On the statistics of wind gusts

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Abstract. Velocity measurements of wind blowing near the North Sea border of Northern Germany and velocity measurements under local isotropic conditions of a turbulent wake behind a cylinder are compared. It is shown that wind gusts – measured by means of velocity increments – do show similar statistics to the laboratory data, if they are conditioned on an averaged wind speed value.
Clear differences between the laboratory data and the atmospheric wind velocity measurement are found for the waiting time statistics between successive gusts above a certain threshold of interest.

Keywords: Gusts, Intermittency, Statistics, Turbulence, Waiting time distribution

1. Introduction

The wind in the atmospheric boundary layer is known to be distinctively turbulent and instationary. As a consequence the wind speed varies rather randomly on many different time scales. These time scales range from long-term variations (years) to very short ones (minutes down to less than a second). The latter are commonly considered to correspond to small-scale (microscale) turbulence. These small-scale fluctuations are superimposed to the mean velocity varying on diurnal or even larger scales. This distinction between a mean flow and superimposed turbulence is justified by the existence of a spectral gap which means that there is only little wind speed variation on time scales between about 10 minutes and 10 hours (Hau, 2000). One of the main challenges in turbulence research is the so-called *intermittency of small-scale turbulence*, which corresponds to an unexpected high probability of large velocity fluctuations. (Note that there are also other independent definitions of intermittency related to other phenomena.) For atmospheric winds large velocity fluctuations on small scales correspond to gusts. A more precise statistical description of the occurrences of gusts is important for many applications.

The aim of this paper is to find a possible relation between laboratory turbulence and the atmospheric small-scale one. The mechanisms ruling the atmospheric turbulence are quite complex. Roughly the origins of atmospheric turbulence can be divided into friction and thermal effects. The influence of both generally depends on many different factors like the thermal stability, the topography, the geographical position and so on (Burton, 2001). In laboratory experiments the situation is less complicated and easier to control. Here turbulence may be generated only by flow disturbances, speed and direction of the mean flow can be kept constant and no buoyancy effects have to be considered. Despite all these differences between different types of turbulence it is a common concept (Frisch, 1995) that cascade-like processes lead to universal statistical features of small-scale turbulence.

In this work we analyze two data sets, (a) one from a wake flow behind a cylinder recorded in a wind tunnel and (b) data from atmospheric wind.

(a) The velocity of the laboratory data was measured in the plane of the cylinder at a great distance (100 times the diameter D of the cylinder, $D = 0.02\ m$) to it (Lueck et al, 1999). At these distances the periodical flow patterns (Karman-street) arising directly beyond the cylinder are vanished and the turbulent wake flow can be considered to be rather isotropic and fully developed. The Reynolds number is $Re \approx 30000$.

(b) The atmospheric data set we use was recorded near the German

coastline of the North Sea in Emden (Hohlen & Liersch, 1998). The velocity was measured by means of an ultrasonic anemometer at 20 m height. The sampling frequency was 4 Hz. The measuring period took about one year (1997-1998). The distance to the sea is between some kilometers and a few tens of kilometers - depending on the direction. After careful investigation of the quality of the data we examine a representative 275-hour-excerpt of October 1997 and focus on the velocity component in direction of the mean wind (Boettcher et al., 2001). During this period the velocity was recorded continously without significant breaks. It is clear that over such a long period meteorological conditions are changing.
In the first part of this paper the statistics of atmospheric wind gusts as a small-scale turbulence phenomenon are examined using the statistics of the horizontal velocity increments. The results are compared to the velocity increments of the laboratory data. In the second part we examine the waiting time distributions of successive wind gusts to resolve their temporal structure. We find evidence that wind gusts are connected to the instationarity of the wind but can be reduced to stationary laboratory turbulence if a proper condition on a constant mean velocity is done.

2. Probabilistic description of wind gusts

Due to the large size of atmospheric flow dimensions the atmospheric wind is distinctively turbulent leading to very large Reynolds number Re. Although a characteristic length scale L for atmospheric turbulent flow structures depends on parameters like the surface roughness z_0 or the height z above ground, L may be of the order of about 100 m (Burton, 2001). With wind velocities of $U \approx 10\ ms^{-1}$ and more Reynolds numbers of larger than 10^8 are obtained.
Turbulent velocity time series are commonly decomposed into a mean speed value $\overline{u}(t)$ and random fluctuations (turbulence) around it $u(t)$:

$$U = \overline{u} + u \quad . \tag{1}$$

In wind energy research an averaging period of ten minutes is commonly used, even though the smallest possible averaging period is in general time-dependent (Treviño & Andreas, 2000). In this paper we are interested in the small-scale fluctuations, thus we only need a lower bound of such an averaging time. The presented analysis was performed with different averaging times ranging from one minute up to ten minutes. No significant changes in our results were observed. Thus we proceed with an averaging period of ten minutes.

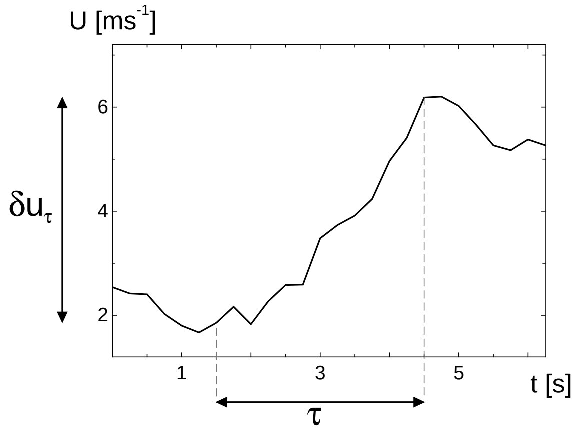

Figure 1. The picture shows an arbitrary excerpt of the horizontal wind speed time series including a strong wind gust.

The larger the fluctuation values u the more turbulent the wind field becomes. In wind energy research this is often expressed by means of the *turbulence intensity* I which is defined as the standard deviation σ in relation to the mean velocity $\bar{u}$ (Burton, 2001):

$$I \;=\; \frac{\sigma}{\overline{u}} \quad . \tag{2}$$

Nevertheless the value of I does not contain any dynamical or time-resolved information about the fluctuation field itself. To achieve a deeper understanding of wind gusts we investigate how far wind gusts are related to the well known features of small-scale turbulence.
As a natural and simple measure of wind gusts we use the statistics of *velocity increments* δu_τ:

$$\delta u_\tau \;=\; u(t+\tau) - u(t) \quad , \tag{3}$$

commonly used for intermittency analysis of small-scale turbulence in laboratory data (Lueck et al, 1999). The increments directly measure the velocity difference after a characteristic time τ as illustrated in Fig.1. So a large increment exceeding a certain threshold S ($\delta u_\tau > S$) can be defined as a gust.
For a statistical analysis we are interested in how frequent a certain increment value occurs and whether this frequency depends on τ. Therefore we first calculate the *probability density functions* (pdfs) $P(\delta u_\tau)$ of the increments of the atmospheric velocity fluctuations. In Fig. 2 the pdfs for 5 different values of τ are shown. These distributions are all characterized by marked fat *tails* and a peak around the mean value. Such pdfs are called *intermittent* and differ extremely from a *Gaussian*

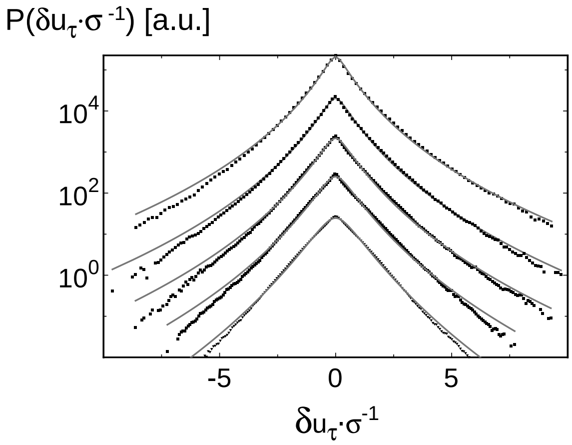

Figure 2. The pdfs of the increments of the atmospheric velocity fluctuations (normalized with σ) for τ being 0.25 s, 1.0 s, 6.8 s, 32 s and 2074 s (full symbols from the top to the bottom) are drawn in. They are shifted in vertical direction against each other for a clearer presentation and the corresponding fits according to eq. (5) are shown as solid curves. $P(\delta u_\tau \cdot \sigma^{-1})$ is given in **a**rbitrary **u**nits (a.u.).

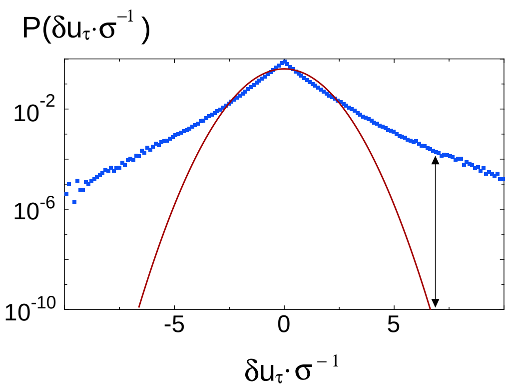

Figure 3. The distribution of the increments for $\tau = 4s$ is represented by the squares, a Gaussian distribution with the same standard deviation σ by the solid line (parabola due to the semilogarithmic presentation). Both distributions are normalized with $\sigma = 0.8\ ms^{-1}$.

distribution that is commonly considered to be the suitable distribution for continuous random processes.

A Gaussian or *normal* distribution is uniquely defined by its mean value μ and its standard deviation σ. Thus every distribution can be compared to a Gaussian distribution in a quantitative way. In Fig. 3 we compare one of the measured pdfs ($\tau = 4\ s$) with a normal distribution with the same σ. In this presentation the different behaviour of the tails of both distributions becomes evident. Note that the large increments –

located in the tails of the pdfs – correspond to strong gusts. For instance the value of $\delta u_\tau = 7 \cdot \sigma$ corresponds to a velocity ascending of 5.6 ms^{-1} during 4 s. As shown in Fig. 3 (arrow) the measured probability density of the increments of our wind data is about 10^6 times higher than for a corresponding Gaussian distribution! The value 10^6 – for instance – means that a certain gust which is observed about five times a day should be observed just once in 500 years if the distribution were a Gaussian instead of the observed intermittent one.

But intermittent distributions seem to appear quite often in natural or economical systems like in earthquake- (Schertzer & Lovejoy, 1994), foreign exchange market- (Ghashghaie et al., 1996) or even in some traffic-statistics (Vassilicos, 1995).

What kind of statistics do we get in the case of local isotropic and stationary laboratory experiments? The typical pdfs in laboratory turbulence – as shown in Fig. 4 a) – change from intermittent ones for small values of τ to rather Gaussian shaped distributions with increasing τ ($\tau \approx T$), where T is the *correlation* or *integral time*:

$$T = \int_0^\infty R(\tau) d\tau \quad , \tag{4}$$

well defined for laboratory data. $R(\tau)$ is the autocorrelation function of the fluctuations. The correlation time is about $6 \cdot 10^{-3} s$.

For the atmospheric wind data eq. (4) does not converge properly. To fix a large time T we take 1800 s as the upper limit of the integral and thus obtain $T = 34$ s.

For the pdfs of the atmospheric velocity field this characteristic change of shape, even for τ-values higher than $T = 34$ s (as shown in Fig. 2) is not observed.

As already mentioned a fundamental difference between atmospheric and laboratory turbulence is that the latter is stationary. In laboratory experiments one usually deals with fixed speed and direction of the mean wind $\bar{u}$, which is obviously never the case for atmospheric wind fields. Therefore in a second step we calculate the pdfs of the atmospheric increments only for certain mean velocity intervals. That means that only those increments are taken into account with $\bar{u}$ ranging in a narrow velocity interval with a width of typically 1 ms^{-1}. These *conditioned* pdfs $P(\delta u_\tau)_{cond}$ all show a similar qualitative change of shape like those of the laboratory experiment[1]. This is illustrated in Fig. 4 b) for one exemplary mean velocity interval.

To quantify this similarity we use a well established fit by an empirical

[1] Only for very small values of $\bar{u}$ ($u < 1$ ms^{-1}) this change of shape is not observed.

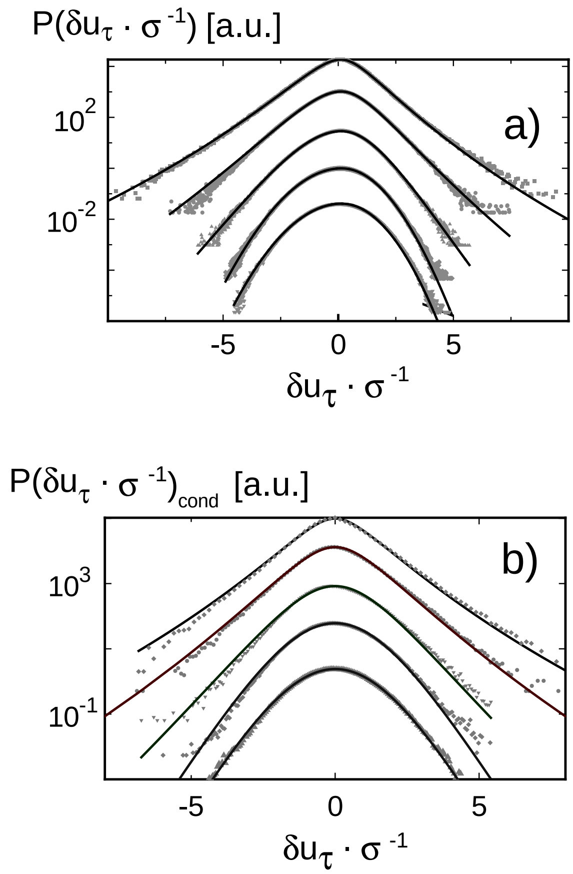

Figure 4. In a) the symbols represent the pdfs $P(\delta u_\tau)$ of the laboratory increments for different values of τ. From the top to the bottom τ takes the values: 0.005 T, 0.02 T, 0.17 T, 0.67 T and 1.35 T. In b) the conditioned pdfs of the atmospheric data are presented, here τ is 0.008 T, 0.03 T, 0.2 T, 0.95 T and 1.9 T. The mean wind interval on which the increments are conditioned is $[4.5\ ; 5.6]\ ms^{-1}$. In both cases the solid lines are the corresponding fits according to eq. (5). The distributions and their fits are shifted in vertical direction against each other for a clearer presentation.

explicit function for the pdf. This formula was derived in (Castaing et al., 1990) on the basis of Kolmogorov's understanding of a turbulent cascade:

$$P(\delta u_\tau) \;=\; \frac{1}{2\pi\lambda_\tau}\int_0^\infty exp(-\frac{\delta u_\tau^2}{2s^2})\cdot exp(-\frac{ln^2(s/s_0)}{2\lambda_\tau^2})\frac{d(lns)}{s} \quad . \tag{5}$$

In Fig. 2 and Fig. 4 these functions are represented by the solid lines. λ_τ^2 is the fundamental parameter in equation (5) and determines the shape of the probability distribution. As it can easily be seen equation

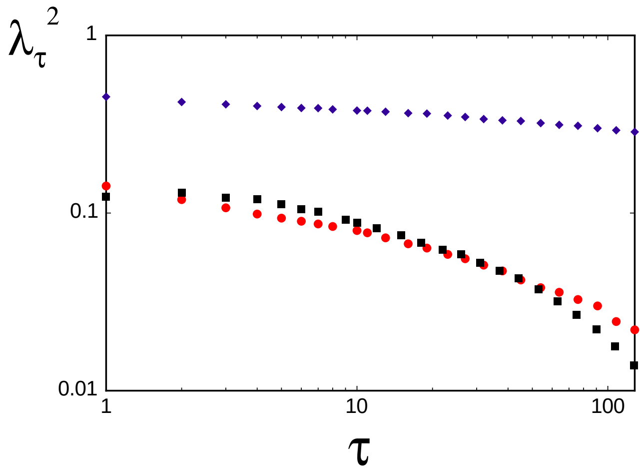

Figure 5. λ_τ^2 as a function of the increment distance τ is shown for the pdfs of the laboratory measurement (squares) and for the unconditioned (diamonds) as well as for the conditioned pdfs (circles) of the wind data. The axes both are logarithmic. For the laboratory data $\tau = 1$ corresponds to 10^{-5} s and for the wind data to 0.25 s.

(5) reduces to a Gaussian distribution if λ_τ^2 goes to zero:

$$\lim_{\lambda_\tau^2 \to o} P(\delta u_\tau) \;=\; \frac{1}{s_0\sqrt{2\pi}} exp(-\frac{\delta u_\tau^2}{2s_0^2}) \quad . \tag{6}$$

On the other hand the more λ_τ^2 increases the more intermittent the distributions become. In this way the parameter λ_τ^2 may serve to compare the pdfs with each other in a more quantitative way. In Fig. 5 the evolution of this parameter as a function of the increment distance τ is shown.

Other laboratory measurements (Castaing et al., 1990; Chabaud et al., 1994) of λ_τ^2 have shown evidence that it saturates at approximately 0.2. As shown in Fig. 5 λ_τ^2 of the conditioned wind increments as well as of the laboratory ones is approximately 0.2 for small $\tau-$values. Furthermore it tends to zero with increasing τ. None of these two features is observed in the case of the unconditioned increments, λ_τ^2 is rather independent from τ with a value of about 0.7. A constant behaviour of λ_τ^2 means that the shape of the pdfs remains unchanged, while its variance may change.

Thus we have shown that the anomalous statistics of wind fluctuations on discrete time intervals – which are obviously related to wind gusts – can be reduced to the well known intermittent (anomalous) statistics of local isotropic turbulence. This result deviates from results of wind data reported in (Ragwitz & Kantz, 2001), where it is claimed that their unconditioned wind pdfs behave like those from laboratory measurements.

3. Waiting time distribution

So far we have shown how the atmospheric turbulence is related to the laboratory one in a statistical way. This probabilistic approach describes the frequency with which certain gusts occur but it is not clear how they are distributed in time. In this sense we now examine the waiting times between successive wind gusts.

The marked fat tail behaviour of the unconditioned pdfs – as illustrated in Fig. 2 – points at an interesting effect. In (Schertzer & Lovejoy, 1993) the equivalence between the divergence of the moments $< x^q >$ and the hyperbolic (intermittent) form of pdfs which leads to a power law behaviour of the probability distribution is emphasized:

$$p(x \geq S) \propto S^{-q} \quad , S >> 1 \quad . \tag{7}$$

A famous example of such a natural power law behaviour is the *Gutenberg-Richter-law* (Gutenberg & Richter, 1956) that describes the frequency N of earthquakes with a magnitude being greater than a certain threshold M (magnitude):

$$\begin{aligned} log(N(m \geq M)) &= a - bM \\ \Leftrightarrow N(m \geq M) &\propto 10^{-bM} \quad . \end{aligned} \tag{8}$$

But also the waiting time distribution of fore- and after shocks obey a power-law, what is known as the *Omori-law* (Omori, 1894).

In this sense we now examine the waiting time distribution of wind gusts. Therefore we refer to the gust illustrated in Fig. 1 choosing different thresholds S and different increment distances τ (see eq. (3)). Always when the condition $\delta u_\tau > S$ is fulfilled a gust event is registered. To avoid that one event is counted several times we use the condition that the temporal distance between two successive events ΔT is at least τ.

The waiting time distributions $P(\Delta T)$ for $S = 4.0\ ms^{-1}$ and $\tau = 10\ s$ and for $S = 1.5\ ms^{-1}$ and $\tau = 65\ s$ are shown in Fig. 6 a) and 6 b), respectively. Due to the double-logarithmic presentation the distributions in Fig. 6 a) and 6 b) both follow a power law but with different exponents. The exponents depend on S and τ. This power law behaviour of the waiting time distributions is only observed for the atmospheric wind data and not for the stationary laboratory one.

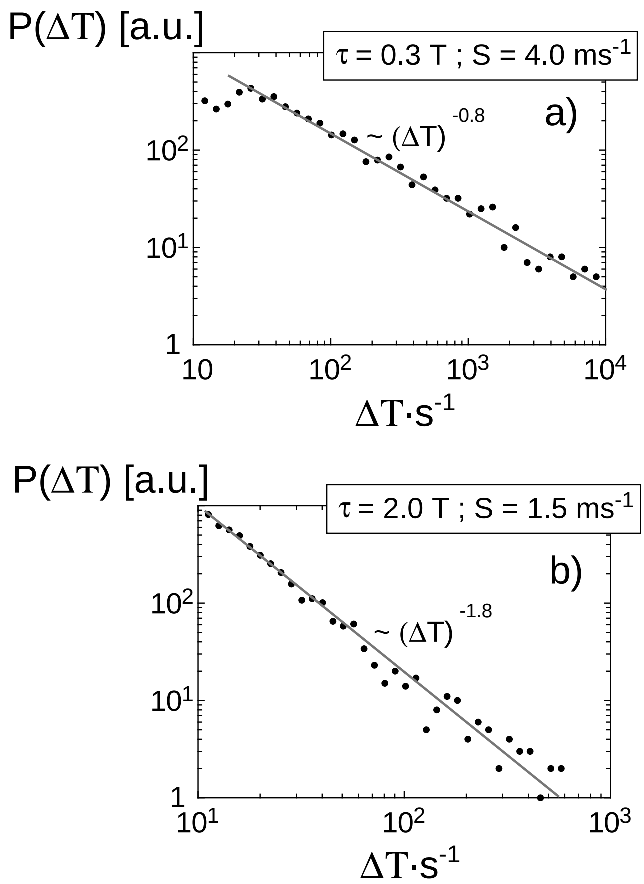

Figure 6. The filled symbols illustrate the waiting time distributions $P(\Delta T)$ – given in arbitrary units – between successive gusts. Additionally two power-law functions are drawn in (solid lines). a) and b) refer to two different combinations of τ and S. The corresponding exponents are -0.8 and -1.8 respectively.

4. Discussion and summary

On the basis of well defined velocity increments an analogous analysis of measured wind data and measured data from a turbulent wake was performed. The statistics of velocity increments, as related to the occurrence frequency of wind gusts, showed that they are highly intermittent. These anomalous (not Gaussian distributed) statistics explain an increased high probability of finding strong gusts. This could be set in analogy with turbulence measurements of an idealized, local isotropic laboratory flow if a proper condition on a mean wind speed was performed. This result is rather astonishing, insofar as solely the condition on the mean velocity leads to a good agreement between the pdfs of

the wind velocity increments and those of the laboratory wake flow. At least for our data it seems to be not necessary to introduce further conditions which take special meteorological situations into account. A possible explanation is the proposed universality of small-scale turbulence which means that the statistics of the small-scale fluctuations become independent of the driving large scale structures.
As a further statistical feature of wind gusts we have investigated the waiting times between successive gusts exceeding a certain strength. Here we find power-law-statistics (fractal statistics) – similar to earthquake statistics – that can not be reproduced in laboratory measurements.
To conclude we have shown two important aspects of wind gusts. The overall occurrence statistics could be set into analogy to the anomalous statistics of velocity increments in local isotropic turbulence. The time structure of successive gust events displays fractal behaviour. We think that these results may be helpful for a better characterization and understanding of gust events.
Of course we have to note that these results are obtained from one single wind data set. It should be very interesting to explore not only the effect of conditioning on a mean wind velocity but also on different flow situations or different boundary layer or other environmental conditions.

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