eq. (6) follows for either percolation or turbulent bottom friction (Shemdin *et al.*, 1978). Which mechanism actually dominates depends on the sediment mean grain size. This size is relatively small (0.125–0.25 mm) near the measurement site (de Reus, private communication) which suggests that turbulent bottom friction dominates. Unfortunately, bottom friction largely depends on the bottom roughness due to ripples—especially during storm events—in a rather changeable sediment, which makes it difficult to determine its magnitude from first principles. In fact, the work of Shemdin *et al.* (1978) has shown that the actual friction may vary by as much as an order of magnitude, depending on the precise bottom conditions.

The interaction of waves and currents can be neglected, because the phase speed of the energy-carrying waves is more than one order of magnitude greater than the current-speed component in the mean wave direction.

Advection is not negligible. This became clear from a hindcast of the *Texel* storm with the numerical wave-prediction model GONO (de Voogt *et al.*, 1983). Typically the gradient in significant wave height was about 1 m per 100 km. This gives for the total energy advection

$$\nabla \cdot \int df\, c_g F(f) \approx \langle c_g\rangle(\Delta E/\Delta L) \approx 10^{-4}\ {\rm m^2\ s^{-1}}, \qquad (7)$$

which is of the same magnitude as the other terms in the energy balance. The nonhomogeneity of the wave field is mainly the result of the depth variation (from 30 m depth at the actual measurement site to 45 m at 100 km to the northwest). Vincent (1982) and Rosenthal (private communication) have found that the depth dependence of the spectrum can be described by the so-called Kitaigorodskii factor

$$\Phi(kd) = \tanh^2 kd\left(1 + \frac{2kd}{\sinh 2kd}\right)^{-1} \qquad (8)$$

as follows:

$$F(f, \theta; d) = \Phi(kd)F(f, \theta; \infty). \qquad (9)$$

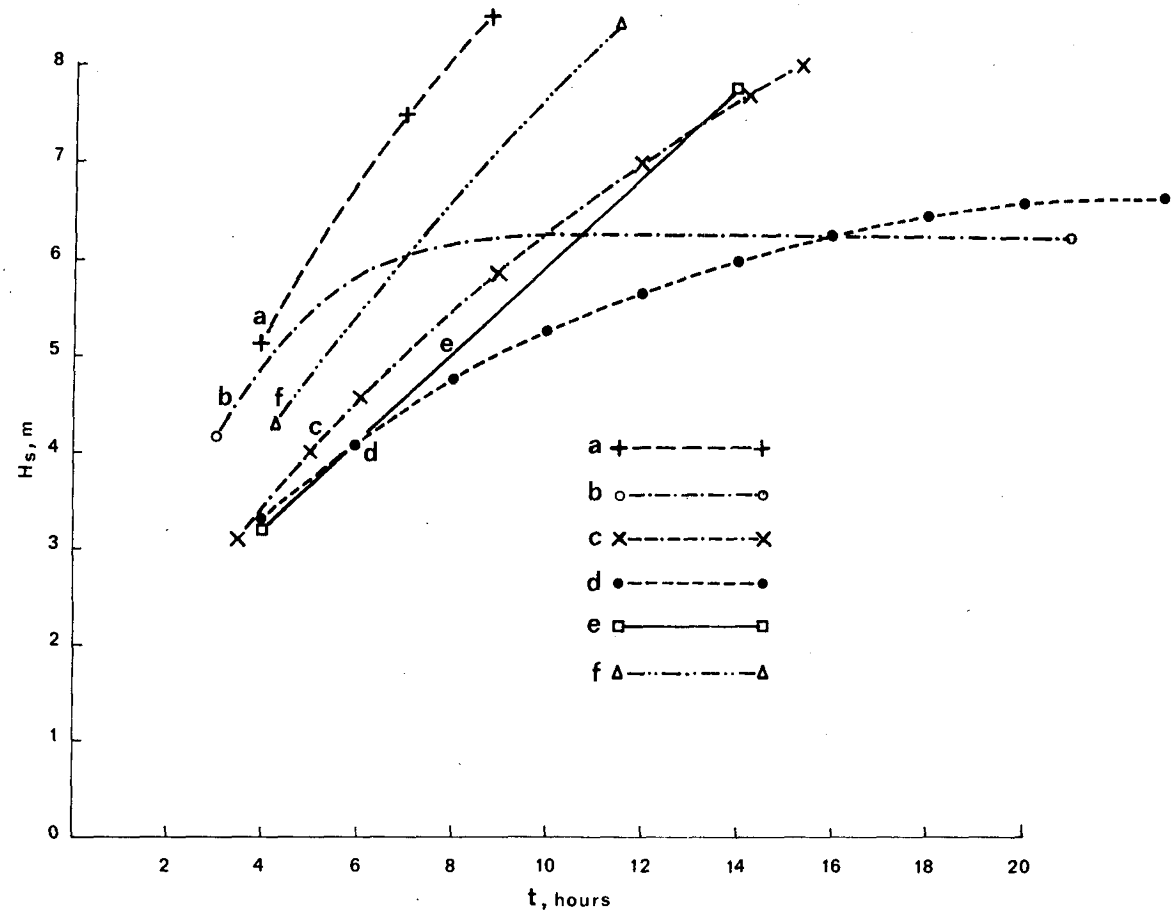

FIG. 4. Duration-limited growth curves for $U = 25$ m s^{-1}. (a) GONO deep water, (b) GONO, $d = 35$ m, (c) Kahma, deep water (d) CERC, modified, $d = 35$ m, (e) HYPA, (f) Hasselmann and Hasselmann. [For (c) and (d), see Appendix.]

This factor was introduced by Kitaigorodskii *et al.* (1975) to explain the high-frequency shape of shallow-water spectra. However, it turned out from the work of Vincent and Rosenthal that (9) is quite realistic over the full spectral range. Therefore, we used (9) to determine the advection as follows:

$$\nabla \cdot (\mathbf{c}_g F) = \frac{c_g}{\Delta L} [\Phi(kD_2) - \Phi(kD_1)]F, \qquad (10)$$

with $\Delta L = 10^5$ m, $D_1 = 30$ m and $D_2 = 45$ m. The result is given in Fig. 3c.

To study the combined effect of the six source terms, we first considered the balance of Eq. (1) with the constants η as given by Hasselmann (1974), and with c_2 equal to its JONSWAP value. For these values no equilibrium was found. Therefore we considered η and c_2 as free parameters. To fix their values we required that

$$\sum S_{\text{tot}}^2(f_i), \quad 0.8f_m < f_i < 2f_m \qquad (11)$$

be minimal and

$$\int_{0.8f_m}^{2f_m} S_{\text{tot}} df = 0. \qquad (12)$$

This gave the values

$$\left.\begin{aligned} \eta &= 1.9 \times 10^{-4}\ \text{s} \\ c_2 &= 0.067\ \text{m}^2\ \text{s}^{-3} \end{aligned}\right\}.$$

The value of η is about 60% of the value given by (5). This is remarkably close. Hasselmann's determination of (5) was based on the assumption that Miles's result can be extrapolated to $f > 3f_m$, an assumption for which no direct experimental evidence existed. On the other hand, our estimate assumes the validity of the whitecapping equation over the whole spectral range. The order of magnitude agreement between both determinations is a measure of the consistency as well as an indication of the accuracy to which dissipation can be determined at present. The value of c_2 is nearly twice as large as the mean JONSWAP value (0.038). This may look troublesome. However, it has been pointed out earlier that bottom friction has a large variability. Also the JONSWAP observations displayed a considerable scatter. A selection of 25% of the JONSWAP swell cases with relatively large swell energy flux would give a c_2 value equal to the value found in our analysis.

Fig. 3 shows the contribution of the various source terms as estimated for the *Texel* spectrum. They add up to S_{tot} which is also given. The cancellation is not perfect, but the deviation of S_{tot} from zero is so small that the induced change in the *Texel* spectrum would be well inside measurement errors. Finally, it is of some interest to note the net rate of change in energy due to the various source terms:

$$\int S_{in}df = 2.9 \times 10^{-4}\ \text{m}^2\ \text{s}^{-1},$$

$$\int S_{diss}df = 2.1 \times 10^{-4}\ \text{m}^2\ \text{s}^{-1},$$

$$\int S_{bot}df = 1.6 \times 10^{-4}\ \text{m}^2\ \text{s}^{-1},$$

$$\int \nabla\cdot(\mathbf{c}_g F)df = 0.9 \times 10^{-4}\ \text{m}^2\ \text{s}^{-1}.$$

The integrals all extend from $0.8f_m$ to $2f_m$.

3. Discussion

We have shown how the observed *Texel* spectrum can be used to study the energy-balance equation governing wave evolution. Our estimates have been fairly rough. It is difficult to quote errors. One error source is the natural variability of the spectral estimate, but this effect is quite small since we have averaged several observed spectra to obtain "the *Texel* spectrum." A more important error source is undoubtedly related to the parametrization of the dissipation, but at present it does not seem possible to quantify this error. An experimental confirmation of Eq. (4) would be most welcome. A third uncertainty comes from the accuracy to which the constant B in Eq. (2) is known. Snyder *et al.* (1981) quote an error of about 50%.

The dominant term was the wind input, but it is more than balanced by dissipation and bottom friction. For the dissipation we took an expression which takes whitecapping as the dominant physical mechanism. The magnitude of the constant η in Eq. (4) was found to be smaller than the value estimated by Hasselmann (1974), who studied the energy balance in the range $f > 3f_m$. This may indicate that viscous effects are important in that frequency range. His neglect of viscosity would lead to an overestimate of η. As to the value of the bottom-friction constant, we found a value which agrees well with the JONSWAP cases with high swell energy flux. Advection was found to be rather small although not quite negligible. Because of this relative smallness we could make a rough estimate, based on empirical Kitaigorodskii scaling. A striking feature was the relative smallness of the nonlinear transfer. The total (nonlinear) transfer of energy from the spectral range $0.8f_m < f < 2f_m$ to higher and lower frequencies is about 0.2×10^{-4} m^2 s^{-1}, an order of magnitude lower than the wind input. Also the transfer across the peak is relatively small. This is directly related to the smallness of γ as compared to the mean JONSWAP value. It has often been assumed that the nonlinear interaction is important in maintaining the spectral shape. True as this may be for young wind sea in deep water, this does not appear to be the case for the *Texel* spectrum.

Acknowledgments. We thank Klaus and Susanne Hasselmann, and Willem de Voogt for stimulating discussions. Special thanks go to the Hasselmanns for putting their nonlinear transfer routines at our disposition.

APPENDIX

Wave Growth With and Without Bottom Influence

In the first stage of wave growth, bottom influence can be neglected. For this stage we use relations that are taken from Kahma (1983) which are based in part on JONSWAP; some minor adjustments have been made here. Restricting ourselves to $\tilde{x} < 2000$ we have

$$\epsilon = 2.09 \times 10^{-7}\tilde{x}, \quad \text{(A1)}$$

$$\nu = 3.5\tilde{x}^{-1/3}, \quad \text{(A2)}$$

here ϵ denotes dimensionless wave variance $(=g^2EU^{-4})$, $E = m_0$, the zeroth moment of the wave spectrum (the total wave variance); $\nu = f_mUg^{-1}$, the dimensionless peak frequency of the wind-wave spectrum; $\tilde{x} = gXU^{-2}$ the dimensionless fetch; and g is the acceleration of gravity.

The influence of bottom depth becomes noticeable after some time when the wavelength associated with the peak frequency λ_m exceeds $0.5d$, defining the shallow-water condition as $k_md < \pi$. For $d = 35$ m this becomes $k_m \equiv 2\pi\lambda_m^{-1} < 0.09$, and because of $\omega^2 \simeq gk$, $f_m < 0.15$ Hz; for $U = 25$ m s^{-1} this corresponds to $\nu < 0.38$. Expressed in terms of $\tilde{x}$, using (A2), this becomes

$$\tilde{x} > 781\ (d = 35\ \text{m},\ U = 25\ \text{m s}^{-1}).$$

A convenient growth curve for shallow water is given in the *CERC Shore Protection Manual* [Vol. I, Chap. 3, Eq. (3.25)], originally developed by Bretschneider (constants slightly changed):

$$\tilde{H}_s = 0.30D^* \tanh[6.1 \times 10^{-3}\tilde{x}^{1/2}(D^*)^{-1}], \quad \text{(A3)}$$

with depth parameter

$$D^* = \tanh[0.656\tilde{d}^{0.75}], \quad \text{(A4)}$$

in which $\tilde{d} = gdU^{-2}$ is dimensionless depth. For deep water ($D^* = 1$) and short fetches, Eq. (A3) is in agreement with (A1). The most common parameter for wave-growth relations of the type (A3) is dimensionless fetch $\tilde{x}$. However, we are interested here in duration-limited rather than in fetch-limited wave growth. Carter (1982) has presented relations between dimensionless duration $\tilde{t} = gtU^{-1}$ and dimensionless fetch, based on JONSWAP results. We take

$$\tilde{t} = 60.0\tilde{x}^{0.7}. \quad \text{(A5)}$$

Using (A5), we note that Eq. (A3) can be transformed into a shallow-water waveheight–duration relation.

REFERENCES

Bouws, E., 1980: Spectra of extreme wave conditions in the southern North Sea considering the influence of water depth. *Climatologie de la Mer,* Collection colloques en séminaires, No. 34, Technip, Paris, 51–71.

Carter, D. J. T., 1982: Prediction of wave height and period for a constant wind velocity using the JONSWAP results. *Ocean Eng.,* **9,** No. 1, 17–33.

Coastal Engineering Research Center, (CERC), Fort Belvoir, VA, 1977: *Shore Protection Manual,* 3 Volumes.

Hasselmann, D. E., M. Dunckel and J. A. Ewing, 1980: Directional wave spectra observed during JONSWAP 1973. *J. Phys. Oceanogr.,* **10,** 1264–1280.

Hasselmann, K., 1974: On the spectral dissipation of ocean waves due to white capping. *Bound.-Layer Meteor.,* **6,** 107–127.

——, T. P. Barnett, E. Bouws, H. Carlson, D. E. Cartwright, K. Enke, J. A. Ewing, H. Gienapp, D. E. Hasselmann, P. Kruseman, A. Meerburg, P. Müller, D. J. Olbers, K. Richter, W. Sell and H. Walden, 1973: Measurement of wind wave growth and swell decay during the Joint North Sea Wave Project (JONSWAP). *Dtsch. Hydrogr. Z.,* Suppl. A, No. 12.

Hasselmann, S., and K. Hasselmann, 1981: A symmetrical method of computing the nonlinear transfer in a gravity wave spectrum. *Hamburger Geophys. Einzel.,* No. 52.

——, and ——, 1983: Integration of the spectral transport equation with exact and parametrical computation of the nonlinear energy transfer. *Proc. Symp. Wave Dynamics and Radio Probing of the Ocean Surface,* Miami, 1981, Plenum Press (in press).

Kahma, K. K., 1983: On prediction of the fetch-limited wave spectrum in a steady wind. Submitted to *Finn. Mar. Res.*

Kitaigorodskii, S. A., V. P. Krasitskii and M. M. Zaslavskii, 1975: On Philips' theory of equilibrium range in the spectra of wind-generated gravity waves. *J. Phys. Oceanogr.,* **5,** 410–420.

Sanders, J. W., and J. Bruinsma, 1983: Some aspects of the development of a wind driven sea on shallow water. *Proc. Symp. Wave Dynamics and Radio Probing of the Ocean Surface,* Plenum Press (in press).

Shemdin, O., K. Hasselmann, S. V. Hsiao and K. Herterich, 1978: Nonlinear and linear bottom interaction effects in shallow water. *Turbulent Fluxes through the Sea Surface, Wave Dynamics and Prediction,* NATO Conference Ser. V, Vol. 1, Plenum Press, 647–665.

Snyder, R. L., F. W. Dobson, J. A. Elliott and R. B. Long, 1981: Array measurements of atmospheric pressure fluctuations above surface gravity waves. *J. Fluid Mech.,* **102,** 1–59.

SWAMP, 1983: An intercomparison study of wind wave prediction models. *Proc. Symp. Wave Dynamics and Radio Probing of the Ocean Surface,* Plenum Press (in press).

Vincent, C. L., 1982: Shallow water wave modelling. *Preprints 1st Int. Conf. Meteorology and Air/Sea Interaction of the Coastal Zone,* The Hague, Amer. Meteor. Soc., 87–95.

De Voogt, W., G. J. Komen and J. Bruinsma, 1983: The KNMI operational wave prediction model GONO. *Proc. Symp. Wave Dynamics and Radio Probing of the Ocean Surface,* Plenum Press (in press).