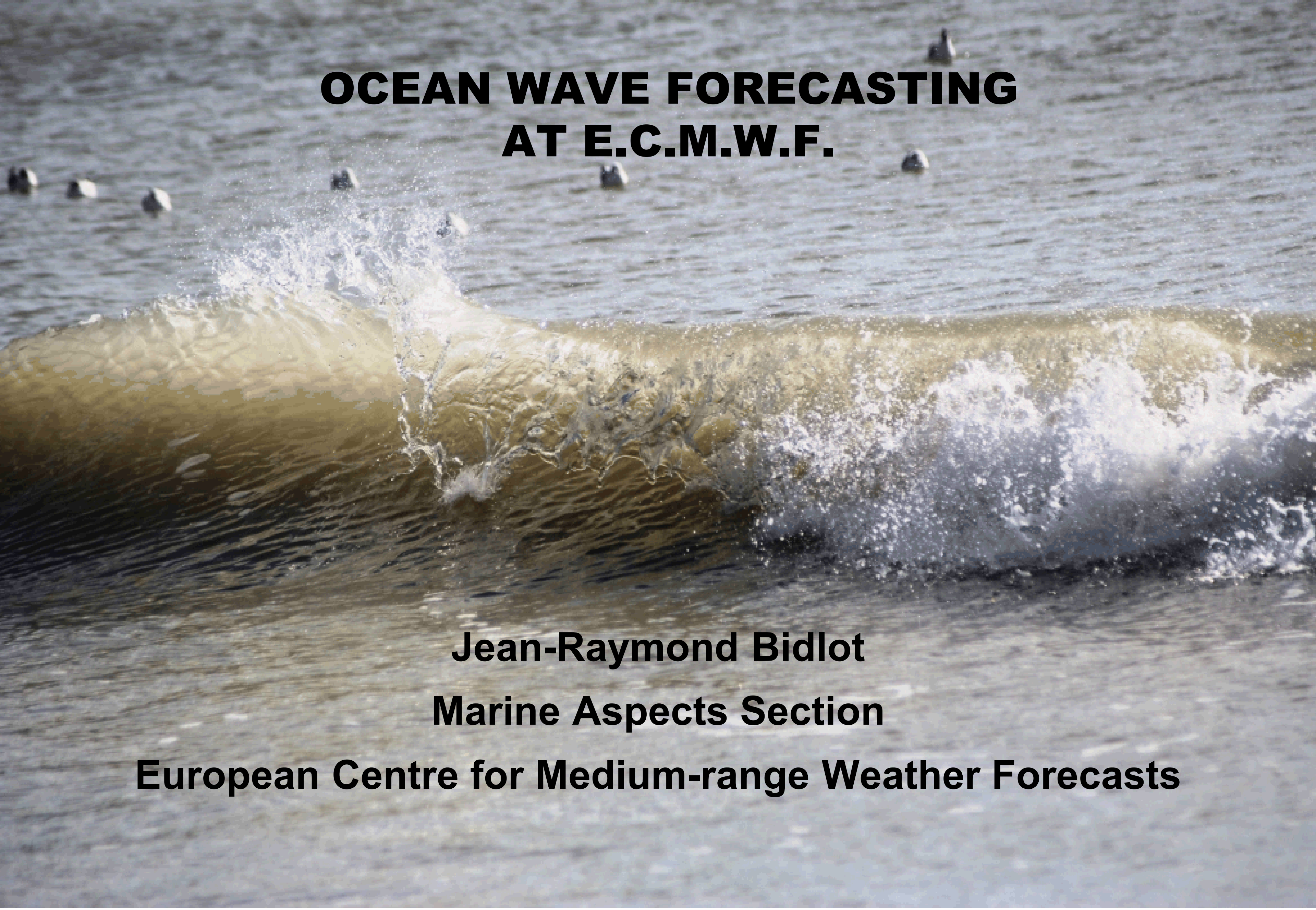


OCEAN WAVE FORECASTING AT E.C.M.W.F.

Jean-Raymond Bidlot

Marine Aspects Section

European Centre for Medium-range Weather Forecasts

Resources:

Lecture notes available at:

http://www.ecmwf.int/newsevents/training/lecture_notes/pdf_files/NUMERIC/Wave_mod.pdf

Model documentation available at:

http://www.ecmwf.int/research/ifsdocs/CY38r1/index.html

Directly Related Books:

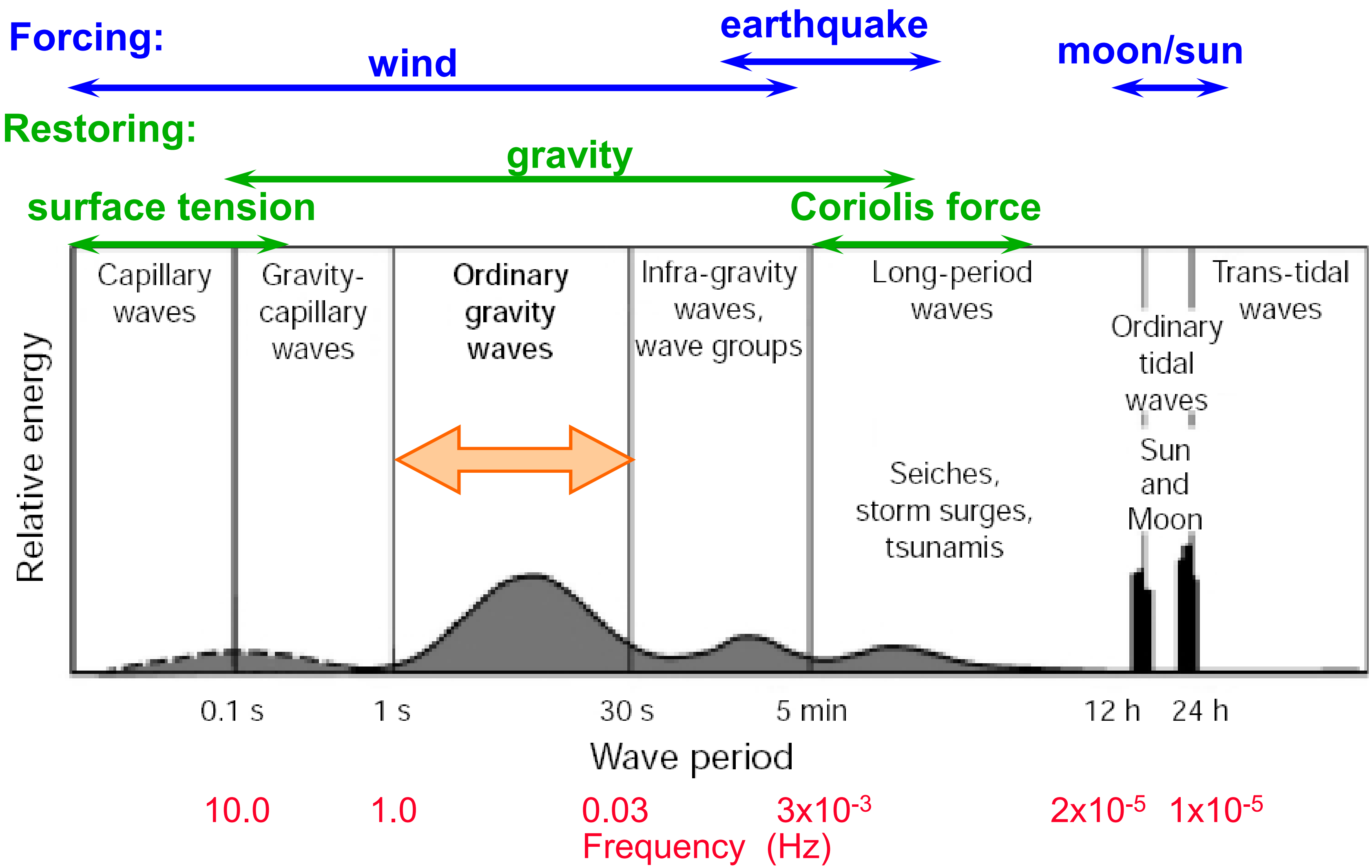
- "Dynamics and Modelling of Ocean Waves".
  by: G.J. Komen, L. Cavaleri, M. Donelan,
  K. Hasselmann, S. Hasselmann,
  P.A.E.M. Janssen.
  Cambridge University Press, 1996.

- "The Interaction of Ocean Waves and Wind".
  By: Peter Janssen
  Cambridge University Press, 2004.

INTRODUCTION

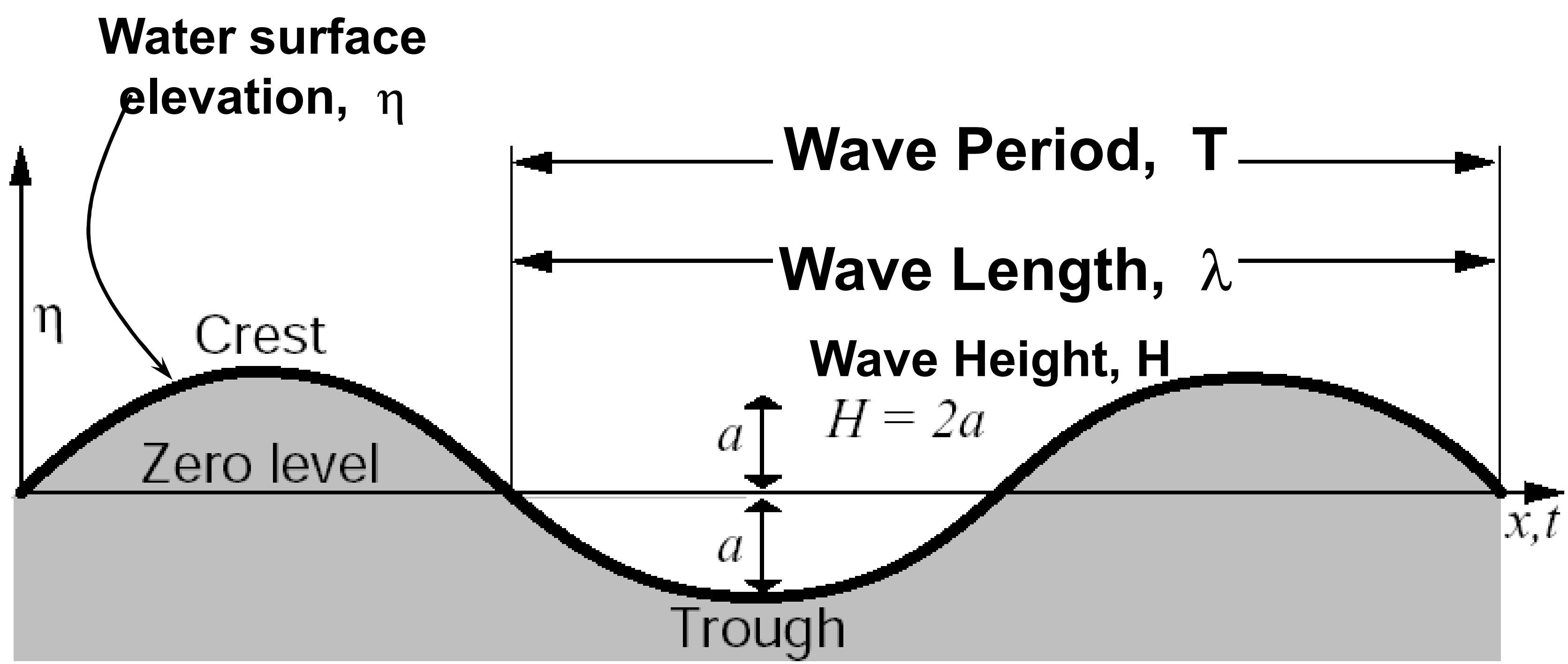
ECMWF

What we are dealing with



ECMWF

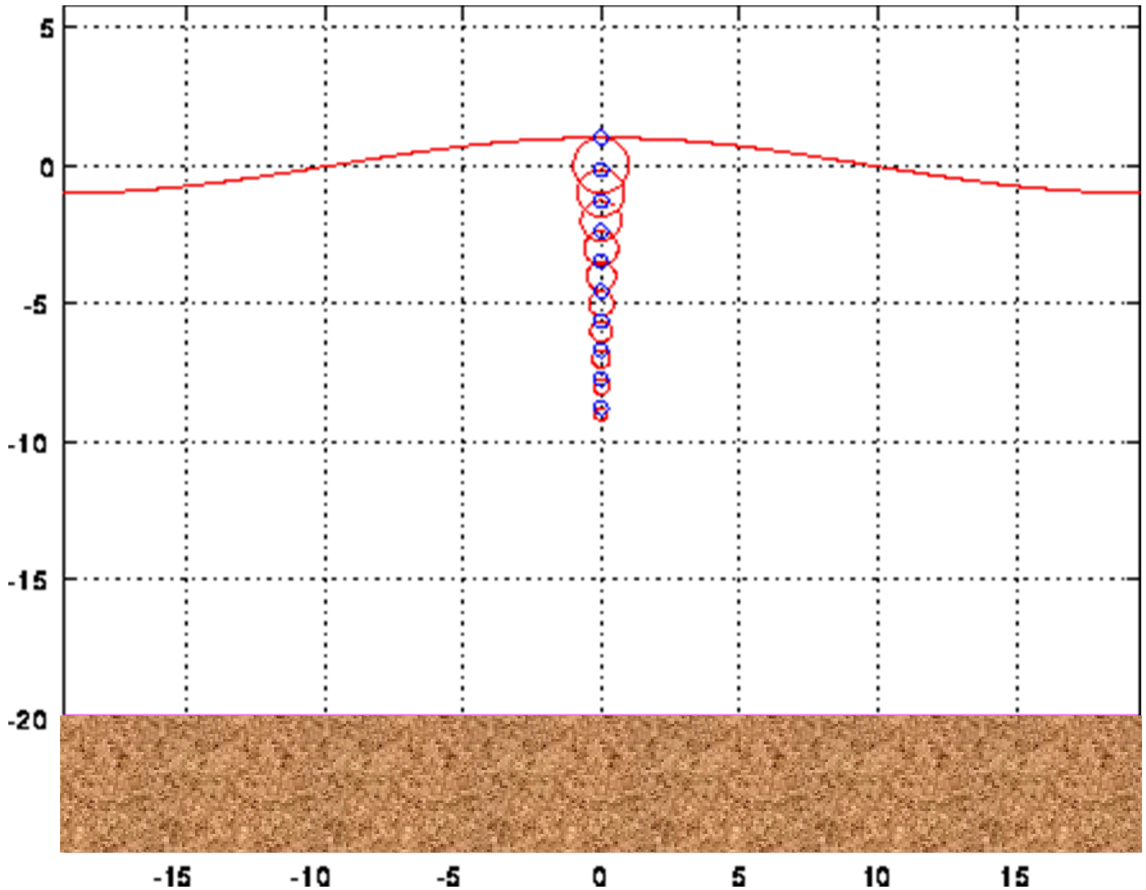
What we are dealing with



Wave number: $k = 2\,\pi\,/\,\lambda$

Wave (angular) frequency: $\omega = 2\,\pi\,/\,T$

What we are dealing with



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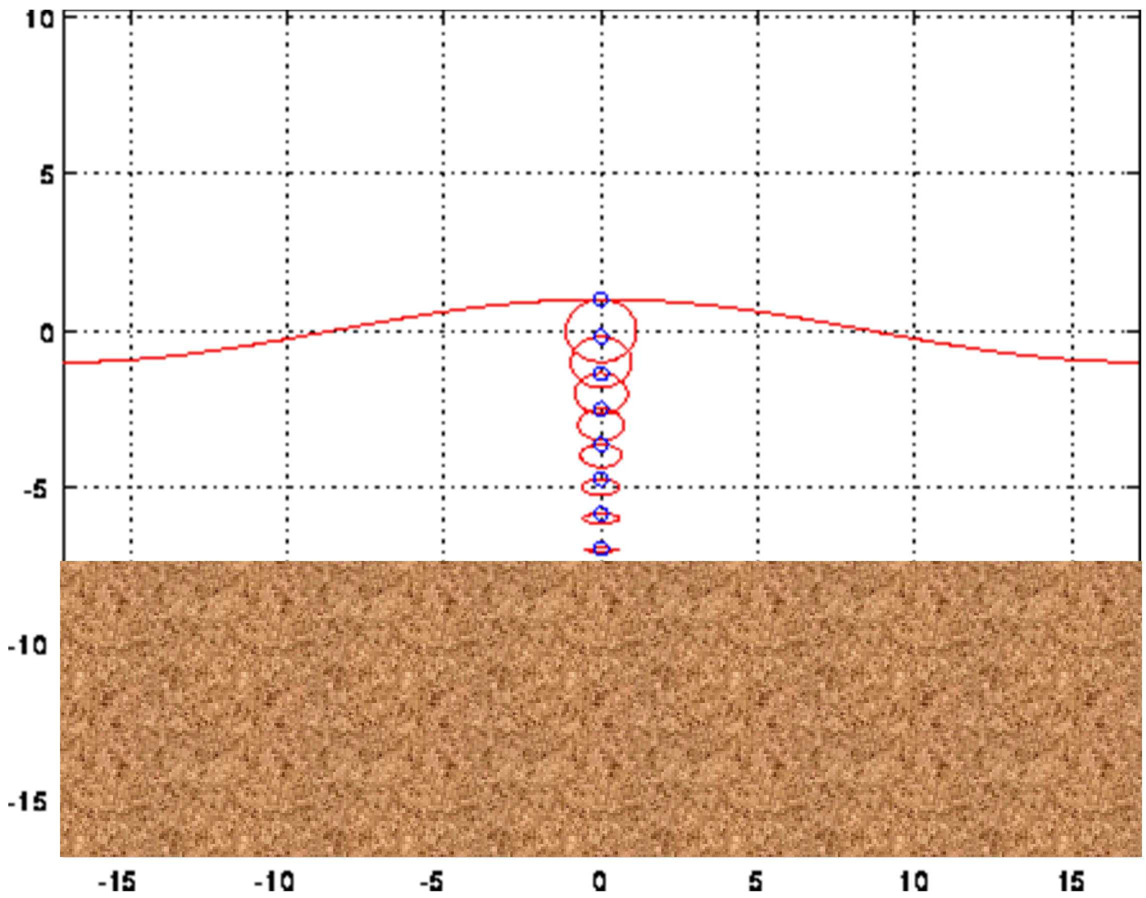
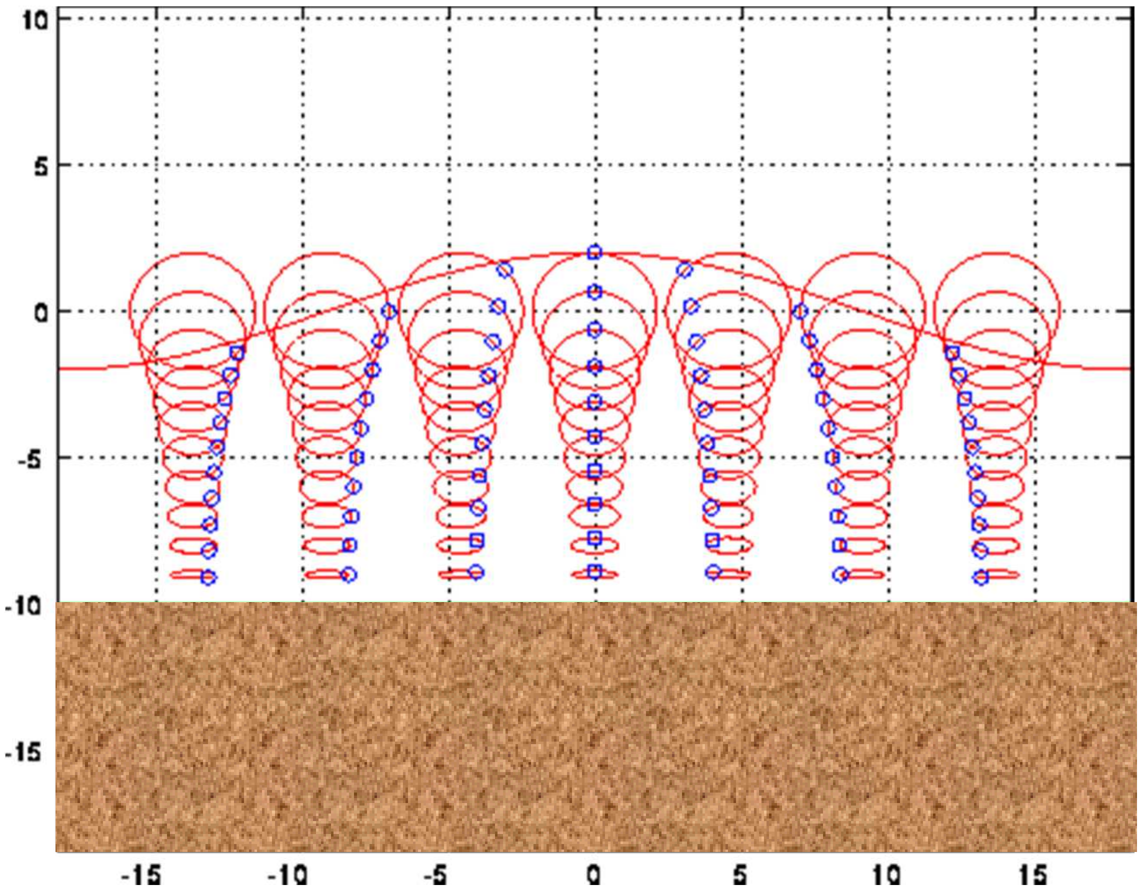
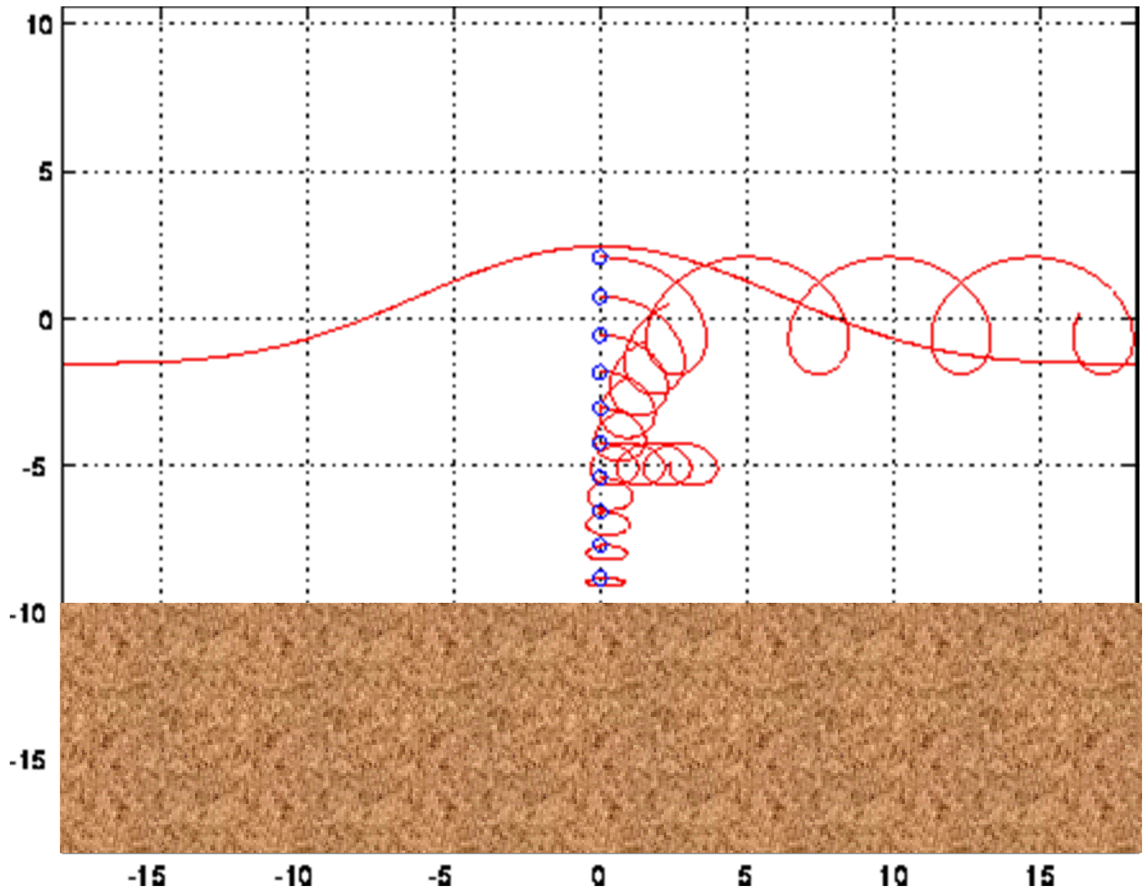


What we are dealing with

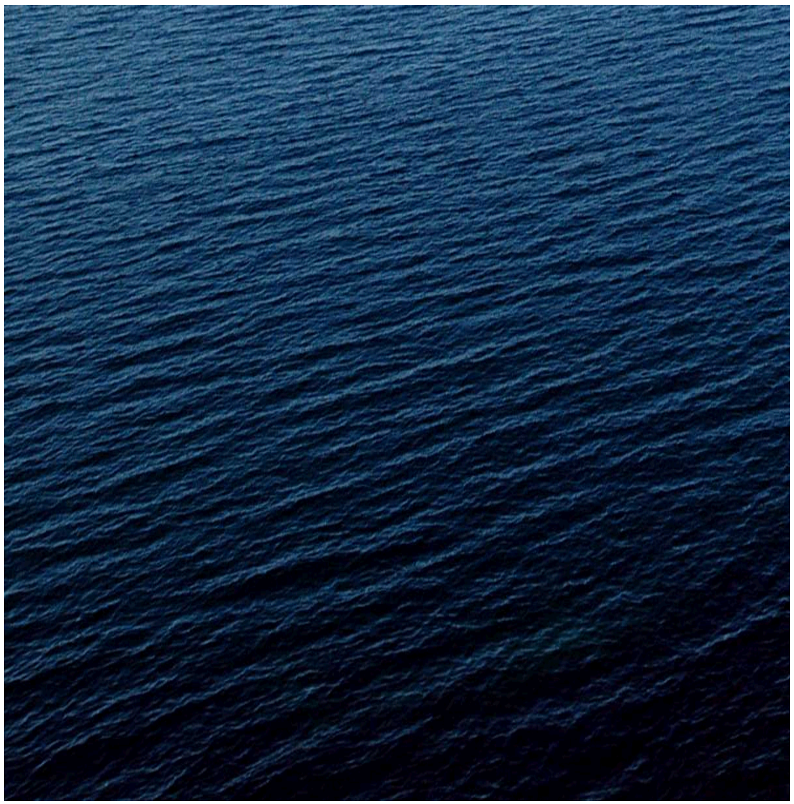


What we are dealing with

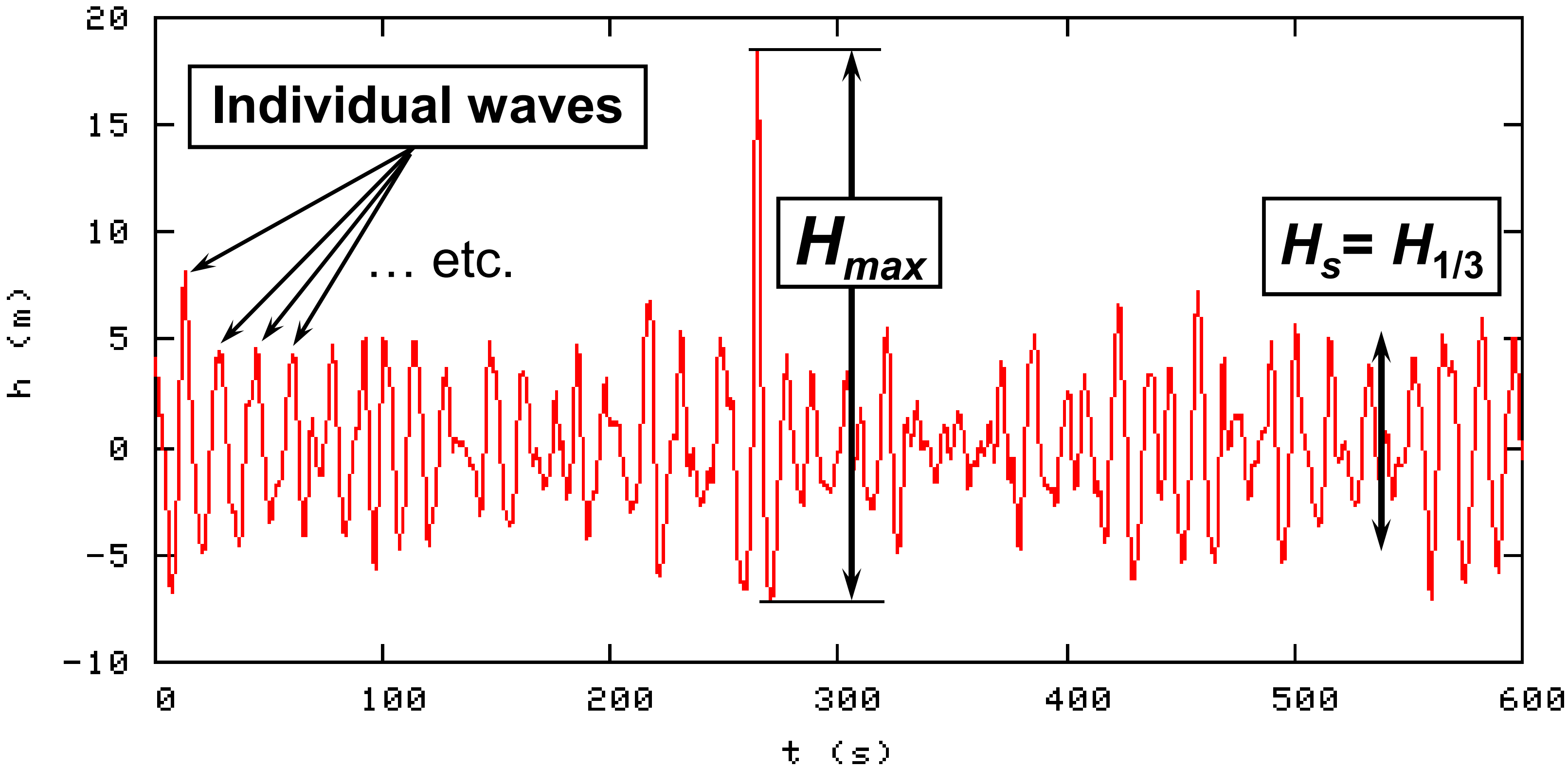


Ocean waves:

We are dealing with wind generated waves at the surface of the oceans, from gentle to rough …

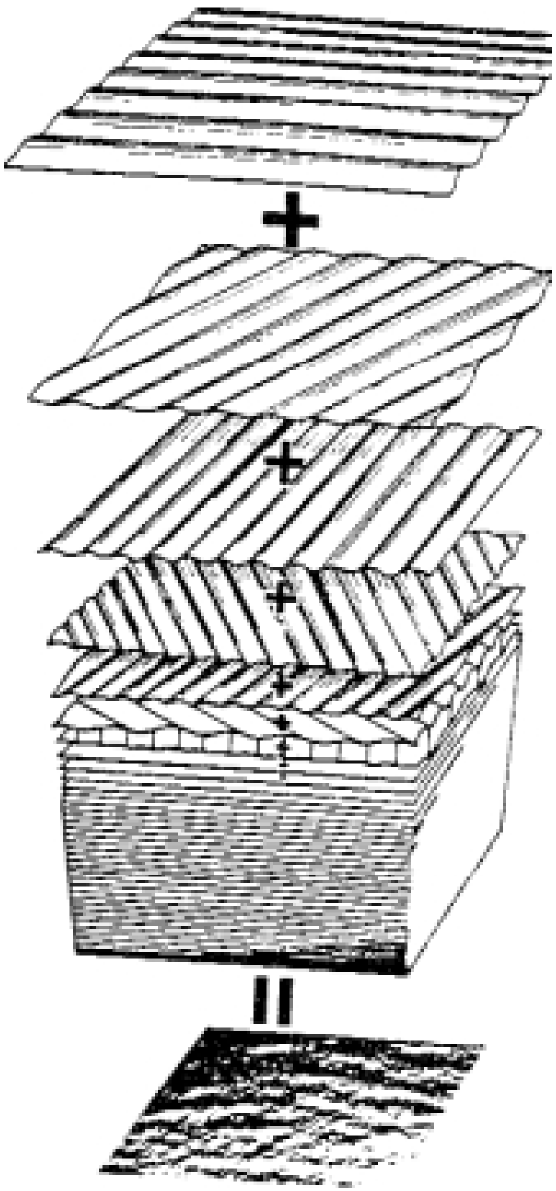


A Wave Record
Individual Waves,
Significant Wave Height, H_s,
Maximum Individual Wave Height, H_{max}



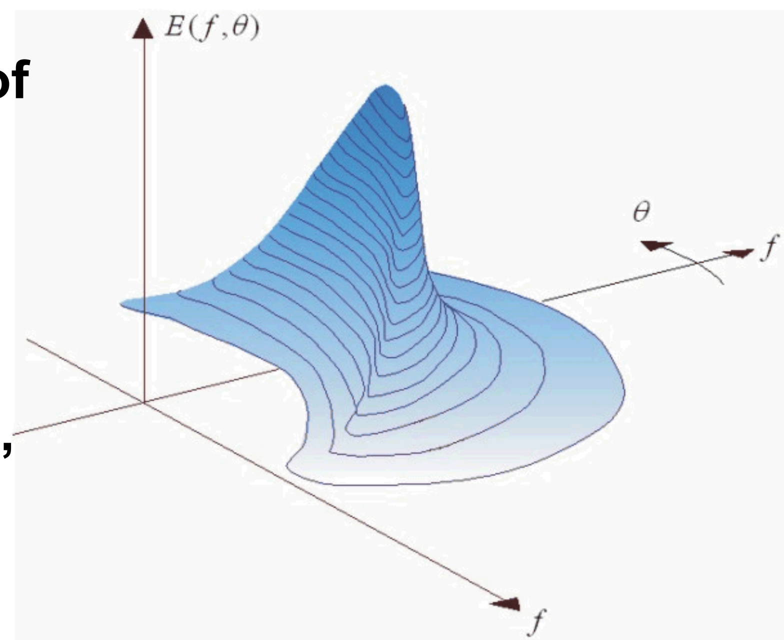
Surface elevation time series from platform Draupner in the North Sea

Wave Spectrum



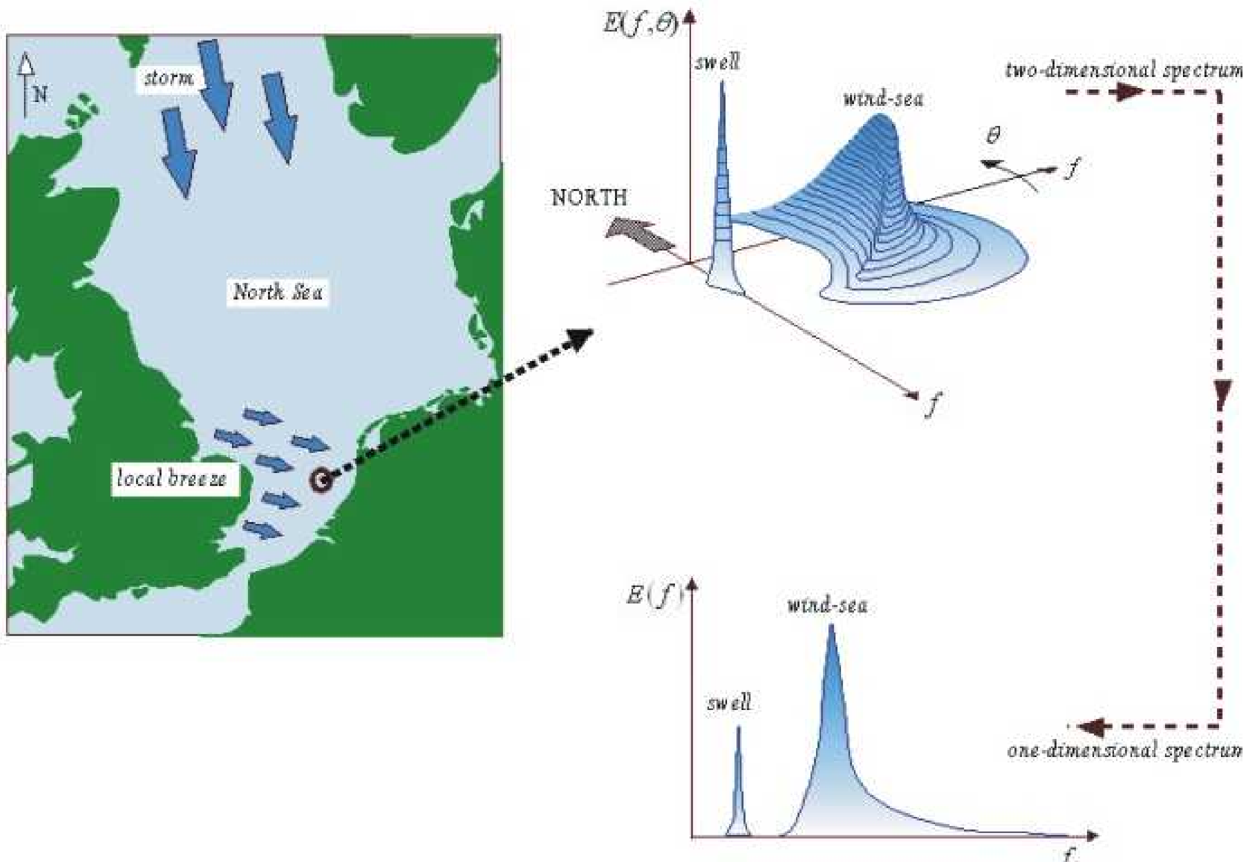
- The irregular water surface can be decomposed into (*infinite*) number of simple sinusoidal components with different frequencies (*f*) and propagation directions (θ).

- The distribution of wave energy among those components is called: "wave spectrum", F(*f*,θ).



- **Modern ocean wave prediction systems are based on statistical description of oceans waves (i.e. ensemble average of individual waves).**

- **The sea state is described by the two-dimensional wave spectrum F(*f*,θ).**



ECMWF

What we are dealing with

What we are dealing with

ECMWF

What we are *(not)* **dealing with** *(this week)*

PROGRAM
OF
THE LECTURES

Program of the lectures:

1. Derivation of energy balance equation

1.1. Preliminaries

- Basic Equations
- Dispersion relation in deep & shallow water.
- Group velocity.
- Energy density.
- Hamiltonian & Lagrangian for potential flow.
- Average Lagrangian.
- Wave groups and their evolution.

Program of the lectures:

1. Derivation of energy balance equation

1.2. Energy balance Eq. - Adiabatic Part

- Need of a statistical description of waves: the wave spectrum.
- Energy balance equation is obtained from averaged Lagrangian.
- Advection and refraction.

Program of the lectures:

1. Derivation of energy balance equation

1.3. Energy balance Eq. - Physics

Diabatic rate of change of the spectrum determined by:

- energy transfer from wind (S_{in})
- non-linear wave-wave interactions (S_{nonlin})
- dissipation by white capping (S_{dis}).

Program of the lectures:

1. Derivation of energy balance equation

1.4. Energy balance for wind sea

- Wind sea and swell.
- Empirical growth curves.
- Energy balance for wind sea.
- Evolution of wave spectrum.
- Comparison with observations (JONSWAP).

Program of the lectures:

2. Wave Forecasting at ECMWF

2.1 Operational configurations

- Limited area model
- Global, coupled to atmospheric model

Program of the lectures:

2. Wave Forecasting at ECMWF

2.2 ECWAM

- Wind input
- Feedback to the atmosphere (two-way coupling)
- Swell damping
- Non linear source term
- Bottom effects
- Feedback on SST.

Program of the lectures:

3. Validation

3.1. Use as a diagnostic tool

3.2. Comparison to observations

- in-situ
- Altimeter
- Buoy spectra

Program of the lectures:

4. Future developments

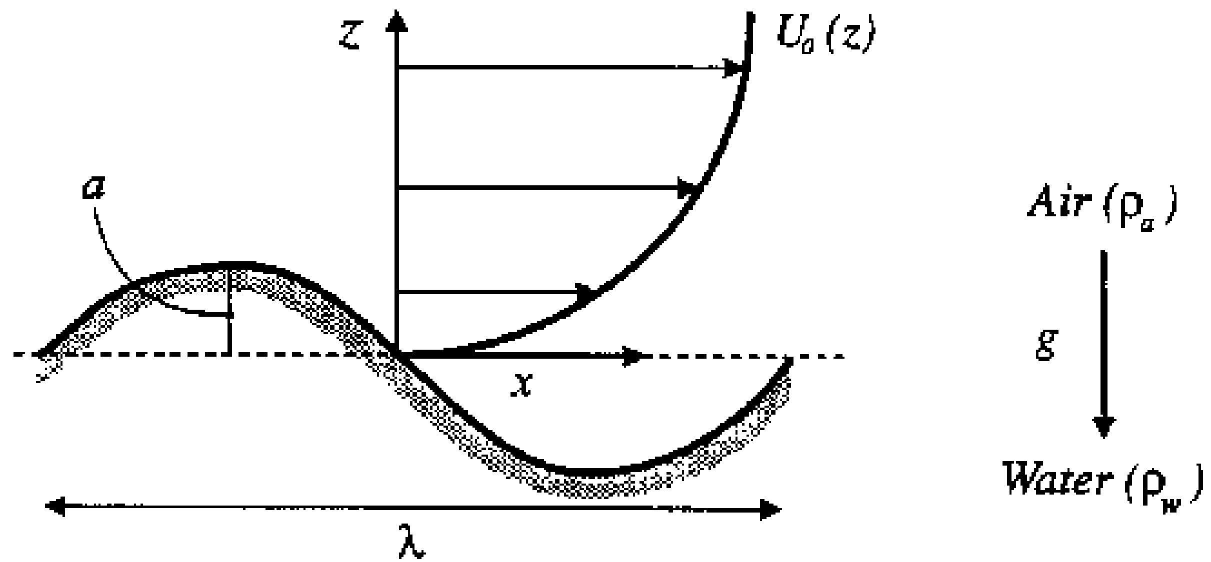
4.1. Impact on ocean circulation

4.2. Unstructured grid

4.3. wave – sea ice interaction

1
DERIVATION
OF THE
ENERGY BALANCE EQUATION

1. Derivation of the Energy Balance Eqn:



$$\textit{Steepness } s \equiv ka = \frac{2\pi a}{\lambda}$$

Solve problem with <u>perturbation</u> methods:

(i) $\rho_a / \rho_w \ll 1$ **(ii)** $s \ll 1$

Lowest order: ➔ <u>free gravity waves.</u>

Higher order effects:

➔ wind input, nonlinear transfer & dissipation

- Result: **Deterministic Evolution Equations**

Application to wave forecasting is a problem:

1. Do not know the phase of waves
   → Spectrum $F(k) \sim a^*(k)\, a(k)$
   → Statistical description

2. Direct Fourier Analysis gives too many scales:
   Wavelength: 1 - 250 m
   Ocean basin: 10^7 m
   2D → 10^{14} equations
   → Multiple scale approach
   - short-scale, $O(\lambda)$, solved analytically
   - long-scale related to physics.

Result: Energy balance equation that describes large-scale evolution of the wave spectrum.

1.1
PRELIMINARIES

1.1. Preliminaries:

- Interface between air and ocean: $\eta(\vec{x}, t)$
- Incompressible two-layer ideal fluid
- Navier-Stokes: $\nabla \cdot \vec{u} = 0$

$$\left(\frac{\partial}{\partial t} + \vec{u} \cdot \nabla\right)\vec{u} = -\frac{1}{\rho}\nabla P - \vec{g} + \nabla \cdot \underline{\underline{\tau}}$$

Here $\rho = \begin{cases} \rho_a\,, & z > \eta \\ \rho_w\,, & z < \eta \end{cases}$

and surface elevation follows from: $\frac{\partial}{\partial t}\eta + \vec{u} \cdot \nabla \eta = w$

- Oscillations should vanish for:

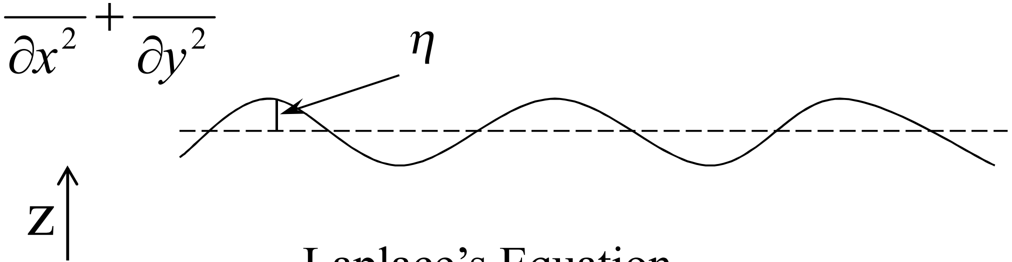
  $z \to \pm\infty$ and $z = -D$ (bottom)

- No stresses; $\rho_a \to 0$; irrotational ➔ potential flow:

  $\vec{u} = \nabla \phi$ (velocity potential ϕ)

Then, ϕ obeys potential equation.

- $$\Delta\phi + \frac{\partial^2}{\partial z^2}\phi = 0 \quad , \quad \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$



Laplace's Equation

- Conditions at surface:

$$z = \eta \;\; ; \;\; \begin{cases} \dfrac{\partial \eta}{\partial t} + \nabla\phi \cdot \nabla\eta = \dfrac{\partial \phi}{\partial z} \\[2ex] \dfrac{\partial \phi}{\partial t} + \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}\left(\dfrac{\partial \phi}{\partial z}\right)^2 + g\eta = 0 \qquad [Bernoulli] \end{cases}$$

- Conditions at the bottom: $z = -D \;\; ; \;\; \dfrac{\partial \phi}{\partial z} = 0$
- Conservation of total energy: $E = \int d\vec{x}\,\varepsilon$

with the <u>energy density</u> $\varepsilon = \frac{1}{2}\rho g \eta^2 + \frac{1}{2}\rho \int_{-D}^{\eta} d\vec{x} \left[(\nabla\phi)^2 + \left(\dfrac{\partial \phi}{\partial z}\right)^2 \right]$

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Hamilton Equations:

Choose as variables:

η and $\psi(\vec{x},t)=\phi(\vec{x},z=\eta,t)$

Boundary conditions then follow from Hamilton's equations:

$$\boxed{\frac{\partial\eta}{\partial t}=\frac{\delta E}{\delta\psi} \quad ; \quad \frac{\partial\psi}{\partial t}=-\frac{\delta E}{\delta\eta}}$$

Advantage of this approach:

Solve Laplace equation with boundary conditions:

$$\phi(\vec{x},z=\eta)=\psi \quad ; \quad \frac{\partial}{\partial z}\phi(\vec{x},z=-D)=0$$

➔ $\phi = \phi(\eta,\psi)$.

Then evolution in time follows from Hamilton equations.

Lagrange Formulation

Variational principle:

$$\delta \int dx\, dy\, dt\, \mathrm{L} = 0$$

with:

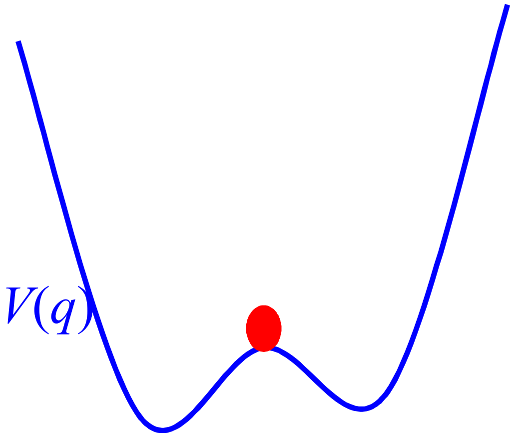
$$\mathrm{L} = -\rho \int_{-D}^{\eta} dz \left[\frac{\partial \phi}{\partial t} + \tfrac{1}{2}\left\{ \left(\nabla \phi\right)^2 + \left(\frac{\partial \phi}{\partial z}\right)^2 \right\} + g\, z \right]$$

gives Laplace’s equation & boundary conditions.

- Intermezzo

Classical mechanics:

Particle (p,q) in potential well $V(q)$



Total energy: $$E = \tfrac{1}{2}\frac{p^2}{m} + V(q)$$

Regard p and q as canonical variables.
Hamilton's equations are:

$$\dot{q} = \frac{\partial q}{\partial t} = \frac{\partial E}{\partial p} = \frac{p}{m}$$

$$\dot{p} = \frac{\partial p}{\partial t} = -\frac{\partial E}{\partial q} = -\frac{\partial V}{\partial q}$$

Eliminate p ➔ Newton's law

$$m\,\ddot{q} = -\frac{\partial V}{\partial q}$$ = Force

Principle of “least” action. Lagrangian:

$$\mathrm{L} = T - V = \tfrac{1}{2} m \dot{q}^2 - V(q) = \Lambda(q, \dot{q})$$

Newton’s law $\Leftrightarrow$ Action is extreme, where

$$action = \int_{t_1}^{t_2} dt\, \mathrm{L}(q, \dot{q})$$

Action is extreme if δ(action) = 0, where

$$\delta(action) = \int dt \left[\mathrm{L}(q + \delta q, \dot{q} + \delta \dot{q}) - \mathrm{L}(q, \dot{q}) \right]$$

$$\cong \int dt \left[\delta q \tfrac{\partial}{\partial q} \mathrm{L} + \delta \dot{q} \tfrac{\partial}{\partial \dot{q}} \mathrm{L} \right]$$

$$= \int dt \left[\tfrac{\partial}{\partial q} \mathrm{L} - \tfrac{\partial}{\partial t} \tfrac{\partial \mathrm{L}}{\partial \dot{q}} \right] \delta q$$

This is applicable for arbitrary δq hence

(Euler-Lagrange equation)

$$\mathrm{L}_q - \tfrac{\partial}{\partial t} \mathrm{L}_{\dot{q}} = 0 \quad \Rightarrow \quad m \ddot{q} = -\tfrac{\partial}{\partial q} V$$

Define momentum, *p*, as: $p \equiv L_{\dot q} \quad \Rightarrow \quad \dot q = \dot q\,(p)$

and regard now *p* and *q* are independent, the Hamiltonian, *H* , is given as:

$$H(p,q) \equiv \dot q\, L_{\dot q} - L = \tfrac{1}{2}\frac{p^2}{m} + V(q)$$

Differentiate *H* with respect to *q* gives

$$\frac{\partial H}{\partial q} = -L_q \doteq -\frac{\partial}{\partial t} L_{\dot q} = -\frac{\partial p}{\partial t} \qquad \Rightarrow \qquad \boxed{\frac{\partial p}{\partial t} = -\frac{\partial H}{\partial q}}$$

The other Hamilton equation:

$$\frac{\partial H}{\partial p} = \frac{\partial \dot q}{\partial p} L_{\dot q} + \dot q \frac{\partial}{\partial p} L_{\dot q} - L_{\dot q} \frac{\partial}{\partial p} \dot q = \dot q$$

All this is less straightforward to do for a continuum. Nevertheless, Miles obtained the Hamilton equations from the variational principle.

<u>Homework:</u> Derive the governing equations for surface gravity waves from the variational principle.

- Linear Theory

Linearized equations become:

$$\Delta\phi + \frac{\partial^2}{\partial z^2}\phi = 0$$

$$z = 0 \;\; ; \;\; \begin{cases} \dfrac{\partial\eta}{\partial t} = \dfrac{\partial\phi}{\partial z} \\ \dfrac{\partial\phi}{\partial t} = -g\,\eta \end{cases}$$

$$z = -D \;\; ; \;\; \frac{\partial\phi}{\partial z} = 0$$

Elementary sines $\eta = a\,e^{i\theta} \;\; ; \;\; \phi = Z(z)\,e^{i\theta} \;\; ; \;\; \theta = \vec{k}\cdot\vec{x} - \omega\,t$

where a is the wave amplitude, θ is the wave phase.

Laplace: $Z'' - k^2\,Z = 0 \;\; ; \;\; k = \left|\vec{k}\right| = \sqrt{k_x^2 + k_y^2}$

Constant depth: $z = -D$ ➔ $Z'(-D) = 0$ ➔ $Z \sim \cosh[\, k\,(z+D)]$

with $\dfrac{\partial\phi}{\partial t} = -\,g\,\eta$ and $\eta = a\,e^{i\theta}$ ➔ $\phi = \dfrac{-i\,g}{\omega}\;a\;\dfrac{\cosh\left[k\,(z+D)\right]}{\cosh\,(kD)}\,e^{i\theta}$

finally $\dfrac{\partial\eta}{\partial t} = \dfrac{\partial\phi}{\partial z}$ ➔ Dispersion Relation

$$\boxed{\omega^{2} = g\,k\,\tanh\,(k\,D)}$$

	Deep water $D \to \infty$	Shallow water $D \to 0$
dispersion relation:	$\omega^{2} = g\,k$	$\omega = \pm\,k\,\sqrt{g\,D}$
phase speed:	$c = \dfrac{\omega}{k} = \dfrac{g}{\omega}$	$c = \sqrt{g\,D}$
group speed:	$v_g = \dfrac{\partial\omega}{\partial k} = \dfrac{1}{2}\,\dfrac{g}{\omega} = \dfrac{1}{2}\,c$	$v_g = \sqrt{g\,D}$
Note:	**low freq. waves faster !**	**No dispersion.**

energy: $\varepsilon = 2\,\rho\,g\,a\,a^{*}$; $\bar{\varepsilon} = \frac{1}{\lambda}\int dx\,\varepsilon$

Slight generalisation:

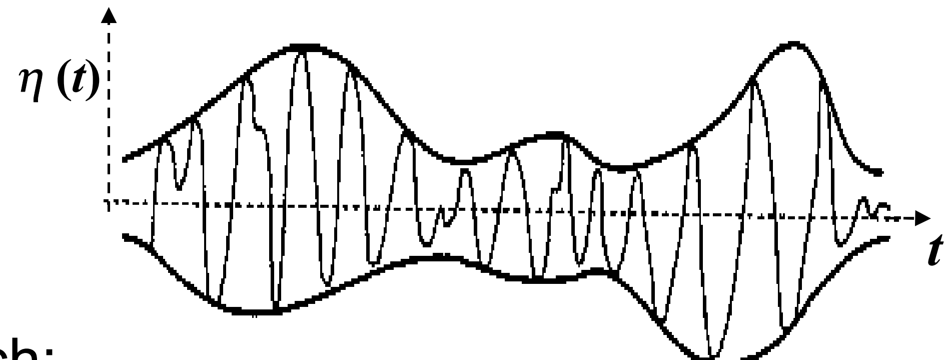
Slowly varying depth and/or current, $\vec{U}(\vec{x},t)$

$$\omega = \vec{k}\cdot\vec{U} \pm \sigma \quad , \quad \sigma = \text{intrinsic frequency}$$

$$\sigma = \sqrt{g\, k \tanh(k\, D)}$$

- Wave Groups

So far a single wave. However, waves come in groups.



Long-wave groups may be described with geometrical optics approach:

$$\boxed{\eta = a(\vec{x},t)\, e^{i\theta(\vec{x},t)} + c.c.}$$

Amplitude and phase vary slowly:

$$\frac{1}{a}\nabla a << k \quad , \quad \frac{1}{a}\frac{\partial a}{\partial t} << \omega$$

Local wave number and phase

(recall wave phase $\theta = \vec{k}\cdot\vec{x} - \omega\, t$!)

$$\omega \equiv -\frac{\partial \theta}{\partial t} \quad ; \quad \vec{k} = \nabla\theta$$

Consistency: conservation of number of wave crests

$$\frac{\partial}{\partial t}\vec{k} + \nabla\omega = 0$$

Slow time evolution of amplitude is obtained by averaging the Lagrangian over *rapid* phase, θ.

$$\langle \mathrm{L} \rangle = \tfrac{1}{2\pi}\int d\theta\ \mathrm{L}$$

For water waves we get *(after dropping the brackets ⟨ ⟩)*

with $\mathrm{L} = \frac{1}{2}\varepsilon\left\{\frac{(\omega - \vec{k}\cdot\vec{U}_o)^2}{g\,k\,T} - 1\right\} - \frac{1}{2}\frac{k^2\,\varepsilon^2}{\rho\, g}\left\{\frac{9T^4 - 10T^2 + 9}{8\,T^4}\right\} + O(\varepsilon^3)$

$$\varepsilon = 2\,\rho\, g\,|a|^2 \quad , \quad T = \tanh(k\,D) \quad , \quad \vec{U}_o = \text{mean current}$$

In other words, we have $\mathrm{L} = \mathrm{L}\,(\omega, \vec{k}, a)$

(omitting the brackets ⟨ ⟩)

Evolution equations then follow from the average variational principle $\delta \int d\vec{x}\; dt\; \mathrm{L}\,(\omega, \vec{k}, a) = 0$

We obtain: δ*a* : $\boxed{\frac{\partial}{\partial a}\mathrm{L} = 0}$ *(dispersion relation)*

δθ : $\frac{\partial}{\partial t}\mathrm{L}_\omega - \frac{\partial}{\partial \vec{x}}\mathrm{L}_{\vec{k}} = 0$ *(evolution of amplitude)*

plus consistency: $\boxed{\frac{\partial}{\partial t}\vec{k} + \nabla \omega = 0}$ *(conservation of crests)*

Finally, if we introduce a transport velocity $\vec{u} = -\mathrm{L}_{\vec{k}} / \mathrm{L}_\omega$

then $\boxed{\frac{\partial}{\partial t}\mathrm{L}_\omega + \frac{\partial}{\partial \vec{x}}\vec{u}\,\mathrm{L}_\omega = 0}$

L_ω is called the <u>action density</u>.

Apply our findings to gravity waves.

Linear theory, write L as $L = \frac{1}{2}\varepsilon\, D_i(\omega, \vec{k})$

where $$D_i = \frac{(\omega - \vec{k}\cdot\vec{U}_o)^2}{g\,k\,T} - 1$$

Dispersion relation follows from $\frac{\partial}{\partial a} L = 0 \quad \Rightarrow \quad D_i = 0$

hence, with $\sigma = \sqrt{g\,k\,T}$,

$$\boxed{\omega = \vec{k}\cdot\vec{U}_o \pm \sigma}$$ ★★★

Equation for action density, N , $N \equiv \mathrm{L}_\omega = \varepsilon / \sigma$

becomes $$\boxed{\frac{\partial}{\partial t} N + \nabla\cdot\vec{v}_g\, N = 0}$$ ★★★

with $\vec{v}_g = \partial\omega / \partial\vec{k}$

Closed by $$\boxed{\frac{\partial\vec{k}}{\partial t} + \nabla\,\omega = 0}$$ ★★★

Consequencies

Zero flux through boundaries → $N_{tot} = \int d\vec{x}\; N = const.$

hence, in case of slowly varying bottom and currents, the wave energy $E = \int d\vec{x}\; \varepsilon$

is **not** constant.

Of course, the total energy of the system, including currents, ... etc., is constant. However, when waves are considered in isolation (regarded as "the" system), energy is not conserved because of interaction with current (and bottom).

The total action density is N_{tot} is conserved.

The action density is called an adiabatic invariant.

Homework: Adiabatic Invariants (study this)

Consider once more the particle in potential well.

Externally imposed change $\lambda(t)$. We have $\delta \int dt \, \mathrm{L}(q, \dot{q}, \lambda) = 0$

Variational equation is $\frac{d}{dt}\mathrm{L}_{\dot{q}} - \mathrm{L}_q = 0$

Calculate average Lagrangian with λ fixed.
If period is $\tau = 2\pi/\nu$, then $\langle \mathrm{L} \rangle = \frac{\nu}{2\pi}\int_0^\tau \mathrm{L} \, dt$

For periodic motion (λ = const.) we have conservation of energy $E = \dot{q}\,\mathrm{L}_{\dot{q}} - \mathrm{L}$
thus $\dot{q} = \dot{q}(q, E, \lambda)$
also momentum $p = \mathrm{L}_{\dot{q}}$ is $p = p(q, E, \lambda)$

The average Lagrangian becomes

$$\langle \mathrm{L} \rangle = \frac{\nu}{2\pi}\oint p(q, E, \lambda)\, dq - E$$

Allow now slow variations of λ which give consequent changes in E and ν. Average variational principle

$$\delta \int dt \, \mathrm{L}(\nu, E, \lambda) = 0$$

Define again $\nu = \dot{\theta}$

Variation with respect to E & θ gives

$$\mathrm{L}_E = 0 \quad , \quad \frac{d}{dt}\mathrm{L}_\nu = 0$$

The first corresponds to the dispersion relation while the second corresponds to the action density equation. Thus

$$\mathrm{L}_\nu = \frac{1}{2\pi}\oint p \, dq = \text{const.}$$

which is just the classical result of an adiabatic invariant. As the system is modulated, ν and E vary individually but

$$N(\nu, E) = \frac{1}{2\pi}\oint p \, dq$$

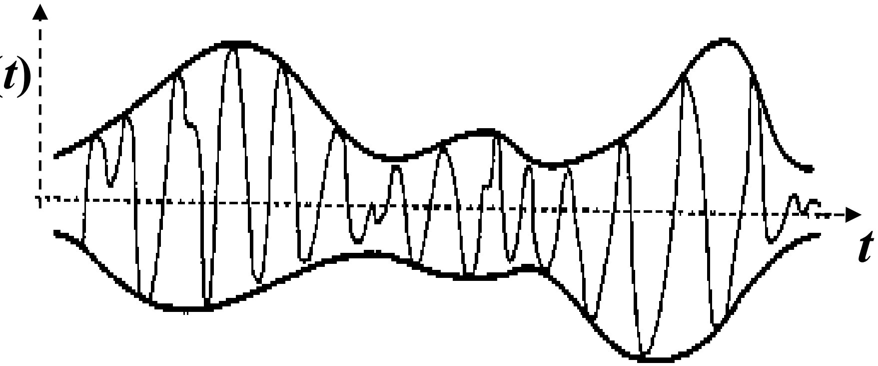
remains constant: Analogy $\Lambda_\nu \leftrightarrow \Lambda_\omega$ waves.

Example: Pendulum with varying length!

1.2
ENERGY BALANCE EQUATION: THE ADIABATIC PART

Wave packet evolution:

$$\boxed{\eta = a(\vec{x},t)\, e^{i\theta(\vec{x},t)} + c.c.}$$



$D(\vec{x},t) = \text{water depth}$,

$\vec{U}_o(\vec{x},t) = \text{mean current}$

Local wave number and phase:

$$\omega \equiv -\frac{\partial \theta}{\partial t} \quad ; \quad \vec{k} = \nabla \theta$$

Consistency:

$$\boxed{\frac{\partial \vec{k}}{\partial t} + \nabla \omega = 0}$$

Dispersion relation:

$$\boxed{\omega = \vec{k} \cdot \vec{U}_o \pm \sigma} \qquad \boxed{\sigma^2 = g\, k \tanh(k\, D)}$$

Equation for action density:

$$\boxed{\frac{\partial}{\partial t} N + \nabla \cdot \vec{v}_g\, N = 0} \qquad \boxed{N = \frac{\tilde{\varepsilon}}{\sigma}} \qquad \vec{v}_g = \partial \omega / \partial \vec{k}$$

$$\tilde{\varepsilon} = 2\, \rho\, g\, |a|^2$$

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1.2. Energy Balance Eqn (Adiabatic Part):

- Together with other perturbations

$$\frac{\partial}{\partial t}N + \nabla \cdot \vec{v}_g\ N = \left.\frac{\partial N}{\partial t}\right|_{wind} + \left.\frac{\partial N}{\partial t}\right|_{nonlin} + \left.\frac{\partial N}{\partial t}\right|_{dissip}$$

where wave number and frequency of wave packet follow from

$$\frac{\partial \vec{k}}{\partial t} + \nabla \omega = 0 \qquad \omega = \vec{k} \cdot \vec{U}_o \pm \sqrt{g\, k \tanh(k\, D)}$$

So far we were solving for $\eta = a(\vec{x},t)\, e^{i\theta(\vec{x},t)} + c.c.$

- Statistical description of waves: **concept of the wave spectrum**
  - Random phase ➔ Gaussian surface
  - Covariance $R = \langle \eta(\vec{x}_1)\eta(\vec{x}_2) \rangle$ homogeneous
    with ⟨ ⟩ = ensemble average

➔ $\boxed{R(\vec{\xi}) = \langle \eta(\vec{x}+\vec{\xi})\eta(\vec{x}) \rangle}$ depends only on $\vec{\xi} = \vec{x}_2 - \vec{x}_1$

- By <u>definition</u>, wave number spectrum is the Fourier transform of correlation R

$$F(\vec{k}) = \frac{1}{(2\pi)^2} \int d\vec{\xi}\; e^{i\vec{k}\cdot\vec{\xi}}\; R(\vec{\xi})$$

We now take a <u>continuum</u> of waves. Realising that we have two modes :

$$\eta(\vec{x},t) = \int_{-\infty}^{\infty} d\vec{k}\, \hat{\eta}_{+}(\vec{k})\, e^{i(\vec{k}\cdot\vec{x}-\omega_{+}t)} + \int_{-\infty}^{\infty} d\vec{k}\, \hat{\eta}_{-}(\vec{k})\, e^{i(\vec{k}\cdot\vec{x}-\omega_{-}t)}$$

η <u>is real</u> $\quad \hat{\eta}_{-}(\vec{k}) = \hat{\eta}_{+}^{*}(-\vec{k})$

$$\rightarrow \quad \eta(\vec{x},t) = \int_{-\infty}^{\infty} d\vec{k}\, \hat{\eta}(\vec{k})\, e^{i(\vec{k}\cdot\vec{x}-\omega t)} + c.c.$$

Use this in homogeneous two point correlation, we must have

$$\left\langle \hat{\eta}(\vec{k})\hat{\eta}(\vec{k}')\right\rangle = 0 \qquad \left\langle \hat{\eta}(\vec{k})\,\hat{\eta}^{*}(\vec{k}')\right\rangle = \left|\hat{\eta}(\vec{k})\right|^{2} \delta(\vec{k}-\vec{k}')$$

and the correlation becomes

$$R(\vec{\xi}) = \int d\vec{k}\left|\hat{\eta}(\vec{k})\right|^{2} e^{i(\vec{k}\cdot\vec{\xi})} + c.c.$$

The wave spectrum becomes

$$\boxed{F(\vec{k}) = 2 \left|\hat{\eta}(\vec{k})\right|^2}$$

Normalisation: for $\xi = 0$ → $\left\langle \eta^2 \right\rangle = R(0) = \int d\vec{k}\, F(\vec{k})$

For linear, propagating waves:

potential and kinetic energy are equal

→ wave energy $\langle E \rangle$ is

$$\left\langle E \right\rangle = \rho_w\, g \left\langle \eta^2 \right\rangle = \rho_w\, g \int d\vec{k}\, F(\vec{k})$$

<u>Note</u>: Wave height is the distance between crest and trough

: given $F(\vec{k})$ ➔ wave height. $H_s = 4\sqrt{\left\langle \eta^2 \right\rangle}$

- Let us now derive the evolution equation for the action density, now defined as

$$\boxed{N(\vec{k}) = \frac{F(\vec{k})}{\sigma}} \qquad \sigma = \sqrt{g\, k \tanh(k\, D)}$$

Starting point is the discrete case.

One pitfall:

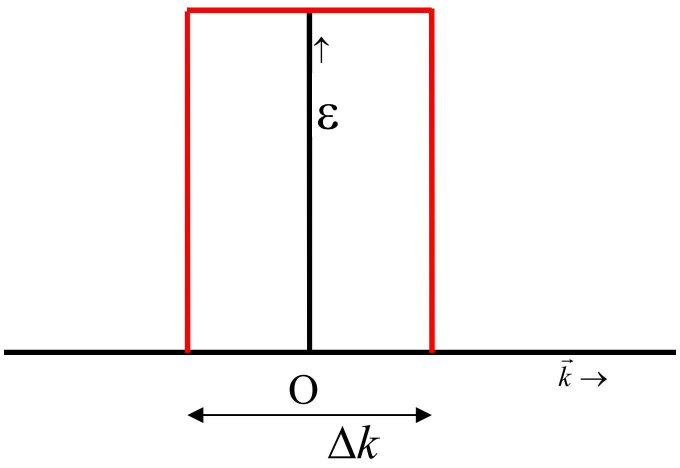
continuum: $\vec{x}$, t and $\vec{k}$ independent.

discrete: $\vec{k} = \vec{k}(\vec{x},t)$, local wave number.

Connection between discrete and continuous case:

Define

$$\varepsilon(\vec{k}) = \begin{cases} 1 & , \quad if \ \tfrac{-1}{2}\Delta\vec{k} < \vec{k} < \tfrac{1}{2}\Delta\vec{k} \\ 0 & , \quad otherwise \end{cases}$$



then

$$N(\vec{k})\Delta\vec{k} = \sum_{\vec{k}'} N_{\vec{k}'}\, \varepsilon(\vec{k}-\vec{k}') \quad ; \qquad N_{\vec{k}'} = \frac{2}{\sigma}\left|a_{\vec{k}'}\right|^2$$

Note: $\vec{k}' = \vec{k}'(\vec{x},t)$!

Then, evaluate $\frac{\partial}{\partial t} N(\vec{k}, \vec{x}, t)$

using the discrete action balance <u>and</u> the connection.

The result:

$$\boxed{\frac{\partial}{\partial t} N + \nabla_{\vec{x}} \cdot (\vec{v}_g N) - \nabla_{\vec{k}} \cdot (\nabla_{\vec{x}} \Omega N) = S_{wind} + S_{nonlin} + S_{dissip} = S}$$

This is the action balance equation.

Note that refraction term stems from time and space dependence of local wave number! Furthermore,

$$\Omega = \vec{k} \cdot \vec{U} + \sigma \quad , \qquad \sigma = \sqrt{g\, k \tanh(k\, D)}$$

and $\vec{v}_g = \partial \Omega / \partial \vec{k}$

- Further discussion of adiabatic part of action balance:

  Slight generalisation: $N(x_1, x_2, k_1 , k_2, t)$

  writing $\vec{z} = (x_1, x_2, k_1, k_2)$

  then the most fundamental form of the transport equation for N in the absence of source terms:

$$\boxed{\frac{\partial}{\partial t} N + \frac{\partial}{\partial z_i}(\dot{z}_i N) = 0}$$

  where $\dot{z}$ is the propagation velocity of wave groups in z-space.

  This equation holds for any rectangular coordinate system.

- Lets go back to the general case and apply to spherical geometry (non-canonical)

Choosing $\hat{N}\,(\omega,\theta,\phi,\lambda,t)$

(ω: angular frequency, θ: direction (clockwise relative to true north), ϕ: latitude, λ:longitude)

then

$$\frac{\partial}{\partial t}\hat{N}+\frac{\partial}{\partial \phi}(\dot{\phi}\hat{N})+\frac{\partial}{\partial \lambda}(\dot{\lambda}\hat{N})+\frac{\partial}{\partial \omega}(\dot{\omega}\hat{N})+\frac{\partial}{\partial \theta}(\dot{\theta}\hat{N})=0$$

Normally, we consider action density in local Cartesian frame (*x*, *y*): $\hat{N}=N\,R^2\cos\phi$

R is the radius of the earth.

Result:

$$\boxed{\frac{\partial}{\partial t}N+\frac{1}{\cos\phi}\frac{\partial}{\partial\phi}(\dot{\phi}\cos\phi N)+\frac{\partial}{\partial\lambda}(\dot{\lambda}N)+\frac{\partial}{\partial\omega}(\dot{\omega}N)+\frac{\partial}{\partial\theta}(\dot{\theta}N)=0}$$

With v_g the group speed, we have

$$\dot{\phi}=(v_g\cos\theta-U_o)/R$$

$$\dot{\lambda}=(v_g\sin\theta-V_o)/(R\cos\phi)$$

$$\dot{\theta}=v_g\sin\theta\tan\phi/R+(\dot{\vec{k}}\times\vec{k})/k^2$$

$$\dot{\omega}=\partial\Omega/\partial t$$

$$\Omega=\vec{k}\cdot\vec{U}+\sigma\quad,\qquad\sigma=\sqrt{g\,k\tanh(k\,D)}$$

$\vec{U}$:

U_o Northerly direction

V_o Easterly direction

$$\dot{k}_i=-\partial\Omega/\partial x_i$$

- Properties:

1. Waves propagate along great circle.
2. Shoaling:
   Piling up of energy when waves, which slow down in shallower water, approach the coast.
3. Refraction:
   * Waves bend towards shallower water!
   * Sea mountains (shoals) act as lenses.
4. Current effects:
   Blocking

   $\Rightarrow$ v_g vanishes for $k = g / (4\ U_o^2)$!

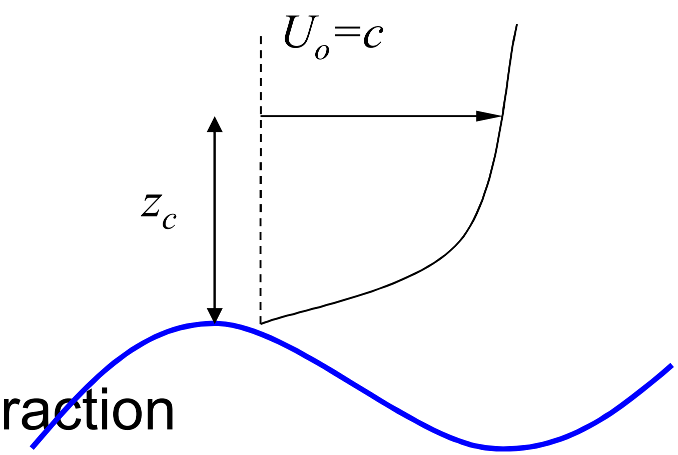
$$\Omega = \sqrt{g\,k} - k\,U_o$$

1.3
ENERGY BALANCE EQUATION: PHYSICS

1.3. Energy Balance Eqn (Physics):

- Discuss wind input and nonlinear transfer in some detail.
- Common feature: Resonant Interaction

- Wind:
  - Critical layer: $c(k) = U_o(z_c)$.
  - Resonant interaction between air at z_c and wave



- Nonlinear: $\sum \theta_i = 0$
  3 and 4 wave interaction

Wind input in pictures

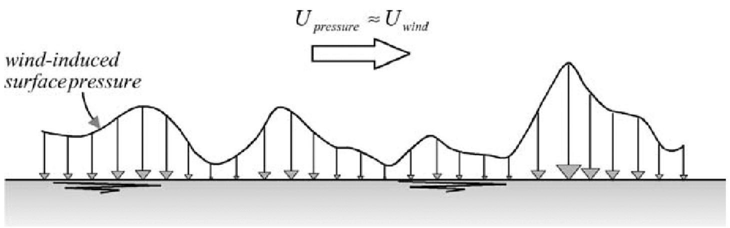


Figure 6.15 The normal wind-induced pressure moving as a (nearly) frozen distribution across the water surface.

Linear growth

But once waves are present, they distort the air flow above:

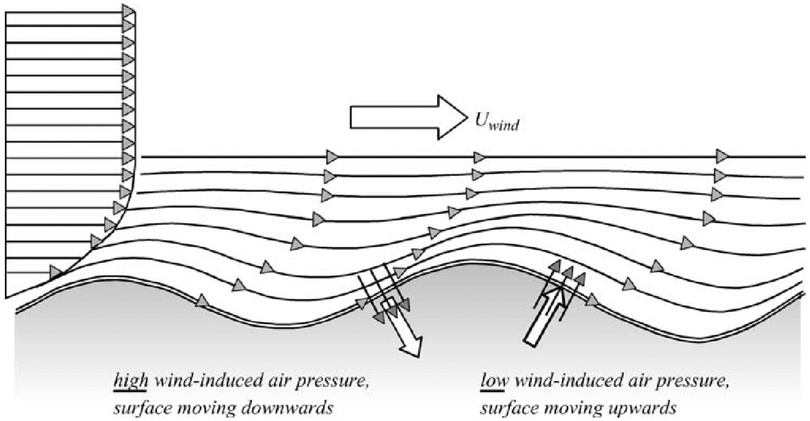


Figure 6.16 The wave-induced wind-pressure variation over a propagating harmonic wave.

the wave grows by this mechanism, the mechanism becomes more effective, so the wave can therefore grow faster, which in turn makes the mechanism even more effective, etc.

exponential growth

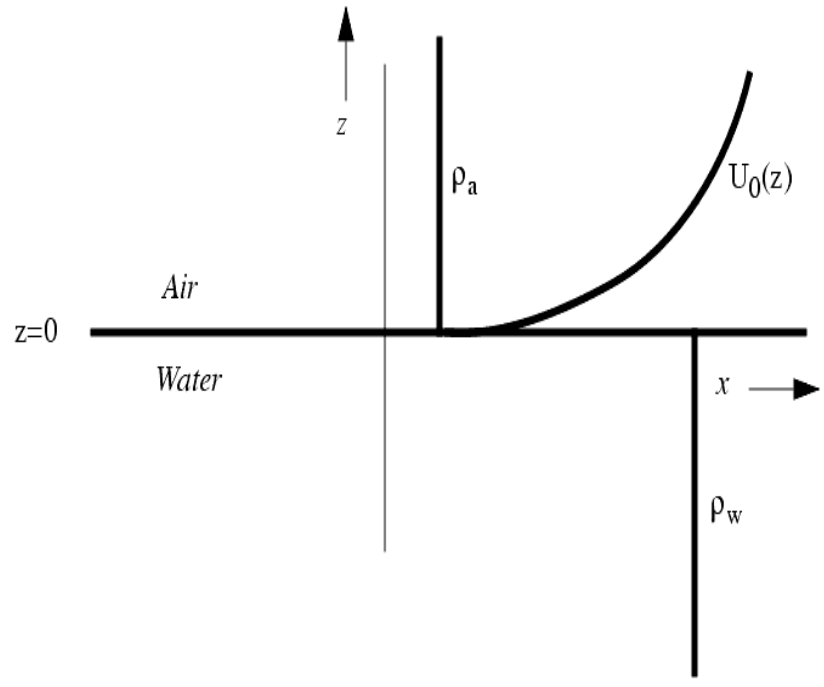
Figures from "Waves in Oceanic and Coastal Waters" by Leo Holthuijsen. Cambridge University Press

Transfer from Wind:

Instability of plane parallel shear flow (2D)

$$\text{"Equilibrium"}: \begin{cases} \vec{U}_o = U_o(z)\,\hat{e}_x \quad , \quad \vec{g} = -g\,\hat{e}_z \\ \rho_o = \rho_o(z) \quad , \quad P_o(z) = g \int dz\, \rho_o(z) \end{cases}$$

$$\underline{\text{Adiabatic fluid}}: \begin{cases} \nabla \cdot \vec{u} = 0 \\ \dfrac{\mathrm{d}}{\mathrm{d}t}\vec{u} = -\dfrac{1}{\rho}\nabla P + \vec{g} \\ \dfrac{\mathrm{d}}{\mathrm{d}t}\rho = 0 \end{cases}$$



Perturb equilibrium: Displacement of streamlines $\boxed{\psi = w/W}$

$W = U_o - c \qquad (c = \omega/k)$

Linearization and taking normal model

$$u = U_o(z) + u_1$$

$$w = w_1$$

$$\rho = \rho_0(z) + \rho_1$$

$$u_1, w_1, \rho_1 \propto e^{i\theta} = e^{i(kx-\omega t)} = e^{ik(x-ct)} = e^{ik(x-c_r t)} e^{kc_i t}$$

Possible growth

$$\text{growth rate}: \ \gamma = kc_i$$

$$\frac{\mathrm{d}}{\mathrm{d}z}(\rho_o W^2 \frac{\mathrm{d}}{\mathrm{d}z}\psi) = (k^2 \rho_o W^2 + g\,\rho_o')\psi \qquad W = U_o(z) - c$$

$$\psi \to 0\,, \quad |z| \to \infty \qquad ; \qquad \rho_o' = \frac{\mathrm{d}}{\mathrm{d}z}\rho_o(z)$$

give Im(c) → possible growth of the wave

Simplify by taking no current and constant density in water and air.

Result:

$$\begin{cases} \frac{\mathrm{d}}{\mathrm{d}z} W^2 \frac{\mathrm{d}}{\mathrm{d}z}\psi_a = k^2 W^2 \psi_a\,, \quad z > 0 \\ \psi_a(0) = 1 \quad , \quad c^2 = g(1-\varepsilon)/[k - \varepsilon\psi_a'(0)] \\ z \to \infty \quad , \quad \psi_a \to 0 \end{cases}$$

Here, $\varepsilon = \rho_a / \rho_w \sim 10^{-3} << 1$, hence for $\varepsilon \to 0$, $\boxed{c^2 = g/k}$!

Perturbation expansion

$$c = c_o + \varepsilon c_1 \to c_o = \sqrt{g/k} \quad , \qquad c_1 = \tfrac{1}{2} c_o (\tfrac{1}{k}\psi_a' - 1)$$

growth rate γ **= Im(*k c*$_1$)**

Further '*simplification*' gives for $\chi = w/w(0)$:

$$\begin{cases} W_o\left(\dfrac{\mathrm{d}^2}{\mathrm{d}z^2}-k^2\right)\chi = W_o''\chi \qquad \text{Rayleigh Eq. (singular at } W_o = 0) \\ \chi(0)=1 \quad , \qquad W_o = \lim\limits_{c\to c_o}(U_o - c) \\ z\to\infty \quad , \quad \chi\to 0 \end{cases}$$

Growth rate: $$\frac{\gamma}{\varepsilon\,\omega_o} = \frac{1}{4k}\mathrm{W}(\chi,\chi^*)\Big|_{z=0}$$

Wronskian: $$\mathrm{W} = -i\,(\chi'\,\chi^* - \chi\,\chi'^*)$$

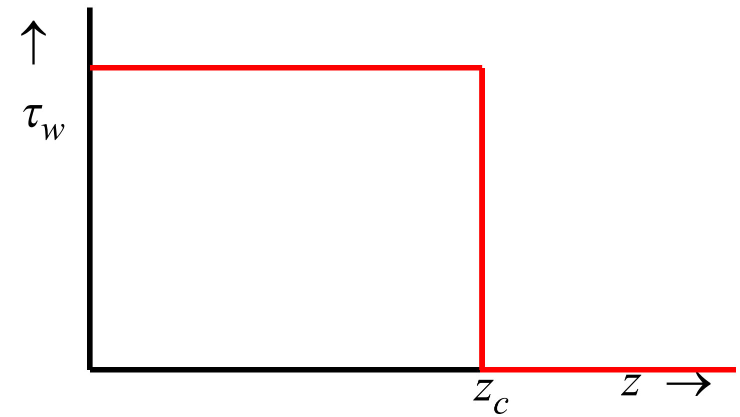
1. Wronskian Ω is related to wave-induced stress

$$\tau_w = -\langle u_1 \, w_1 \rangle$$

Indeed, with $\nabla \cdot \vec{u} = 0$ and the normal mode formulation for u_1, w_1: (e.g. $u_1 = u\, e^{i\theta} + c.c.$)

$$\tau_w = -\frac{i}{k}\left(w^* w' - w w'^*\right)$$

2. Wronskian is a simple function, namely constant except at critical height z_c



To see this, calculate dΩ/dz using Rayleigh equation with proper treatment of the singularity at z=z_c ➔

$$\frac{\mathrm{d}}{\mathrm{d}z}\mathrm{W} = 2\pi \, W''_{oc} \left|\chi_c\right|^2 \delta(W_o)$$

where subscript “c” refers to evaluation at critical height z_c (W_o = 0)

This finally gives for the growth rate (by integrating dΩ/dz to get $\Omega_{(z=0)}$)

$$\frac{\gamma}{\varepsilon\,\omega} = -\frac{\pi}{2k}\,\frac{W''_{oc}}{\left|W'_{oc}\right|}\,\left|\chi_c\right|^2$$

$$W_o = U_o(z) - c$$

$$W_o = 0 \ \text{ when } \ U_o(z_c) = c$$

W'_{oc} and W''_{oc} first and second derivative of W_0 at the critical height z_c

$$\chi_c = \frac{w(z_c)}{w(0)} \qquad \textit{and } \varepsilon = \rho_a / \rho_w$$

Miles (1957): waves grow for which the curvature of wind profile at z_c is negative (e.g. logarithmic profile).

Consequence: waves grow → slowing down wind:
Force = dτ_w / dz ~ $\delta(z)$ (*step function*)
For a single wave, this is singular!
➔ Nonlinear theory.

$$\tau_w = -\left\langle u_1\, w_1 \right\rangle$$

Linear stability calculation

Choose a logarithmic wind profile (*neutral stability*)

$$U_o(z) = \frac{u_*}{\kappa} \ln\left(1 + \frac{z}{z_o}\right)$$

κ = 0.41 (*von Karman*), u_* = friction velocity, a measure of the momentum flux at the surface: $\tau = u_*^2$

Roughness length, z_o : Charnock (1955), modelling to loss of momentum

$z_o = \alpha u_*^2 / g$, $\alpha \cong 0.015$ *(for now)*

Note:

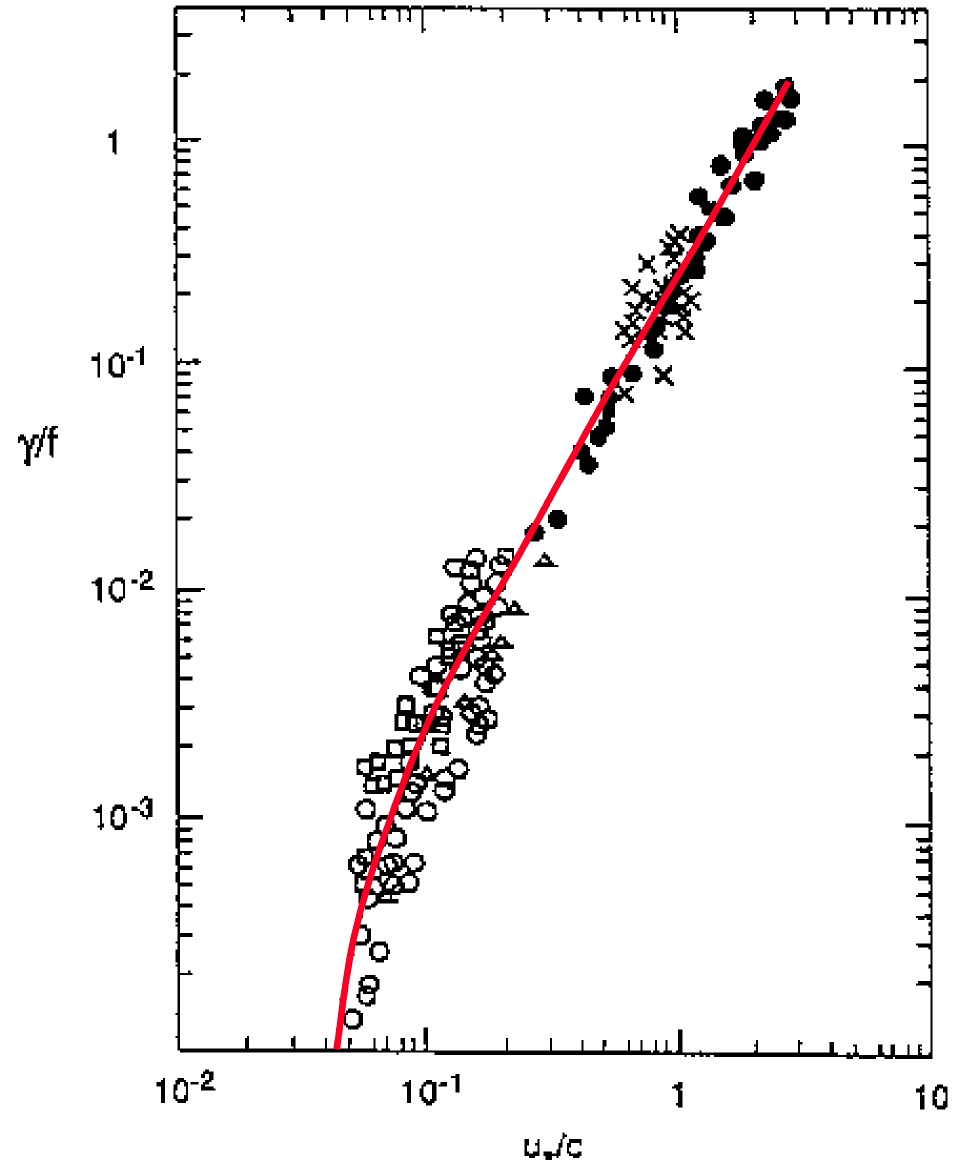
➔ Growth rate, γ depends on $\varepsilon = \rho_a / \rho_w$ and $\frac{u_*}{c} = \frac{\omega u_*}{g}$

so, short waves have the largest growth.

For a continuum of waves:

Action balance equation

$$\boxed{\left.\frac{\partial}{\partial t} N\right|_{wind} \equiv S_{wind} = 2\gamma N \quad , \quad \gamma \sim \left(\frac{u_*}{c}\right)^2}$$



Wave growth versus phase speed
(comparison of Miles' theory and observations)

ECMWF

Nonlinear effect: slowing down of wind

$$\left.\frac{\partial}{\partial t}U_o\right|_{waves} = \frac{\partial}{\partial z}\tau_w$$

Continuum: τ_w is nice function, because of continuum of critical layers

$$\tau_w = -\langle \delta u\, \delta w \rangle$$

τ_w is wave induced stress

(δu , δw): wave induced velocity in air (from Rayleigh Eqn).

. . . $$\boxed{\left.\frac{\partial}{\partial t}U_o\right|_{waves} = D_w \frac{\partial^2}{\partial z^2}U_o}$$

with $D_w = \dfrac{\pi\, c^3\, k^3\, |\chi|^2}{|c - v_g|} N(k)$ (sea-state dep. through *N*!).

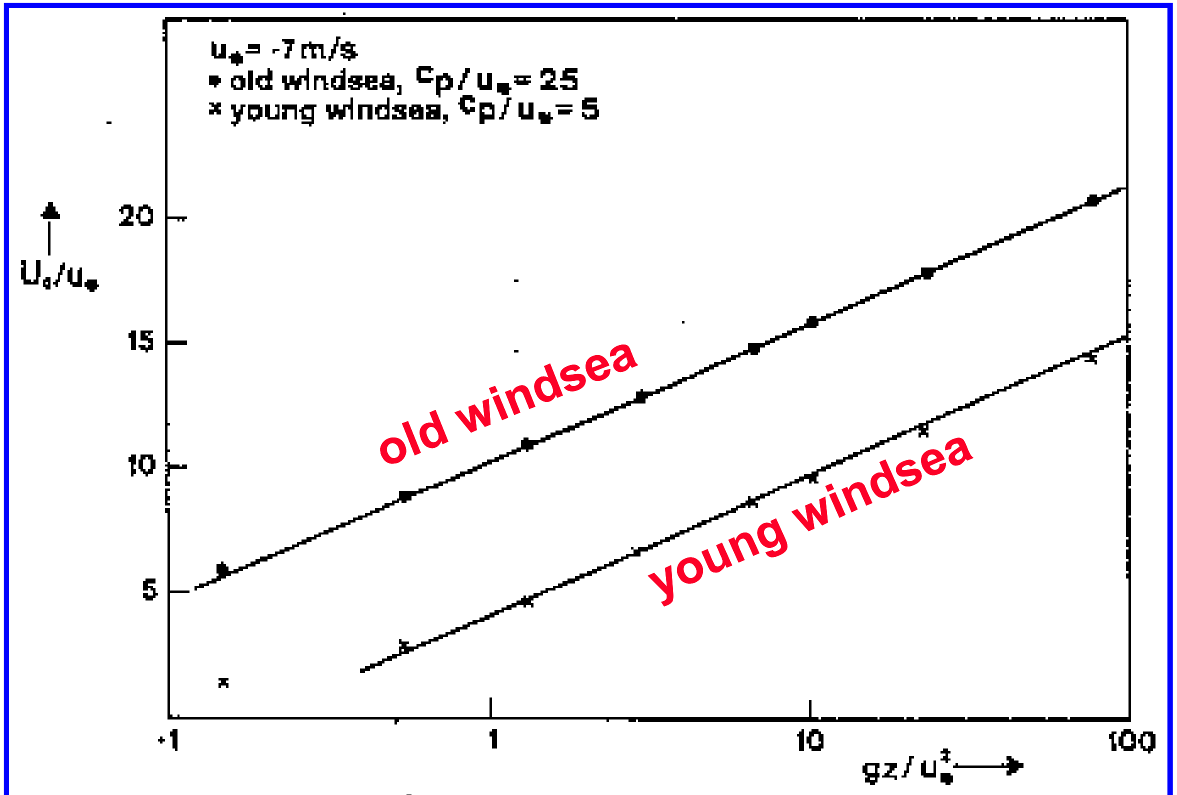
$D_w > 0$ ➔ slowing down the wind.

Example:

- Young wind sea ➔ steep waves.
- Old wind sea ➔ gentle waves.

Charnock parameter depends on sea-state!

(variation of a factor of 5 or so).



Wind speed as function of height for young & old wind sea

ECMWF

Charnock parameter (α) depends on sea-state,

Janssen (1991), extended Miles theory:

roughness length, z_o : modelling to loss of momentum

$$z_0 = \frac{\alpha\, u_*^2}{g} \qquad \tau = u_*^2$$

$$\alpha = \frac{\tilde{\alpha}}{\sqrt{1 - \frac{\tau_w}{\tau}}}$$

Wave induced stress:

$$\tau_w = \frac{g}{\varepsilon} \int \mathrm{d}\omega\, d\theta\, k\, S_{wind}$$

Non-linear Transfer *(finite steepness effects)*

Briefly describe procedure how to obtain: $\left.\frac{\partial N}{\partial t}\right|_{nonlin}$

1. $\varepsilon = \frac{1}{2}\rho g \eta^2 + \frac{1}{2}\rho \int_{-\infty}^{\eta} dz \left[(\nabla \phi)^2 + \phi_z{}^2 \right]$

2. Express ϕ in terms of canonical variables η and $\psi = \phi\,(z = \eta)$ by solving

$$\left.\begin{aligned} &\Delta\phi + \phi_{zz} = 0 \\ &z = \eta \quad , \quad \phi = \psi \\ &z = -\infty \quad , \quad \phi_z = 0 \end{aligned}\right\}$$

   iteratively using Fourier transformation.

3. Introduce complex action-variable $a(\vec{k})$

$$\hat{\eta}(\vec{k}) = \frac{1}{\sqrt{2}} \left(\frac{k}{\omega}\right)^{1/2} \left[a(\vec{k}) + a^*(-\vec{k}) \right]$$

$$\omega = \omega(k) = \sqrt{g\,k}$$

$$\hat{\psi}(\vec{k}) = \frac{-i}{\sqrt{2}} \left(\frac{\omega}{k}\right)^{1/2} \left[a(\vec{k}) - a^*(-\vec{k}) \right]$$

gives energy of wave system:

$$E = \int \mathrm{d}\vec{x}\,\varepsilon$$

$$= \int \mathrm{d}\vec{k}_1 \omega_1 a_1 a_1^* +$$

$$\int \mathrm{d}\vec{k}_1 \mathrm{d}\vec{k}_2 \mathrm{d}\vec{k}_3\, \delta(\vec{k}_1 - \vec{k}_2 - \vec{k}_3)\, V_{1,2,3} [a_1^* a_2 a_3 + c.c.] + \ldots$$

with $a_1 = a(\vec{k}_1)$, … etc.

Hamilton equations: $\begin{cases} \dfrac{\partial \eta}{\partial t} = \dfrac{\delta E}{\delta \psi} \\ \dfrac{\partial \psi}{\partial t} = -\dfrac{\delta E}{\delta \eta} \end{cases}$ become $\dfrac{\partial}{\partial t} a(\vec{k}) = -i \dfrac{\delta E}{\delta a^*}$

… Result:

$$\frac{\partial}{\partial t} a_1 + i\omega_1 a_1 = -i \int \mathrm{d}\vec{k}_2 \mathrm{d}\vec{k}_3 V_{1,2,3}\, a_2 a_3\, \delta(\vec{k}_1 - \vec{k}_2 - \vec{k}_3) + \ldots$$

$$-i \int \mathrm{d}\vec{k}_2 \mathrm{d}\vec{k}_3 \mathrm{d}\vec{k}_4 W_{1,2,3,4}\, \delta(\vec{k}_1 + \vec{k}_2 - \vec{k}_3 - \vec{k}_4) a_2^* a_3 a_4$$

Here, V and W are known functions of $\vec{k}$

At lowest order:

$$\frac{\partial}{\partial t} a_1 + i\omega_1 a_1 = 0 \qquad a_1 \propto e^{i\omega_1 t}$$ i.e. oscillates with frequency $-\omega_1$

oscillates with frequency $-(\omega_2 + \omega_3)$

$$\frac{\partial}{\partial t} a_1 + i\omega_1 a_1 = -i \int \mathrm{d}\vec{k}_2 \mathrm{d}\vec{k}_3 V_{1,2,3} a_2 a_3 \delta(\vec{k}_1 - \vec{k}_2 - \vec{k}_3) + \ldots$$
$$- i \int \mathrm{d}\vec{k}_2 \mathrm{d}\vec{k}_3 \mathrm{d}\vec{k}_4 W_{1,2,3,4} \delta(\vec{k}_1 + \vec{k}_2 - \vec{k}_3 - \vec{k}_4) a_2^* a_3 a_4$$

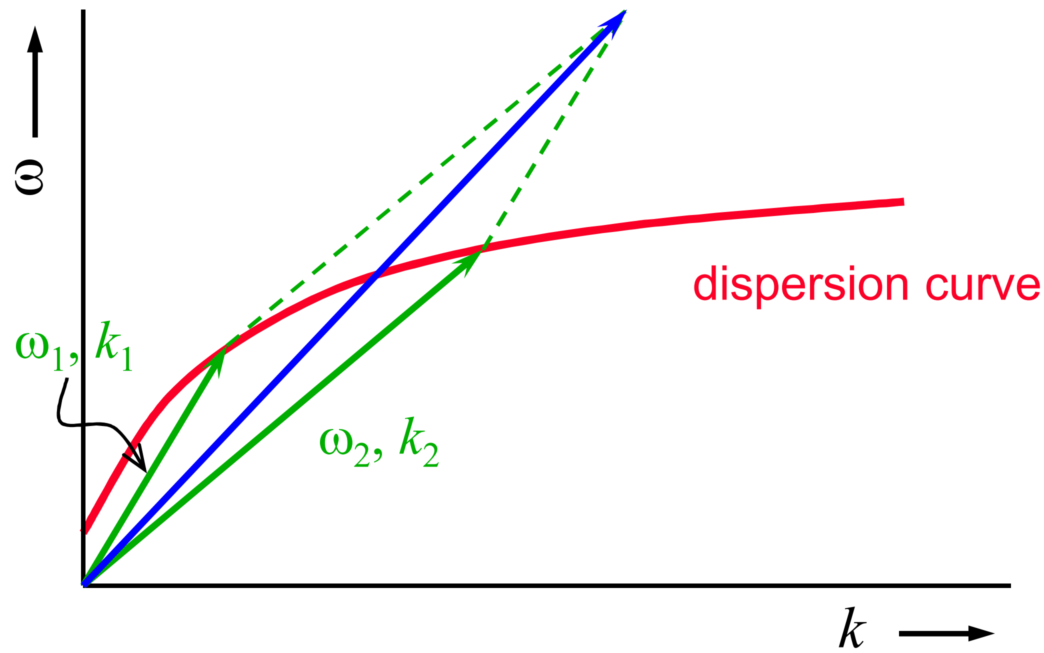
oscillates with frequency $\omega_2 - \omega_3 - \omega_4$

If it matches $-\omega_1$ then potentially resonance !

<u>Three</u>-<u>wave</u> interactions: $\begin{cases} \omega_1 \pm \omega_2 \pm \omega_3 = 0 \\ \vec{k}_1 \pm \vec{k}_2 \pm \vec{k}_3 = 0 \end{cases}$

<u>Four</u>-<u>wave</u> interactions: $\begin{cases} \omega_1 + \omega_2 = \omega_3 + \omega_4 \\ \vec{k}_1 + \vec{k}_2 = \vec{k}_3 + \vec{k}_4 \end{cases}$

Gravity waves: No three-wave interactions possible.



Sum of two waves does not end up on dispersion curve.

Non-linear inter-action in pictures

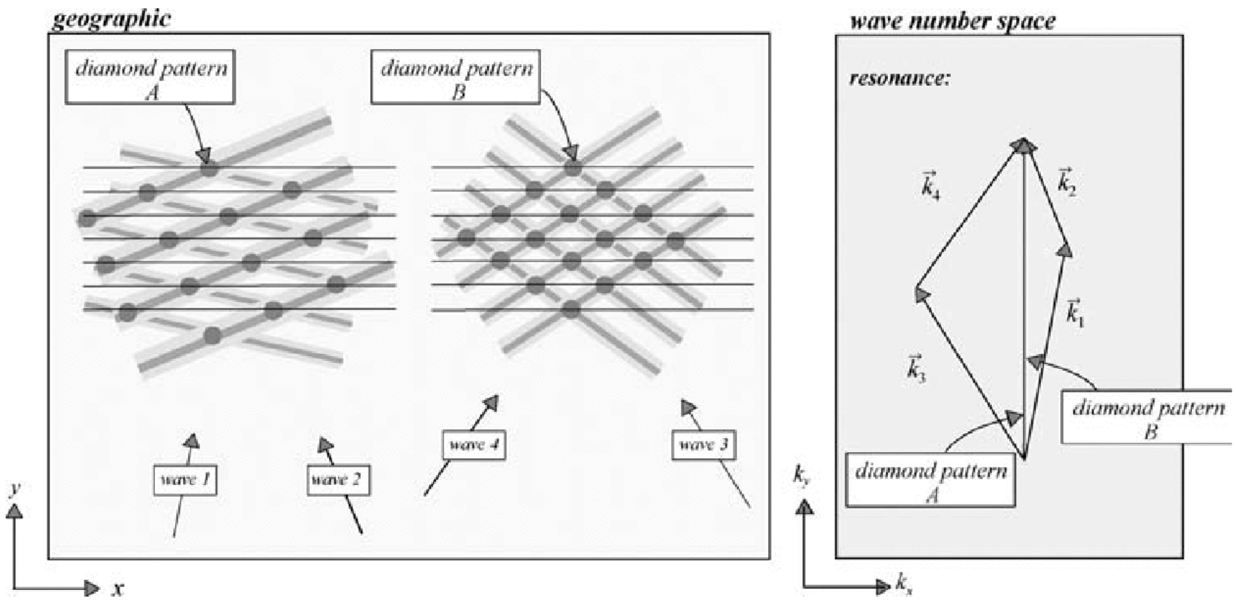


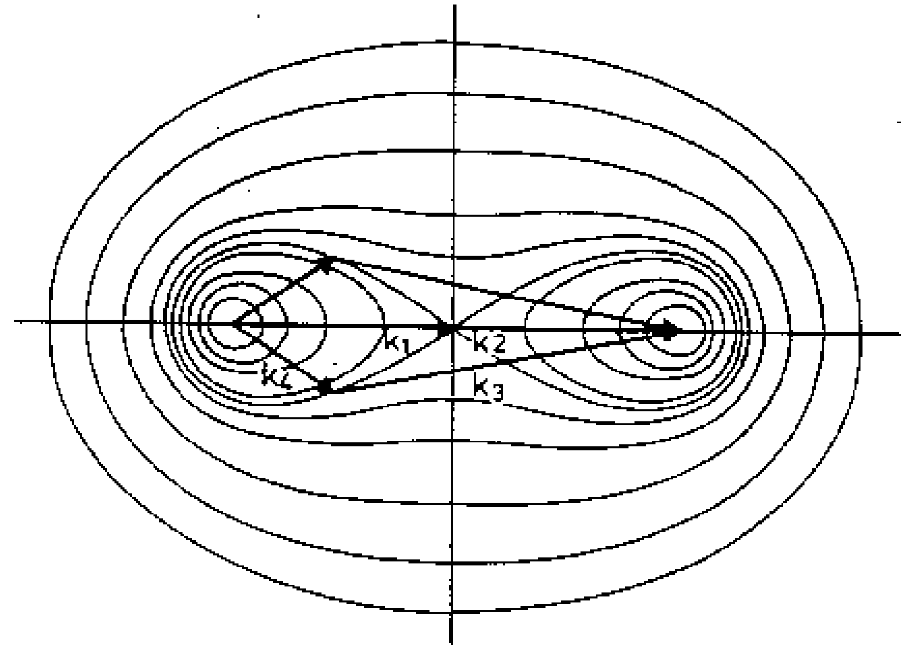
Figure 6.20 *Quadruplet* wave–wave interactions (realisable in deep water). Two pairs of wave components can create two diamond patterns with identical wave lengths and directions and therefore identical wave numbers. When the four waves are superimposed (not shown here), they can thus resonate. The wave-number vectors of the four wave components are shown in the right-hand panel in wave-number space with $\vec{k}_1 + \vec{k}_2 = \vec{k}_3 + \vec{k}_4$.

4-waves interaction (quadruplet) possible in deep water

Figures from "Waves in Oceanic and Coastal Waters" by Leo Holthuijsen. Cambridge University Press

Phillips (1960) has shown that 4-wave interactions do exist!

Phillips' <u>figure of 8</u>



Next step is to derive the statistical evolution equation for $\langle a_1 \, a_2^* \rangle = N_1 \, \delta(\vec{k}_1 - \vec{k}_2)$ with N_1 is the action density.

<u>Nonlinear Evolution</u> Equation ➔

$$\frac{\partial}{\partial t}\langle a_1 \, a_2 \rangle \quad \doteq\longrightarrow \quad \langle a_1 \, a_2 \, a_3 \rangle \quad \& \quad \langle a_1 \, a_2 \, a_3^* \, a_4^* \rangle$$

$$\frac{\partial}{\partial t}\langle a_1 \, a_2 \, a_3 \rangle \quad \doteq\longrightarrow \quad \langle a_1 \, a_2 \, a_3 \, a_4 \rangle \quad \& \quad \langle a_1 \, a_2 \, a_3 \, a_4 \, a_5 \rangle$$

$$\vdots \qquad \text{Infinite hierarchy}$$

Closure is achieved by consistently utilising the assumption of Gaussian probability

Near-Gaussian ➔

$$\left\langle a_1\, a_2\, a_3^*\, a_4^* \right\rangle = \left\langle a_1\, a_3^* \right\rangle \left\langle a_2\, a_4^* \right\rangle + \left\langle a_1\, a_4^* \right\rangle \left\langle a_2\, a_3^* \right\rangle + R$$

Here, *R* is zero for a Gaussian.

Eventual result:

$$\left.\frac{\partial}{\partial t} N_1 \right|_{nonlin} = 4\pi \int \mathrm{d}\vec{k}_2\, \mathrm{d}\vec{k}_3\, \mathrm{d}\vec{k}_4\, T^2_{1,2,3,4}\, \delta(\vec{k}_2 + \vec{k}_3 - \vec{k}_4 - \vec{k}_1) \times$$
$$\delta(\omega_2 + \omega_3 - \omega_4 - \omega_1) \times$$
$$[N_2\, N_3\, (N_1 + N_4) - N_1\, N_4\, (N_2 + N_3)]$$

obtained by Hasselmann (1962).

Properties:

1. *N* never becomes negative.

2. Conservation laws:

action: $\int \mathrm{d}\vec{k}\; N(\vec{k})$

momentum: $\int \mathrm{d}\vec{k}\; \vec{k}\; N(\vec{k})$

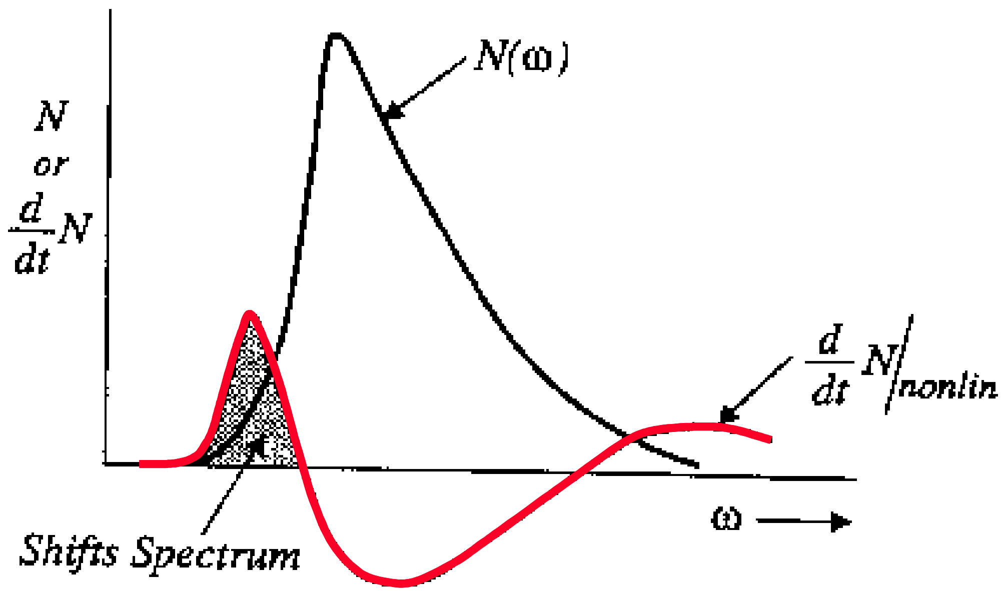
energy: $\int \mathrm{d}\vec{k}\; \omega\; N(\vec{k})$

Wave field cannot gain or loose energy through four-wave interactions.

Properties: *(Cont'd)*

3. Energy transfer

   Conservation of two scalar quantities has implications for energy transfer $\frac{\mathrm{d}}{\mathrm{d}t}N(\omega)$



Two lobe structure  is impossible because if action is conserved, energy ~ ω N cannot be conserved!

ECMWF

Wave Breaking

Dissipation due to Wave Breaking

Define:

$$\overline{\omega} \equiv \frac{\int \mathrm{d}\vec{k}\ \omega\ N(\vec{k})}{\int \mathrm{d}\vec{k}\ N(\vec{k})}$$

$$\overline{k} \equiv \frac{\int \mathrm{d}\vec{k}\ k\ N(\vec{k})}{\int \mathrm{d}\vec{k}\ N(\vec{k})}$$

$$S_{dissip} = -\beta\ \overline{\omega} \left[\left(\overline{k}\right)^2 m_o \right]^2 \left[(1-\delta)\, X + \delta\, X^2 \right] N$$

with $X = k / \overline{k}$, $m_o = \langle \eta^2 \rangle$

Quasi-linear source term: dissipation increases with increasing integral wave steepness $\sqrt{\left(\overline{k}\right)^2 m_o}$

ECMWF

1.4
Energy Balance for Wind Sea

1.4. Energy Balance for Wind Sea

- Summarise knowledge in terms of empirical growth curves.

- Idealised situation of <u>duration-limited waves</u>
  '*Relevant*' parameters:

  ω , $u_{10}(u_*)$, g , ν , surface tension, ρ_a , ρ_w , f_o , t

  Physics of waves: reduction to

  $$\boxed{\omega \, , \; u_{10}(u_*) \, , \; g \, , \; t}$$

  Duration-limited growth not feasible: In practice <u>fully-developed</u> and <u>fetch-limited</u> situations are more relevant.

ECMWF

- Connection between theory and experiments:
  wave-number spectrum: $F(\vec{k}) = \sigma\ N(\vec{k})$

  - $m_o = \left\langle \eta^2 \right\rangle = \int \mathrm{d}\vec{k}\ F(\vec{k})$
  - $H_s = 4\sqrt{\left\langle \eta^2 \right\rangle}$
  - Old days $H_{1/3}$

    $H_{1/3} \cong H_{\mathrm{s}}$ (exact for narrow-band spectrum)

  - 2-D wave number spectrum is hard to observe

    ➔ frequency spectrum

    $$F_2(\omega,\theta)\,\mathrm{d}\omega\,\mathrm{d}\theta = F(\vec{k})\,\mathrm{d}\vec{k} = F(k,\theta)\,k\,\mathrm{d}k\,\mathrm{d}\theta$$

    $$\Rightarrow \boxed{F_2(\omega,\theta) = \frac{k}{v_g} F(k,\theta)}$$

  - One-dimensional frequency spectrum $F_1(\omega) = \int \mathrm{d}\theta\ F_2(\omega,\theta)$

    Use same symbol, *F* , for: $F(\vec{k})$, $F(\omega,\theta)$, $F(\omega)$

- Let us now return to analysis of wave evolution:

Fully-developed:

$$\tilde{F} \equiv g^3 F(\omega) / u_{10}^5 = f(\omega u_{10} / g)$$

$$\tilde{\varepsilon} \equiv g^2 m_o / u_{10}^4 = \text{constant}$$

$$\nu \equiv \omega_p u_{10} / g = \text{constant}$$

Fetch-limited:

$$\tilde{F} \equiv g^3 F(\omega) / u_{10}^5 = f(\omega u_{10} / g\,,\; g X / u_{10}^2)$$

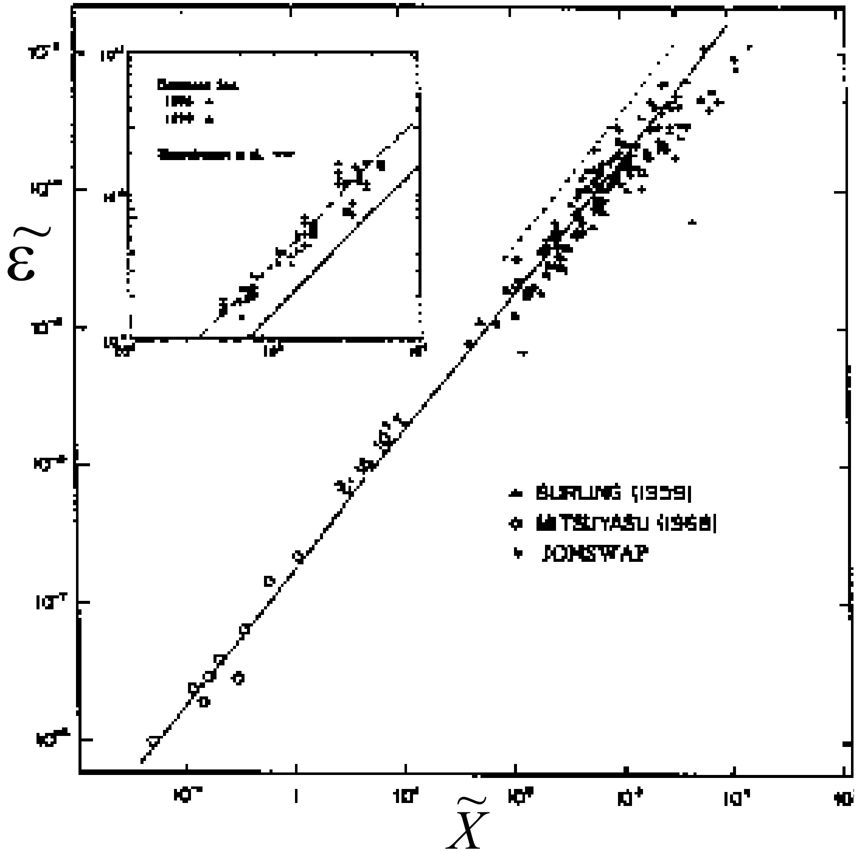
$$\tilde{\varepsilon} \equiv g^2 m_o / u_{10}^4 = f(g X / u_{10}^2)$$

$$\nu \equiv \omega_p u_{10} / g = f(g X / u_{10}^2)$$

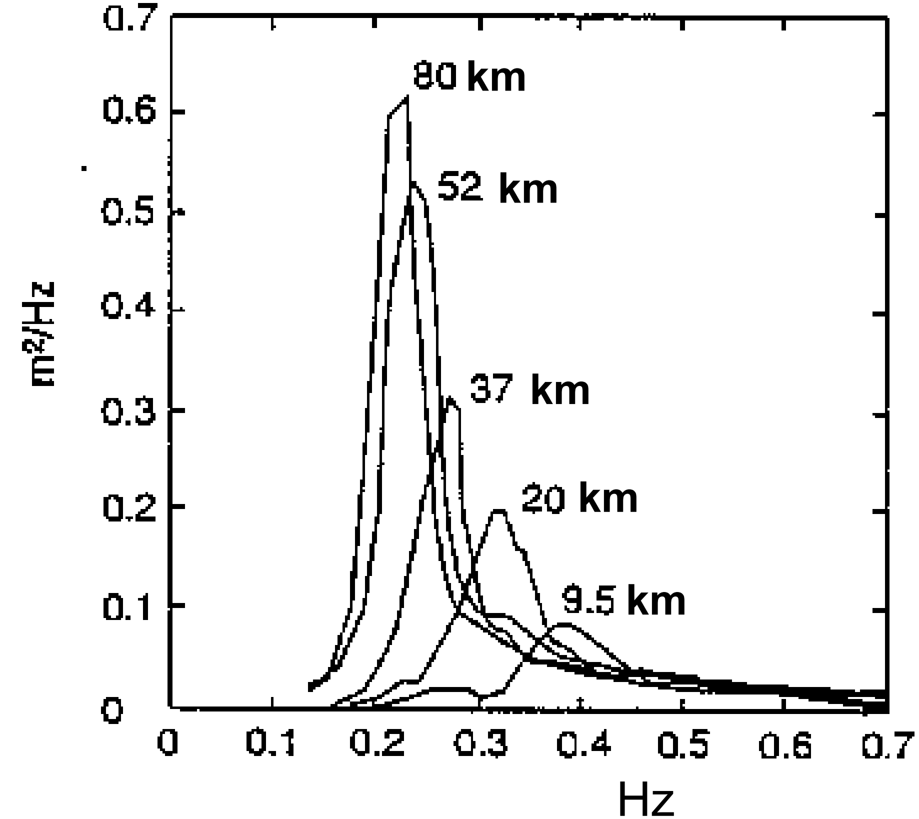
However, scaling with friction velocity u_* is to be preferred over u_{10} since u_{10} introduces an additional length scale, z = 10 m , which is not relevant. In practice, we use u_{10} (as u_* is not available).

- JONSWAP fetch relations (1973):

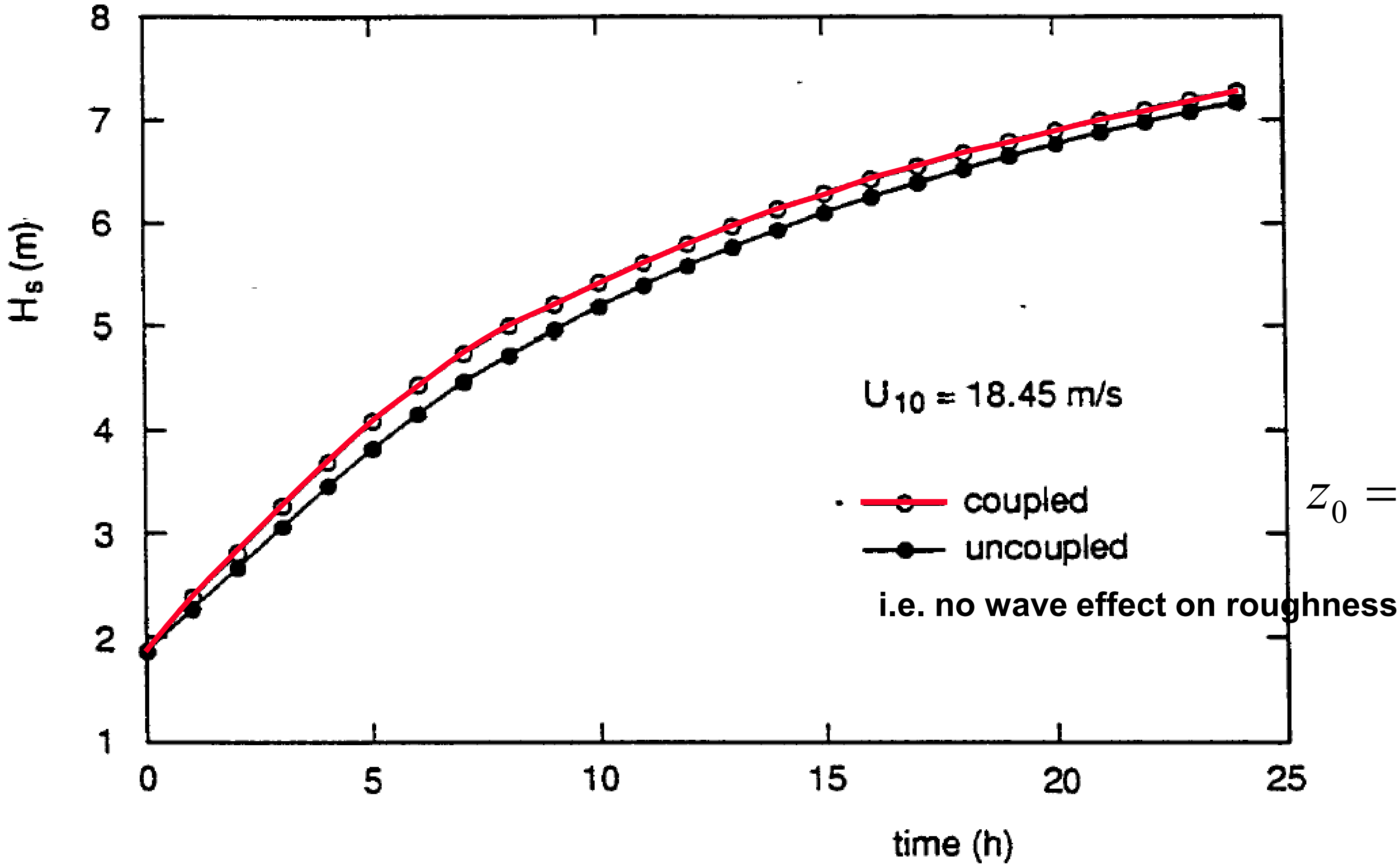
$$\tilde{\varepsilon} \equiv g^2 m_o / u_{10}^4 \quad , \quad \tilde{X} \equiv g X / u_{10}^2 \quad , \quad \nu \equiv \omega_p u_{10} / g$$



Empirical growth relations against measurements



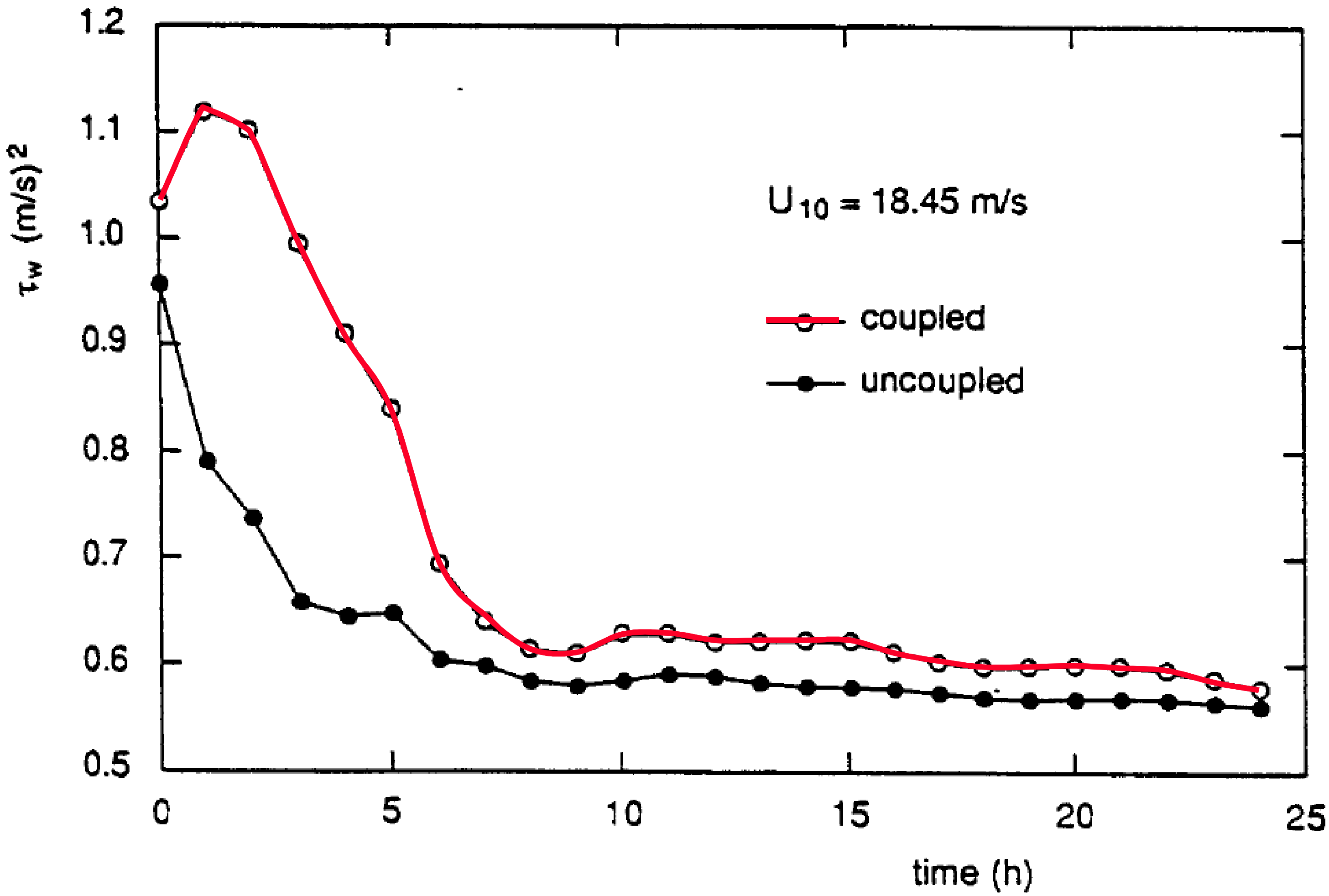
Evolution of spectra with fetch (JONSWAP)



$$z_0 = \frac{\alpha\, u_*^2}{g}$$

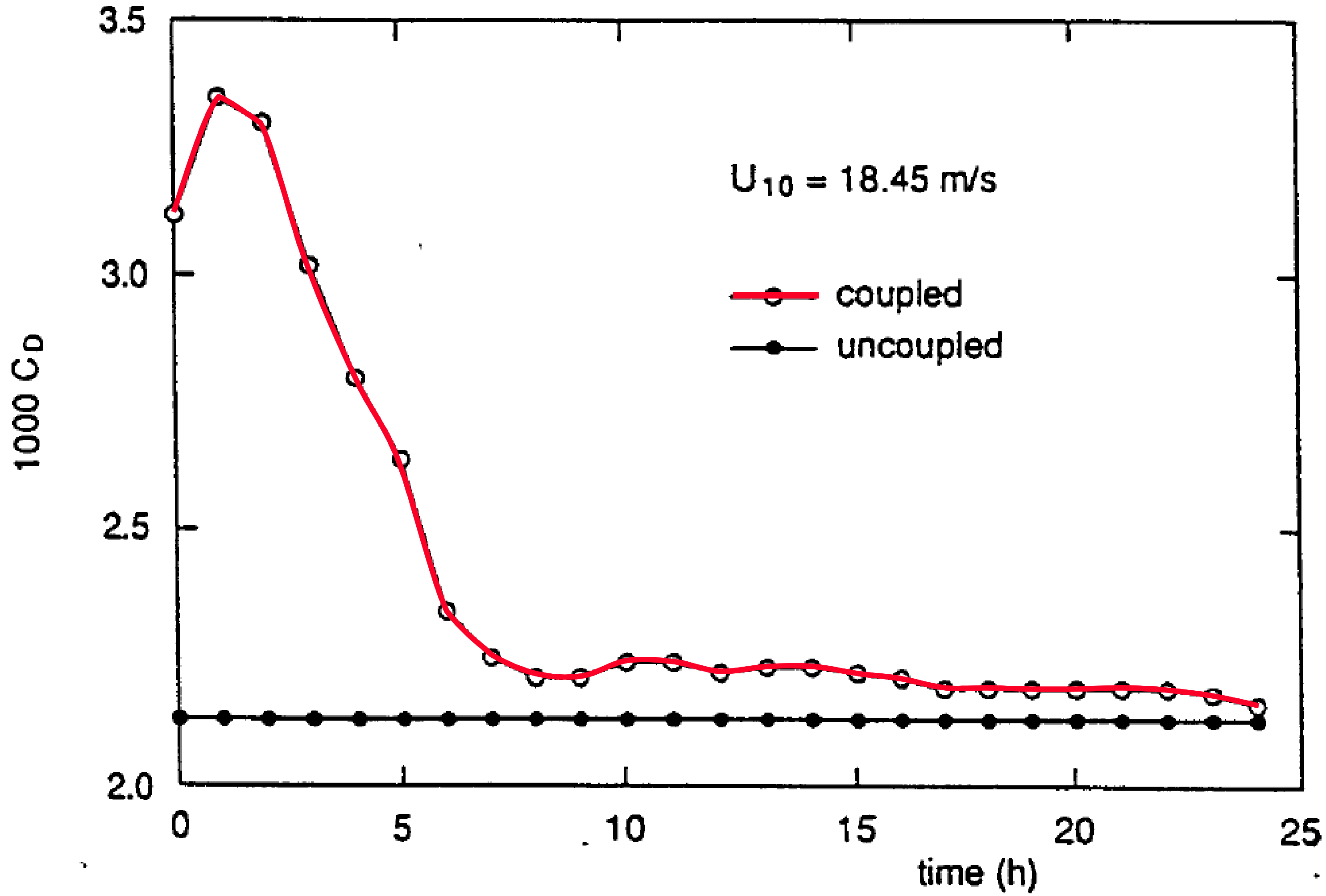
Time dependence of <u>wave height</u> for a reference run and a coupled run (one grid point simulation).

ECMWF



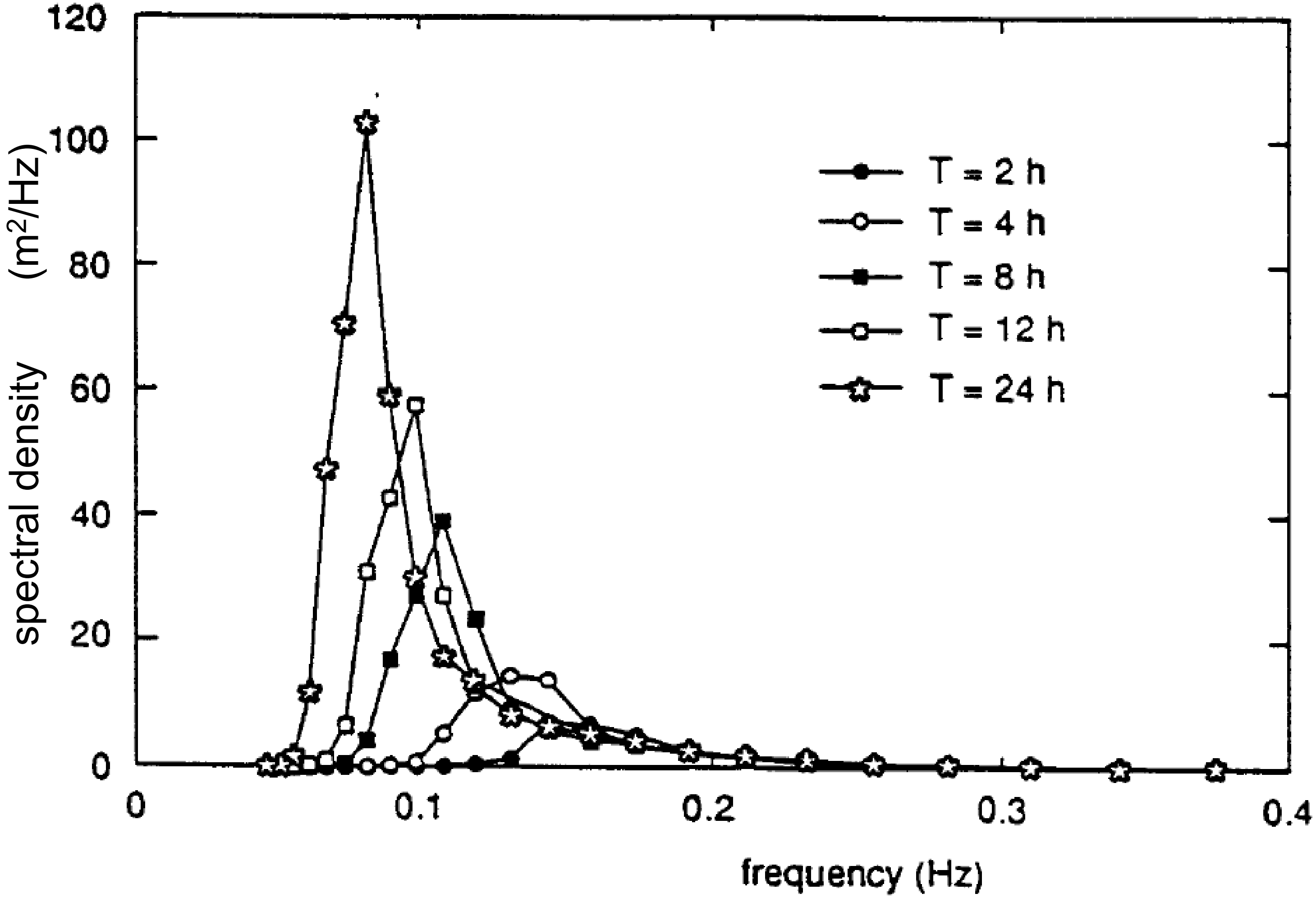
Time dependence of wave-induced stress for a reference run and a coupled run.

ECMWF

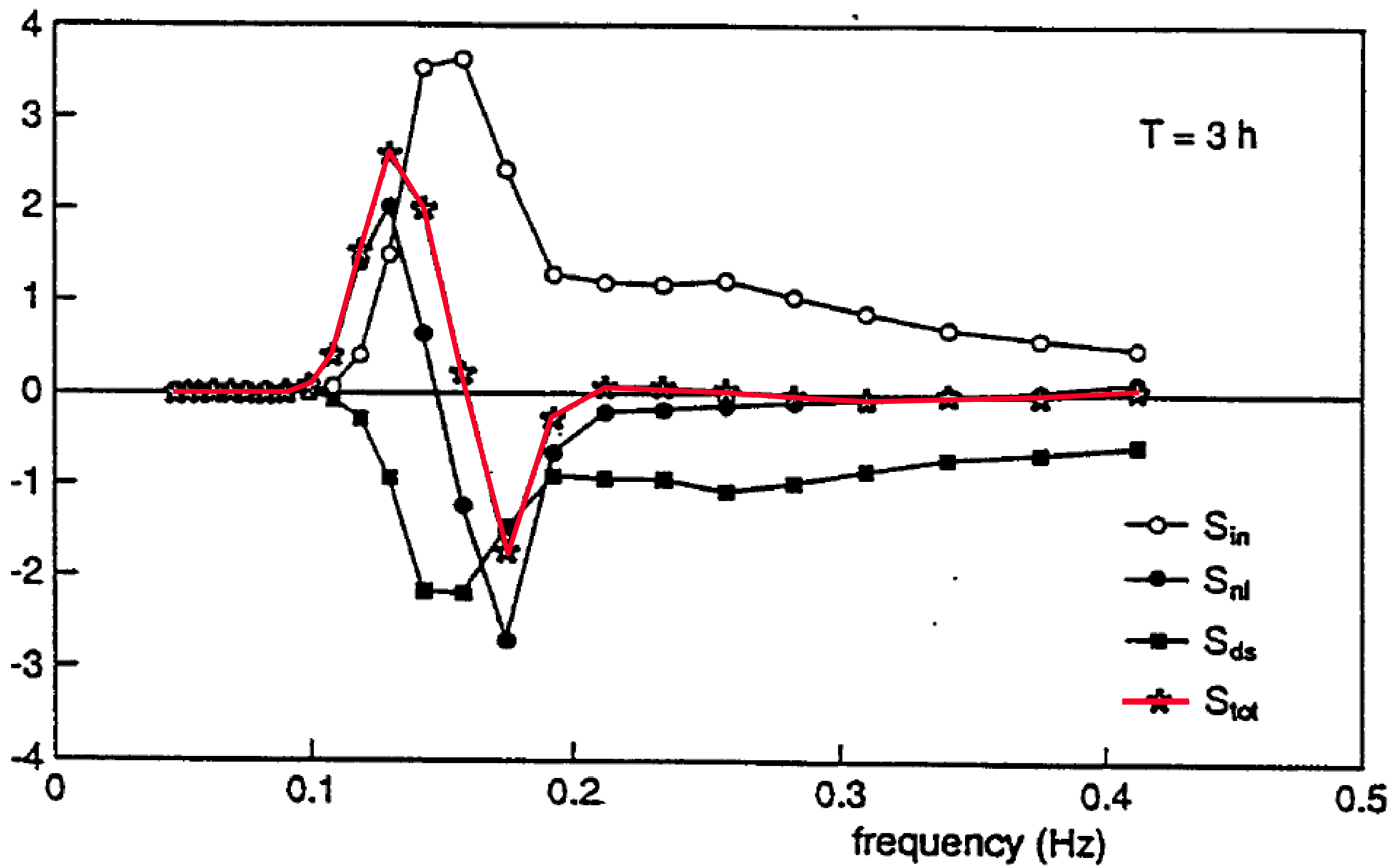


Time dependence of drag coefficient, C_D , for a reference run and a coupled run.

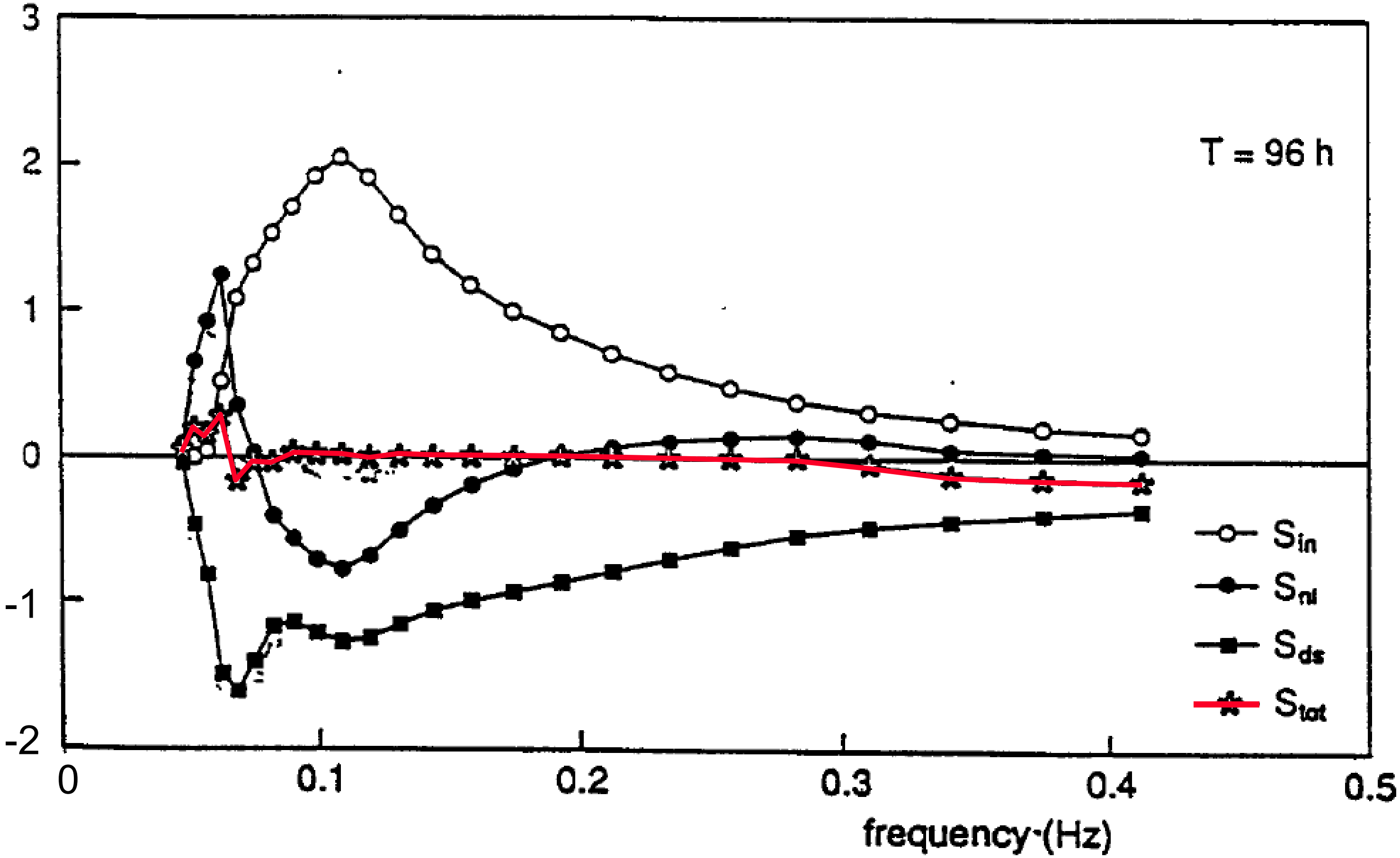
ECMWF



Evolution in time of the one-dimensional frequency spectrum for the coupled run.



The energy balance for young wind sea.



The energy balance for old wind sea.

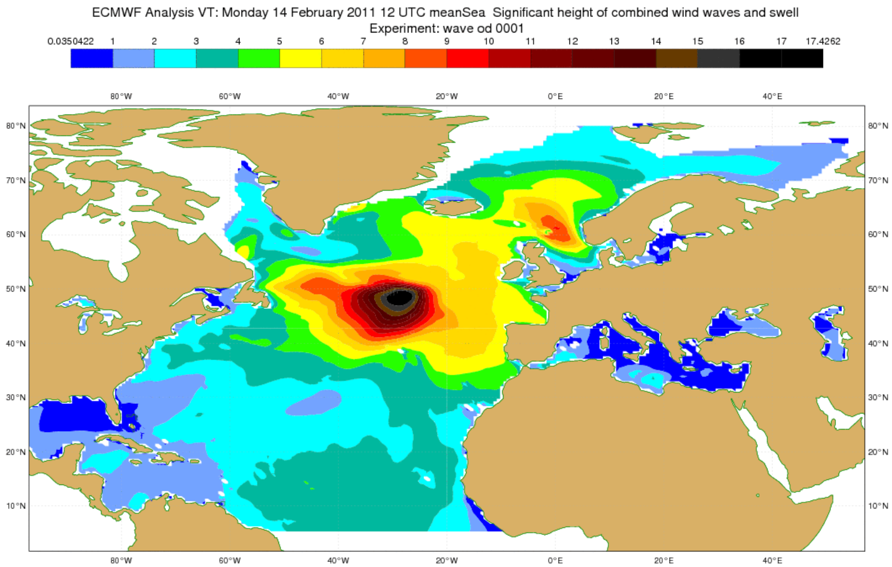
2

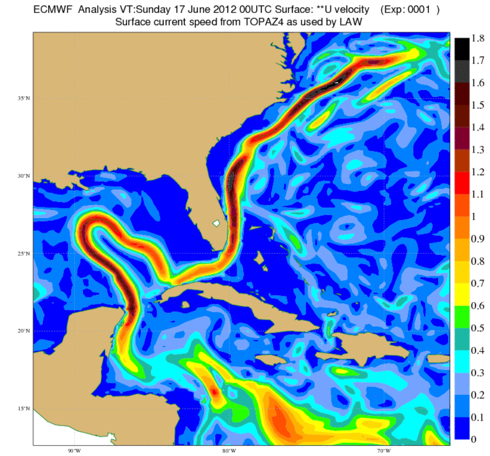
THE ECWAM MODEL

ECMWF Wave Model Configurations

1) Limited Area Wave model (LAW)

- Limited extend.
- 11 km grid spacing.
- Stand alone.
- Forced by 10m neutral wind fields.
- Use surface currents from TOPAZ4.
- Data assimilation of altimeter data.
- 2 daily forecasts extending to day 5.
- Output every hour, including spectra (*).

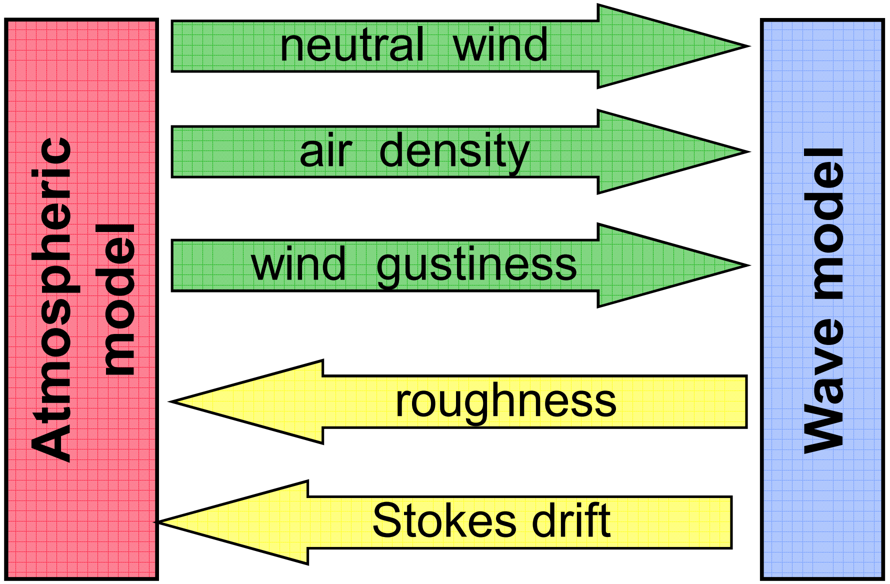


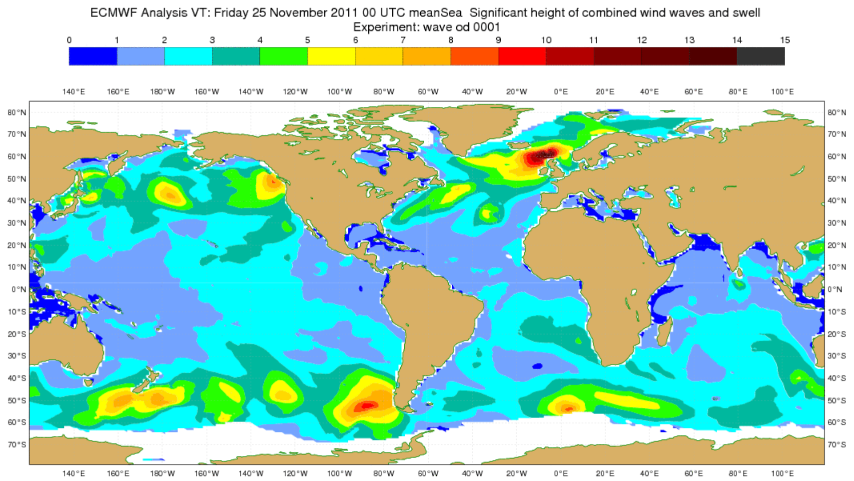


TOPAZ4
surface currents

ECMWF Wave Model Configurations
2) Global models

- Global.
- Coupled to the atmospheric model.
- Data assimilation of altimeter data.
- Part of all forecasting components (high resolution, ensemble, monthly, seasonal, re-analyses)





Global from 81°S to 90°N

ECMWF Wave Model Configurations

High resolution

- 28 km grid spacing.
- 36 frequencies.
- 36 directions.
- Coupled to the TL1279 model (16km).
- Analysis every 6 hrs and 10 day forecasts from 0 and 12Z.

Ensemble forecasts

- 55 km grid spacing.
- 30→ 25 frequencies *.
- 24 → 12 directions *.
- Coupled to TL639 (32 km) → TL319 model *.
- (50+1) (10+5) day forecasts from 0 and 12Z (monthly twice a week).

* Change in resolutions after 10 days

NB: also in seasonal forecast at lower resolutions

ECMWF Wave Model Configurations

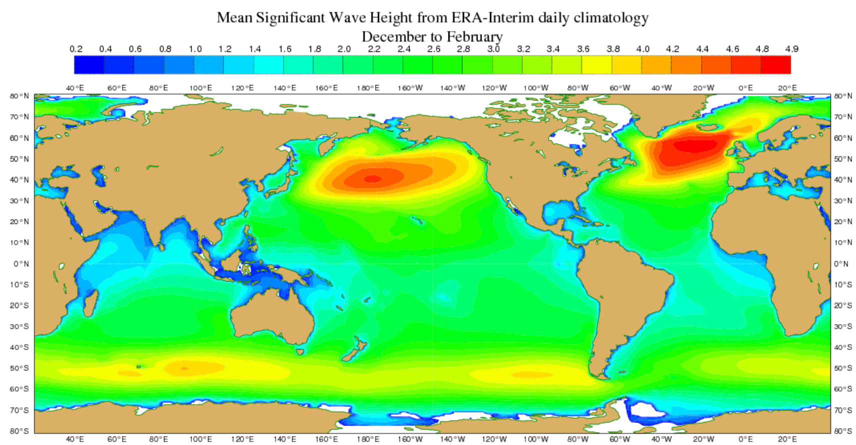
Interim reanalysis (1979 to present)

(as a follow-up to ERA40 (45 year reanalysis)

- **1.0°x1.0°.**
- **30 frequencies.**
- **24 directions.**
- **Coupled to TL255 model.**
- **Production is ongoing.**
- **Data access:**

http://data-portal.ecmwf.int/data/d/interim_full_daily/

Mean wave height for Northern Hemisphere winter:



- http://www.ecmwf.int/research/era/do/get/era-interim
- ECMWF Newsletters No. 110 (Winter 2006/07) & 111 (Spring 2007) http://www.ecmwf/publications/newsletters

ECWAM:

We solve the problem on a sphere (radius R).assuming steady ocean currents (U) and fixed water depth (D). The action balance equation reduces to the energy balance equation for the 2-D frequency and direction spectrum: $F(f,\theta)$

$$\boxed{\frac{\partial}{\partial t}F+\frac{1}{\cos\phi}\frac{\partial}{\partial\phi}(\dot{\phi}\cos\phi N)+\frac{\partial}{\partial\lambda}(\dot{\lambda}N)+\frac{\partial}{\partial\theta}(\dot{\theta}N)=S}$$

$$\sigma = 2\pi\, f$$

(f : frequency, θ : direction (clockwise relative to true north), ϕ: latitude, λ:longitude)

$$\dot{\phi}=(v_g\cos\theta-U_o)/R$$

$$\Omega=\vec{k}\cdot\vec{U}+\sigma\,,\sigma=\sqrt{g\,k\tanh(k\,D)}$$

$$\dot{\lambda}=(v_g\sin\theta-V_o)/(R\cos\phi)$$

$$\vec{U}(\phi,\lambda)=(U_o,V_o)\qquad D(\phi,\lambda)$$

$$\vec{v}_g=\partial\sigma/\partial\vec{k}$$

$$\dot{\theta}=v_g\sin\theta\tan\phi/R+(\dot{\vec{k}}\times\vec{k})/k^2$$

$$\dot{k}_i=-\partial\Omega/\partial x_i$$

ECMWF

ECWAM:

$$\frac{\partial}{\partial t}F + \frac{1}{\cos\phi}\frac{\partial}{\partial\phi}(\dot{\phi}\cos\phi N) + \frac{\partial}{\partial\lambda}(\dot{\lambda}N) + \frac{\partial}{\partial\theta}(\dot{\theta}N) = S$$

$$S = S_{in} + S_{nl} + S_{diss} + S_{bottom} + S_{bbrk}$$

S_{in}: wind input source term (generation).

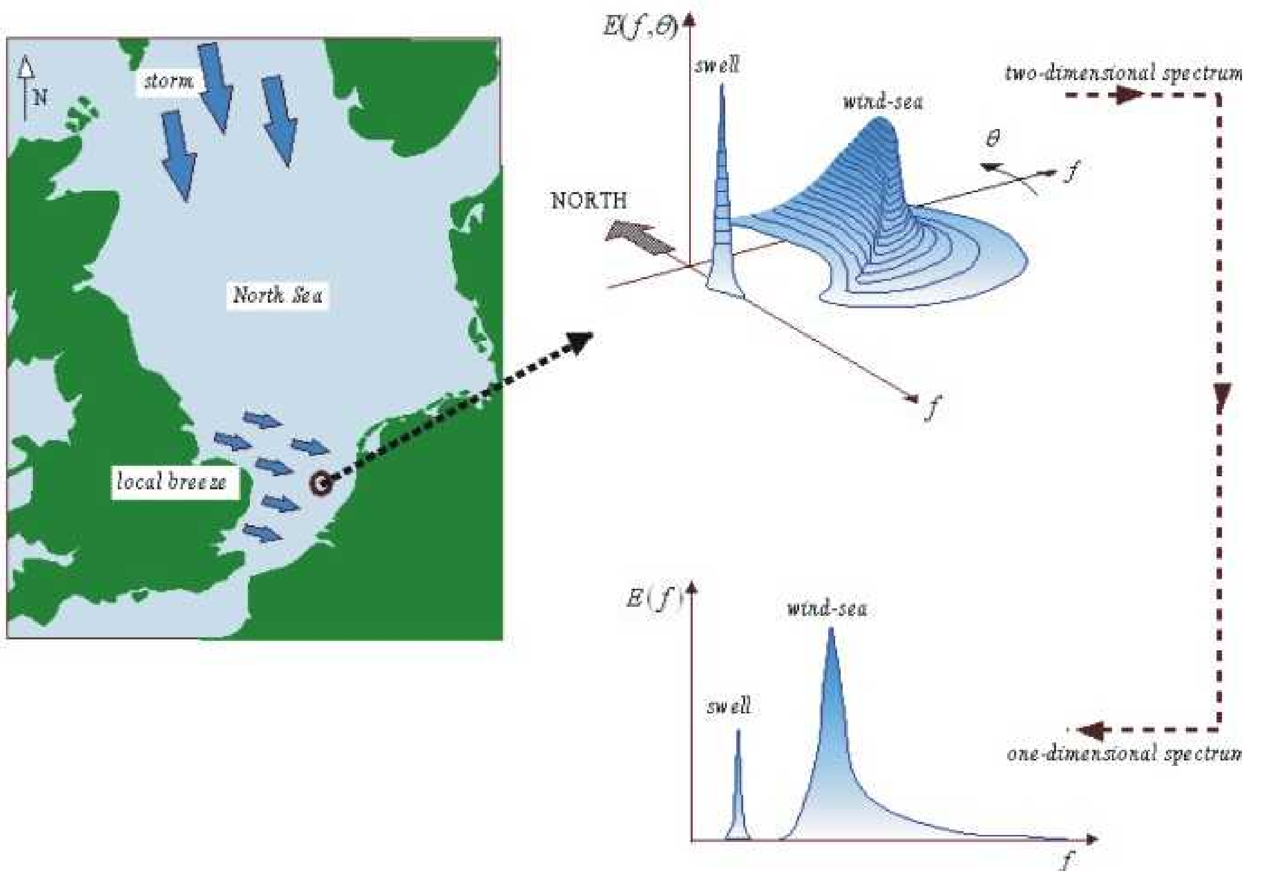
S_{nl}: non-linear 4-wave interaction (redistribution).

S_{diss}: dissipation term due to white capping (dissipation).

S_{bottom}: dissipation dues to bottom effect (dissipation).

S_{bbrk}: dissipation due to bottom induced wave breaking (dissipation).

- **The sea state is then fully described by the two-dimensional wave spectrum F(f,θ).**

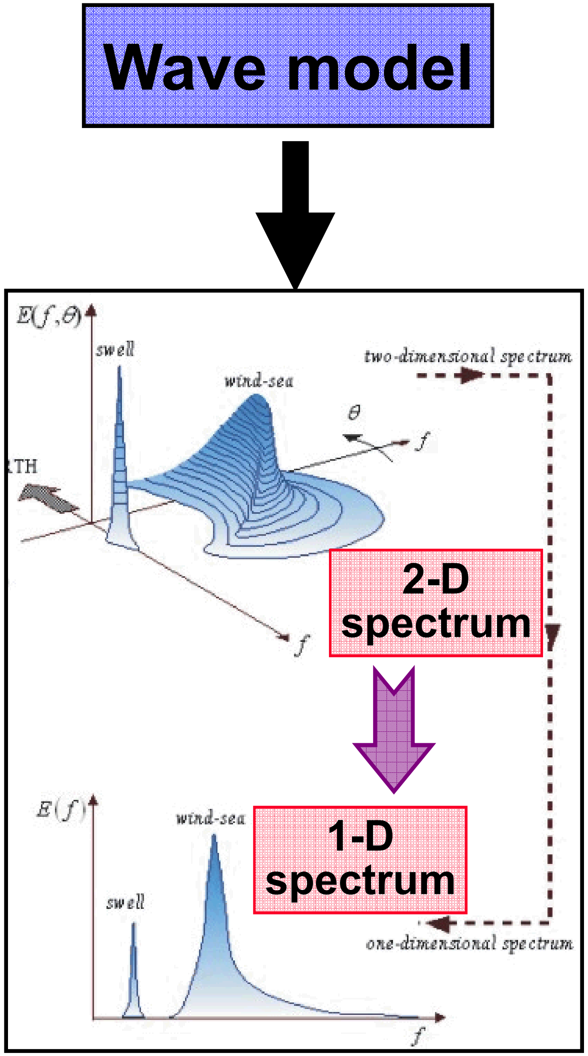


Wave Model Products

The complete description of the sea state is given by the 2-D spectrum, however, it is a fairly large amount of data *(e.g. 1296 values at each grid point in the global model (36x36)***.**

It is therefore reduced to integrated quantities:

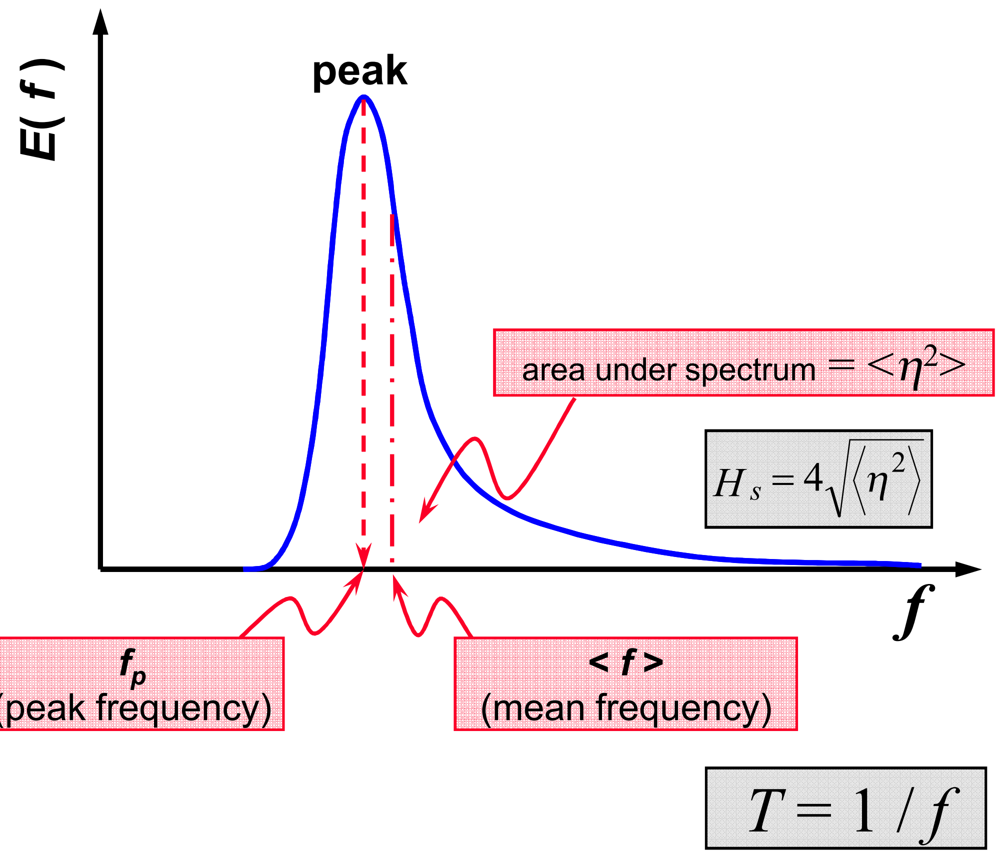
- **1-D spectrum obtained by integrating the 2-D spectrum over all directions and/or over a frequency range.**



Wave Model Products

When simple numbers are required, the following parameters are available:

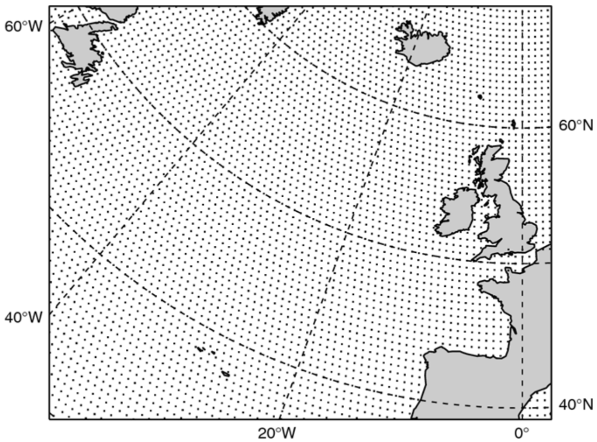
- **The significant wave height (H_s).**
- **The peak period (period of the peak of the 1-D spectrum).**
- **Mean period(s) obtained from weighted integration of the 2-D spectrum.**
- **Integrated mean direction.**
- ***Few others.***



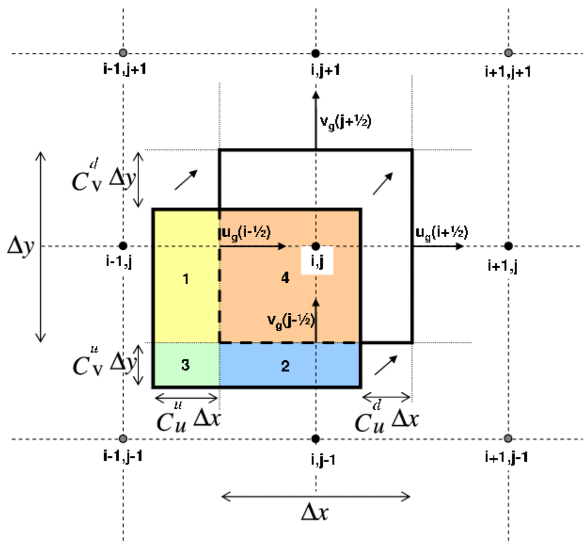
Complete list at: http://www.ecmwf.int/services/archive/d/parameters/order=/table=140/

Grid and Advection:

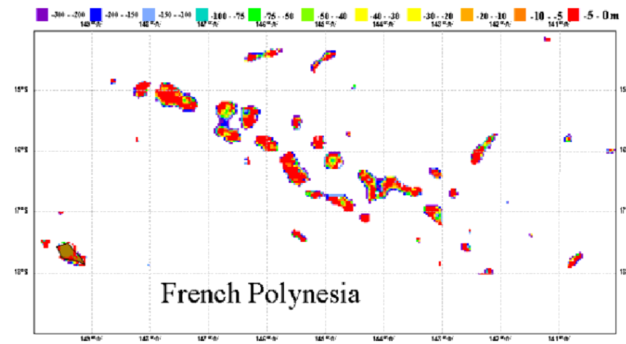
Irregular lat-lon grid to keep the distance between grid points roughly constant

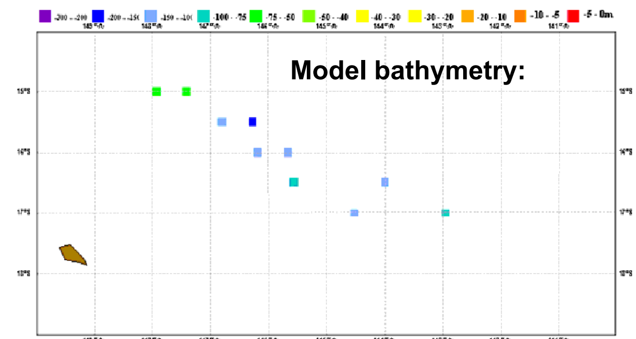


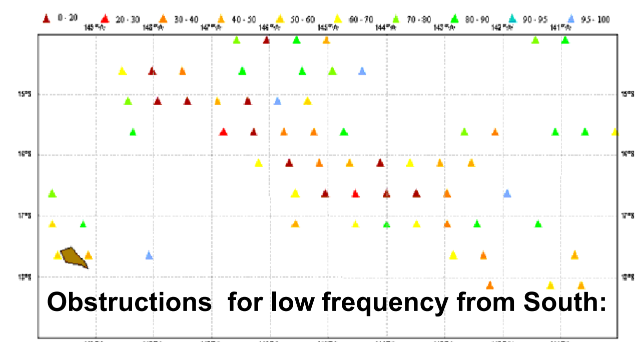
Corner Transport Upstream scheme:



Unresolved bathymetry obstructions:







Wind input S_{in}: based on Janssen (1991):

neutral 10m wind U_{10}

$$U_{10} = \frac{u_*}{\kappa} \ln\left(\frac{10}{z_o}\right)$$

air density

$$S_{in} = \frac{\rho_a}{\rho_w} \frac{\beta_{max}}{\kappa^2} e^Z Z^4 \left(\frac{u_*}{c} \max(\cos(\theta-\phi),0)\right)^2 \omega F$$

κ von Karman constant

u_* friction velocity

z_o surface roughness

ρ_a air density

ρ_w water density

ϕ wind direction, $F(f,\theta)$

c phase speed, g

$\beta_{max} = 1.2$

where Z is the effective wave age:

$$Z = \ln\left(\frac{g z_o}{c^2}\right) + \frac{\kappa}{\cos(\theta-\phi)(\frac{u_*}{c} + z_\alpha)}$$

z_α : wave age tuning parameter:

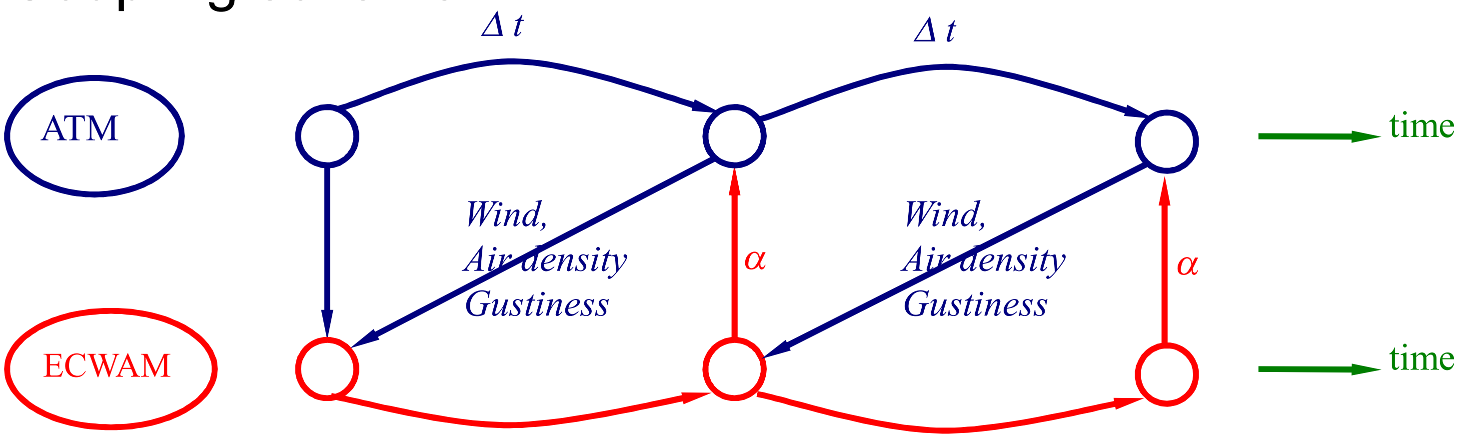
New: $z_\alpha = 0.008$ (0.011 before CY38R1)

roughness

z_o sea state dependent

3.2. Coupled Wind-Wave Modelling

- Coupling scheme:

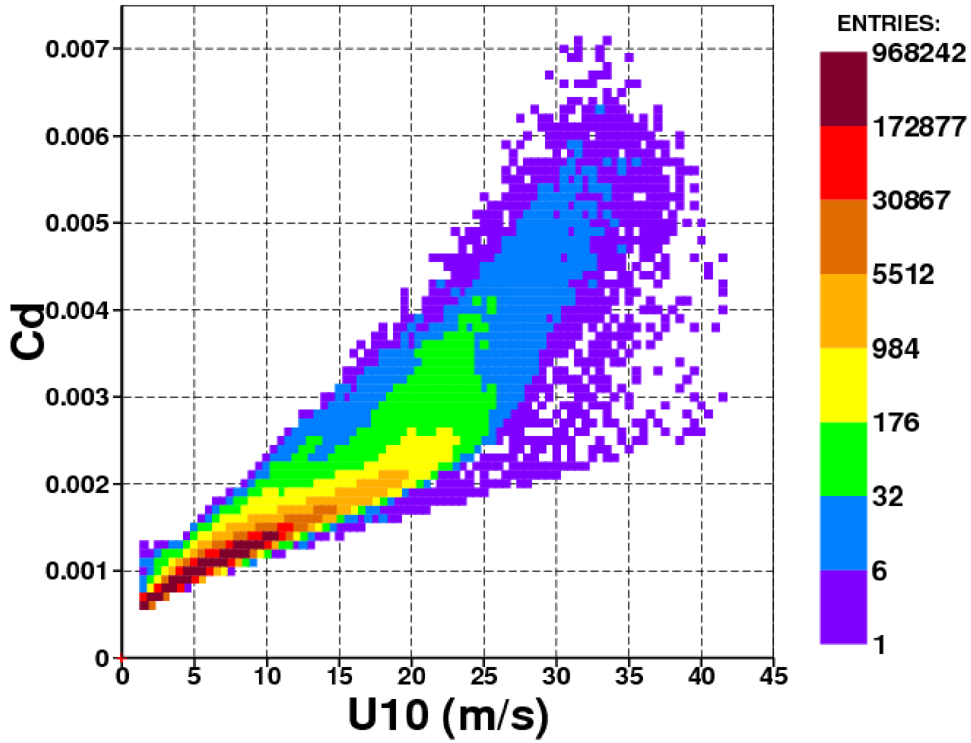


- $\alpha = \tilde{\alpha} \, / \, \sqrt{1 - \tau_w / \tau} \quad , \quad \tilde{\alpha} = 0.006 \quad , \quad \tau_w = \int \mathrm{d}\vec{k} \, \vec{k} \, S_{in}$
- Impact on depression.
- Impact on climate [extra tropic].
- Impact on tropical wind field → ocean circulation.
- Impact on weather forecasting.

$$z_0 = \frac{\alpha \, u_*^2}{g}$$

Sea state dependant Drag Coefficient

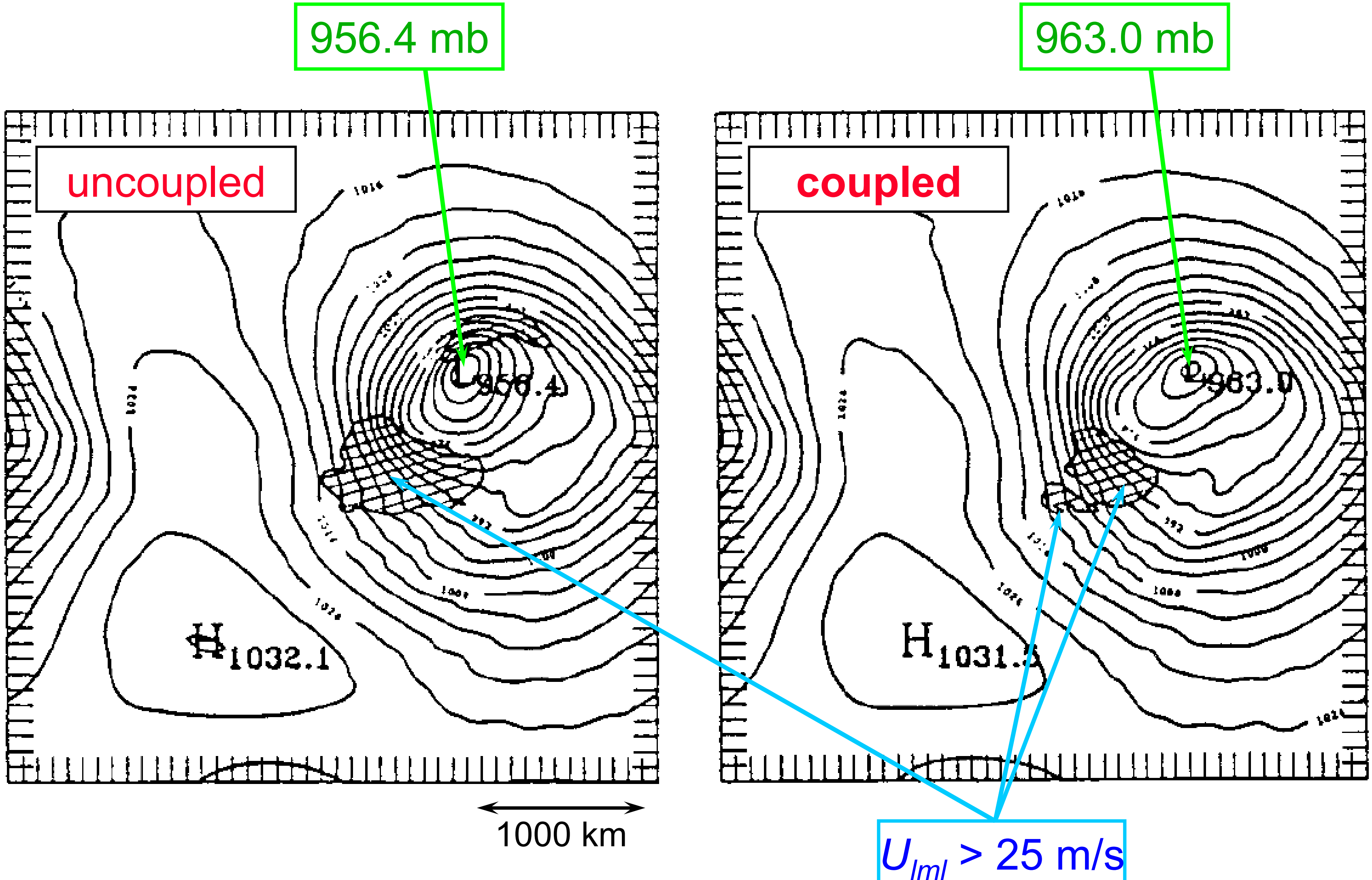
Drag Coefficient versus wind speed (coupled runs):



Forecast data every hour from 2011082600 up to day 2.
All grid points over latitudinal band 20S to 40N

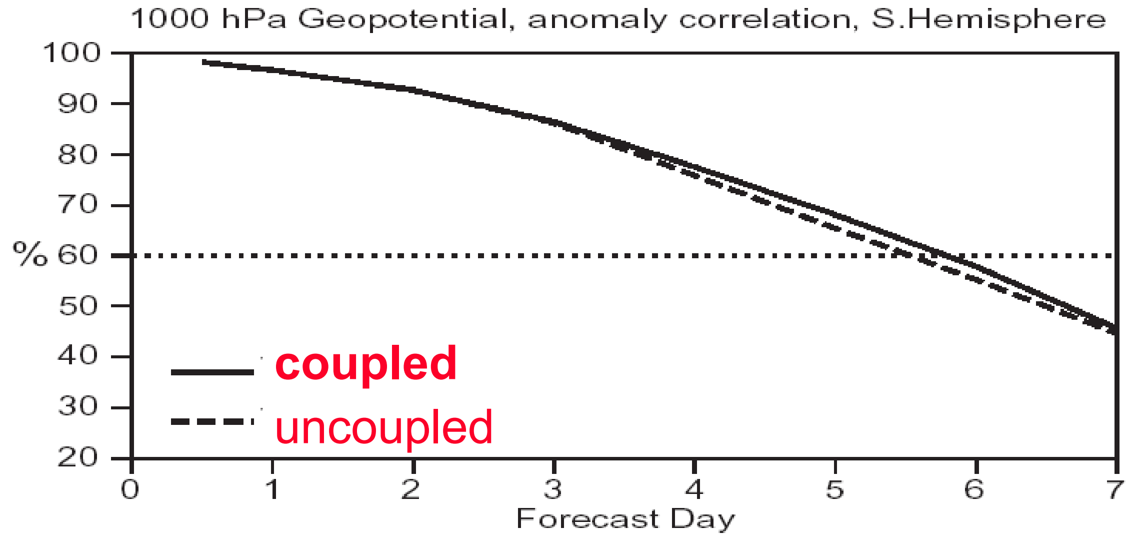
ECMWF

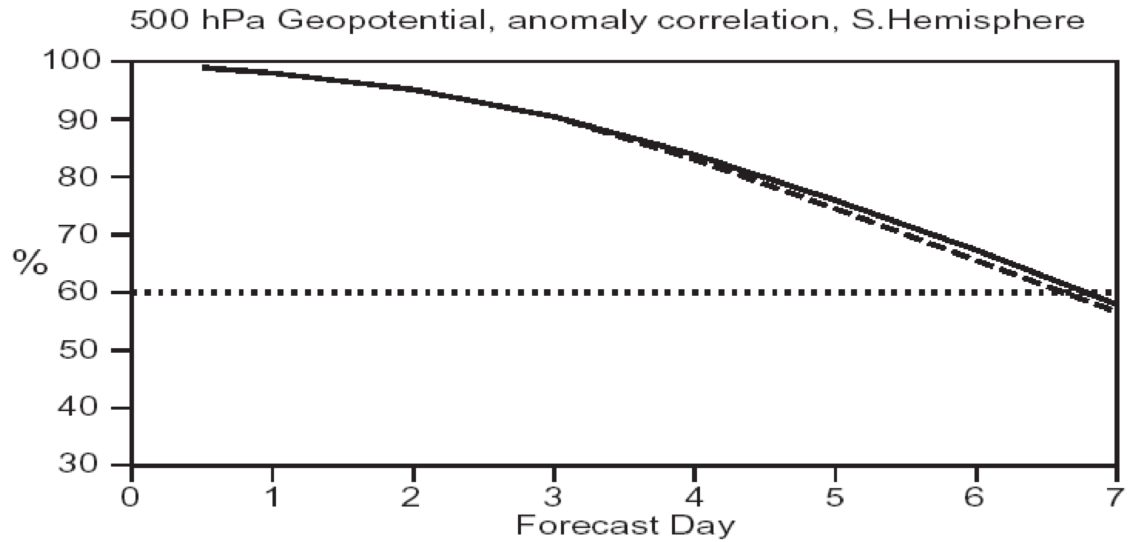
Simulated sea-level pressure for uncoupled and coupled simulations for the 60 h time



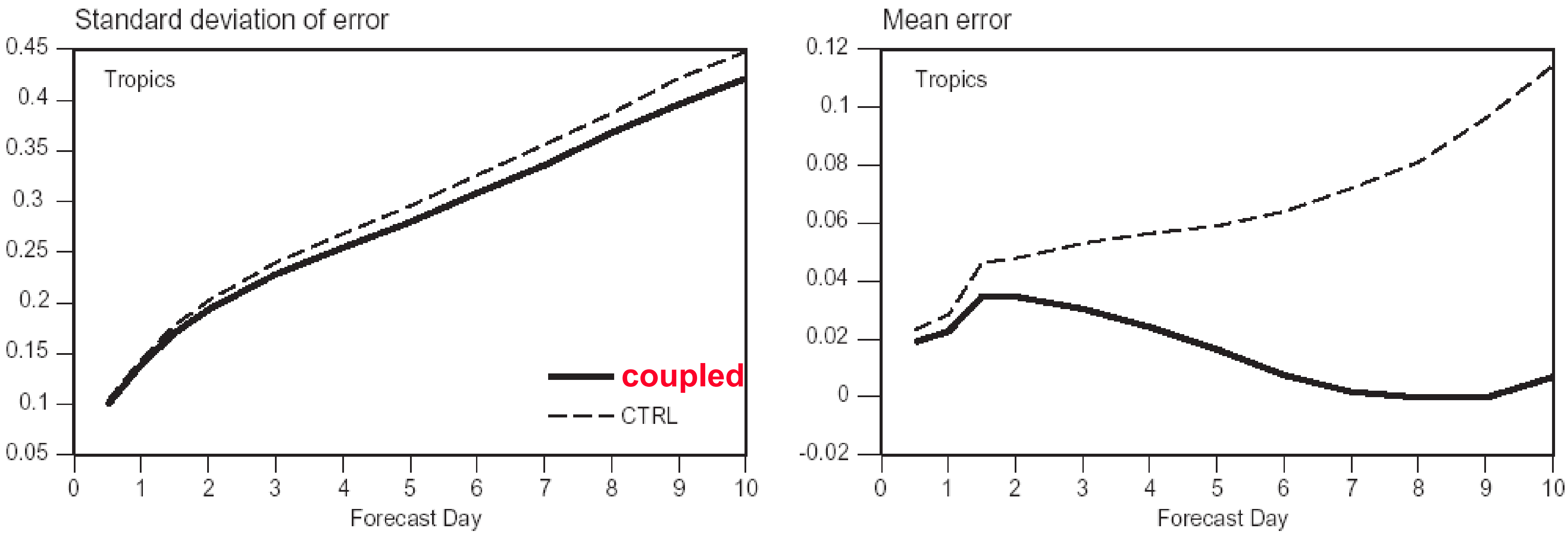
ECMWF

Scores of FC 1000 and 500 mb geopotential for SH (28 cases in ~ December 1997)



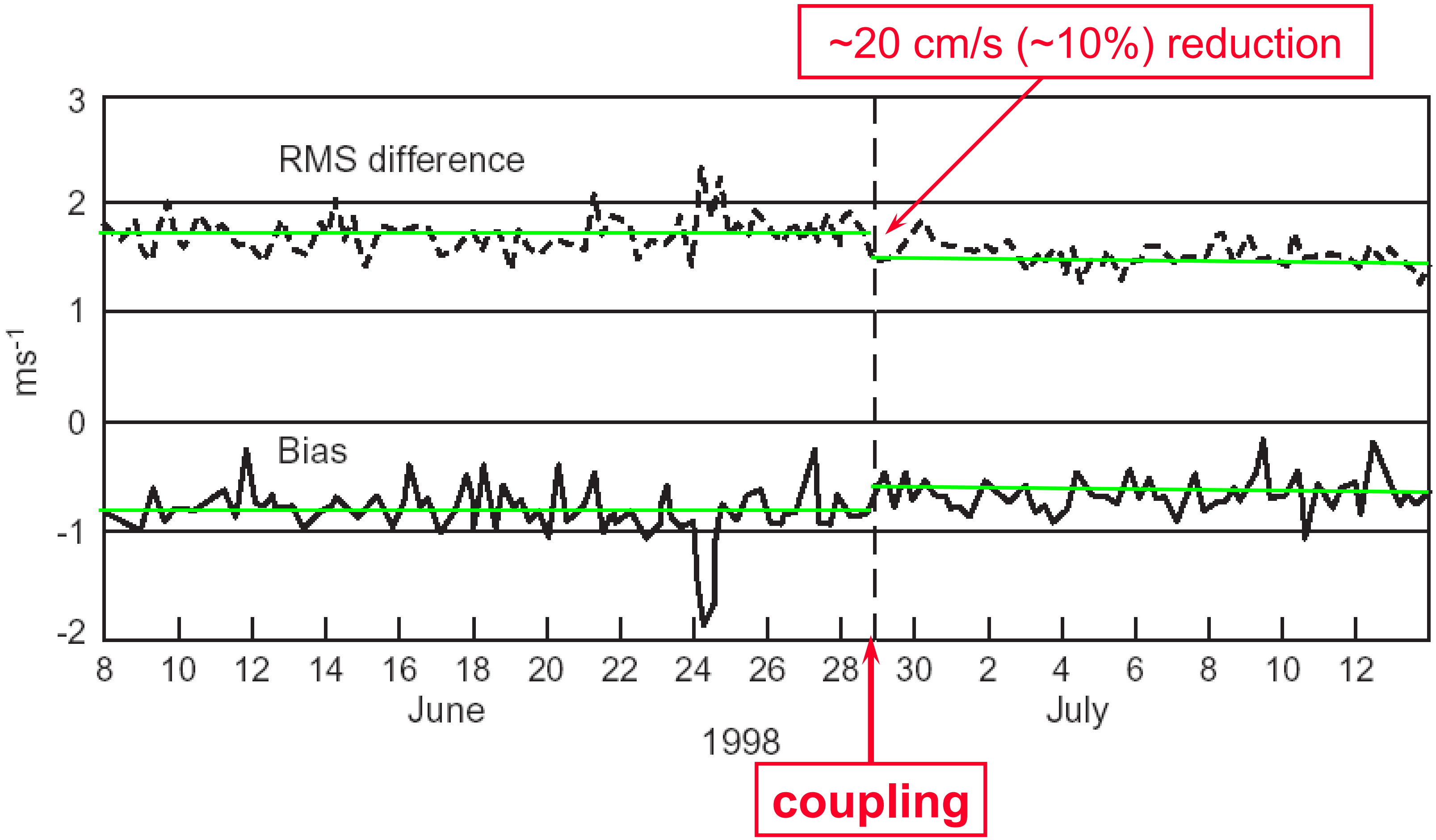


Standard deviation of error and systematic error of forecast wave height for Tropics (74 cases: 16 April until 28 June 1998).



ECMWF

Global RMS difference between ECMWF and ERS-2 scatterometer winds (8 June – 14 July 1998)



Wind input S_{in}: gustiness parameterisation

$$S_{in} = \gamma\, F$$

wind gustiness

$$\bar{\gamma}(u_*) = \frac{1}{\sigma_* \sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left\{ -\frac{(u_* - \bar{u}_*)^2}{2\sigma_*^2} \right\} \gamma(u_*)\, du_*$$

σ_* : standard deviation of u*

$$\bar{\gamma}(u_*) \approx 0.5 \left[\gamma(u_* + \sigma_*) + \gamma(u_* - \sigma_*) \right]$$

$$\sigma_* = \frac{u_*}{U_{10}} \left(1 + \frac{0.5\, U_{10}\, 0.08\, 10^{-3}}{C_d} \right) \left\{ b + 0.5 \left(\frac{z_i}{-L} \right) \right\}^{1/3}$$

from the atmospheric model :

z_i Inversion height

L Monin Obukhov length

$$b = 0$$

$$u_* = \sqrt{C_d}\; U_{10} \qquad C_d = (0.8 + 0.08) 10^{-3}\, U_{10}$$

ECMWF

Wind input S_{in}: linear swell damping

$$S_{in} = \gamma F$$

Following Janssen (2004), the small effect of turbulent eddies on the waves can be modelled as

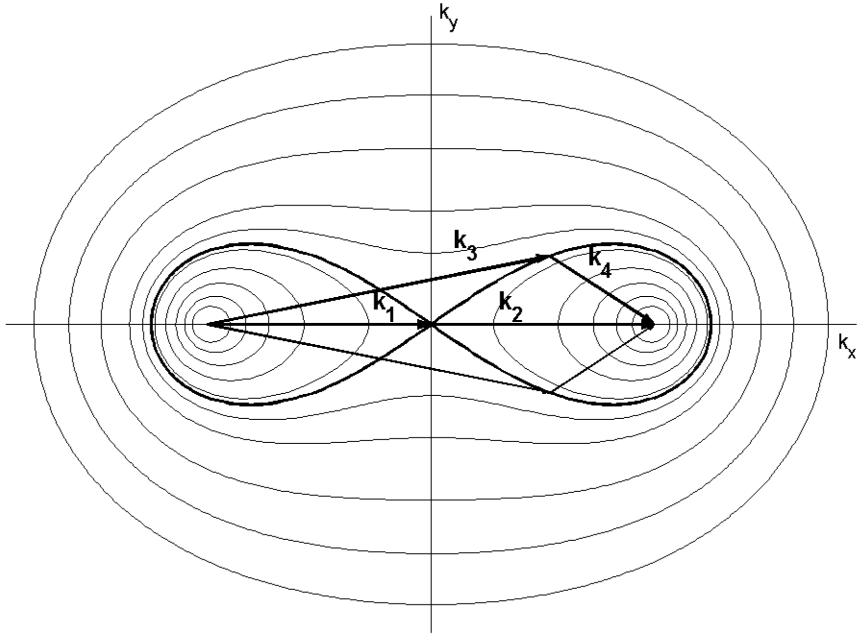
$$\frac{\gamma}{\omega} = \frac{\rho_a}{\rho_w} \left\{ \beta \left(\frac{u_*}{c} \max(\cos(\theta - \phi), 0) \right)^2 + 2\kappa \left(\frac{u_*}{c} \right)^2 \left(\cos(\theta - \phi) - \frac{c}{V} \right) \right\}$$

V : wind speed at height z = $1/\mathrm{k}$

Snl:

- **The calculation of the non linear source term is still based on the Discrete Interaction approximation (DIA).**

Only two configurations of quadruplets are considered in the DIA



$$\vec{k}_1 = \vec{k}_2 = \vec{k},$$

$\vec{k}_3$ and $\vec{k}_4$ at an angle to satisfy the resonance

Specifically:

$$\omega_1 = \omega_2 = \omega, \qquad \text{with } \lambda = 0.25,$$

$$\omega_3 = \omega(1+\lambda), \qquad \theta_3 = 11.48^{\circ},$$

$$\omega_4 = \omega(1-\lambda) \qquad \theta_4 = -33.56^{\circ}$$

and the mirror Image:

$$\theta_3 = -11.48^{\circ},$$

$$\theta_4 = 33.56^{\circ}$$

NB: as in the exact case, DIA also conserves action, energy, and momentum

Snl:

- For shallow water, the transfer coefficients are re-scaled:

$$Transf_{nl}(\text{shallow}) = f(k,h)\, Transf_{nl}(\text{deep})$$

- Following Janssen and Onorato (2005), using the narrow band approximation, it was shown that the scaling factor could be written as

$$f(k,h) = \frac{R^2}{T^8 \dfrac{\partial v_g}{\partial k}}$$

where $$\frac{\partial v_g}{\partial k} = [T - kh(1-T^2)]^2 + 4(kh)^2 T^2 (1-T^2)$$

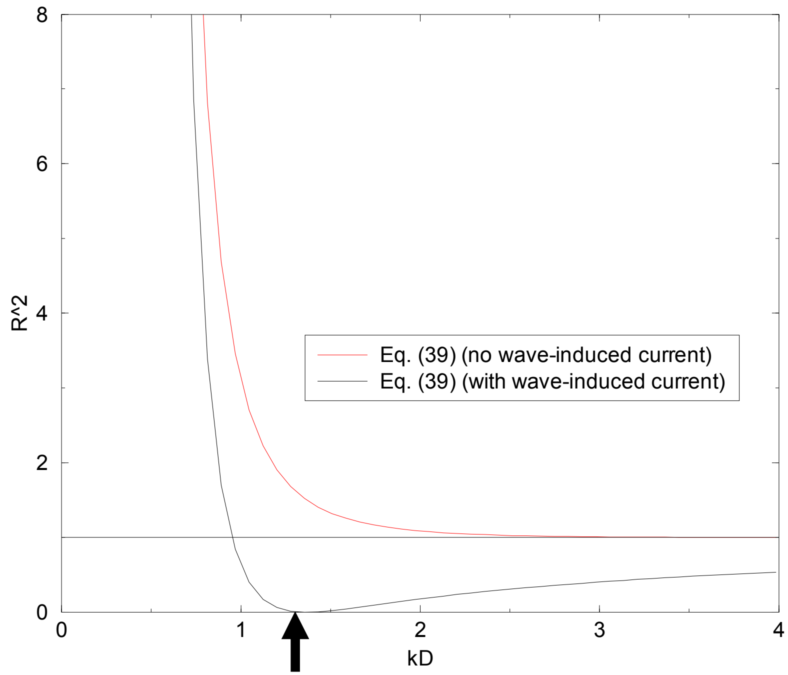
$$T = \tanh(kh), \quad v_g = 0.5c(1 + \frac{2kh}{\sinh(2kh)}), \quad c = \frac{\omega}{k}, \quad c_s = \sqrt{gh}, \quad \omega^2 = gkT$$

Shallow water Snl

$$f(k,h) = \frac{R^2}{T^8 \frac{\partial v_g}{\partial k}} \qquad R = \frac{9T^4 - 10T^2 + 9}{8T^3} - \frac{1}{kh}\left\{\frac{(2v_g - c/2)^2}{c_s^{\,2} - v_g^{\,2}} + 1\right\}$$

Depth dependent correction factor Kinetic Equation

R^2



kh=1.363

$$T = \tanh(kh),$$

$$v_g = 0.5c\left(1 + \frac{2kh}{\sinh(2kh)}\right),$$

$$c = \frac{\omega}{k}, \quad c_s = \sqrt{gh},$$

$$\omega^2 = gkT$$

Sdiss

S_{diss} following Bidlot, Janssen and Abdalla (BJA) 2007, back to Komen et al. 1994 form:

$$S_{diss} = -C_{ds}\,\omega_{mean}\left(k_{mean}^2 m_0\right)^2 \left[(1-\delta)\frac{k}{k_{mean}} + \delta\left(\frac{k}{k_{mean}}\right)^2\right] F(f,\theta)$$

$$C_{ds} = 1.33 \qquad \delta = 0.5$$

$$\omega_{mean} = \frac{\int \omega F\, df d\theta}{m_0} \qquad \sqrt{k_{mean}} = \frac{\int \sqrt{k}\, F\, df d\theta}{m_0} \qquad m_0 = \int F\, df d\theta$$

Sbottom:

S_{bottom} (see Komen et al. 1994):

$$S_{bottom} = -2\,C_{bot}\,\frac{k}{\sinh(2\,k\,h)}\,F(f,\theta)$$

$$C_{bot} = \frac{0.038}{g}$$

Bottom induced wave breaking:

Dissipation due to bottom induced wave breaking was added to the source terms. Following Battjes, Janssen and Beji:

$$S_{dis} = -C_{BJ} \alpha Q_b \langle f \rangle F(f, \theta)$$

$$H_{\max} = \gamma h \qquad \alpha = 2 \frac{H_{\max}^2}{H_s^2}$$

Q_b : fraction of breaking waves $\qquad Q_b = \exp\{-\alpha(1 - Q_b)\}$

$$breaker\ parameter\ \gamma = 0.6$$

$$C_{BJ} = 1 \qquad \langle f \rangle = mean\ frequency$$

Coupling to the waves: Warm skin layer model

- **Following Takaya et al. (JGR 2010), a skin layer model is used to represent the Daily SST Amplitude.**

Stokes drift

- **In this scheme, the temperature profile is controlled by the turbulent diffusivity $K_w(z)$:**

$$K_w(z) = \frac{-\kappa z u_w^* f(L_a)}{\phi_h(z/L)}$$

Langmuir number $$L_a = \sqrt{\frac{u_w^*}{U_{Stokes}}}$$

- **Following Grant and Belcher (JPO 2009),**

for stable condition only and for $f(L_a) > 1$:

$$f(L_a) = \frac{1}{L_a^{2/3}}$$

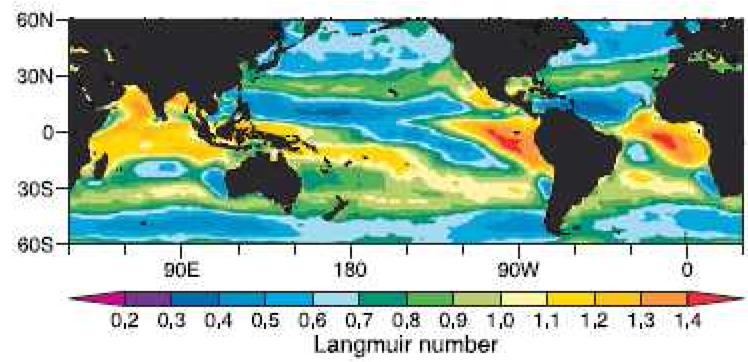
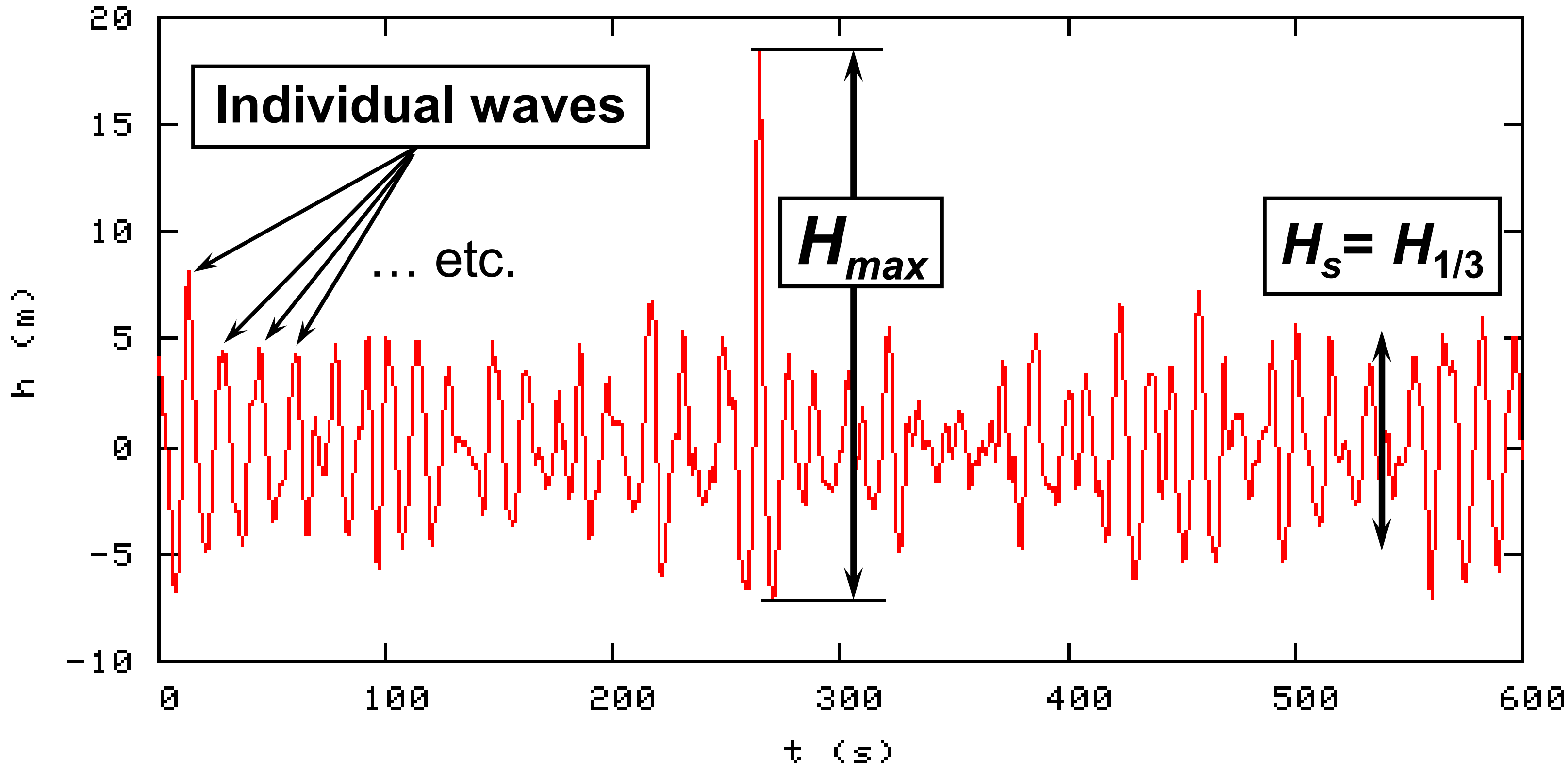


Figure 4. The average of the Langmuir number computed with forecasts starting from 1 January 1990–2007.

u_w^* : friction velocity in water $\phi_h(z/L)$: similarity function, L: Obukhov length

We are not always dealing with nice ‘predictable’ waves:

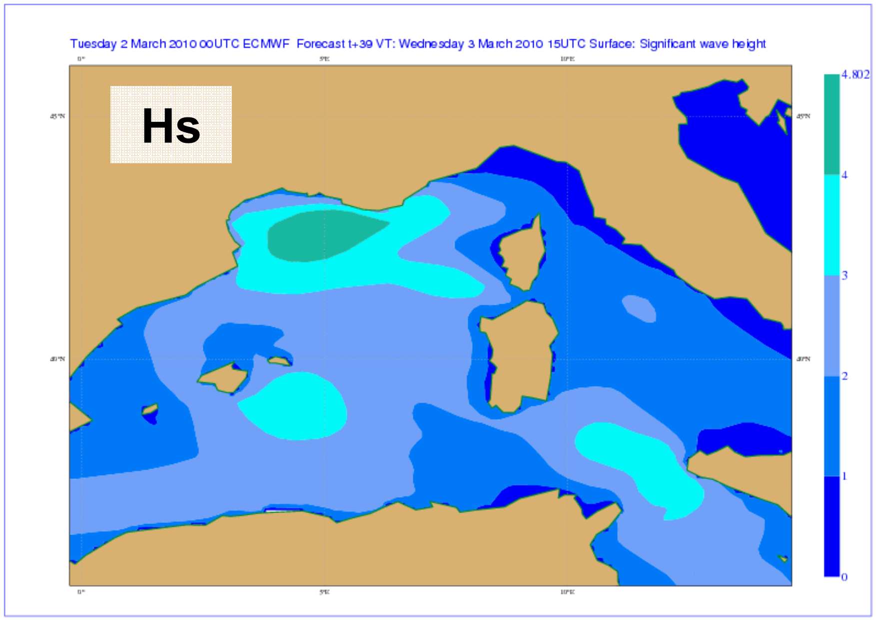
Individual Waves, Significant Wave Height, H_s, Maximum Individual Wave Height, H_{max}, and Freak Wave

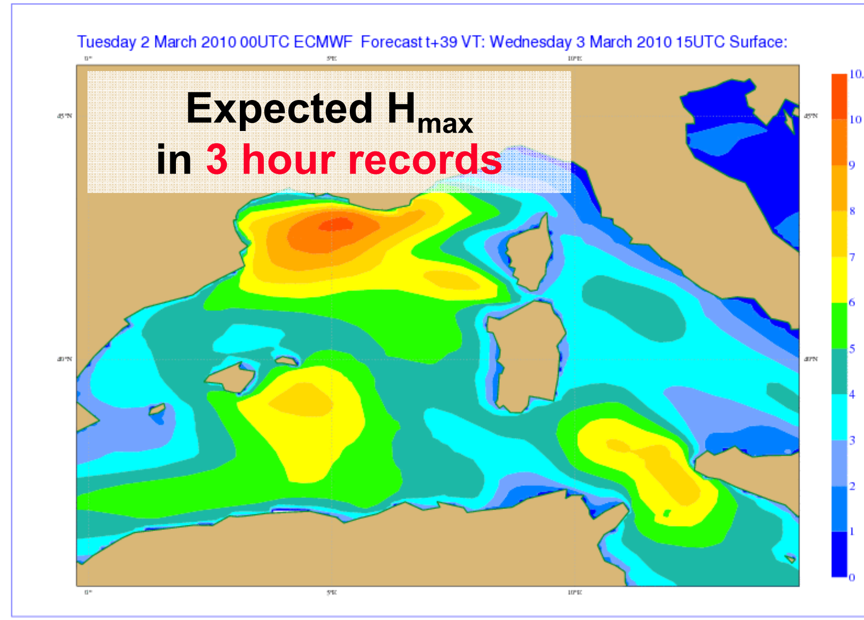


If $H_{max} > 2.2\ H_s$ → freak wave event

Wave Model Products: Extreme Waves

We have recently introduced a new parameter to estimate the height of the highest individual wave (H_{max}) one can expect. Its value can be derived from the 2d wave spectrum:





See ECMWF Tech Memo 288 for derivation and discussion
http://www.ecmwf.int/publications/library/do/references/list/14
or

http://www.ecmwf.int/research/ifsdocs/CY38r1/index.html

3
Validation

2.2. Wave Forecasting

- <u>Sensitivity to wind-field errors</u>.

  For *fully developed* wind sea:

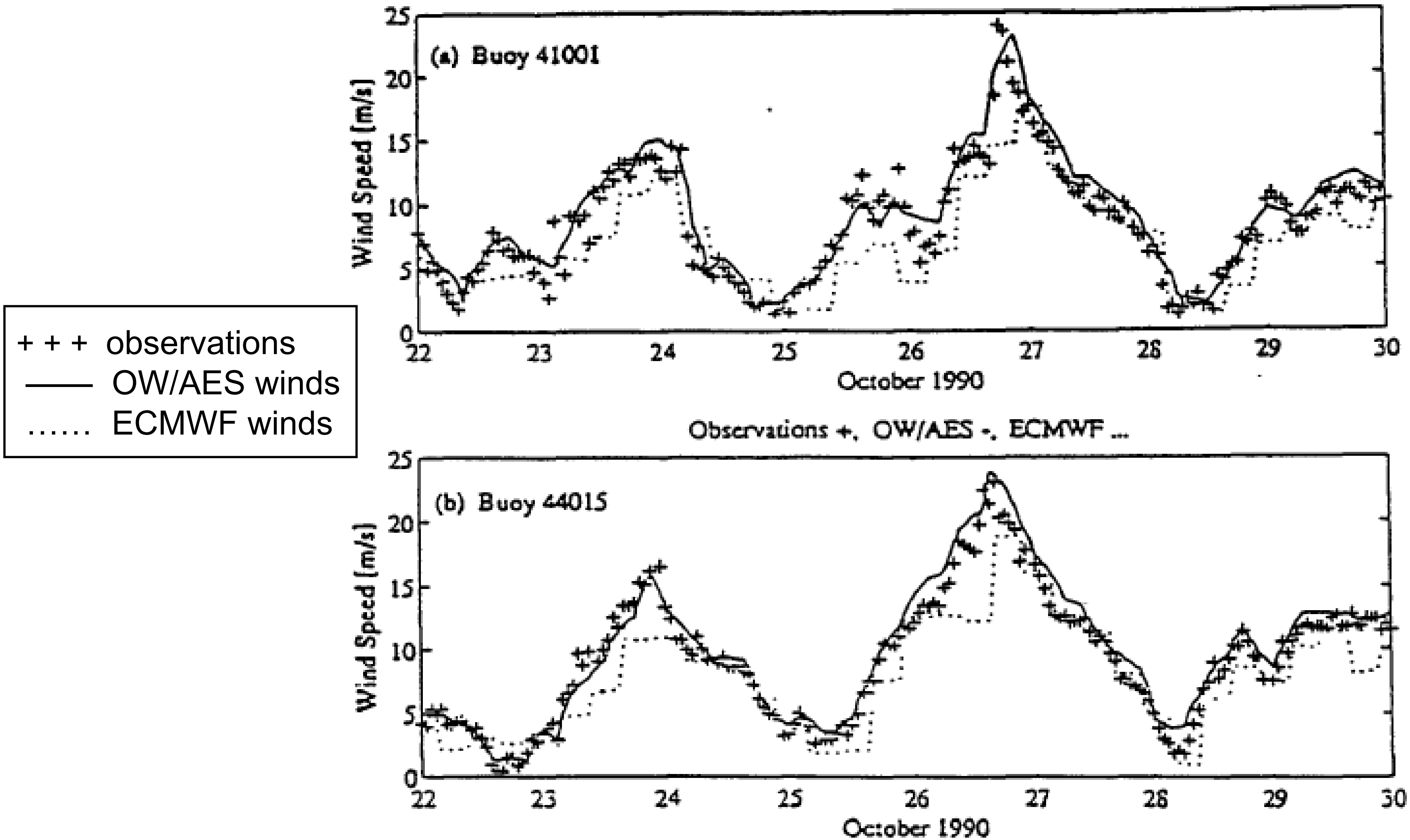
  $$H_s = 0.22\, u_{10}^2 / g$$

  10% error in u_{10} → 20% error in H_s

  ➔ from observed H_s

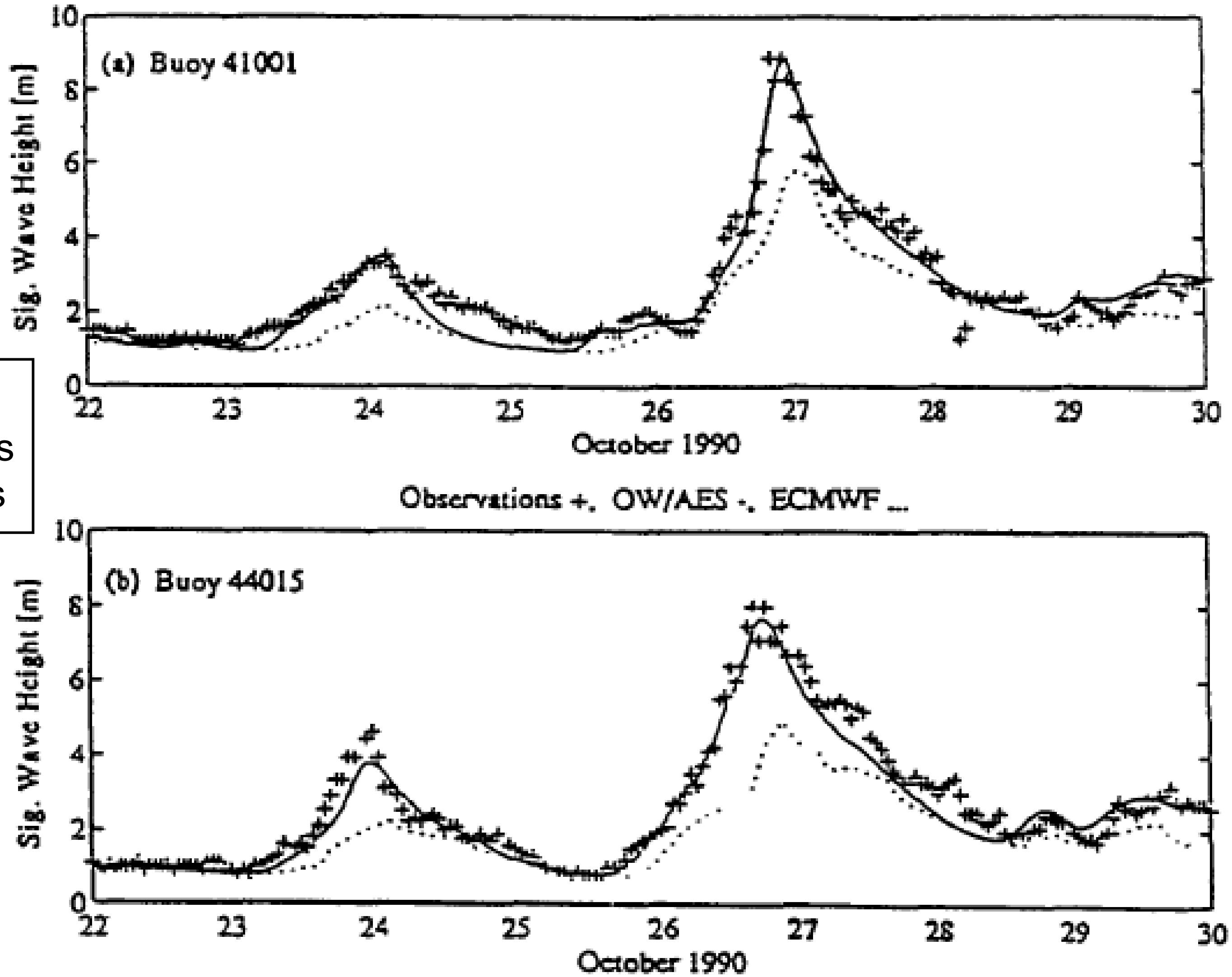
  → Atmospheric state needs reliable wave model.

SWADE case ➔ WAM model is a reliable tool.



Verification of model wind speeds with observations

(OW/AES: Ocean Weather/Atmospheric Environment Service)



+ + + observations
—— OW/AES winds
...... ECMWF winds

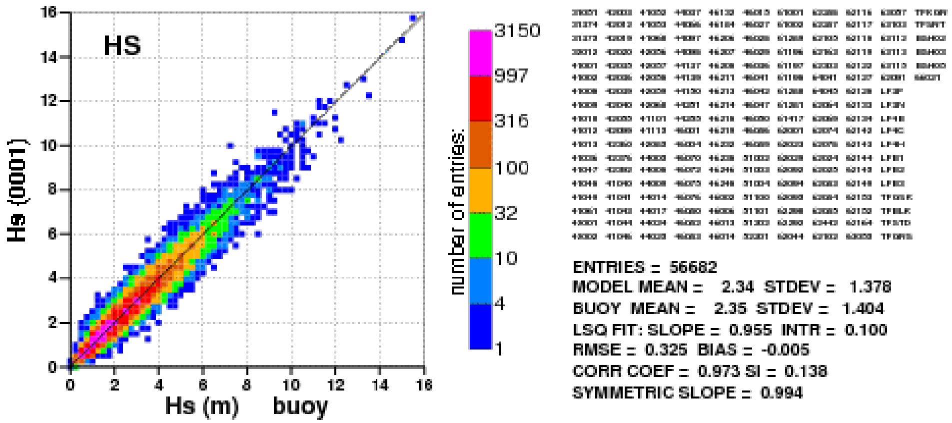
Verification of WAM wave heights with observations

(OW/AES: Ocean Weather/Atmospheric Environment Service)

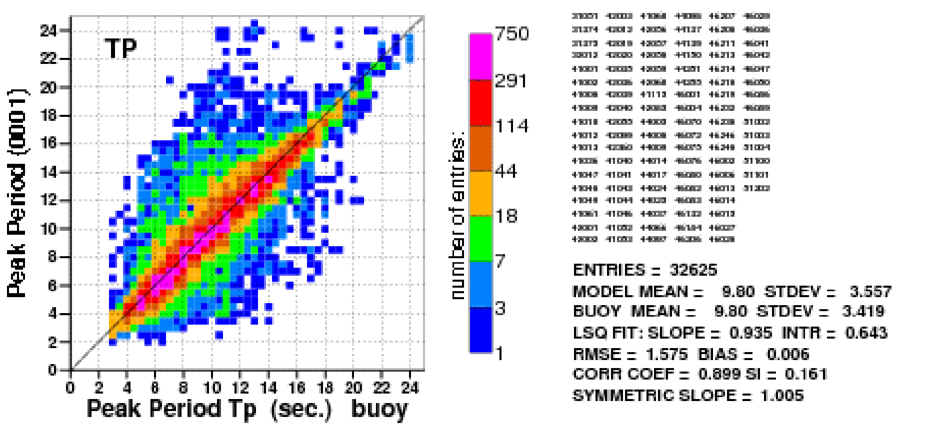
Continued general improvement of model forecasts

For example: ECMWF forecast wave height against buoy measurements:

http://www.ecmwf.int/products/forecasts/wavecharts/index.html#verifications

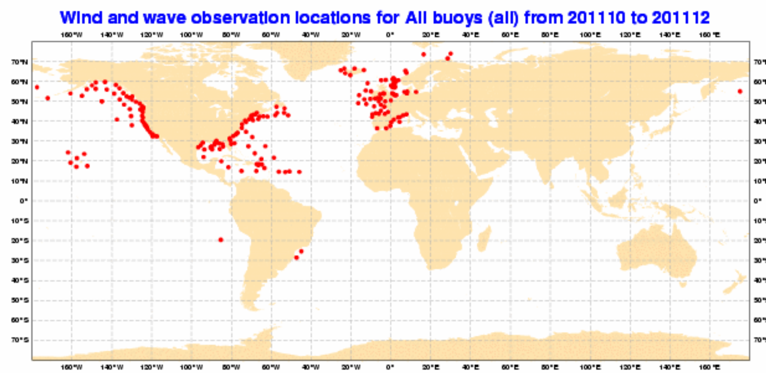


Comparison of analysed ECMWF wave heights with averaged buoy data.



Comparison of analysed ECMWF peak periods with averaged buoy data.

In-situ wave observations

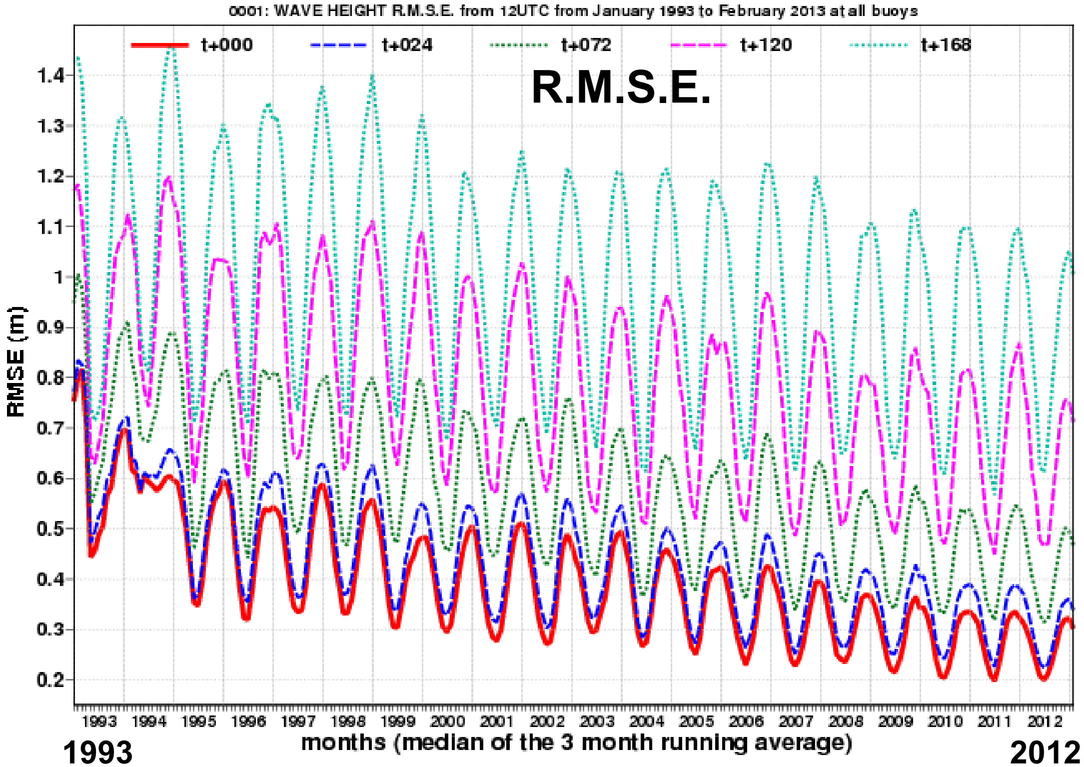


Note: GTS wave data now also available for Korea, Australia, Italy and Greece.

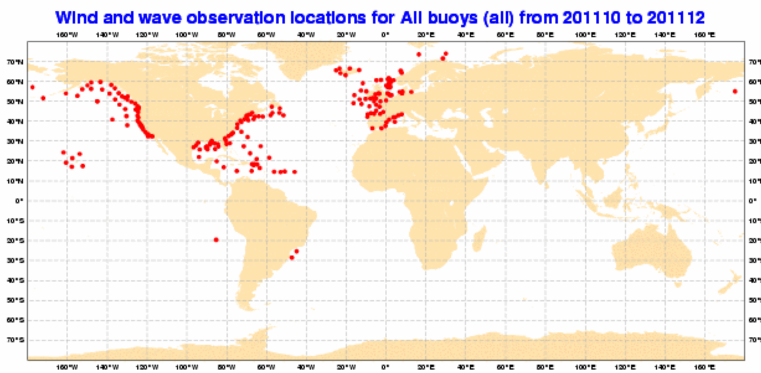
Continued general improvement of model forecasts

For example: ECMWF forecast wave height against buoy measurements:

http://www.ecmwf.int/products/forecasts/wavecharts/index.html#verifications



In-situ wave observations



Note: GTS wave data now also available for Korea, Australia, Italy and Greece.

7 day FC

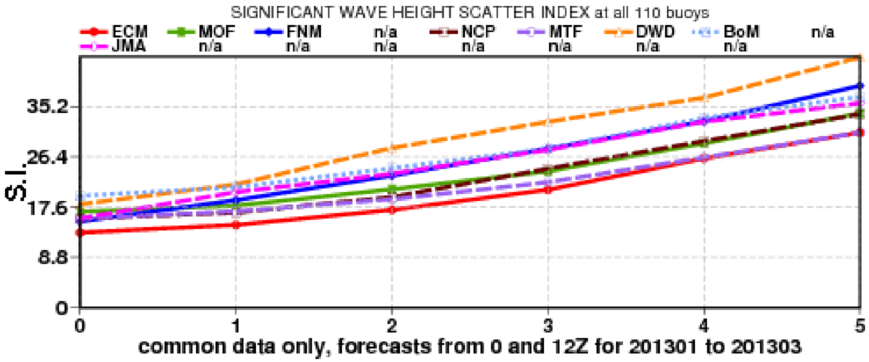
5 day FC

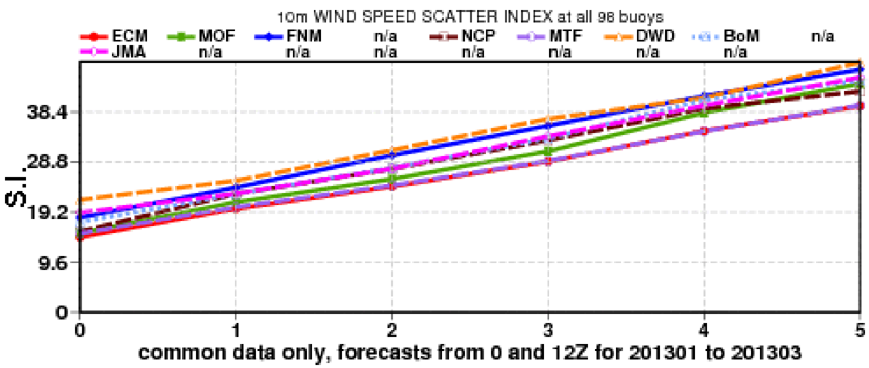
3 day FC

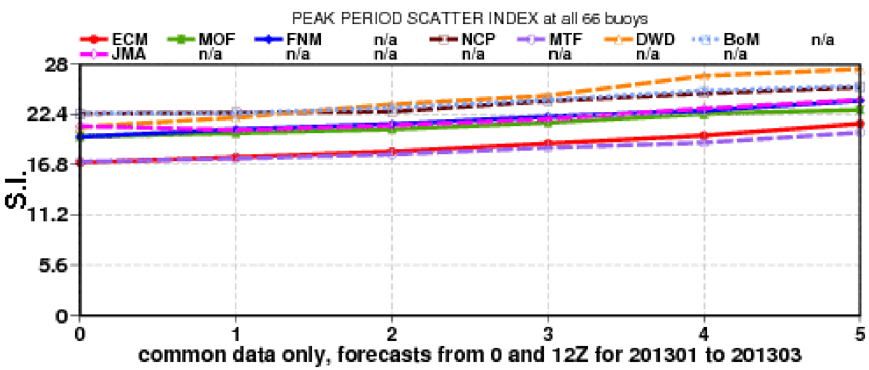
1 day FC

analysis

Continued general improvement of model forecasts







See also the Wave Forecast Verification Project maintained on behalf of the Expert Team on Waves and Coastal Hazards of the WMO-IOC Joint Technical Commission for Oceanography and Marine Meteorology (JCOMM)

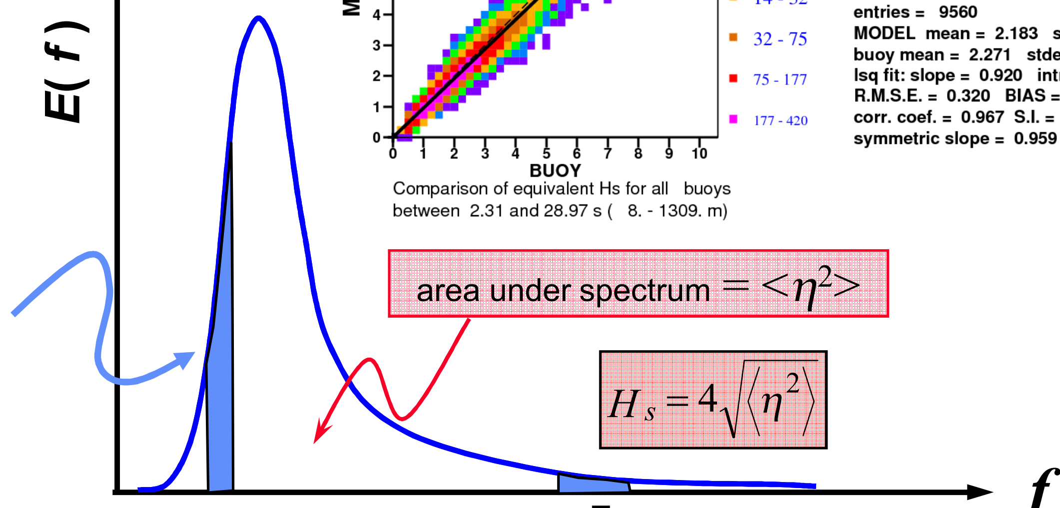
http://www.jcomm.info/index.php?option=com_content&task=view&id=131&Itemid=37

Verification against buoy frequency spectra

EWH,
Equivalent Wave Height for a given frequency bin:

$$EHW = 4\sqrt{A}$$

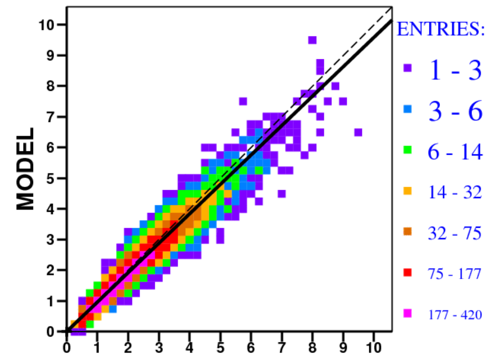
where A is the area under curve for a given frequency bin



area under spectrum $= \langle \eta^2 \rangle$

$$H_s = 4\sqrt{\langle \eta^2 \rangle}$$

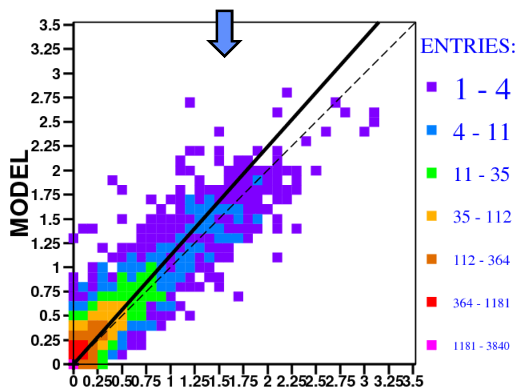
2011030100_2011033118 0001 od wave



Comparison of equivalent Hs for all buoys between 2.31 and 28.97 s (8. - 1309. m)

Hs, significant wave height.

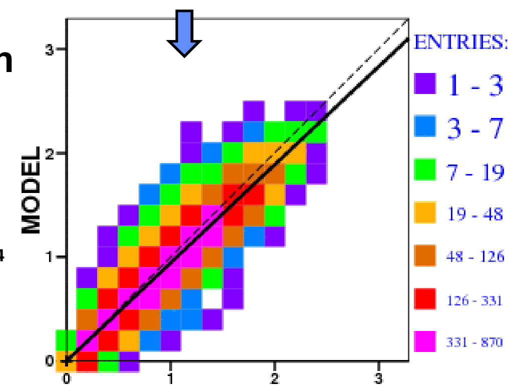
entries = 9560
MODEL mean = 2.183 stdev = 1.140
buoy mean = 2.271 stdev = 1.198
lsq fit: slope = 0.920 intr = 0.094
R.M.S.E. = 0.320 BIAS = -0.088
corr. coef. = 0.967 S.I. = 0.135
symmetric slope = 0.959



Comparison of equivalent Hs for all buoys between 17.13 and 18.84 s (458. - 554. m)

EWH for waves with periods between 17s and 19s.

entries = 9560
MODEL mean = 0.216 stdev = 0.334
buoy mean = 0.165 stdev = 0.314
lsq fit: slope = 0.969 intr = 0.056
R.M.S.E. = 0.147 BIAS = 0.051
corr. coef. = 0.911 S.I. = 0.836
symmetric slope = 1.120

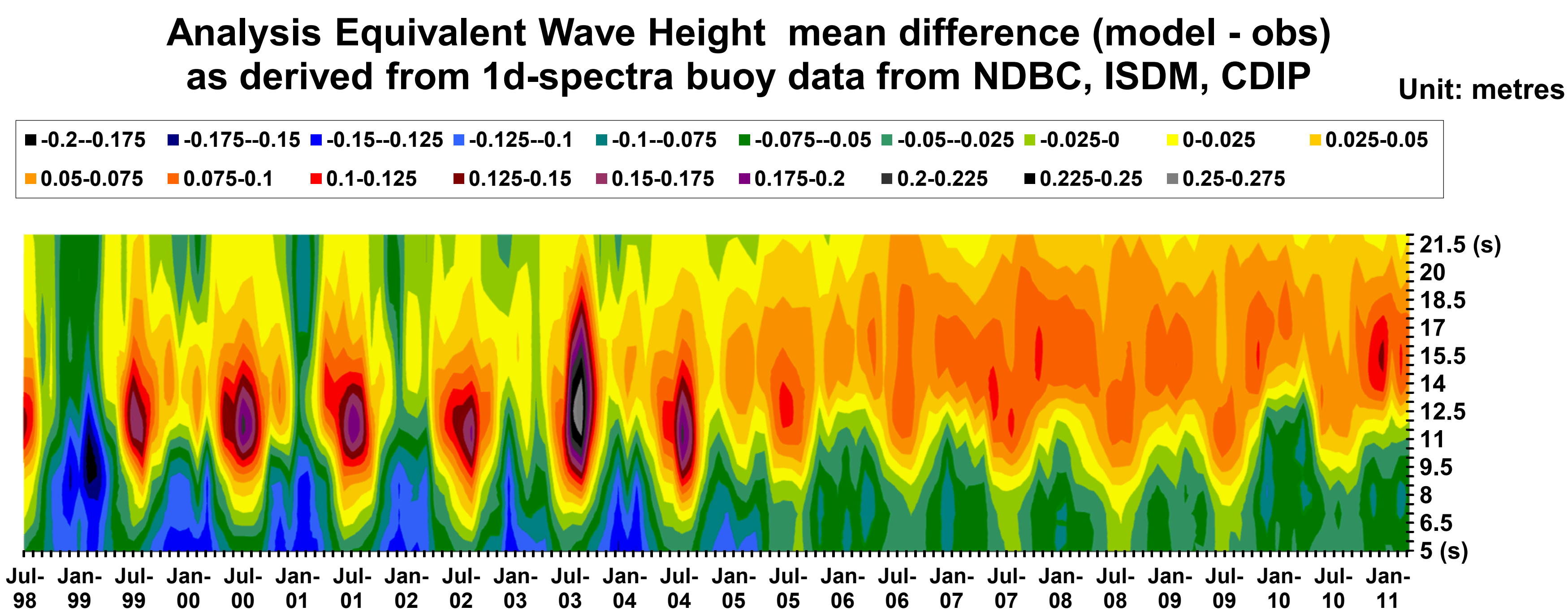


Comparison of equivalent Hs for all buoys between 5.46 and 6.00 s (46. - 56. m)

EWH for waves with periods between 5.5 s and 6.0 s.

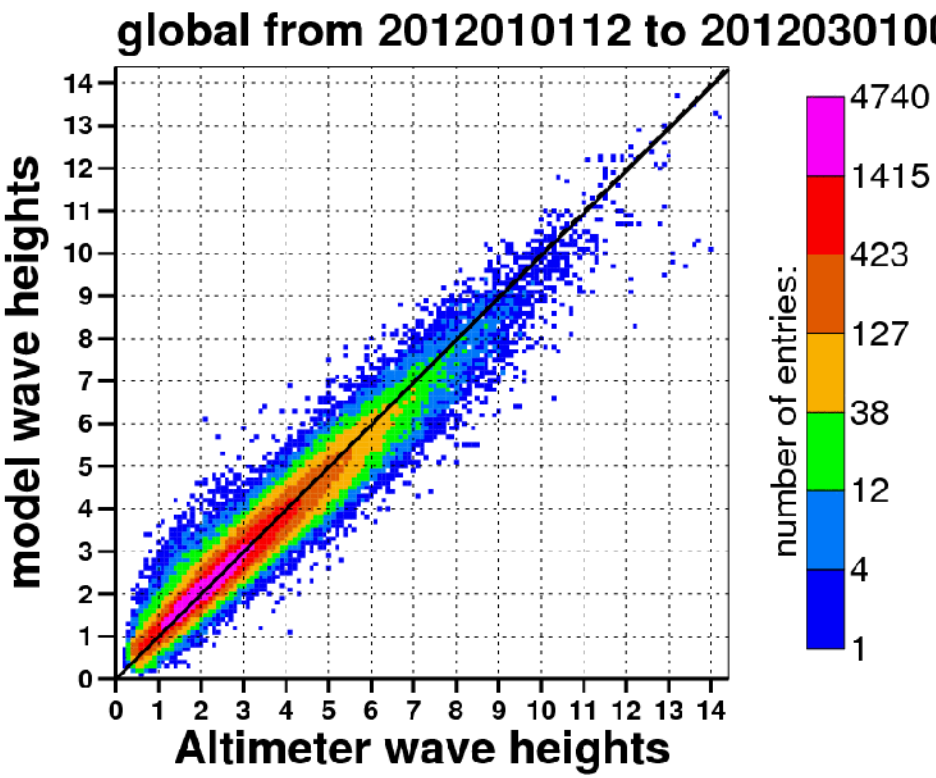
entries = 9560
MODEL mean = 0.894 stdev = 0.442
buoy mean = 0.953 stdev = 0.456
lsq fit: slope = 0.905 intr = 0.031
R.M.S.E. = 0.176 BIAS = -0.060
corr. coef. = 0.933 S.I. = 0.173
symmetric slope = 0.944

wave spectra



NDBC: National Data Buoy Center (US)
ISDM: Integrated Science Data Management (Canada)
CDIP: Coastal Data Information Program (US)

Space borne altimeters:

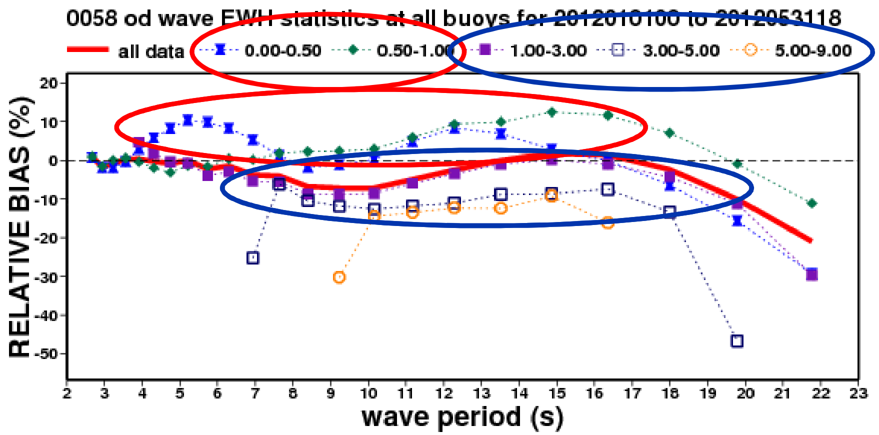


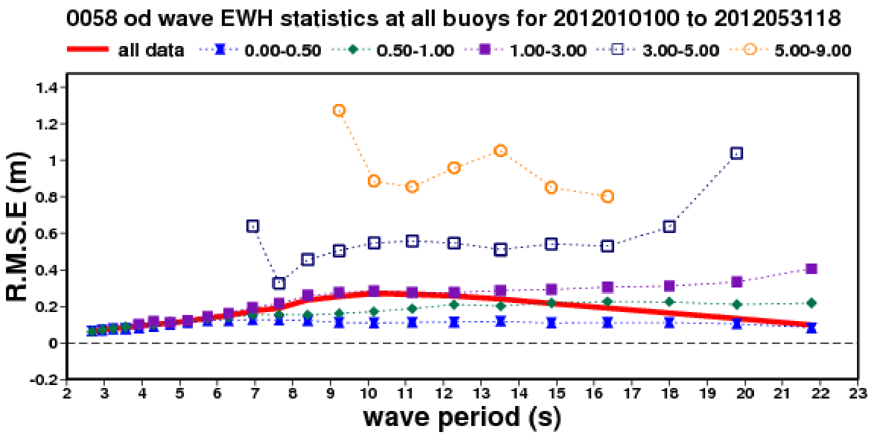
entries = 434376
alt. mean = 2.591 stdev = 1.266
model mean = 2.609 stdev = 1.190
lsq fit: slope = 0.917 intr = 0.234
R.M.S.E. = 0.285 BIAS = 0.018
corr. coef. = 0.975 S.I. = 0.110
symmetric slope = 0.994
kurtosis of difference = 1.984

ENVISAT and Jason 2

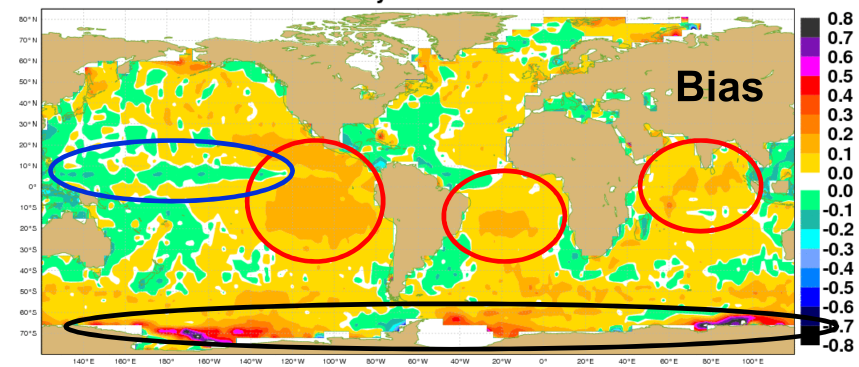
Can we still improve ?

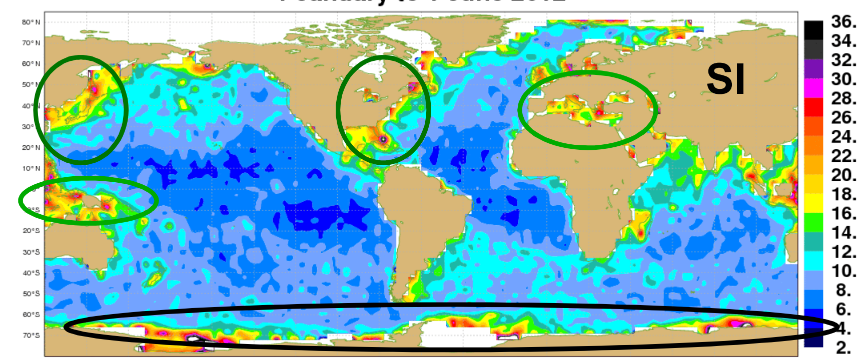
Comparison to buoy spectra, stratified by energy level:





Comparison to altimeter wave heights :





Latest CY38R1 compared to observations, January to May 2012

4

Future developments

Ongoing developments:

- Integration of the atmosphere-wave-ocean models.
- Unstructured grid option.
- Sea ice – wave interaction.
- Useful products to users.

Enhanced coupling

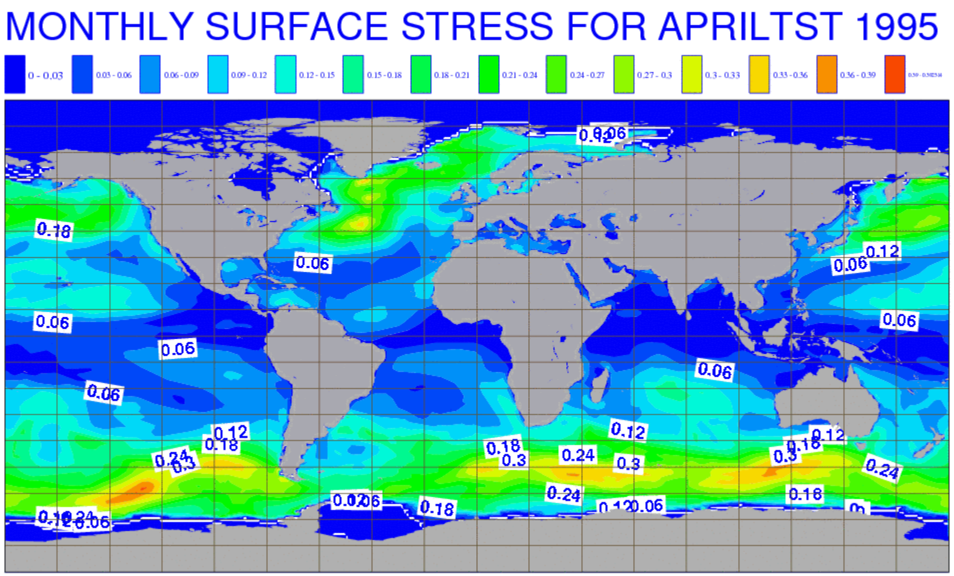
- The momentum and energy fluxes between the oceans and the atmosphere is actually controlled by the waves.
- For instance, we already have a wave dependent surface stress (see coupling between atmosphere and waves):

$$\tau = \rho_a u_*^2$$

ρ_a : air density

u_* : friction velocity

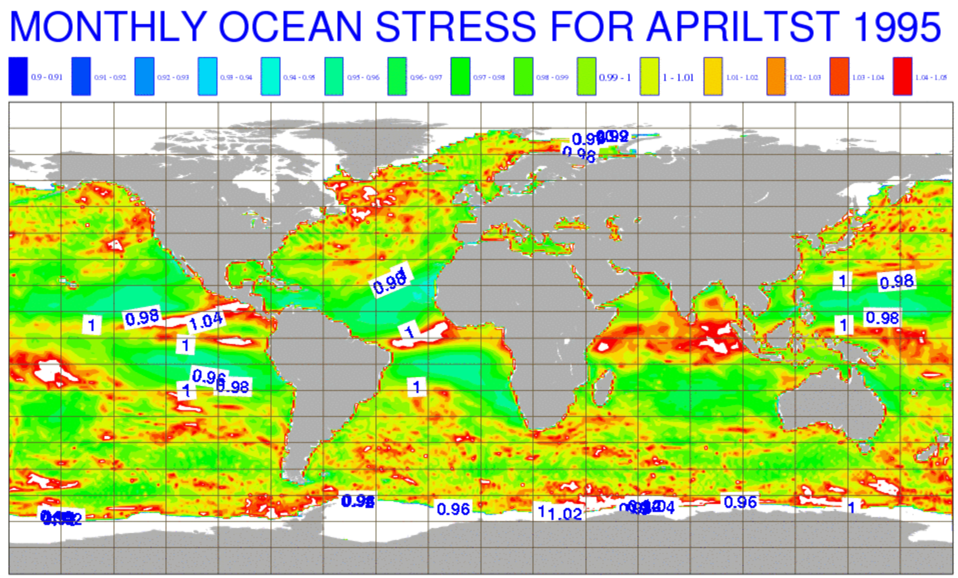
$$u_*^2 = C_D U_{n10}^2$$



Monthly mean as derived from ERA-Interim data

Enhanced coupling

- But actually a small portion of the stress is retained by the wave field to be released later.
- Hence, one can compute the stress that is actually acting on the oceans.

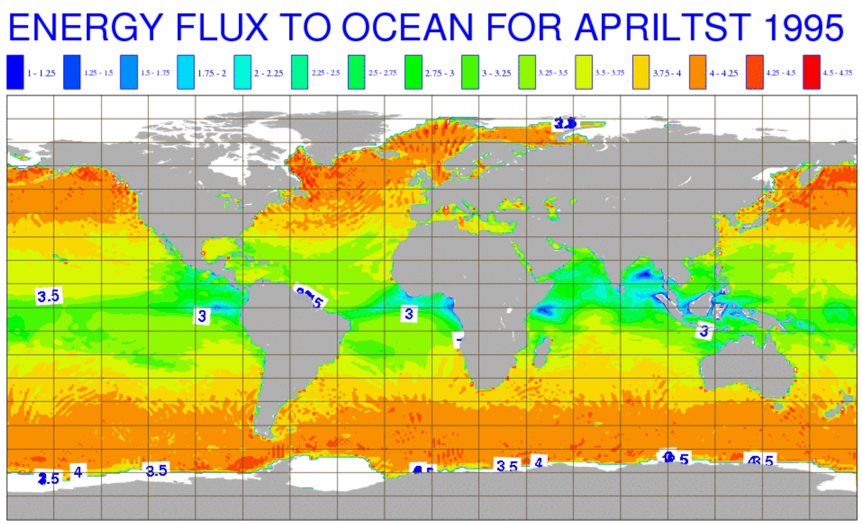


Monthly mean of the <u>normalised</u> stress into the ocean as derived from ERA-Interim data

It is normalised by $\rho_a u_*^2$

Enhanced coupling

- Similar consideration can be made for the energy fluxes passing first into the waves.
- It is then dissipated by the waves and transferred into the upper oceans where it will contribute to the mixing of the top of the oceans.
- Both quantities are connected to the wave model source terms.

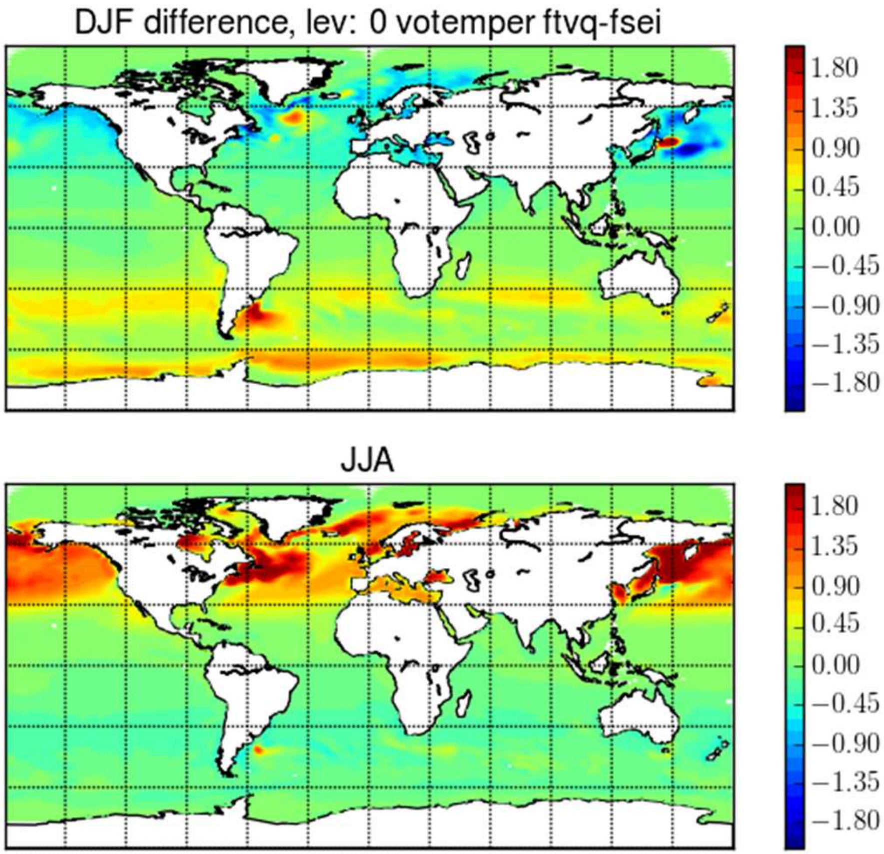


Monthly mean of the <u>normalised</u> energy flux into the ocean as derived from ERA-Interim data.

It is normalised by $\rho_a u_*^3$

Enhanced coupling between waves and oceans

- Global impact: plot showing the average difference of the sea surface temperature over the period 1989-2008 between two experiments with the 42-level configuration of NEMO ORCA1 (10 m upper level). The differences are:
- Modified energy flux into the ocean (PHIOC) vs alpha=100 (corresponding to fully developed wind sea)
- Modified water-side stress (TAUOC)
- Drag coefficient from WAM (only relevant for stand-alone runs)
- Coriolis-Stokes drift



ECMWF

Surface currents also influence the winds and the waves:

T1279 (16km) assimilation/forecasts impact study

- ✓Use daily Mercator currents, averaged in 8 day moving average window.
- ✓22 December 2009 to 28 February 2010.

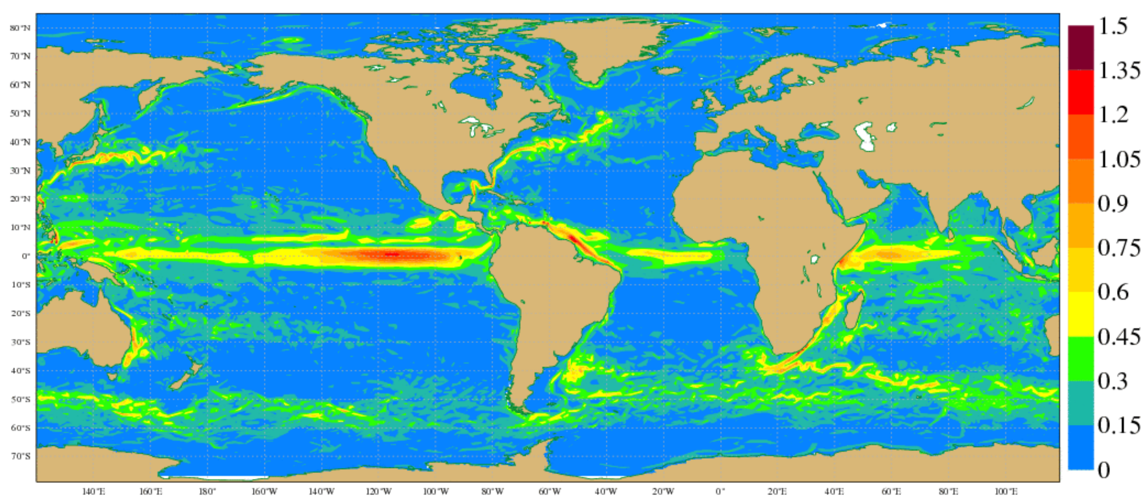
MERCATOR surface currents from the global PSY3V3 system:

- ✓NEMO ocean model.
- ✓Horizontal resolution of the products: 1/4°.
- ✓Data assimilation system.

Atmospheric forcing is from ECMWF.

(we have just started receiving PSY4V1 products 1/4° and 1/12° native)

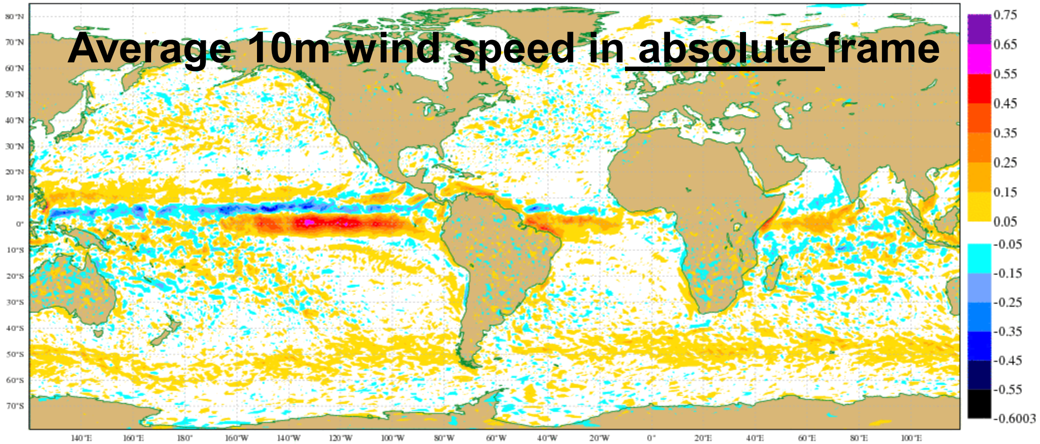
Mean analysis surface current speed: rd feb8 dcda
from 20091222 to 20100228



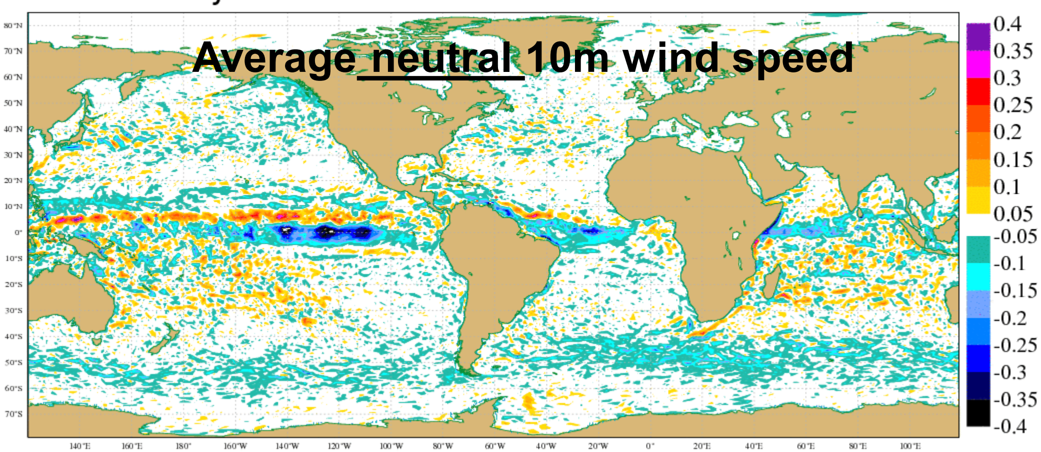
ECMWF

Average effect of surface currents on surface winds

Mean analysis difference in 10m wind speed: rd feb8 dcda - rd febp dcda
from 20091221 to 20100228

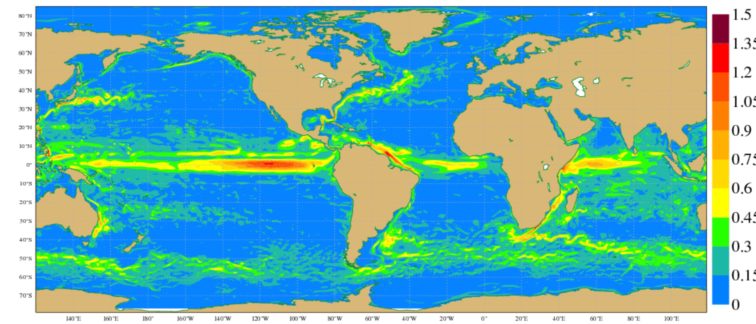


Mean neutral wind speed difference (feb8 dcwv - febp dcwv)
analysis from 20091221 0Z to 20100228 18Z



Currents – no currents

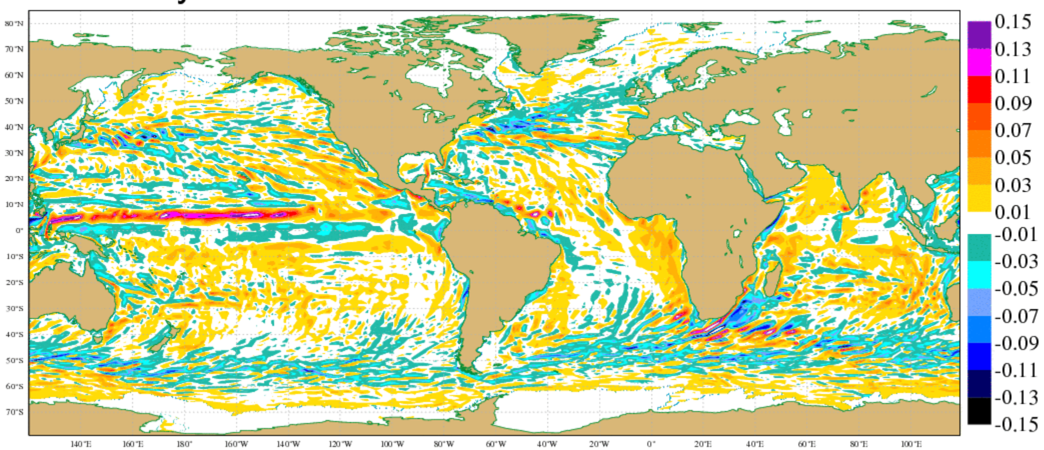
Mean analysis surface current speed: rd feb8 dcda
from 20091222 to 20100228



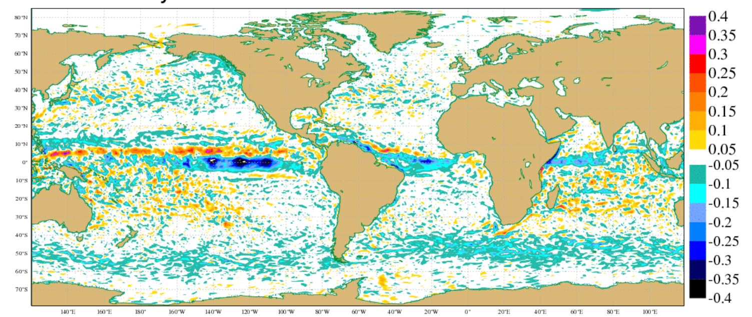
- **Absolute winds receive about 50% from ocean currents**

Average effect on ocean waves

Mean wave height difference (feb8 dcwv - febp dcwv)
analysis from 20091221 0Z to 20100228 18Z



Mean neutral wind speed difference (feb8 dcwv - febp dcwv)
analysis from 20091221 0Z to 20100228 18Z

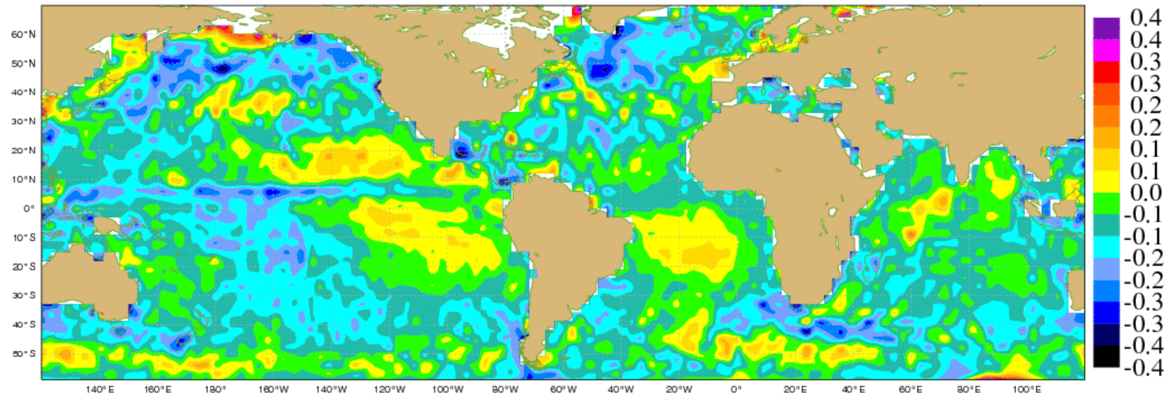


Currents – no currents

Averaged effect on waves: comparison to altimeter wave heights

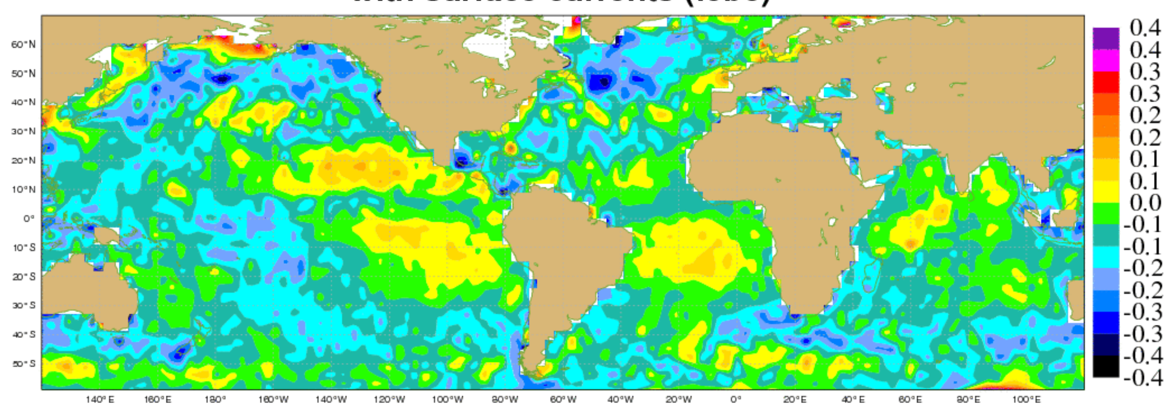
Wave height bias with respect to ENVISAT and Jason's (model - alt)
IFS/WAM T1279 dcda first guess, 2009122-20100228
reference, NO surface currents (febp)

No currents

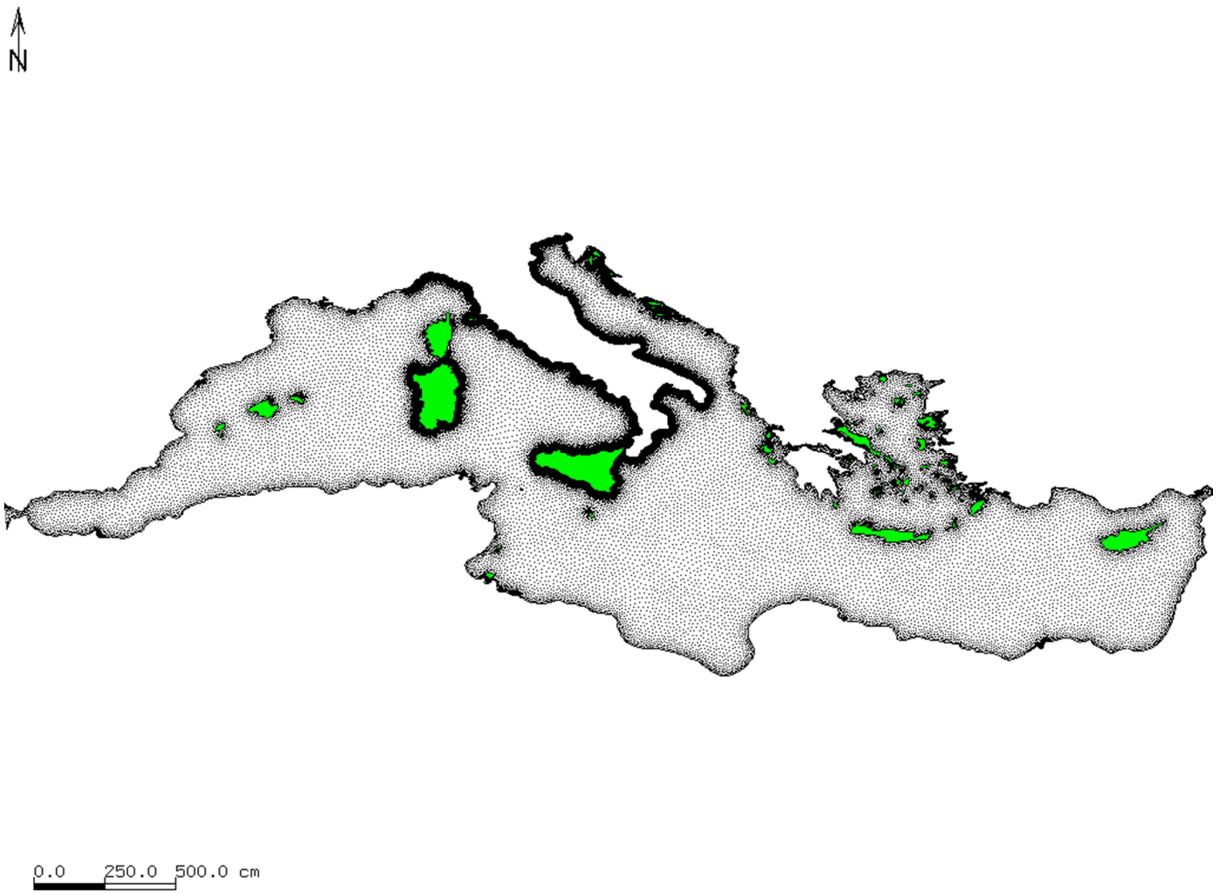


Wave height bias with respect to ENVISAT and Jason's (model - alt)
IFS/WAM T1279 dcda first guess 20091222-20100228
with surface currents (feb8)

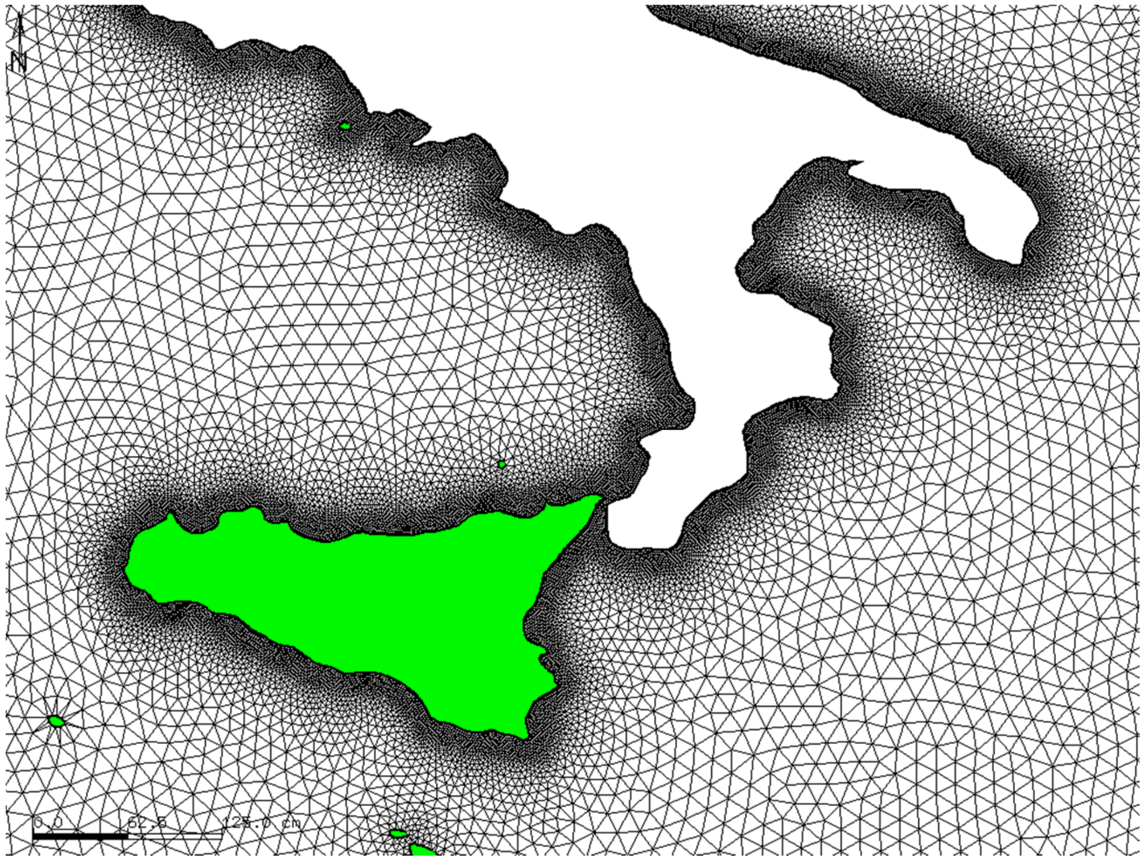
With currents



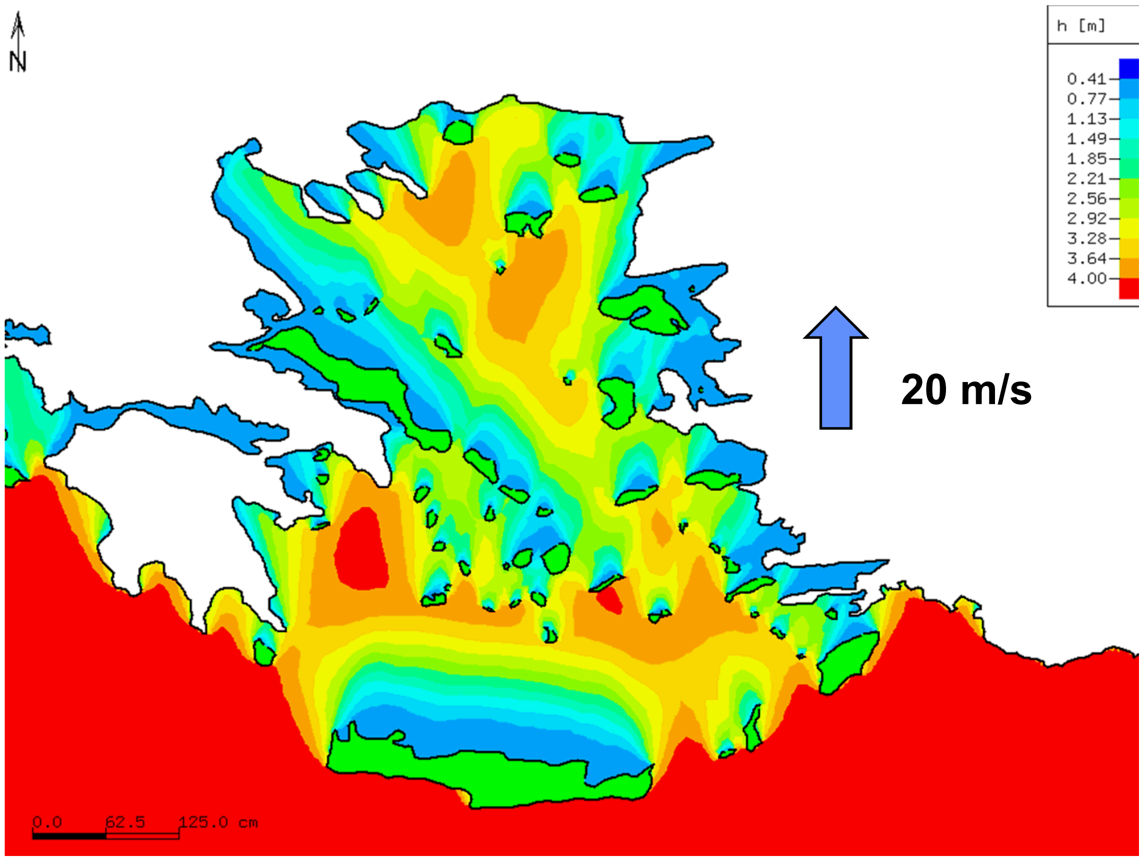
Future developments: unstructured grid



Future developments: unstructured grid

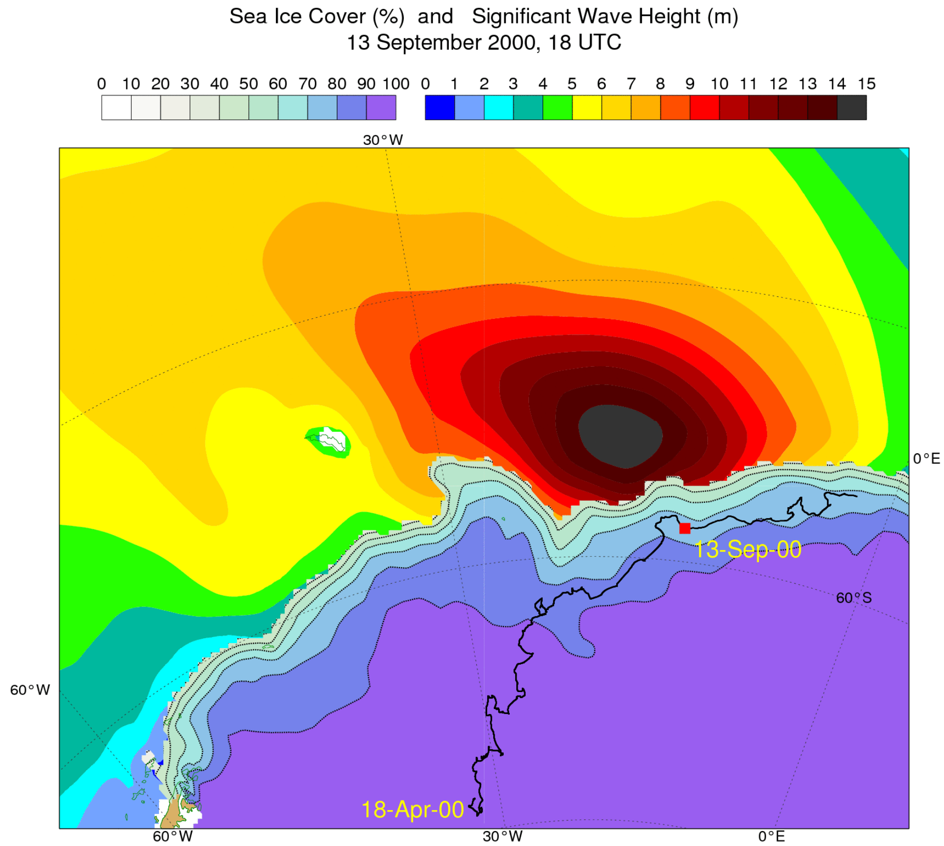


Future developments: unstructured grid



Situation leading to the break-up:

13th 18UTC



ECMWF

Sea ice attenuation:

Dumont et al. (JGR, 2011) applied the Kohout and Meylan (JGR 2008) wave scattering model to an ice model with parameterised waves spectra:

$$F(\mathbf{x}, f, \theta, t+\Delta t) = F(\mathbf{x}, f, \theta, t)\exp(-\alpha\, c_g \Delta t)$$

c_g : group speed $\quad$ Δt : model time step

$$\alpha = ci\,\frac{a}{\overline{D}}$$

a : non dimensional attenuation, function (ci, ice thickness h)

ci : sea ice fraction

$\overline{D}$: mean size of ice floes

assuming that ice floes will break up into ξ pieces : $\overline{D} = \dfrac{\sum_{m=0}^{M} (\xi^2 fr)^m \xi^{-m} D_{max}}{\sum_{m=0}^{M} (\xi^2 fr)^m}$

$M = \log_\xi \left(\dfrac{D_{max}}{D_{min}}\right) \qquad D_{min} = 20\text{m} \qquad D_{max} = 200\text{m}$

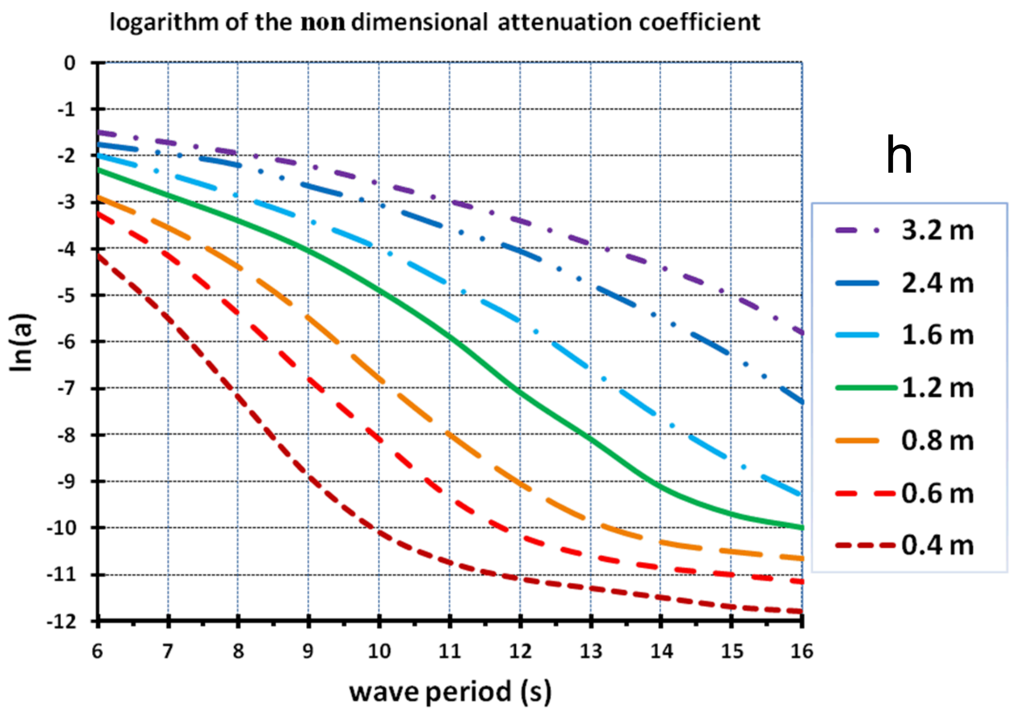
$\xi = 2 \qquad fr = 0.9$: ice floes fragility $\quad \longrightarrow \quad \overline{D} = 36\text{m}$

The Kohout and Meylan is based on elastic bedding of floes and does not contain other physics.

Sea ice attenuation:

$$\alpha = c_i \frac{a}{\overline{D}}$$

The non-dimensional attenuation coefficient “a” was found to depend only on wave period and sea ice thickness “h”



From Kohout and Meylan (JGR 2008), Figure 6.

Sea ice attenuation:

Kohout et al. (Annals Glaciology 2011) added the attenuation due to the **bottom roughness of the ice floes** to account for wave energy loss in compact MIZ:

For the portion of the grid box covered by sea ice

$$F(\mathbf{x}, f, \theta, t+\Delta t) = F(\mathbf{x}, f, \theta, t)\exp(-\alpha\, \mathrm{c_g}\, \Delta \mathrm{t})$$

$\mathrm{c_g}$: group speed $\quad\quad$ $\Delta\mathrm{t}$: model time step

$$\alpha = \mathrm{C_d}\, H\, k^2$$

H : wave height for each spectral component

k : wave number of each spectral component

C_d : ice - water drag coefficient

C_d : 1×10^{-3} to 35×10^{-3}

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H : wave height for each spectral component

k : wave number of each spectral component

C_d : ice - water drag coefficient

C_d : 1×10^{-3} to 35×10^{-3}

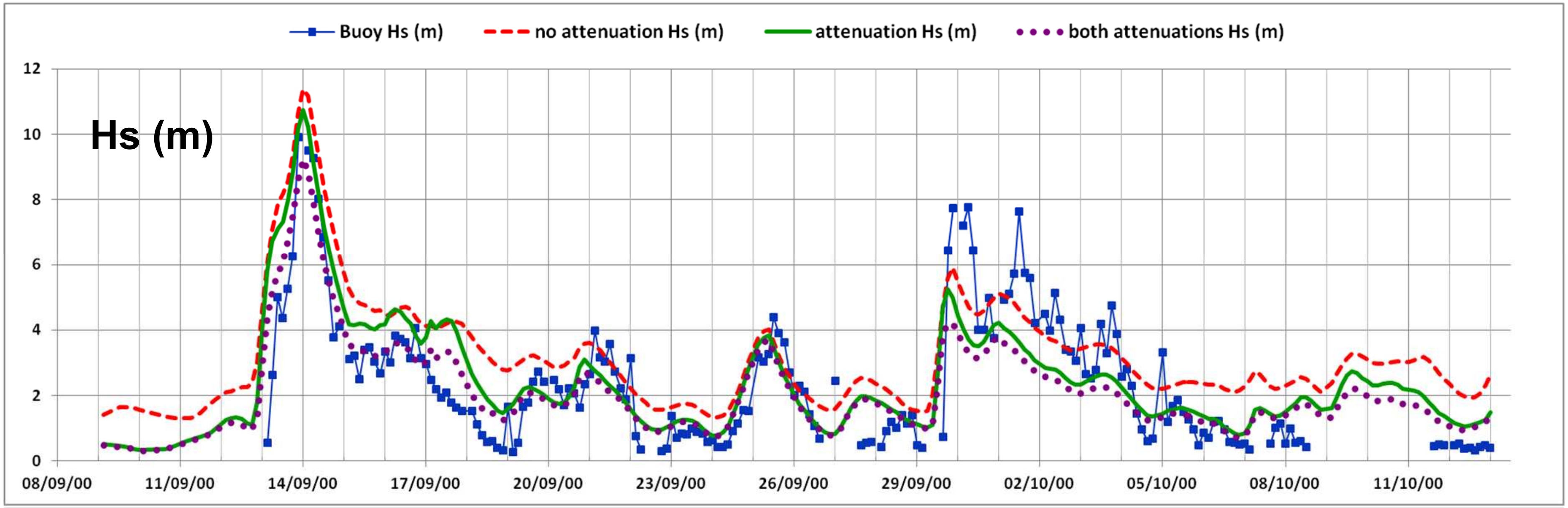
Sea ice attenuation in ECWAM

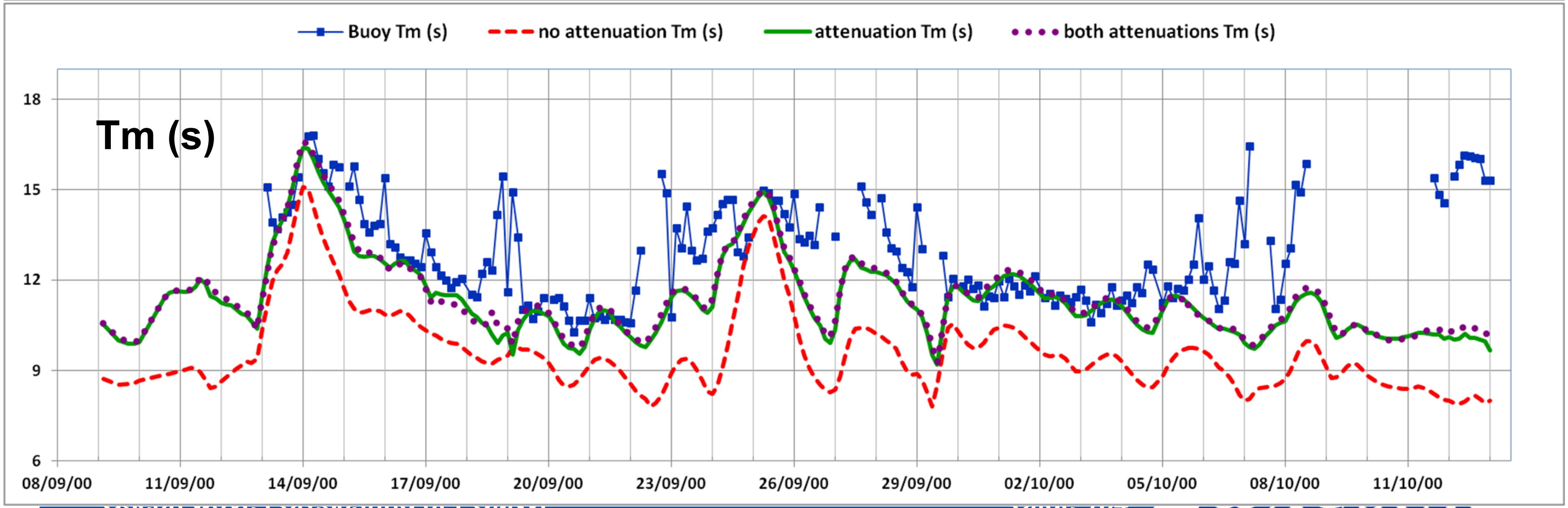
So, we have a model for wave spectra but we do not (yet) have an operational sea ice model sea ice thickness "h".

Could we instead parameterise "*h*" in term of *ci* ?

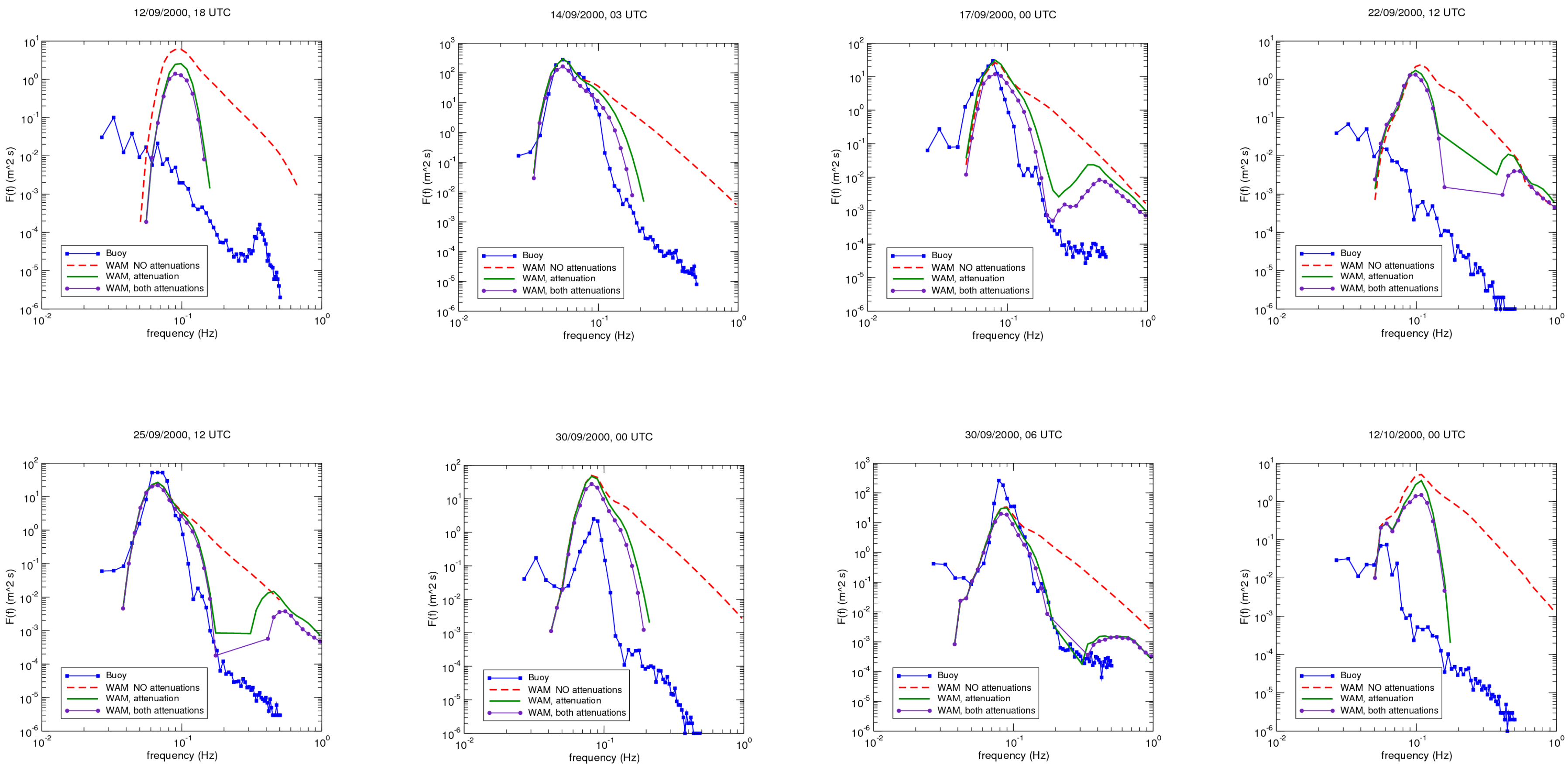
$$\mathrm{h} = 0.2 + 0.4 * ci$$

Sea ice attenuations in ECWAM



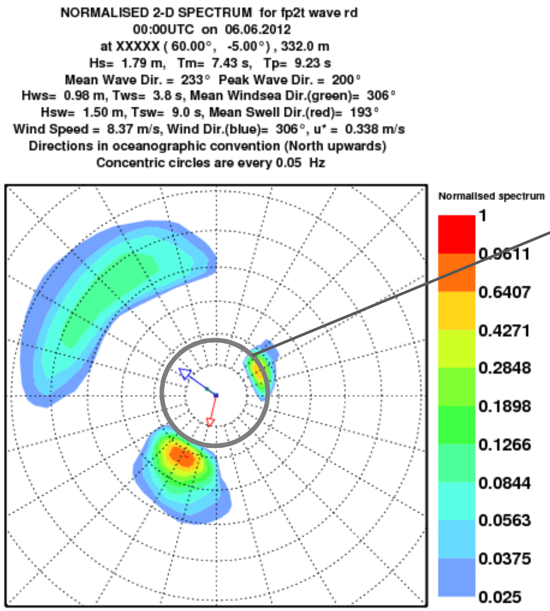


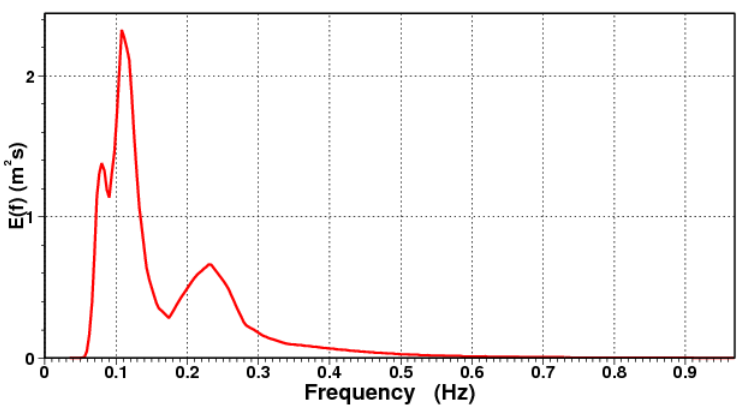
Sea ice damping modeling



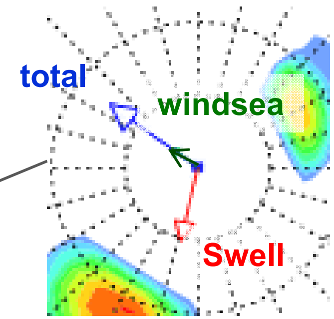
ECMWF

Future developments: spectral partitioning

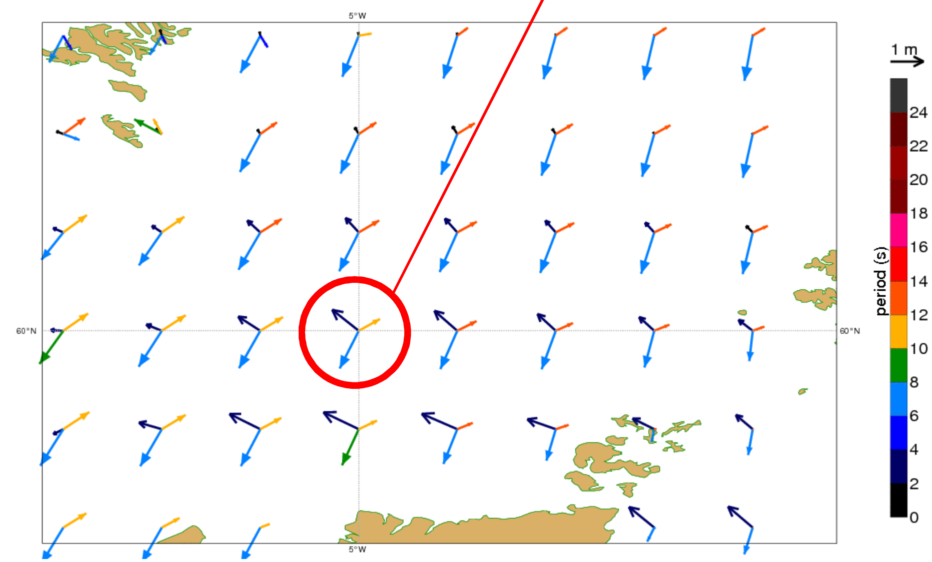




Operational:



New decomposition:



ECMWF

Questions/comments?

ECMWF