Proc. R. Soc. Lond. B **240**, 591–605 (1990)
Printed in Great Britain

Energy absorption from ocean waves: a free ride for cetaceans

By N. Bose[1] and J. Lien[2]

[1] *Naval Architectural Engineering, Faculty of Engineering and Applied Science, Memorial University of Newfoundland, St John's, Newfoundland, A1B 3X5, Canada*

[2] *Ocean Studies Centre and Department of Psychology, Memorial University of Newfoundland, St John's, Newfoundland, A1C 5S7, Canada*

(*Communicated by Sir James Lighthill, F.R.S. – Received* 11 *December* 1989)

Flukes of cetaceans are capable of absorbing energy from ocean waves for propulsion. The extent of this energy absorption is demonstrated by considering the flukes of an immature fin whale, *Balaenoptera physalus*. In a fully developed seaway corresponding to a wind speed of 20 knots† (around Beaufort force 5) and at a low swimming speed, of 2.5 m s^{-1}, this whale was able to absorb up to 25% of its required propulsive power in head seas and 33% of propulsive power in following seas.

Consequences of wave-energy absorption for energetics of cetacean migrations are discussed.

Abbreviations used in the text

a	wave amplitude
b	x-coordinate of pitching axis
$C(k)$	Theodorsen function
C_T	thrust coefficient, $C_T = 2\bar{T}/\rho U^2 S$
c	fluke chord length
D	frictional drag
d	depth of immersion
E	mean wave energy per unit area
g	gravitational acceleration
h	heave amplitude of fluke
i	$\sqrt{-1}$
k	wave number
L	lift
M	moment
P	mean power
P_{nw}	mean delivered power in calm water
P_s	mean percentage power saved at a single wave frequency
P_{st}	mean percentage power saved in a given wave spectrum
P_w	mean delivered power in waves
$P(\xi)$	normalized mean percentage power saved at a single wave frequency, $P(\xi) = P_s/a^2$
S	fluke-planform area
$S(\xi)$	wave spectrum
$\bar{T}$	mean thrust
U	forward speed
U_w	wind speed
V_m	mean velocity of inflow
w	vertical velocity of hydrofoil
W_d	work done on frictional drag
x	horizontal axis and distance from mid-chord
α	pitch amplitude of fluke
β_i	induced angle of attack at three-quarter-chord point
η_{nw}	efficiency in calm water
η_w	efficiency in waves
λ	heading angle to seas
ρ	water density
σ	reduced frequency
ω	wave encounter frequency and frequency of oscillation
ξ	wave frequency

† 1 knot ≈ 0.51 m s^{-1}.

1. Wave energy for propulsion

For years inventors have proposed using the energy of ocean waves for propulsion of boats (see, for example, Schult (1974); Isshiki (1982)). Recently, use of restrained horizontal hydrofoils has been shown to be a means of extracting wave energy for propulsion of ships and boats (Jakobsen 1981; Isshiki 1984; Jaeger 1986) and this has been demonstrated on both models and small craft (Terao 1982; The Motor Ship 1983; Berg 1985; Isshiki 1986; Lai & McGregor 1989). One study shows that for a small ship of 70 m length with a hydrofoil area of 25.5 m^2, fuel savings would be in the range 9–42% for speeds of 15–9 knots (Korbijn 1985). If such savings are possible for boats, it is likely that cetacea use wave energy for propulsion to minimize their energy consumption during swimming.

Whalers knew that the action of the sea caused the carcass of a dead whale to propel itself forwards at about one nautical mile per hour. When whale boats were propelled by oars, this motion was used to help tow carcasses to the whale ship (Doane 1987, p. 70). If left floating unattended between capture and processing, a dead whale could move considerable distances and be lost; to avoid this their flukes would be removed (Ash 1962, p. 63). If whales' flukes act effectively as passive wave propulsors when an animal is dead, they might work even more effectively as active wave propulsors when the whale is alive.

Studies done on wave surfing by small cetacea (Newman & Wu 1974; Fejer & Backus 1960; Hayes 1953) show that the whole body of a whale gains energy from riding in the crest of waves or in the bow wave of a ship. Lighthill (1969) noted that reports of extraordinary speeds in dolphins may be due to these animals riding currents, wakes or waves. The action considered in this paper is that where whales swimming below waves use only their flukes as wave energy absorbers.

2. Method of analysis

The theory of how a horizontal hydrofoil, oscillating in heave and pitch, extracts energy from surface waves has been described by Wu (1972). Wu & Chwang (1974) described how this principle could be used by fish and birds to extract energy from a wavy stream. The method was extended by Isshiki (1982) to allow approximately for surface effects that lead to thrust changes in waves. A more rigorous approach to the problem of a foil oscillating very close to a free surface, which includes the generation of waves by the foil, was developed by Grue *et al.* (1988). Wu (1972) neglects free surface effects, but explains that these effects are secondary if the hydrofoil operates at least two chord lengths below the surface.

To explore possible wave-energy absorption by cetacea, Wu's (1972) methods and conclusions were studied and a simplified theoretical approach based on his conclusions was developed. This was justified because the approach represents a first estimate of the order of wave-energy absorption based only on an assumed motion of a whale's flukes. An exhaustive approach would consider the motion of the whole animal, and the calculated relative motion between water particles and the flukes would be found.

Wu (1972) explains that wave-energy absorption by an oscillating hydrofoil, travelling below the water surface and the waves, is greatest when the hydrofoil oscillates at the same frequency as the wave-encounter frequency. He then finds the optimum motion at this frequency, which minimizes values of energy imparted by the hydrofoil to the water; these values are negative when energy is absorbed from the waves. At the optimum motion, there is a phase difference of $\frac{1}{2}\pi$ between heave and pitch motions; the vertical, wave-particle, orbital velocity can be considered to be constant over the hydrofoil chord, because useful energies are absorbed by waves of relatively long wave length compared with the hydrofoil chord length; and, at lower values of reduced frequency, σ, ($\sigma = \omega c/2U$) below about 0.2, the heaving motion of the hydrofoil lags (or leads) the vertical, wave-particle, orbital velocity by very nearly π. This range of reduced frequency corresponds to the best propulsive efficiencies for an oscillating hydrofoil in a uniform stream and covers the most interesting range for a hydrofoil oscillating in waves. Wu shows how, for the optimum motion, the vertical component of the wave orbital velocity is used to enhance the effective angle of attack of the hydrofoil, which leads to a thrust gain by the hydrofoil from its interaction with the waves (Wu 1972, fig. 10).

If these optimum conditions apply at all times, then the vertical velocity of a two-dimensional hydrofoil relative to the water is:

$$w = \Re\,[\mathrm{i}(\omega h - U\alpha) + \omega\alpha(x-b) + \xi a\,\mathrm{e}^{-kd}]\,\mathrm{e}^{\mathrm{i}\omega t}, \tag{1}$$

where the hydrofoil section lies in the x–z plane with the origin at the mid-chord, the positive z-axis vertically upward and the positive x-axis towards the trailing edge; $\Re$ denotes, 'the real part of'; ω is the radian frequency of oscillation and the encounter frequency of the waves; h is the amplitude of heave relative to the fixed x–z axes; the point $x = b$, $z = 0$ is the pitching axis; and t denotes time. Equation (1) is similar to the equation given by Lighthill (1970) for a hydrofoil oscillating in uniform flow, with the addition of the last term in the square brackets, which represents the orbital velocity of the water particles in waves. The equation is only valid if, the foil oscillates at the wave-encounter frequency; the vertical, wave-particle, orbital velocity is constant over the hydrofoil chord; and the heaving motion of the hydrofoil lags the vertical, wave-particle, orbital velocity by π. The latter restriction was assumed throughout this analysis; other phases are possible by introducing an orbital velocity term composed of both real and imaginary parts.

The wave orbital velocity is for a sinusoidal, deep-water, two-dimensional, progressive wave. Both head and following seas are accounted for by equation (1), so long as the appropriate encounter frequency, ω, is incorporated (see, for example, Bishop & Price (1979, p. 109)):

$$\omega = \xi - (U\xi^2/g)\cos\lambda, \tag{2}$$

where, g is gravitational acceleration and λ is the heading angle of the whale to the waves (0° for following seas, 180° for head seas). However, the phase of the heaving motion of the hydrofoil to the wave's amplitude will be a lag of $\frac{1}{2}\pi$ for head seas; a lead or a lag of $\frac{1}{2}\pi$ for following seas, dependent on whether the hydrofoil is travelling faster or slower than the wave phase velocity. As in Wu's method, the amplitude of the wave orbital velocities are considered to be small in relation to the forward speed, U, of the hydrofoil, and the horizontal component of the orbital velocity is neglected in the formulation.

The wave term in equation (1) represents an additional oscillatory vertical velocity of amplitude $\xi a\,\mathrm{e}^{-kd}$. By using the results given by von Kármán & Sears (1938, p. 385), this vertical velocity leads to an additional lift, L, and moment, M, of

$$L = \rho\,\mathrm{e}^{\mathrm{i}\omega t}\xi a\,\mathrm{e}^{-kd}\pi c[(\mathrm{i}c\omega/4)+UC(k)], \tag{3}$$

and

$$M = (1/4)\rho\,\mathrm{e}^{\mathrm{i}\omega t}\xi a\,\mathrm{e}^{-kd}\pi c^2 UC(k), \tag{4}$$

which are added to the lift and moment equations for the hydrofoil in uniform flow (Lighthill 1970; Bose & Lien 1989, equations (2) and (3)). In the above equations, ρ is the fluid density; c is the hydrofoil chord length; $C(k)$ is the Theodorsen function (Theodorsen 1935); positive lift acts in the z-direction, and positive moments tend to turn the leading edge in the z-direction.

The solution follows the format described by Bose & Lien (1989), for an oscillating hydrofoil in uniform flow, with the following additions.

1. The equations for mean rate of working and thrust (Bose & Lien 1989, equations (13) and (14)) are unchanged, but the modified lift and moment values are used in their solution. This is because addition of vertical, wave-particle, orbital velocity to the equation of water flow relative to the hydrofoil does not change the actual velocities of the hydrofoil relative to fixed axes in space. However, the leading-edge, suction force is changed, because the flow velocity at the leading edge is influenced by the vertical, wave-particle, orbital velocity; this is accounted for by a change to the equation for term A (Bose & Lien 1989, equation (15)) in the equation for thrust:

$$A = -U[\{\omega\alpha(b-(c/4))+\mathrm{i}(U\alpha-\omega h)-\xi a\,\mathrm{e}^{-kd}\}C(k)+(\omega\alpha c/4)]. \tag{5}$$

2. The induced angle of attack at the three-quarter-chord point is given by

$$\beta_{\mathrm{i}} = C(k)\{\omega\alpha c/4U-\mathrm{i}\alpha+\mathrm{i}\omega h/U-\omega\alpha b/U+\xi a\,\mathrm{e}^{-kd}/U\}\,\mathrm{e}^{\mathrm{i}\omega t}. \tag{6}$$

3. The approximation for frictional drag (Bose & Lien 1989, equations (26), (29) and (30)) is modified to give an approximate mean velocity of inflow:

$$V_{\mathrm{m}} = [((\xi a\,\mathrm{e}^{-kd}-\omega\alpha b)^2+\omega^2h^2)/2+U^2]^{\frac{1}{2}}; \tag{7}$$

angle of inflow:

$$\phi \approx [(\mathrm{i}\omega h-\omega\alpha b+\xi a\,\mathrm{e}^{-kd})/U]\,\mathrm{e}^{\mathrm{i}\omega t}; \tag{8}$$

and, an increase in mean rate of working due to frictional drag:

$$W_{\mathrm{d}} = (D/2U)[(\xi a\,\mathrm{e}^{-kd}-\omega\alpha b)^2+\omega^2h^2], \tag{9}$$

where, D is the frictional drag (Bose & Lien 1989, equation (28)).

Calculations for a whale's flukes in regular waves were obtained by using the strip theory described by Bose & Lien (1989), with the above changes.

Isshiki (1982, figs 7 and 9) presents values of thrust, for a two-dimensional hydrofoil (fixed against heave and pitch, but moving at forward speed) in a regular wave of frequency 2.78 rad s^{-1}, calculated by using Wu's (1972) method. The hydrofoil has a chord length of 0.328 m and it is submerged at a depth of 0.492 m;

the wave has an amplitude of 0.2 m and thrust is calculated for both head and following seas. For this hydrofoil in these waves, the above theory was used to predict the two-dimensional thrust in non-viscous flow. A comparison of the two methods is shown in figure 1. There is good agreement between the methods for this condition.

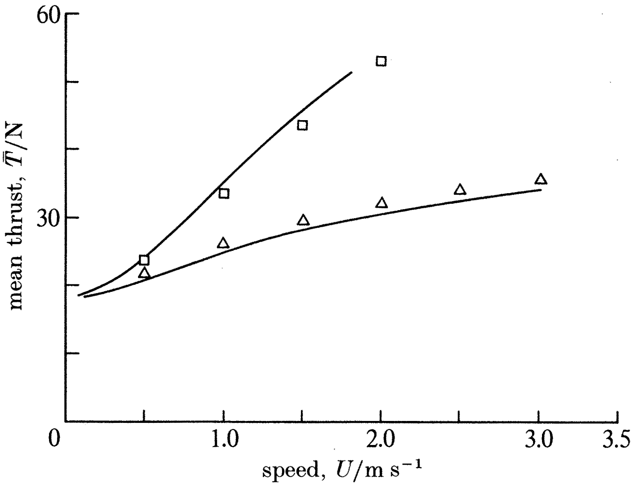

FIGURE 1. Thrust in waves of a two-dimensional hydrofoil fixed against heave and pitch, but moving at forward speed. Hydrofoil chord length is 0.328 m; depth of submergence is 0.492 m; wave frequency is 2.78 rad s^{-1}, and wave amplitude is 0.2 m. The upper curve is for following seas, the lower curve is for head seas. The lines are Isshiki's (1982) results from Wu's (1972) theory; the points are calculations by using the present method: (□), for following seas; (△), for head seas.

3. A FIN WHALE IN WAVES

Morphological data for a 14.5 m long immature fin whale (*Balaenoptera physalus*), was used by Bose & Lien (1989) to predict propulsive efficiency and thrust during swimming. Although the possibility of wave-energy absorption exists for many cetacean species, the fin whale was used as an example because of the availability of relatively complete morphological data for this animal.

Typically, fin whales swim on a relatively straight course with surfacing and blows at regular intervals (Watkins 1981 *b*). A whale swimming near the surface is in a region subject to water-particle, orbital motions of the waves. Water-particle motions will cause a whale to oscillate in waves, although, because of exponential decay (see, for example, Newman 1977, p. 242), this is reduced for bodies below the surface. In waves that are much longer than the whale, the action causes the whale to move up and down with the wave motion. In smaller waves, ranging down from wave lengths several times the length of the whale, the whale's body is either steady or oscillates in such a fashion that the flukes are in water oscillating vertically at a different phase and amplitude to the motion of the body. The differential vertical velocity of the flukes can be used to extract energy from the waves.

In this analysis the flukes are considered to be oscillating in a vertical plane in pitch and heave while moving forward at uniform speed through waves at some distance below the surface. Motions of the whale body are ignored, except in so far as they influence the range of wave lengths, or frequencies, from which energy can be extracted. The analysis was done by using the methods described in §2 and geometrical measurements of the fluke planform (Bose & Lien 1989, table 2). Values of thrust coefficient and efficiency calculated here include corrections for finite span, finite-amplitude motion and frictional drag. Table 1 shows the drag of the animal, estimated by using the method described in Bose & Lien (1989, §5), and the corresponding required thrust coefficients from the flukes, $C_T = 2\bar{T}/\rho U^2 S$. The mean thrust from the flukes is assumed to be equal to the drag at steady forward speed. The effects of wake from the body on the flukes, and thrust deduction on the body as a result of reduced pressure just ahead of the flukes, are assumed to be small because of the large span of the flukes relative to the size and form of the body.

TABLE 1. DRAG AND REQUIRED THRUST COEFFICIENT OF THE FIN WHALE

(The drag estimate assumes a turbulent boundary layer.)

speed/(m s^{-1})	Reynolds number ($\times 10^{-7}$)	friction coefficient ($\times 10^3$)	drag/kN	required thrust coefficient, C_T	effective power/kW
0.5	0.54	3.33	0.03	0.169	0.02
1.0	1.07	2.97	0.11	0.143	0.11
1.5	1.61	2.79	0.23	0.134	0.35
2.0	2.15	2.67	0.39	0.128	0.78
2.5	2.69	2.58	0.59	0.124	1.48
3.0	3.22	2.51	0.82	0.120	2.46
3.5	3.76	2.45	1.09	0.118	3.82
4.0	4.30	2.40	1.40	0.115	5.60

Figure 2*a* shows the variation of percentage power saved by the whale with depth of submergence of the flukes. Here the flukes are moving at 2.5 m s^{-1} into regular, long-crested, head seas of frequency 1.43 rad s^{-1} (wave length 30 m) and amplitude 0.48 m. The encounter frequency is 1.95 rad s^{-1}, thrust coefficient is 0.124 and advance ratio, J, is 5.0 ($J = \pi U/\omega h$). Amplitude of pitch is adjusted to give the required thrust coefficient of 0.124, and varies from 28.1° at a depth of submergence of 1.0 m, to 27.3° at a depth of submergence of 4.0 m (figure 2*b*). Mean percentage power saved is defined as:

$$P_s = (1-(P_w/P_{nw}))\,100 = (1-(\eta_{nw}/\eta_w))\,100, \tag{10}$$

where, P_w is the mean power delivered to the flukes in waves; P_{nw} is the mean power delivered when there are no waves; η_w and η_{nw} are the corresponding efficiencies (efficiency, $\eta = \bar{T}U/P$). The effective power of the animal (drag times forward speed) is assumed to be the same in waves as in calm water, and is 1.48 kW at 2.5 m s^{-1} (table 1); the efficiency in calm water is $\eta_{nw} = 0.832$ and mean delivered power is 1.78 kW. The mean percentage power saved drops from 45% at a depth of submergence of 1 m to 13% at a depth of submergence of 4 m; at a depth of 2 m it is about 30%. At a depth of only 1 m, the flukes may breach the surface.

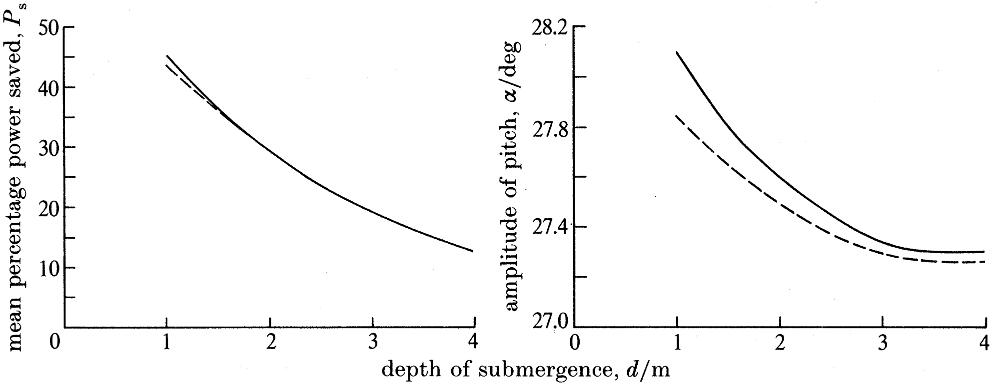

FIGURE 2(*a*). Mean percentage power saved against depth of fluke submergence for an immature fin whale. Flukes are moving at 2.5 m s^{-1} into head seas of 1.43 rad s^{-1} and amplitude of 0.48 m; thrust coefficient is 0.124 and advance ratio is 5. The dotted line includes correction for Stokes drift velocity. (*b*) Amplitude of pitch against depth of fluke submergence for an immature fin whale. Flukes are moving at 2.5 m s^{-1} into head seas of 1.43 rad s^{-1} and amplitude of 0.48 m; thrust coefficient is 0.124 and advance ratio is 5. The dotted line includes correction for Stokes drift velocity.

Figure 3 shows the variation of mean percentage power saved with wave amplitude when travelling at 2.5 m s^{-1}, and a depth of submergence of 2 m, into regular head seas of the same frequency (1.43 rad s^{-1}). Again, the advance ratio is 5.0 and thrust coefficient is 0.124. Mean percentage power saved rises as the wave amplitude increases, at first in a parabolic manner (as would be expected because wave energy is proportional to wave amplitude squared), but later almost linearly (when the vertical, wave-particle, orbital velocity is no longer small compared with the forward speed). At a wave amplitude of around 0.9 m, the whale would be completely wave propelled. Power savings of over 100 % indicate that the flukes are absorbing wave power over that which is required for propulsion. This would be unlikely to occur in practice for several reasons: the whale would reduce heave

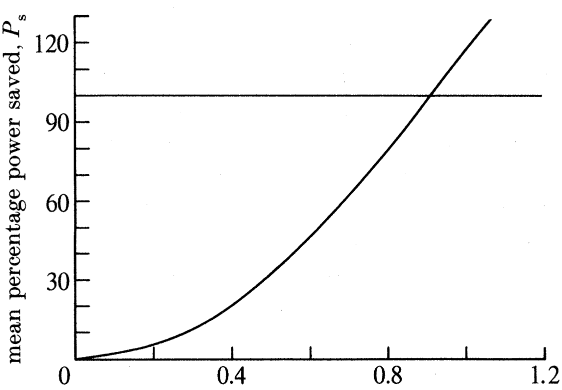

FIGURE 3. Variation of mean percentage power saved with wave amplitude. Flukes are moving at 2.5 m s^{-1} into head seas of 1.43 rad s^{-1}; thrust coefficient is 0.124 and advance ratio is 5.

amplitude of its flukes; waves of this amplitude at this frequency are steep and are uncommon except in shallow water, wind against current or restricted water situations; a positive energy cost is described as being incurred by an animal even when it does negative work (i.e. absorbs power) (Chopra & Kambe 1977); or, the whale would accelerate.

3.1. *Effect of Stokes drift velocity in waves*

Results discussed in the previous section neglect the effect of mean horizontal velocity of the water particles in waves known as 'Stokes drift', the magnitude of which is $\xi a^2 k\, e^{-2kd}$ (see, for example, Lighthill (1978, p. 280); Lamb (1932, p. 419)). This velocity will have only a small influence on the forces on the flukes and this has been neglected, however, it will lead to an effective increase in overall whale drag in head seas and a reduction in drag in following seas.

For the whale swimming into a head sea of frequency 1.43 rad s^{-1} and amplitude 0.48 m, the example shown in figure 2, the reduction of mean percentage power saved due to this increase in drag was obtained. This was done by calculating the Stokes drift velocity at different depths: 0.045 m s^{-1} at a depth of 1 m; 0.030 m s^{-1} at a depth of 2 m; and 0.013 m s^{-1} at a depth of 4 m. From this the increment in drag was estimated by using the method to calculate drag used for table 1. The required thrust coefficients were found at each depth by assuming that the full drag applied at the swimming speed of 2.5 m s^{-1}; the mean percentage power saved was found as before. The dotted lines in figure 2 show the results over a range of depth of submergence. The Stokes drift velocity in head seas reduces the mean power saved by about 3% at a depth of submergence of 1 m, but by a depth of submergence of 2 m, the mean power saved is almost the same as the uncorrected value (figure 2*a*). The corresponding amplitudes of pitch of the flukes are shown in figure 2*b*.

Figure 2 shows that despite small changes in required thrust coefficient from the flukes (from 0.128 at a depth of submergence of 1 m, to 0.125 at a depth of submergence of 4 m, compared with 0.124 in calm water), which are accommodated by a change in amplitude of pitch, the mean percentage power saved does not change by a similar amount. The mean percentage power saved is calculated from the efficiency of the flukes and this quantity is relatively insensitive to alterations in the angle of pitch to make small adjustments to the thrust coefficient (see Bose & Lien (1989), fig. 6). A similar result would apply in following seas, although here the mean power saved would be increased slightly.

Correction for Stokes drift velocity has not been applied in the analysis that follows. In a sea state corresponding to a wind speed of 20 knots, the Stokes drift velocity at a depth of 2 m is in the range 0.0 to 0.06 m s^{-1}; in a sea state corresponding to a wind speed of 30 knots it lies in the range up to 0.2 m s^{-1}. These levels would change the overall estimated power savings presented by a few percent reduction in head seas and a few percent increase in following seas. The changes would be very small in a wind speed of 20 knots and somewhat larger in a wind speed of 30 knots; they are within the level of accuracy of prediction implied by other assumptions in the method of calculation. Fin whales are known to swim near to the surface during transit (Watkins 1981*b*) and would be subject to these drift velocities whether or not they are able to extract energy from waves; overall the drag represented by drift velocities would cancel if the animals spend equivalent periods on upwind and downwind headings

3.2. *Power saving at sea*

Watkins (1981 *a*) continuously tracked a 20 m fin whale that swam 2095 km in 226 h, often in the company of other fin whales. Watkins does not report the sea conditions in detail throughout this period, but he does record that the sea varied from calm to a maximum wave height of 5–6 m (Watkins 1981 *a*; Watkins *et al.* 1984). The whale swam west from Iceland, across the Denmark Strait, to an area about 130 km from Greenland, and then moved back and forth in Greenland waters. The maximum day's travel was 292 km; the whale maintained an overall average speed of 2.6 m s^{-1} (5.0 knots) and an average of 3.4 m s^{-1} (6.6 knots) on its maximum day's run. Actual swimming speeds could be a little different because of the influence of currents.

In the absence of more detailed information, it was assumed that a speed of 2.5 m s^{-1} is a reasonable average swimming speed for the smaller immature fin whale. Wave conditions for this open ocean location were assumed to be represented by a Pierson–Moskowitz spectrum for uni-directional, fully developed seas (see, for example, Newman (1977), p. 315). This, in conjunction with the wave-height estimates, show that wind speed must have varied from calm to about 30 knots (Beaufort force 0–7) during the radio tracking event.

The Pierson–Moskowitz spectrum indicates the level of wave energy associated with different wave frequencies for fully developed seas in a given windspeed. The mean energy per unit area for a regular wave is:

$$E = \rho g a^2/2, \tag{11}$$

and for a wave system represented by a Pierson–Moskowitz spectrum is:

$$E = \rho g \int_0^\infty S(\xi)\, \mathrm{d}\xi, \tag{12}$$

where ξ is wave frequency and $S(\xi)$ is the spectrum,

$$S(\xi) = \frac{0.0081 g^2}{\xi^5} \exp\left[-0.74\left(\frac{g}{U_\mathrm{w}\,\xi}\right)^4\right]. \tag{13}$$

U_w is the windspeed at a height of 19.5 m above the water surface. Table 2 shows the values of $S(\xi)$ in different wind speeds for wave frequencies of 2.03, 1.66, 1.43

Table 2. Wave-spectrum amplitudes, $S(\xi)$, at different wind speeds and wave frequencies, ξ

wave frequency/rad s^{-1}		2.03	1.66	1.43	1.17
windspeed/ knots	windspeed/ (m s^{-1})	$S(\xi)$/ (m^2s)	$S(\xi)$/ (m^2s)	$S(\xi)$/ (m^2s)	$S(\xi)$/ (m^2s)
10	5.14	0.013	0.017	0.013	0.002
20	10.28	0.022	0.057	0.113	0.260
30	15.42	0.023	0.061	0.127	0.330

and 1.17 rad s^{-1}, which corresponds to wavelengths of 15, 22.5, 30 and 45 m, or about 1, 1.5, 2 and 3 times the length of the whale. Figure 4 is a plot of the Pierson–Moskowitz spectra for wind speeds of 10, 20 and 30 knots.

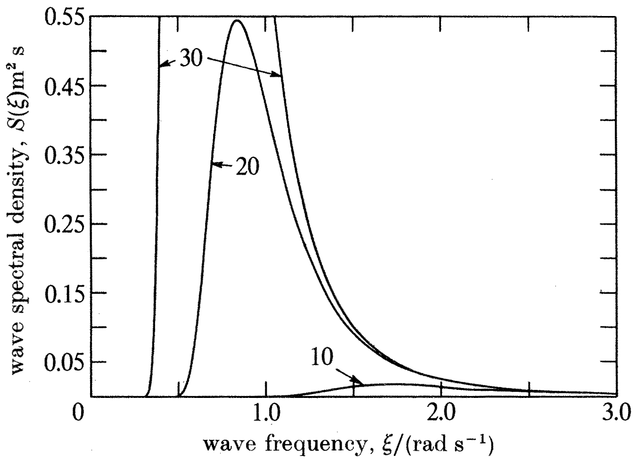

FIGURE 4. Pierson–Moskowitz wave frequency spectrum. Wind speeds in knots are shown on the curves.

Over the range of validity of the analysis and for realistic wave amplitudes, mean percentage power saved at each wave frequency was assumed to be proportional to wave amplitude squared (figure 3). The average percentage of power saved in a given wave spectrum was then assumed to be given by:

$$P_{st} = 2\int_0^\infty S(\xi)\,P(\xi)\,d\xi, \tag{14}$$

where $P(\xi) = P_s/a^2$.

In practice, as the ratio of wave length to whale length increases, the ability of the whale to absorb wave energy must reduce. This is illustrated by thinking of a whale swimming in very long waves, where the vertical, water-particle, orbital motions are almost constant along the length of the animal and the associated vertical accelerations are low. Here there is little relative vertical, water-particle, motion between the flukes and surrounding water, arising from the waves, and consequently, little wave energy can be absorbed. It is possible that a whale could optimize this relative motion by controlling body motions using its pectoral fins.

To get an approximation for overall power saving of the 14.5 m whale, it is assumed that the full power saving can be achieved from a wave of three times the length of the whale, but that the power saving drops (linearly) to zero in waves of four times the length of the whale. This assumption is arbitrary, but it is expected to give a result on the low side of the power saving that is possible. Long whales are likely to make larger propulsive power savings than short whales.

Table 3 and table 4 show the average percentage power saved when swimming at 2.5 m s^{-1} (with advance ratio $J = 5$, depth 2 m, pitching amplitude to give $C_T = 0.124$) in head and following seas described by Pierson–Moskowitz spectra for

TABLE 3. AVERAGE PERCENTAGE POWER SAVED IN HEAD SEAS FOR WIND SPEEDS OF 10, 20 AND 30 KNOTS

(Integrations are done by trapezoidal rule; swimming depth is 2.0 m; forward speed 2.5 m s^{-1}; advance ratio, J, is 5; pitching amplitude is adjusted to give a thrust coefficient, C_T, of 0.124.)

	windspeed/knots		10	20	30
wave frequency/ rad s^{-1}	wave length/ m	$P(\xi)$/ % m^{-2}	$S(\xi)P(\xi)$/ % s	$S(\xi)P(\xi)$/ % s	$S(\xi)P(\xi)$/ % s
1.01	60	0	0.0	0.0	0.0
1.17	45	123	0.2	32.0	40.6
1.43	30	126	1.6	14.2	16.0
1.66	22.5	112	1.9	6.4	6.8
2.03	15	45	0.6	1.0	1.0
	power saved		2.2%	24.6%	29.3%

TABLE 4. AVERAGE PERCENTAGE POWER SAVED IN FOLLOWING SEAS FOR WIND SPEEDS OF 10, 20 AND 30 KNOTS

(Integrations are done by trapezoidal rule; swimming depth is 2.0 m; forward speed is 2.5 m s^{-1}; advance ratio, J, is 5; pitching amplitude is adjusted to give a thrust coefficient, C_T, of 0.124.)

	windspeed/knots		10	20	30
wave frequency/ rad s^{-1}	wave length/ m	$P(\xi)$/ % m^{-2}	$S(\xi)P(\xi)$/ % s	$S(\xi)P(\xi)$/ % s	$S(\xi)P(\xi)$/ % s
1.01	60	0	0.0	0.0	0.0
1.17	45	145	0.3	37.7	47.9
1.43	30	174	2.3	19.7	22.1
1.66	22.5	175	3.0	10.0	10.7
2.03	15	153	2.0	3.4	3.5
	power saved		3.8%	32.7%	38.7%

wind speeds of 10, 20 and 30 knots (Beaufort forces 3, 5 and 7, approximately). In seas corresponding to 10 knots of wind, the whale absorbs little wave energy. In seas corresponding to 20 knots of wind, around 25% and 33% power savings are possible in head and following seas, respectively; the values are 29% and 39% in head and following seas corresponding to 30 knots of wind. Although wave energy increases considerably as the wind increases from 20 to 30 knots (figure 4), the increase in wave energy is concentrated in waves of frequency less than 1 rad s^{-1}. It has been assumed that the whale cannot absorb energy from these low frequency waves; this explains why the increase in power saving is greater between seas corresponding to 10 and 20 knots of wind, than from 20 to 30 knots of wind.

The calculations have been restricted to head and following seas. Absorption of wave energy would be possible at other headings, although it is likely to be a minimum in beam seas, where the vertical, wave-particle, orbital velocity would not vary greatly over the length of the whale. The directional spread of waves in a real seaway would aid energy absorption in beam seas.

4. Other sizes and species

It has been shown that in seas corresponding to a windspeed of 20 knots, a 14.5 m long fin whale could make a propulsive power saving of around 25% in head seas and 33% in following seas. Wind speeds of this level have an occurrence of about 30% in the North Atlantic; for almost 50% of the time they are less than this level (see, for example, Bishop & Price (1979), p. 344). Total wave energy input to the energy budget of whales could be significant and may be especially relevant to studies of migration energetics such as those described by Kshatriya & Blake (1988) for the blue whale (*Balaenoptera musculus*).

Wave-energy absorption potential is higher for whales with wide flukes (large span) and for slow-swimming whales. Slow-swimming species with relatively large flukes (e.g. humpback, *Megaptera novaeangliae*) are expected to make the largest propulsive power savings.

That longer whales potentially gain more propulsive energy from waves may provide a means of testing if cetaceans do use wave propulsion. Sizes of different species of cetaceans may be related to their migration patterns. Careful analysis of propulsive properties of morphological traits may provide evidence to evaluate the hypothesis further.

Generally, female baleen whales are longer than males. Meredith & Campbell (1988), for example, report a 3.5% difference in length between female and male North Atlantic fin whales. Sexual dimorphism in length to this degree would not result in substantial difference in required migratory effort between females and males. However, male sperm whales (*Physeter macrocephalus*) are 30–40% longer than females (Leatherwood *et al.* 1982): such a difference might contribute to the greater mobility attributed to males without substantially increasing the energy costs of such movements.

We have no direct evidence that cetaceans actually do extract energy from waves. Whitehead (1985) found that humpback whales (*Megaptera novaeangliae*) breach more frequently during stronger winds (greater than Beaufort 4), but other behaviours, such as flippering or lobtailing, were not consistently related to wind speed. Also, breaches were more common when the whales were moving faster. Breaching may require greater speeds and it would be less expensive energetically if assisted by wave energy. R. Payne (personal communication) concludes that breaching in right whales (*Eubalaena sustralis*) is contagious; the probability of a whale breaching is related to the presence of whales nearby that are behaving similarly. However, like soaring in birds, clustering of similar behaviours could be due to advantageous environmental conditions rather than social context. The underlying factor might be advantageous propulsive conditions.

Migration routes of many of the larger cetaceans are near coastlines (Gaskin 1982). Such migration patterns could be related to the nature of waves that occur in shallower coastal areas. The highest energy, deep-sea waves, which occur offshore, are too long to effectively contribute to propulsion. However, when passing over shallow coastal areas these long waves shorten, become steeper and potentially provide a more usable energy form for most cetaceans.

Direct evidence is needed to demonstrate the energetic savings that cetaceans receive from wave propulsion. Some of this might come from analysis of movements in radio- or satellite-tagged whales that can be directly related to wave conditions.

5. Conclusions

The possibility of wave-assisted propulsion for whales is discussed. An approximate method is presented to estimate wave-energy absorption by a hydrofoil oscillating in pitch and heave below regular head and following seas. The method is used to study wave-energy absorption by the flukes of an immature fin whale of length 14.5 m. When swimming at a depth of 2 m and a speed of 2.5 m s^{-1} into a regular wave of frequency 1.43 rad s^{-1} and amplitude of 0.5 m, this animal has the potential to make a propulsive power saving of around 30% by absorbing wave energy.

In a fully developed seaway, represented by a Pierson–Moskowitz spectrum for a wind speed of 20 knots (around Beaufort force 5), this animal is estimated to make a propulsive power saving of around 25% in head seas and just over 30% in following seas. In a seaway corresponding to 10 knots of wind (around Beaufort force 3) little wave-energy absorption is possible, but in seas corresponding to 30 knots of wind (around Beaufort force 7) propulsive power saving increases to almost 30% in head seas and approaching 40% in following seas.

Restrictions on wave-energy absorption potential are expected in long waves (low frequencies); it has been assumed that wave-energy absorption drops to zero in waves equal to and longer than four times the length of a whale. The exact wave length at which this occurs is unknown and depends on the relative motion between vertical, water-particle, orbital velocities and the flukes. Pectoral fins might be used for control to optimize these relative velocities. The level of total propulsive power saving is very sensitive to this assumption and any percentage estimates made can only be regarded as approximate. Also, this dependency on wave length indicates that larger whales are likely to absorb larger amounts of wave energy, because in sea states corresponding to wind speeds of 20 knots and greater, the majority of wave energy is concentrated in the long, wave-length range.

Slow-swimming species with wide flukes have potential to absorb larger amounts of wave energy for propulsion than fast swimmers with small flukes. Humpbacks have the potential to make larger power savings than fin whales.

Estimates of propulsive power savings presented here are not intended to be exact. They were done to show that the potential for wave-energy absorption by flukes is substantial. Studies of energetics of cetacea should consider wave energy as an input to the energy budget.

Financial support for this work came from the Natural Sciences and Engineering Research Council of Canada; Fisheries and Oceans, Canada, and the Newfoundland and Labrador Department of Fisheries. Computations were done on a VAX 8530 computer at the Centre for Computer Aided Engineering in the Memorial University of Newfoundland. Both authors work with the Whale Research Group at the Memorial University of Newfoundland.

References

Ash, C. 1962 *Whaler's eye*. London: George Allen and Unwin Ltd.

Berg, A. 1985 *Trials with passive foil propulsion on M/S "Kystfangst"*. Project no. 672.138, Fiskeriteknologisk Forskningsinstitutt, Fartoyseksjon, Marineteknisk senter, Hakon Hakonsensgt 34, 7000, Trondheim. (In Norwegian.)

Bishop, R. E. D. & Price, W. G. 1979 *Hydroelasticity of ships* (423 pp.). Cambridge University Press.

Bose, N. & Lien, J. 1989 Propulsion of a fin whale (*Balaenoptera physalus*): why the fin whale is fast swimmer. *Proc. R. Soc. Lond.* B **237**, 175–200.

Chopra, M. G. & Kambe, T. 1977 Hydromechanics of lunate tail swimming propulsion, part 2. *J. Fluid Mech.* **79**, 49–69.

Doane, B. 1987 *Following the sea* (275 pp.). Halifax, Nova Scotia: Nimbus Publishing and The Nova Scotia Museum.

Fejer, A. A. & Backus, R. H. 1960 Porpoises and the bow-riding of ships under way. *Nature, Lond.* **188**, 700–703.

Gaskin, D. E. 1982 *The ecology of whales and dolphins* (459 pp.). London: Heinemann.

Grue, J., Mo, A. & Palm, E. 1988 Propulsion of a foil moving in water waves. *J. Fluid Mech.* **186**, 393–417.

Hayes, W. D. 1953 Wave riding of dolphins. *Nature, Lond.* **172**, 1060.

Isshiki, H. 1982 A theory of wave devouring propulsion – 1st report – thrust generation by linear Wells turbine. *J. Soc. nav. Arch. Jap.* **151**, 54–64.

Isshiki, H. 1984 A theory of wave devouring propulsion – 4th report – a comparison between theory and experiment in case of a passive-type hydrofoil propulsor. *J. Soc. nav. Arch. Jap.* **156**, 102–114.

Isshiki, H. 1986 Wave power utilization into ship propulsion. Tokyo: In *Fifth offshore mechanics and arctic engineering symposium*. Tokyo: American Society of Mechanical Engineers.

Jaeger, A. 1986 *Prosjektering av en foilpropeller for et ca. 70 meter langt skip*, Fag 805.40. Trondheim, Norway. Norges Tekniske Hogskole. (In Norwegian.)

Jakobsen, E. 1981 The foil propeller, wave power for propulsion. In *Second international symposium on wave and tidal energy*, BHRA Fluid Engineering, pp. 363–368.

Korbijn, F. 1985 *Analysis of a foil propeller as an auxiliary propulsion system*, report no. 85-3326, Veritec, P.O. Box 300, 1322 Hovik, Norway.

Kshatriya, M. & Blake, R. W. 1988 Theoretical model of migration energetics in the blue whale, *Balaenoptera musculus*. *J. theor. Biol.* **133**, 479–498.

Lai, P. S. K. & McGregor, R. C. 1989 *Flexible fin wave propulsion*. Glasgow, Scotland: University of Glasgow, Department of Naval Architecture and Ocean Engineering, report no. NAOE-89-17.

Lamb, H. 1932 *Hydrodynamics*, sixth edn, Cambridge: Cambridge University Press.

Leatherhead, S., Reeves, R. R., Perrin, W. F. & Evans, W. E. 1982 *Whales, dolphins and porpoises of the eastern north Pacific and adjacent Arctic waters: a guide to their identification*, (243 pages), NOAA technical report NMFS Circular 444. Washington, D.C.: U.S. Department of Commerce.

Lighthill, M. J. 1969 Hydromechanics of aquatic animal propulsion. *Ann. Rev. Fluid Mech.* **1**, 413–446.

Lighthill, M. J. 1970 Aquatic animal propulsion of high hydromechanical efficiency. *J. Fluid Mech.* **44**, 265–301.

Lighthill, M. J. 1978 *Waves in fluids*. Cambridge University Press.

Meredith, G. N. & Campbell, R. R. 1988 Status of the fin whale, *Balaenoptera physalus*, in Canada. *Can. Field Natural.* **102** (1), 351–368.

Newman, J. N. 1977 *Marine hydrodynamics*. Cambridge, Massachusetts: MIT Press.

Newman, J. N. & Wu, T. Y. 1974 Hydromechanical aspects of fish swimming. In *Proceedings of the symposium on swimming and flying in nature* (ed. T. Y. T. Wu, C. J. Brokaw & C. Brennen), pp. 615–634. New York: Plenum Press.

Schult, J. (*trans.* I. Moore) 1974 *Curious yachting inventions*. London: Paul Elek.

Terao, Y. 1982 A floating structure which moves toward the waves. *J. Kans. Soc. nav. Arch.* **184**, 51–54. (In Japanese.)

The Motor Ship 1983 Wave power for ship propulsion (ed. D. Woodyard), pp. 67–69. London.

Theodorsen, T. 1935 *General theory of aerodynamic instability and the mechanism of flutter.* National Advisory Committee for Aeronautics Technical report no. 496, pp. 413–433.

von Kármán, T. & Sears, W. R. 1938 Airfoil theory for non-uniform motion. *J. aero. Sci.* **5** (10), 379–390.

Watkins, W. A. 1981*a* Radio tagging of finback whales–Iceland, June–July 1980. *Technical report WHOI-81-2.* Massachusetts: Woods Hole Oceanographic Institution.

Watkins, W. A. 1981*b* Activities and underwater sounds of fin whales. *Sci. Rep. Whales Res. Inst.* **33**, 83–117.

Watkins, W. A., Moore, K. E., Sigurjonsson, J., Wartzok, D. & Notarbartolo di Sciara, G. 1984 Fin whale (*Balaenoptera physalus*) tracked by radio in the Irminger Sea. *Rit Fiskideilar* **8** (1), 1–14.

Whitehead, H. 1985 Why whales leap. *Scient. Am.* **252** (3), 84–93.

Wu, T. Y. & Chwang, A. T. 1974 Extraction of flow energy by fish and birds in a wavy stream. In *Proceedings of the symposium on swimming and flying in nature* (ed. T. Y. T. Wu, C. J. Brokaw & C. Brennan), pp. 687–702.. New York: Plenum Press.

Wu, T. Y. 1972 Extraction of flow energy by a wing oscillating in waves. *J. Ship Res.* **14** (1), 66–78.