BATTEMENT EN PROFONDEUR INFINIE

COORDONNÉES DE LAGRANGE

$$x = x_0 + \left[2a \cos (\Delta kt - \Delta m x_0)\right] e^{-m y_0} \sin (kt - m x_0) + m k e^{-2 m y_0} \int^t \left[2a \cos (\Delta kt - \Delta m x_0)\right]^2 dt - 2 a^2 m e^{-2 \Delta m y_0} \sin (2 \Delta kt - 2 \Delta m x_0)$$

$$y = y_0 - \left[2a \cos (\Delta kt \;\; \Delta m x_0)\right] e^{-m y_0} \cos (kt - m x_0) - \frac{1}{2} m \left[2a \cos (\Delta kt - \Delta m x_0)\right]^2 e^{-2 m y_0} + 2 a^2 m (e^{-2 \Delta m y_0} - e^{-2 m y_0}) \cos (2 \Delta kt - 2 \Delta m x_0)$$

$$\frac{p}{\rho} = g y_0 + 2 g a^2 m (e^{-2 \Delta m y_0} - e^{-2 m y_0}) \cos (2 \Delta kt - 2 \Delta m x_0)$$

COORDONNÉES D'EULER

$$\varphi = - \left[2a \cos (\Delta kt - \Delta m x)\right] \frac{k}{m} e^{-my} \sin (kt - mx) + g a^2 k \frac{m}{k} e^{-2 \Delta m y} \sin (2 \Delta kt - 2 \Delta m x)$$

$$u = \left[2a \cos (\Delta kt - \Delta m x)\right] k e^{-my} \cos (kt - mx) - 4 a^2 m \Delta k e^{-2 \Delta m y} \cos (2 \Delta kt - 2 \Delta m x)$$

$$v = \left[2a \cos (\Delta kt - \Delta m x)\right] k e^{-my} \sin (kt - mx) - 4 a^2 m \Delta k e^{-2 \Delta m y} \sin (2 \Delta kt - 2 \Delta m x)$$

$$\frac{p}{\rho} = g y + g \left[2a \cos (\Delta kt - \Delta m x)\right] e^{-my} \cos (kt - mx) - \frac{1}{2} g \left[2a \cos (\Delta kt - \Delta m x_0)\right]^2 m e^{-2my} - 2 g a^2 m \frac{\Delta k}{k} e^{-2 \Delta m y} \cos (2 \Delta kt - 2 \Delta m x)$$

HOULE RÉGULIÈRE EN PROFONDEUR QUELCONQUE

(Formules données à titre de comparaison.)

COORDONNÉES DE LAGRANGE

$$x = x_0 + a \frac{\operatorname{ch} m(h-y_0)}{\operatorname{sh} mh} \sin (kt - m x_0) - a^2 m \frac{\sin (2kt - 2 m x_0)}{4 \operatorname{sh}^2 mh} \left[1 - \frac{3}{2} \frac{\operatorname{ch} 2m (h-y_0)}{\operatorname{sh}^2 mh}\right] + mk \frac{\operatorname{ch} 2m(h-y_0)}{2 \operatorname{sh}^2 mh} \int^t a^2 dt$$

$$y = y_0 - a \frac{\operatorname{sh} m (h-y_0)}{\operatorname{sh} mh} \cos (kt - m x_0) - a^2 m \frac{\operatorname{sh} 2m (h-y_0)}{4 \operatorname{sh}^2 mh} \left[1 + \frac{3}{2} \frac{\cos (2kt - 2 m x_0)}{\operatorname{sh}^2 mh}\right]$$

$$\frac{p}{\rho} = g y_0 + g a \frac{\operatorname{sh} m y_0}{\operatorname{sh} mh \operatorname{ch} mh} \cos (kt - m x_0) + g a^2 m \frac{\operatorname{sh} m y_0}{4 \operatorname{sh}^2 mh} \left\{ 3 \cos (2kt - 2 m x_0) \left[\frac{\operatorname{ch} m y_0}{\operatorname{sh}^2 mh} - \frac{2 \operatorname{ch} m (h-y_0)}{\operatorname{ch} mh}\right] + \frac{2 \operatorname{ch} m (h-y_0)}{\operatorname{ch} mh} \right\}$$

COORDONNÉES D'EULER

$$\varphi = -a \frac{k}{m} \frac{\operatorname{ch} m (h-y)}{\operatorname{sh} mh} \sin (kt - mx) - \frac{3}{8} a^2 k \frac{\operatorname{ch} 2m (h-y)}{\operatorname{sh}^4 mh} \sin (2kt - 2mx)$$

$$u = a k \frac{\operatorname{ch} m (h-y)}{\operatorname{sh} mh} \cos (kt - mx) + \frac{3}{4} a^2 m k \frac{\operatorname{ch} 2 m (h-y)}{\operatorname{sh}^4 mh} \cos (2kt - mx)$$

$$v = a k \frac{\operatorname{sh} m (h-y)}{\operatorname{sh} mh} \sin (kt - mx) + \frac{3}{4} a^2 m K \frac{\operatorname{sh} 2 m (h-y)}{\operatorname{sh}^4 mh} \sin (2kt - 2mx)$$

$$\frac{p}{\rho} = g y + g a \frac{\operatorname{ch} m (h-y)}{\operatorname{ch} mh} \cos (kt - mx) + \frac{3}{4} g a^2 m \operatorname{th} mh \left[\frac{\operatorname{ch} 2 m (h-y)}{\operatorname{sh}^2 mh} - \frac{1}{3}\right] \left[\frac{\cos (2kt - 2mx)}{\operatorname{sh}^2 mh} - \frac{1}{3}\right] + \frac{1}{4} g a^2 m \operatorname{th} mh \left(\frac{1}{\operatorname{sh}^2 mh} - \frac{1}{3}\right)$$

HOULE RÉGULIÈRE EN PROFONDEUR INFINIE

(Formules données à titre de comparaison.)

COORDONNÉES DE LAGRANGE

$$x = x_0 + a e^{-m y_0} \sin (kt - m x_0) + mk e^{-2 m y_0} \int^t a^2 dt$$

$$y = y_0 - a e^{-m y_0} \cos (kt - m x_0) - \frac{a^2 m}{2} e^{-2 m y_0}$$

$$\frac{p}{\rho} = g y_0$$

COORDONNÉES D'EULER

$$\varphi = -a \frac{k}{m} e^{-my} \sin (kt - mx)$$

$$u = a k e^{-my} \cos (kt - mx)$$

$$v = a k e^{-my} \sin (kt - mx)$$

$$\frac{p}{\rho} = g y + g a e^{-my} \cos (kt - mx) - \frac{1}{2} g a^2 m e^{-2my}$$

CLAPOTIS IMPARFAIT EN PROFONDEUR INFINIE

COORDONNÉES DE LAGRANGE

$$x = x_0 + 2 a e^{-m y_0} \cos (m x_0 - \Delta kt) \sin (kt - \Delta m x_0)$$

$$y = y_0 - 2 a e^{-m y_0} \sin (m x_0 - \Delta kt) \sin (kt - \Delta m x_0) - 2 a^2 m e^{-2 m y_0} \sin^2 (kt - \Delta m x_0)$$

$$\frac{p}{\rho} = g y_0 + 2 g a^2 m \left[e^{-2 \Delta m y_0} - e^{-2 m y_0}\right] \cos (2kt - 2 \Delta m x_0)$$

COORDONNÉES D'EULER

$$\varphi = 2 \frac{ak}{m} e^{-my} \sin (mx - \Delta kt) \cos (kt - \Delta mx) - a^2 k e^{-2 \Delta m y} \sin (2kt - 2 \Delta mx)$$

$$u = 2 a k e^{-my} \cos (mx - \Delta kt) \cos (kt - \Delta mx) + 4 a^2 m \Delta k e^{-2 \Delta m y} \cos (2kt - 2 \Delta mx)$$

$$v = - 2 a k e^{-my} \sin (mx - \Delta kt) \cos (kt - \Delta mx) + 4 a^2 m \Delta k e^{-2 \Delta m y} \sin (2kt - 2 \Delta mx)$$

$$\frac{p}{\rho} = g y + 2 g a e^{-my} \sin (mx - \Delta kt) \sin (kt - \Delta mx) - 2 g a^2 m e^{-2my} \sin^2 (kt - \Delta mx) + 2 g a^2 m (e^{-2 \Delta m y} - e^{-2my}) \cos (2kt - 2 \Delta mx)$$

CLAPOTIS RÉGULIER EN PROFONDEUR QUELCONQUE

(Formules données à titre de comparaison.)

COORDONNÉES DE LAGRANGE

$$x = x_0 + 2a \frac{\operatorname{ch} m (h-y_0)}{\operatorname{sh} mh} \cos m x_0 \sin kt - a^2 m \frac{\sin 2 m x_0}{\operatorname{sh}^2 mh} \left[\sin^2 kt + \frac{\operatorname{ch} 2m (h-y_0)}{4 \operatorname{sh}^2 mh} (3 \cos 2kt + \operatorname{th}^2 mh)\right]$$

$$y = y_0 - 2a \frac{\operatorname{sh} m (h-y_0)}{\operatorname{sh} mh} \sin m x_0 \sin kt - a^2 m \frac{\operatorname{sh} 2 m (h-y_0)}{\operatorname{sh}^2 mh} \left[\sin^2 kt + \frac{\cos 2 m x_0}{4 \operatorname{sh}^2 mh} (3 \cos 2kt + \operatorname{th}^2 mh)\right]$$

$$\frac{p}{\rho} = g y_0 + 2 g a \frac{\operatorname{sh} m y_0}{\operatorname{sh} mh \operatorname{ch} mh} \sin m x_0 \sin kt + g a^2 m \frac{\operatorname{sh} m y_0}{2 \operatorname{sh}^2 mh} \left\{ \operatorname{ch} m (2h - y_0) \left[\frac{\cos 2 m x_0}{\operatorname{ch}^2 mh} + 4 \sin^2 kt\right] + 4 \operatorname{th} mh \operatorname{sh} m (2h - y_0) \left[1 - 3 \sin^2 kt\right] + 3 \cos 2 m x_0 \cos 2kt \left[\frac{\operatorname{ch} m y_0}{\operatorname{sh}^2 mh} - \frac{2 \operatorname{ch} m (h-y_0)}{\operatorname{ch} mh}\right] \right.$$

COORDONNÉES D'EULER

$$\varphi = 2a \frac{k}{m} \frac{\operatorname{ch} m (h-y)}{\operatorname{sh} mh} \sin mx \cos kt - 3 a^2 k \frac{\operatorname{ch} 2m (h-y)}{4 \operatorname{sh}^4 mh} \cos 2mx \sin 2kt$$

$$u = 2ak \frac{\operatorname{ch} m (h-y)}{\operatorname{sh} mh} \cos mx \cos kt + 3 a^2 m k \frac{\operatorname{ch} 2m (h-y)}{2 \operatorname{sh}^4 mh} \sin 2mx \sin 2kt$$

$$v = -2ak \frac{\operatorname{sh} m (h-y)}{\operatorname{sh} mh} \sin mx \cos kt + 3 a^2 m k \frac{\operatorname{sh} 2m (h-y)}{2 \operatorname{sh}^4 mh} \cos 2mx \sin 2kt$$

$$\frac{p}{\rho} = g y + 2 g a \frac{\operatorname{ch} m (h-y)}{\operatorname{ch} mh} \sin mx \sin kt - g a^2 m \frac{\cos^2 kt}{\operatorname{sh} mh \operatorname{ch} mh} \left[\operatorname{ch} 2m (h-y) + \cos 2mx - 1\right] + 3 g a^2 m \frac{\operatorname{ch} 2 m (h-y)}{2 \operatorname{sh}^3 mh \operatorname{ch} mh} \cos 2mx \cos 2kt + 2 g a^2 m \operatorname{th} mh \cos 2kt$$

CLAPOTIS RÉGULIER EN PROFONDEUR INFINIE

(Formules données à titre de comparaison.)

COORDONNÉES DE LAGRANGE

$$x = x_0 + 2 a e^{-m y_0} \cos m x_0 \sin kt$$

$$y = y_0 - 2 a e^{-m y_0} \sin m x_0 \sin kt - 2 a^2 m e^{-2 m y_0} \sin^2 kt$$

$$\frac{p}{\rho} = g y_0 + 2 g a^2 m \left[1 - e^{-2 m y_0}\right] \cos 2kt$$

COORDONNÉES D'EULER

$$\varphi = 2 a \frac{k}{m} e^{-my} \sin mx \cos kt$$

$$u = 2 a k e^{-my} \cos mx \cos kt$$

$$v = 2 a k e^{-my} \sin mx \cos kt$$

$$\frac{p}{\rho} = g y + 2 g a e^{-my} \sin mx \cos kt - 2 g a^2 m e^{-2my} \sin^2 kt + 2 g a^2 m (1 - e^{-2my}) \cos 2kt$$

BATTEMENT EN PROFONDEUR QUELCONQUE

COORDONNÉES DE LAGRANGE

$$x = x_0 + \left[2a\cos(\Delta kt - \Delta m x_0)\right]\frac{\operatorname{ch} m(h-y_0)}{\operatorname{sh} mh}\sin(kt - m x_0) - \left[2a\cos(\Delta kt - \Delta m x_0)\right]^2\frac{m}{4\operatorname{sh}^2 mh}\left[1 - \frac{3}{2}\,\frac{\operatorname{ch} 2m(h-y_0)}{\operatorname{sh}^2 mh}\right]\sin(2kt - 2m x_0) + \frac{mk\operatorname{ch} 2m(h-y_0)}{2\operatorname{sh}^2 mh}\int^t\left[2a\cos(\Delta kt - \Delta m x_0)\right]^2 dt - a^2\frac{mk(\alpha + 2\alpha^2\operatorname{ch}^2 mh)\operatorname{ch} 2\Delta m(h-y_0)}{4\Delta k\operatorname{sh}^2 mh\left[(2\alpha-\alpha^2)\frac{\operatorname{sh} 2\Delta mh}{2\Delta mh}\operatorname{ch}^2 mh - \operatorname{ch} 2\Delta mh\right]}\sin(2\Delta kt - 2\Delta m x_0)$$

$$y = y_0 - \left[2a\cos(\Delta kt - \Delta m x_0)\right]\frac{\operatorname{sh} m(h-y_0)}{\operatorname{sh} mh}\cos(kt - m x_0) - \left[2a\cos(\Delta kt - \Delta m x_0)\right]^2\frac{m\operatorname{sh} 2m(h-y_0)}{4\operatorname{sh}^2 mh}\left[1 + \frac{3}{2}\,\frac{\cos(2kt - 2m x_0)}{\operatorname{sh}^2 mh}\right] - \frac{a^2 m\alpha}{2\operatorname{sh}^2 mh}\operatorname{sh} 2m(h-y_0)\cos(2\Delta kt - 2\Delta m x_0) + a^2\frac{mk(\alpha + 2\alpha^2\operatorname{ch}^2 mh)\operatorname{sh} 2\Delta m(h-y_0)}{4\Delta k\operatorname{sh}^2 mh\left[(2\alpha-\alpha^2)\frac{\operatorname{sh} 2\Delta mh}{2\Delta mh}\operatorname{ch}^2 mh - \operatorname{ch} 2\Delta mh\right]}\cos(2\Delta kt - 2\Delta m x_0)$$

$$\frac{p}{\rho} = g y_0 + g\left[2a\cos(\Delta kt - \Delta m x_0)\right]\frac{\operatorname{sh} m y_0}{\operatorname{sh} mh\operatorname{ch} mh}\cos(kt - m x_0) + g\left[2a\cos(\Delta kt - \Delta m x_0)\right]^2\frac{m\operatorname{sh} m y_0}{4\operatorname{sh}^2 mh}\times\left\{3\cos(2kt - 2m x_0)\left[\frac{\operatorname{ch} m y_0}{\operatorname{sh}^2 mh} - \frac{2\operatorname{ch} m(h-y_0)}{\operatorname{ch} mh}\right] + \frac{2\operatorname{ch} m(h-y_0)}{\operatorname{ch} mh}\right\} + g\frac{a^2 m\alpha}{\operatorname{sh}^2 mh}\operatorname{sh} m y_0\operatorname{ch} m(2h-y_0)\cos(2\Delta kt - 2\Delta m x_0) -$$

$$- g a^2\frac{mk(\alpha + 2\alpha^2\operatorname{ch}^2 mh)\operatorname{sh}\Delta m y_0\operatorname{ch}\Delta m(2h-y_0)}{2\Delta k\operatorname{sh}^2 mh\left[(2\alpha-\alpha^2)\frac{\operatorname{sh} 2\Delta mh}{2\Delta mh}\operatorname{ch}^2 mh - \operatorname{ch} 2\Delta mh\right]}\cos(2\Delta kt - 2\Delta m x_0) + g a^2\frac{m(\alpha + 2\alpha\operatorname{ch}^2 mh)\operatorname{sh}\Delta m y_0\operatorname{sh}\Delta m(2h-y_0)}{\operatorname{sh} mh\operatorname{ch} mh\left[(2\alpha-\alpha^2)\frac{\operatorname{sh} 2\Delta mh}{2\Delta mh}\operatorname{ch}^2 mh - \operatorname{ch} 2\Delta mh\right]}\cos(2\Delta kt - 2\Delta m x_0)$$

COORDONNÉES D'EULER

$$\varphi = -\left[2a\cos(\Delta kt - \Delta mx)\right]\frac{k}{m}\,\frac{\operatorname{ch} m(h-y)}{\operatorname{sh} mh}\sin(kt - mx) - \frac{3}{8}\left[2a\cos(\Delta kt - \Delta mx)\right]^2 k\frac{\operatorname{ch} 2m(h-y)}{\operatorname{sh}^4 mh}\sin(2kt - 2mx) + a^2\frac{k^2(1 + 2\alpha\operatorname{ch}^2 mh)\operatorname{ch} 2\Delta m(h-y)}{4\Delta k\operatorname{sh}^2 mh\left[(2\alpha-\alpha^2)\frac{\operatorname{sh} 2\Delta mh}{2\Delta mh}\operatorname{ch}^2 mh - \operatorname{ch} 2\Delta mh\right]}\sin(2\Delta kt - 2\Delta mx)$$

$$u = \left[2a\cos(\Delta kt - \Delta mx)\right]k\frac{\operatorname{ch} m(h-y)}{\operatorname{sh} mh}\cos(kt - mx) + \frac{3}{4}\left[2a\cos(\Delta kt - \Delta mx)\right]^2\frac{mk\operatorname{ch} 2m(h-y)}{\operatorname{sh}^4 mh}\cos(2kt - 2mx) - a^2\frac{mk(\alpha + 2\alpha^2\operatorname{ch}^2 mh)\operatorname{ch} 2\Delta m(h-y)}{2\operatorname{sh}^2 mh\left[(2\alpha-\alpha^2)\frac{\operatorname{sh} 2\Delta mh}{2\Delta mh}\operatorname{ch}^2 mh - \operatorname{ch} 2\Delta mh\right]}\cos(2\Delta kt - 2\Delta mx)$$

$$v = \left[2a\cos(\Delta kt - \Delta mx)\right]k\frac{\operatorname{sh} m(h-y)}{\operatorname{sh} mh}\sin(kt - mx) + \frac{3}{4}\left[2a\cos(\Delta kt - \Delta mx)\right]^2\frac{mk\operatorname{sh} 2m(h-y)}{\operatorname{sh}^4 mh}\sin(2kt - 2mx) - a^2\frac{mk(\alpha + 2\alpha^2\operatorname{ch}^2 mh)\operatorname{sh} 2\Delta m(h-y)}{2\operatorname{sh}^2 mh\left[(2\alpha-\alpha^2)\frac{\operatorname{sh} 2\Delta mh}{2\Delta mh}\operatorname{ch}^2 mh - \operatorname{ch} 2\Delta mh\right]}\sin(2\Delta kt - 2\Delta mx)$$

$$\frac{p}{\rho} = gy + g\left[2a\cos(\Delta kt - \Delta mx)\right]\frac{\operatorname{ch} m(h-y)}{\operatorname{ch} mh}\cos(kt - mx) + \frac{3}{4}g\left[2a\cos(\Delta kt - \Delta mx)\right]^2 m\operatorname{th} mh\left[\frac{\operatorname{ch} 2m(h-y)}{\operatorname{sh}^2 mh} - \frac{1}{3}\right]\left[\frac{\cos(2kt - 2mx)}{\operatorname{sh}^2 mh} - \frac{1}{3}\right] + \frac{1}{4}g\left[2a\cos(\Delta kt - \Delta mx)\right]^2 m\operatorname{th} mh\left(\frac{1}{\operatorname{sh}^2 mh} - \frac{1}{3}\right) - ga^2\frac{m}{2\operatorname{sh} mh\operatorname{ch} mh}\cos(2\Delta kt - 2\Delta mx) -$$

$$- ga^2 m\frac{(1 + 2\alpha\operatorname{ch}^2 mh)\operatorname{ch} 2\Delta m(h-y)}{2\operatorname{sh} mh\operatorname{ch} mh\left[(2\alpha-\alpha^2)\frac{\operatorname{sh} 2\Delta mh}{2\Delta mh}\operatorname{ch}^2 mh - \operatorname{ch} 2\Delta mh\right]}\cos(2\Delta kt - 2\Delta mx)$$

CLAPOTIS IMPARFAIT EN PROFONDEUR QUELCONQUE

COORDONNÉES DE LAGRANGE

$$x = x_0 + 2a\frac{\operatorname{ch} m(h-y_0)}{\operatorname{sh} mh}\cos(m x_0 - \Delta kt)\sin(kt - \Delta m x_0) - a^2 m\frac{\sin(2m x_0 - 2\Delta kt)}{\operatorname{sh}^2 mh}\left\{\sin^2(kt - \Delta m x_0) + \frac{\operatorname{ch} 2m(h-y_0)}{4\operatorname{sh}^2 mh}\left[3\cos(2kt - 2\Delta m x_0) + \operatorname{th}^2 mh\right]\right\} - a^2 m\frac{\sin(2m x_0 - 2\Delta kt)}{2\operatorname{sh}^2 mh}\left[(\alpha+1)\operatorname{ch} 2\Delta m(h-y_0) - 1 - \left(\frac{B_c}{\Delta k} + \frac{1}{2\operatorname{ch}^2 mh}\right)\operatorname{ch} 2m(h-y_0)\right]$$

$$y = y_0 - 2a\frac{\operatorname{sh} m(h-y_0)}{\operatorname{sh} mh}\sin(m x_0 - \Delta kt)\sin(kt - \Delta m x_0) - a^2 m\frac{\operatorname{sh} 2m(h-y_0)}{\operatorname{sh}^2 mh}\left\{\sin^2(kt - \Delta m x_0) + \frac{\cos(2m x_0 - 2\Delta kt)}{4\operatorname{sh}^2 mh}\left[3\cos(2kt - 2\Delta m x_0) + \operatorname{th}^2 mh\right]\right\} - a^2 m\frac{\cos(2m x_0 - 2\Delta kt)}{2\operatorname{sh}^2 mh}\left[\frac{k}{\Delta k}\operatorname{sh} 2\Delta m(h-y_0) - \left(\frac{B_c}{\Delta k} + \frac{1}{2\operatorname{ch}^2 mh}\right)\operatorname{sh} 2m(h-y_0)\right]$$

$$\frac{p}{\rho} = g y_0 + 2ga\frac{\operatorname{sh} m y_0}{\operatorname{sh} mh\operatorname{ch} mh}\sin(m x_0 - \Delta kt)\sin(kt - \Delta m x_0) + g m a^2\frac{\operatorname{sh} m y_0}{2\operatorname{sh}^2 mh}\left\{\operatorname{ch} m(2h-y_0)\left[\frac{\cos(2m x_0 - 2\Delta kt)}{\operatorname{ch}^2 mh} + 4\sin^2(kt - \Delta m x_0)\right] + 4\operatorname{th} mh\operatorname{sh} m(2h-y_0)\left[1 - 3\sin^2(kt - \Delta m x_0)\right] + 3\cos(2m x_0 - 2\Delta kt)\cos(2kt - 2\Delta m x_0)\left[\frac{\operatorname{ch} m y_0}{\operatorname{sh}^2 mh} - \frac{2\operatorname{ch} m(h-y_0)}{\operatorname{ch} mh}\right]\right\} -$$

$$- g m a^2\frac{\cos(2m x_0 - 2\Delta kt)}{\operatorname{sh}^2 mh}\left[\left(\frac{B_c}{\Delta k} + \frac{1}{2\operatorname{ch}^2 mh}\right)\operatorname{ch} m(2h-y_0)\operatorname{sh} m y_0 - \frac{k}{\Delta k}\operatorname{ch}\Delta m(2h-y_0)\operatorname{sh}\Delta m y_0\right] - g m a^2\frac{\cos(2kt - 2\Delta m x_0)}{\operatorname{sh}^2 mh}\cdot\frac{4\operatorname{ch}^2 mh - 3}{\operatorname{ch} 2\Delta mh}\operatorname{th} mh\operatorname{sh}\Delta m(2h-y_0)\operatorname{sh}\Delta m y_0$$

COORDONNÉES D'EULER

$$\varphi = 2\frac{ak}{m}\,\frac{\operatorname{ch} m(h-y)}{\operatorname{sh} mh}\sin(mx - \Delta kt)\cos(kt - \Delta mx) - 3a^2 k\frac{\operatorname{ch} 2m(h-y)}{4\operatorname{sh}^4 mh}\cos(2mx - 2\Delta kt)\sin(2kt - 2\Delta mx) - a^2 k\frac{4\operatorname{ch}^2 mh - 3}{4\operatorname{sh}^2 mh\operatorname{ch} 2\Delta mh}\operatorname{ch} 2\Delta m(h-y)\sin(2kt - 2\Delta mx) - a^2\frac{B_c}{2\operatorname{sh}^2 mh}\operatorname{ch} 2m(h-y)\sin(2mx - 2\Delta kt)$$

$$u = 2ak\frac{\operatorname{ch} m(h-y)}{\operatorname{sh} mh}\cos(mx - \Delta kt)\cos(kt - \Delta mx) + 3a^2 mk\frac{\operatorname{ch} 2m(h-y)}{2\operatorname{sh}^4 mh}\sin(2mx - 2\Delta kt)\sin(2kt - 2\Delta mx) + a^2\Delta mk\frac{4\operatorname{ch}^2 mh - 3}{2\operatorname{sh}^2 mh\operatorname{ch} 2\Delta mh}\operatorname{ch} 2\Delta m(h-y)\cos(2kt - 2\Delta mx) - a^2 m\frac{B_c}{2\operatorname{sh}^2 mh}\operatorname{ch} 2m(h-y)\cos(2mx - 2\Delta kt)$$

$$v = -2ak\frac{\operatorname{sh} m(h-y)}{\operatorname{sh} mh}\sin(mx - \Delta kt)\cos(kt - \Delta mx) + 3a^2 mk\frac{\operatorname{sh} 2m(h-y)}{2\operatorname{sh}^4 mh}\cos(2mx - 2\Delta kt)\sin(2kt - 2\Delta mx) + a^2\Delta mk\frac{4\operatorname{ch}^2 mh - 3}{2\operatorname{sh}^2 mh\operatorname{ch} 2\Delta mh}\operatorname{sh} 2\Delta m(h-y)\sin(2kt - 2\Delta mx) + a^2 m\frac{B_c}{2\operatorname{sh}^2 mh}\operatorname{sh} 2m(h-y)\sin(2mx - 2\Delta kt)$$

$$\frac{p}{\rho} = gy + 2ga\frac{\operatorname{ch} m(h-y)}{\operatorname{ch} mh}\sin(mx - \Delta kt)\sin(kt - \Delta mx) - g\frac{a^2 m}{\operatorname{sh} mh\operatorname{ch} mh}\left[\operatorname{ch} 2m(h-y) + \cos(2mx - 2\Delta kt) - 1\right]\cos^2(kt - 2\Delta mx) + 3ga^2 m\frac{\operatorname{ch} 2m(h-y)}{2\operatorname{sh}^3 mh\operatorname{ch} mh}\cos(2mx - 2\Delta kt)\cos(2kt - 2\Delta mx) +$$

$$+ 2ga^2 m\operatorname{th} mh\cos(2kt - 2\Delta mx) + ga^2 m\frac{4\operatorname{ch}^2 mh - 3}{2\operatorname{sh} mh\operatorname{ch} mh}\left[\frac{\operatorname{ch} 2\Delta m(h-y)}{\operatorname{ch} 2\Delta mh} - 1\right]\cos(2kt - 2\Delta mx) - ga^2 m\frac{\operatorname{ch} 2\Delta m(h-y) - 1}{2\operatorname{sh} mh\operatorname{ch} mh}\cos(2mx - 2\Delta kt)$$

avec :

$$B_c = k\,\frac{\operatorname{sh} 2\Delta mh - \frac{\Delta k}{k}\operatorname{th} mh\operatorname{ch} 2\Delta mh}{\operatorname{sh} 2mh}$$

ÉQUATIONS GÉNÉRALES

COORDONNÉES DE LAGRANGE

$$x = x_0 + \sum_{i=1}^{n} a_i \frac{\operatorname{ch} m_i (h-y_0)}{\operatorname{sh} m_i h} \sin (k_i t - m_i x_0) - \sum_{i=1}^{n} a_i^2 \frac{m_i}{4 \operatorname{sh}^2 m_i h} \left[1 - \frac{3}{2} \frac{\operatorname{ch} 2 m_i (h-y_0)}{\operatorname{sh}^2 m_i h}\right] \sin (2 k_i t - 2 m_i x_0) + \sum_{i=1}^{n} a_i^2 \frac{m_i k_i \operatorname{ch} 2 m_i (h-y_0)}{2 \operatorname{sh}^2 m_i h} x t$$

$$+ \sum_{i=1}^{n} \sum_{j=1}^{i-1} \frac{a_i a_j}{2 \operatorname{sh} m_i k \operatorname{sh} m_j h} \left\{ \frac{m_i k_i + m_j k_j}{k_i - k_j} \operatorname{ch}\left[(m_i + m_j)(h-y_0)\right] \sin\left[(k_i - k_j)t - (m_i - m_j) x_0\right] - \frac{m_i k_i + m_j k_j}{k_i + k_j} \operatorname{ch}\left[(m_i - m_j)(h-y_0)\right] \sin\left[(k_i + k_j)t - (m_i + m_j) x_0\right] + \frac{m_i + m_j}{k_i + k_j} A_{ij} \operatorname{ch}\left[(m_i + m_j)(h-y_0)\right] \sin\left[(k_i + k_j)t - (m_i + m_j) x_0\right] - \frac{m_i - m_j}{k_i - k_j} B_{ij} \operatorname{ch}\left[(m_i - m_j)(h-y_0)\right] \sin\left[(k_i - k_j)t - (m_i - m_j) x_0\right] \right\}$$

$$y = y_0 - \sum_{i=1}^{n} a_i \frac{\operatorname{sh} m_i (h-y_0)}{\operatorname{sh} m_i h} \cos (k_i t - m_i x_0) - \sum_{i=1}^{n} a_i^2 \frac{m_i}{4 \operatorname{sh}^2 m_i h} \left[1 + \frac{3}{2} \frac{\cos (2 k_i t - 2 m_i x_0)}{\operatorname{sh}^2 m_i h}\right] \operatorname{sh} 2 m_i (h-y_0)$$

$$- \sum_{i=1}^{n} \sum_{j=1}^{i-1} \frac{a_i a_j}{2 \operatorname{sh} m_i h \operatorname{sh} m_j h} \left\{ \frac{m_i k_i - m_j k_j}{k_i - k_j} \operatorname{sh}\left[(m_i + m_j)(h-y_0)\right] \cos\left[(k_i - k_j)t - (m_i - m_j) x_0\right] - \frac{m_i k_i - m_j k_j}{k_i k_j} \operatorname{sh}\left[(m_i - m_j)(h-y_0)\right] \cos\left[(k_i + k_j)t - (m_i + m_j) x_0\right] + \frac{m_i + m_j}{k_i + k_j} A_{ij} \operatorname{sh}\left[(m_i + m_j)(h-y_0)\right] \cos\left[(k_i + k_j)t - (m_i + m_j) x_0\right] - \frac{m_i - m_j}{k_i - k_j} B_{ij} \operatorname{sh}\left[(m_i - m_j)(h-y_0)\right] \cos\left[(k_i - k_j)t - (m_i - m_j) x_0\right] \right\}$$

$$\frac{p}{\rho} = g y_0 + \sum_{i=1}^{n} g a_i \frac{\operatorname{sh} m_i y_0}{\operatorname{sh} m_i h \operatorname{ch} m_i h} \cos (k_i t - m_i x_0) + \sum_{i=1}^{n} g a_i^2 \frac{m_i \operatorname{sh} m_i y_0}{4 \operatorname{sh}^2 m_i h} \left\{ 3 \cos (2 k_i t - 2 m_i x_0) \left[\frac{\operatorname{ch} m_i y_0}{\operatorname{sh}^2 m_i h} - \frac{2 \operatorname{ch} m_i (h-y_0)}{\operatorname{ch} m_i h} \right] + \frac{2 \operatorname{ch} m_i (h-y_0)}{\operatorname{ch} m_i h} \right\}$$

$$+ \sum_{i=1}^{n} \sum_{j=1}^{i-1} \frac{a_i a_j}{2 \operatorname{sh} m_i h \operatorname{sh} m_i h} \left\{ (k_i^2 - k_i k_j + k_j^2) \operatorname{ch}\left[(m_i + m_j)(h-y_0)\right] \cos\left[(k_i - k_j)t - (m_i - m_j) x_0\right] - (k_i^2 + k_i k_j + k_j^2) \operatorname{ch}\left[(m_i - m_j)(h-y_0)\right] \cos\left[(k_i + k_j)t - (m_i + m_j) x_0\right] + (k_i + k_j) A_{ij} \operatorname{ch}\left[(m_i + m_j)(h-y)\right] \cos\left[(k_i + k_j)t - (m_i + m_j) x_0\right] - (k_i - k_j) B_{ij} \operatorname{ch}\left[(m_i - m_j)(h-y_0)\right] \cos\left[(k_i - k_j)t - (m_i - m_j) x_0\right] \right\}$$

$$- g \sum_{i=1}^{n} \sum_{j=1}^{i-1} \frac{a_i a_j}{2 \operatorname{sh} m_i h \operatorname{sh} m_j h} \left\{ \frac{m_i k_i - m_j k_j}{k_i - k_j} \operatorname{sh}\left[(m_i + m_j)(h-y_0)\right] \cos\left[(k_i - k_j)t - (m_i - m_j) x_0\right] - \frac{m_i k_i - m_j k_j}{k_i + k_j} \operatorname{sh}\left[(m_i - m_j)(h-y_0)\right] \cos\left[(k_i + k_j)t - (m_i + m_j) x_0\right] + \frac{m_i + m_j}{k_i + k_j} A_{ij} \operatorname{sh}\left[(m_i + m_j)(h-y_0)\right] \cos\left[(k_i + k_j)t - (m_i + m_j) x_0\right] - \frac{m_i - m_j}{k_i - k_j} B_{ij} \operatorname{sh}\left[(m_i - m_j)(h-y_0)\right] \cos\left[(k_i - k_j)t - (m_i - m_j) x_0\right] \right\}$$

COORDONNÉES D'EULER

$$\varphi = - \sum_{i=1}^{n} \frac{a_i k_i}{m_i} \frac{\operatorname{ch} m_i (h-y)}{\operatorname{sh} m_i h} \sin (k_i t - m_i x) - \sum_{i=1}^{n} \frac{3}{8} a_i^2 k_i \frac{\operatorname{ch} 2 m_i (h-y)}{\operatorname{sh}^4 m_i h} \sin (2 k_i t - 2 m_i x) - \sum_{i=1}^{n} \sum_{j=1}^{i-1} \frac{a_i a_j}{2 \operatorname{sh} m_i h \operatorname{sh} m_j h} \left\{ A_{ij} \operatorname{ch}\left[(m_i + m_j)(h-y)\right] \sin\left[(k_i + k_j)t - (m_i + m_j) x\right] - B_{ij} \operatorname{ch}\left[(m_i - m_j)(h-y)\right] \sin\left[(k_i - k_j)t - (m_i - m_j) x\right] \right\}$$

$$u = \sum_{i=1}^{n} a_i k_i \frac{\operatorname{ch} m_i (h-y)}{\operatorname{sh} m_i h} \cos (k_i t - m_i x) + \sum_{i=1}^{n} \frac{3}{4} a_i^2 \frac{m_i k_i \operatorname{ch} 2 m_i (h-y)}{\operatorname{sh}^4 m_i h} \cos (2 k_i t - 2 m_i x) + \sum_{i=1}^{n} \sum_{j=1}^{i-1} \frac{a_i a_j}{2 \operatorname{sh} m_i h \operatorname{sh} m_j h} \left\{ (m_i + m_j) A_{ij} \operatorname{ch}\left[(m_i + m_j)(h-y)\right] \cos\left[(k_i + k_j)t - (m_i + m_j) x\right] - (m_i - m_j) B_{ij} \operatorname{ch}\left[(m_i - m_j)(h-y)\right] \cos\left[(k_i - k_j)t - (m_i - m_j) x\right] \right\}$$

$$v = \sum_{i=1}^{n} a_i k_i \frac{\operatorname{sh} m_i (h-y)}{\operatorname{sh} m_i h} \sin (k_i t - m_i x) + \sum_{i=1}^{n} \frac{3}{4} a_i^2 \frac{m_i k_i \operatorname{sh} 2 m_i (h-y)}{\operatorname{sh}^4 m_i h} \sin (2 k_i t - 2 m_i x) + \sum_{i=1}^{n} \sum_{j=1}^{i-1} \frac{a_i a_j}{2 \operatorname{sh} m_i h \operatorname{sh} m_j h} \left\{ (m_i + m_j) A_{ij} \operatorname{sh}\left[(m_i + m_j)(h-y)\right] \sin\left[(k_i + k_j)t - (m_i + m_j) x\right] - (m_i - m_j) B_{ij} \operatorname{sh}\left[(m_i - m_j)(h-y)\right] \sin\left[(k_i - k_j)t - (m_i - m_j) x\right] \right\}$$

$$\frac{p}{\rho} = g y + \sum_{i=1}^{n} a_i \frac{k_i^2}{m_i} \frac{\operatorname{ch} m_i (h-y)}{\operatorname{sh} m_i h} \cos (k_i t - m_i x) + \sum_{i=1}^{n} \frac{3}{4} a_i^2 k_i^2 \left[\frac{\operatorname{ch} 2 m_i (h-y)}{\operatorname{sh}^2 m_i h} \frac{1}{3}\right] \left[\frac{\cos (2 k_i t - 2 m_i x)}{\operatorname{sh}^2 m_i h} - \frac{1}{3}\right] + \sum_{i=1}^{n} \frac{a_i^2 k_i^2}{4} \left(\frac{1}{\operatorname{sh}^2 m_i h} - \frac{1}{3}\right)$$

$$- \sum_{i=1}^{n} \sum_{j=1}^{i-1} \frac{a_i a_j}{2 \operatorname{sh} m_i h \operatorname{sh} m_j h} \left\{ k_i k_j \operatorname{ch}\left[(m_i + m_j)(h-y)\right] \cos\left[(k_i - k_j)t - (m_i - m_j) x\right] + k_i k_j \operatorname{ch}\left[(m_i - m_j)(h-y)\right] \cos\left[(k_i + k_j)t - (m_i + m_j) x\right] - (k_i + k_j) A_{ij} \operatorname{ch}\left[(m_i + m_j)(h-y)\right] \cos\left[(k_i + k_j)t - (m_i + m_j) x\right] + (k_i - k_j) B_{ij} \operatorname{ch}\left[(m_i - m_j)(h-y)\right] \cos\left[(k_i - k_j)t - (m_i - m_j) x\right] \right\}$$

Dans ces formules on a posé pour simplifier

$$A_{ij} = \frac{(k_i + k_j)(k_i^2 + k_i k_j + k_j^2) - C'^2_{g,ij} (m_i k_i - m_j k_j)(m_i - m_j)}{(k_i + k_j)^2 \left[1 - \left(\frac{D'_{g,ij}}{D_{g,ij}}\right)^2\right]} \times \frac{\operatorname{ch} (m_i - m_j) h}{\operatorname{ch} (m_i + m_j) h}$$

$$B_{ij} = \frac{(k_i - k_j)(k_i^2 - k_i k_j + k_j^2) - D'^2_{g,ij} (m_i k_i - m_j k_j)(m_i + m_j)}{(k_i - k_j)^2 \left[1 - \left(\frac{C'_{g,ij}}{C_{g,ij}}\right)^2\right]} \times \frac{\operatorname{ch} (m_i + m_j) h}{\operatorname{ch} (m_i - m_j) h}$$

avec

$C'_{g,ij} = \sqrt{\frac{g}{m_i - m_j} \operatorname{th} (m_i - m_j) h}$ — célérité propre des ondes de groupe

$C_{g,ij} = \frac{k_i - k_j}{m_i - m_j}$ — vitesse des groupes

$D'_{g,ij} = \sqrt{\frac{g}{m_i + m_j} \operatorname{th} (m_i + m_j) h}$ — célérité propre des demi-ondes

$D_{g,ij} = \frac{k_i + k_j}{m_i + m_j}$ — vitesse dite des demi-ondes (vitesse moyenne)