NEAR-BOTTOM KINEMATICS OF SHOALING AND BREAKING WAVES : EXPERIMENTAL INVESTIGATION AND NUMERICAL PREDICTION

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Abstract : In order to predict the near-bottom kinematics under shoaling and breaking waves, five possible numerical modeling strategies are reviewed and applied to two test-cases performed under regular waves in a wave flume. These tests correspond to spilling and plunging breakers over a plane and smooth slope of 1:20. By comparing the numerical predictions with LDA velocity measurements close to the bottom, it is found that the Modified Transfer Function Method gives good estimations for the horizontal velocity, but it must be emphasized that this approach requires the knowledge of the free-surface signal on input. If only offshore wave conditions are known, best results are obtained from Nonlinear Deterministic Models, in particular from the extended Boussinesq equations. In the breaking zone, however, the agreement with measurement is better for the spilling breaker case ; the treatment of plunging breaker needs to be improved in the numerical model.

1. INTRODUCTION

The accurate prediction of wave kinematics (orbital velocities, pressure, etc) in the shoaling zone and in the breaking zone is of highest interest both for the design of marine structures (such as breakwaters, platforms, pipelines) and for the prediction of morphodynamical changes (due to sediment transport). The final aim of this research project is to propose numerical methods or tools applicable for the prediction of efforts acting on a pipeline laid on the bottom in the coastal zone. Although some experimental

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results are available on given configurations for the case of a horizontal pipeline exposed to near-breaking and breaking waves (Yuksel and Narayanan, 1994 ; Chevalier *et al.*, 2000), numerical methods to predict these efforts are quite few and remain an open topic for research and development of operational tools.

In this paper, as a first step towards the prediction of these efforts, attention is paid to the estimation of near-bottom kinematics (in the absence of the pipeline). This problem has been addressed by several authors, both on experimental basis or by numerical methods (e.g. Isobe and Horikawa, 1982 ; Hamm, 1996). In the present work, experimental tests are performed in a wave flume on a sloping bottom. In order to highlight the resolving capabilities and limitations of these methods, the measured kinematics are compared to the predictions obtained by various numerical methods.

2. EXPERIMENTAL SET-UP AND SELECTED LABORATORY TEST-CASES

2.1 Experimental set-up

As a first step, in order to build a reference data-base of measurements, a series of experimental tests has been performed in a wave flume at the University of Caen Basse Normandie (France) for a constant and smooth slope of 5 % (1:20).

The wave flume is 22 m long, 0.80 m wide and the maximum water depth is 0.70 m. It is equipped with a wavemaker, able to generate regular and irregular waves, and to absorb waves reflected by the structure placed in the flume.

The present tests were performed under regular wave conditions for an offshore depth (flat bottom part) of 0.50 m. Wave period T and incident wave height H were varied during the tests, so that various breaking type conditions (plunging and spilling breakers) and various positions of the breaking point were considered.

The free-surface elevation was recorded by means of 10 resistive probes (3 "offshore" probes for analysing the incident wave height and 7 probes along the slope to measure the evolution of waves during shoaling). The orbital velocities were measured by using a Laser-Doppler Anemometer (LDA). This technique offers precise measurements of the horizontal and vertical components of the velocity. Vertical profiles were measured at given locations and near-bottom velocities were recorded 2 cm above the bed. Measurements were performed over a duration of at least 90 s, and then averaged for producing the signals over one period.

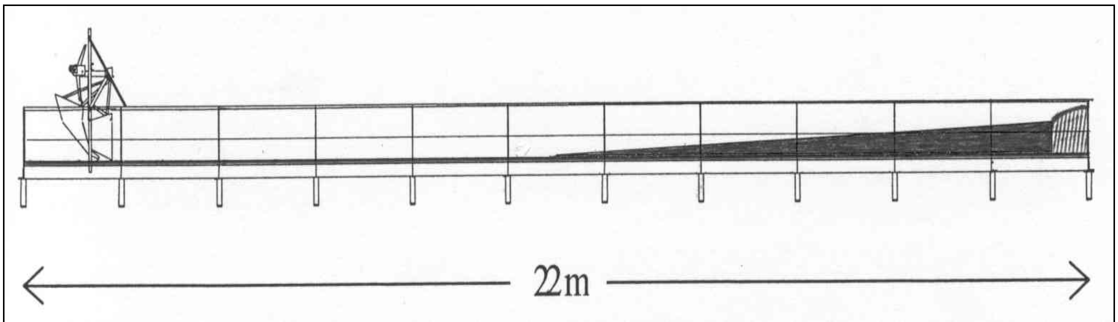

Figure 1 : Experimental set-up in the wave flume

2.2 Selected test-cases for comparison with numerical methods

Among the experiments, two test-cases were selected, corresponding to two different modes of breaking. Their characteristics are given in the table below :

Case		(1) Spilling breaker	(2) Plunging breaker
Incident wave height	H	14.7 cm	17.9 cm
Wave period	T	1.25 s	2.00 s
Breaking wave height	H_b	16.8 cm	23 cm
Breaking water depth	d_b	22 cm	30 cm

In this paper, measurements are presented at three locations (X = 5 m, 6 m and 7 m, with X = 0 corresponding to the beginning of the slope). The corresponding still water depths are 25 cm, 20 cm and 15 cm respectively. Figure 2 shows the positions of the measuring lines, as well as the locations of breaking points (BP) for the two cases.

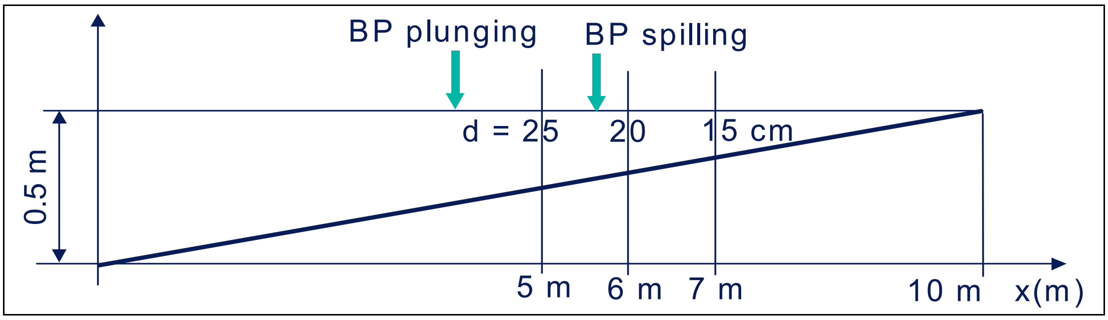

Figure 2 : Locations of the 3 measurements positions and breaking points (BP)

3. NUMERICAL METHODS FOR PREDICTING NEAR-BOTTOM VELOCITIES

3.1 Overview of modeling approaches

Various numerical methods can be applied to the tests described previously. They can be sorted depending on one hand on the level of available information both on wave conditions and bathymetry and, on the other hand, on the assumptions and theoretical grounds they rely on (see figure 3 for a schematic overview).

3.2 Flat-Bottom Theories (FBT)

The first idea is to use existing wave theories or numerical algorithm, which were developed for progressive waves of permanent form over a flat bottom (e.g. Sobey *et al.*, 1987). The input data is simply the wave period (T), the local wave height (H) and the water depth (d). The Airy theory or linear theory for small amplitude waves is the simplest approach of this family. Nonlinear theories include Stokes theories (of order 2, 3 or 5) and cnoidal theories (of order 1, 2, 3 or 5). For shallow-water waves, Stokes theories are not suitable (and thus not considered in this paper) and cnoidal theories are more appropriate. We use here the first order (Isobe, 1985) and third order cnoidal (Horikawa, 1988) theories. The "Stream-Function" approach can also be used (e.g. Sobey *et al.*, 1987), as it provides a more accurate solution of the problem, whatever the relative water depth end the wave steepness are.

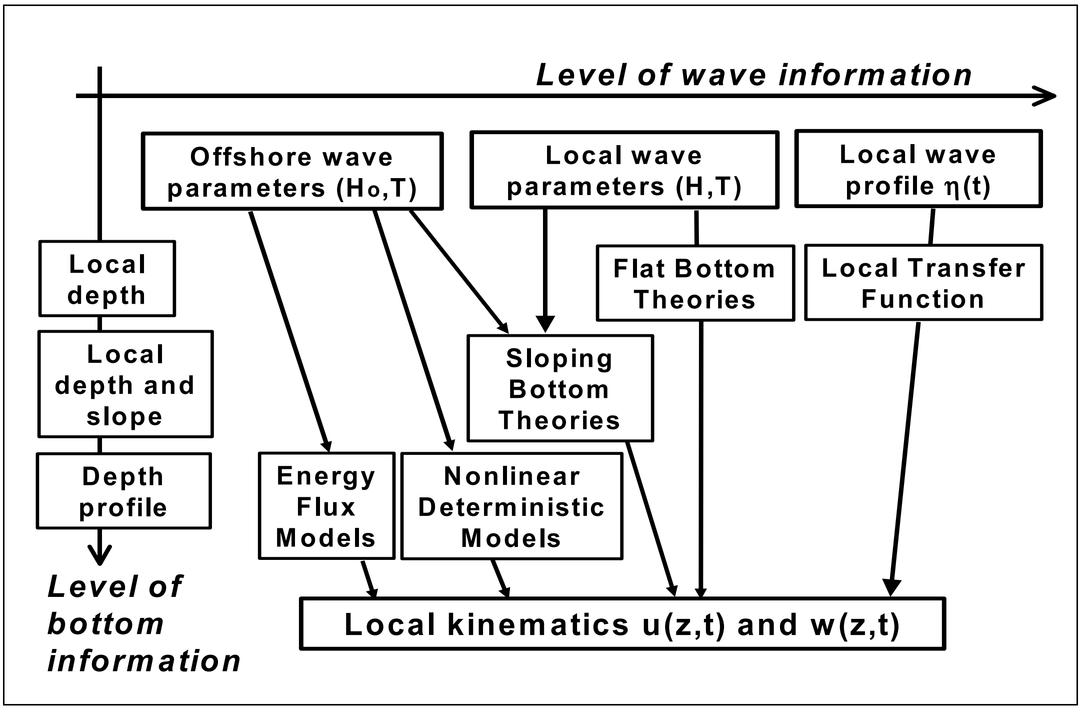

Figure 3 : Overview of modeling strategies for computing local wave kinematics

3.3 Sloping-Bottom Theories (SBT)

Although nonlinear FBT can account for the asymmetry of wave profile with respect to a horizontal plane, they are not able to reproduce the asymmetry with respect to a vertical plane, induced by nonlinear shoaling. So, there have been several attempts to modify FBT methods or to propose alternative semi-empirical methods for including this asymmetry (e.g. Hamm (1996) for a review). For instance, Hattori and Katsuragawa (1990) developed a Modified Stream Function Method by shifting the phases of the harmonics of the wave profile in order to obtain the skewed shape of near-breaking waves. Swart and Crowley (1988) established a so-called “covocoïdal” theory, suitable for sloping bottom profiles. Another approach, widely used for practical applications, is the parameterised method of Isobe and Horikawa (1982) for estimating the horizontal velocity. For SBT methods, the input data are the wave period (T), the local wave height (H), the water depth (d) and the local bottom-slope (m). In this work, we present results obtained with the method of Isobe and Horikawa (1982), which has proven to be efficient for shoaling waves, while remaining quite easy to use (Hamm, 1996).

3.4 Transfer Function Methods (TFM)

In the case where the local time-series of free surface elevation η(t) and local water depth d are known, one may use a Transfer Function Method (TFM) to compute the velocity profiles from the free surface elevation profile (e.g. Koyama and Iwata, 1986):

$$u(z,t) = G_u(z,d,T)\ \eta(t + \varphi_u) \qquad (1.a)$$

$$w(z,t) = G_w(z,d,T)\ \eta(t + \varphi_w) \qquad (1.b)$$

where G_u and G_w are local transfer functions, and φ_u and φ_w time-lags between velocities and free surface signals. Several methods are available for obtaining G and φ. The simplest one is based on the small amplitude wave theory and called the Linear Transfer Function Method (LTFM) :

$$G_u = \omega \frac{cosh(k(d+z))}{sinh(kd)} \qquad \varphi_u = 0 \tag{2.a}$$

$$G_w = \omega \frac{sinh(k(d+z))}{sinh(kd)} \qquad \varphi_w = T/4 \tag{2.b}$$

Koyama and Iwata (1986) observed that LTFM usually predicts correctly the negative values of the horizontal velocity u, but overestimates its positive values. They then developed a Modified Transfer Function Method (MTFM) by which they modify the linear transfer function only under the wave crest. For the positive values of the horizontal velocity, the inclusion of finite amplitude effects was achieved by replacing the water depth d by f.d, where f is a correction term (and keeping $\varphi_u = 0$) :

$$G_u = \omega \frac{cosh(k(d+z))}{sinh\left\{k\left(d+\eta\left(1-\frac{d+z}{d+\eta}\right)\right)\right\}} \qquad for\ \eta > 0 \tag{3.a}$$

$$G_u = \omega \frac{cosh(k(d+z))}{sinh(kd)} \qquad for\ \eta \le 0 \tag{3.b}$$

Dean (1965) also proposed a Stream Function Method (DSFM) which can be used once the free-surface elevation signal is known ; this method is not tested in this work. Recent work by Chevalier *et al.* (2000) confirmed that the MTFM method is suitable in modeling nonlinear effects in the horizontal velocity profile near the bottom.

3.5 Energy Flux Models (EFM)

Starting from offshore wave conditions (wave period T, offshore wave height Ho, direction θo) and knowing the bottom profile (d(x)), another approach is to solve the energy flux equation for wave propagation (with surf-breaking dissipation included) and to combine it with a flat bottom theory (FBT) which is assumed to be locally valid. For the case of a bottom profile constant along the y direction the equation reads :

$$\frac{\partial(F\cos\theta)}{\partial x} = D_b \tag{4}$$

Once F is known at a given location, the local properties of the waves (height H, wave-number k, phase and group celerities C and Cg, etc.) can be computed by the use of the relationships of the corresponding FBT. Then, the velocities and pressure profiles can be evaluated for the local water depth. EFM methods are usually based on the coupling of a Stokes theory (order 1, 3 or 5) for the deeper part of the domain with a cnoidal theory (order 1,2 or 3) for the shallow-water area, the transition being done when the Ursell number Us reaches a given value. In this work, we solve equation (4) by using the linear wave theory and the first order cnoidal theory, with a transition when Us = 25. For the breaking dissipation term D_b, we use the formulation from Izumiya and Horikawa (1984) with its default parameters.

3.6 Nonlinear Deterministic Models (NDM)

When offshore wave conditions and the bathymetric profile are given, a more accurate way of computing wave propagation is to use Nonlinear Deterministic wave Models (NDM). Such models solve the nonlinear equations of motion for wave evolution in space and time (e.g. Laplace, Euler or Boussinesq equations) and each wave is discretized by 20 to 50 mesh points. In this work, we consider two models based on extended Boussinesq equations, with improved dispersion and nonlinear properties. Evaluation of nonlinearity and dispersion characteristics is provided by the following dimensionless parameters $\varepsilon = a/h$ and $\mu = kh$ respectively (where a, h and k are characteristic wave amplitude, water depth and wave-number respectively). Various forms of Boussinesq equations may be obtained, depending on the order of the terms which are retained when developments in ε and μ are performed in the equations put in a non-dimensional form. Two particular sets of such extended Boussinesq equations are used in two versions of a code developed at LNHE :

- equations by Nwogu (1993) including terms or order $O(\mu^2 ; \varepsilon)$ corresponds to version 1.1 of BSQ (BSQ_V1P1)
- equations by Wei *et al.* (1995) including terms or order $O(\mu^2 ; \varepsilon^3.\mu^2)$ corresponds to version 2.0 of BSQ (BSQ_V2P0). These latter equations are fully nonlinear with respect to the order of truncation of the dispersive terms

The equations are solved on a computational mesh (1D case here) for free surface elevation η and the horizontal velocity u_α at a depth $z_\alpha = C_\alpha.d$, where C_α is a constant value chosen to optimise the dispersion properties of the model (here C_α = -0.53, as suggested by Nwogu, 1993). Surf-breaking dissipation is included by adding an eddy viscosity-like term in the momentum equation. For instance, the one-dimensional version of the equations of Nwogu (1993) solved by BSQ (version 1.1) read :

$$\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x}\left((d+\eta)u_\alpha\right) + \frac{\partial}{\partial x}\left[\left(\frac{1}{2}C_\alpha^2 - \frac{1}{6}\right)d^3\frac{\partial^2 u_\alpha}{\partial x^2} + \left(C_\alpha + \frac{1}{2}\right)d^2\frac{\partial^2(du_\alpha)}{\partial x^2}\right] = 0 \qquad (5.a)$$

$$\frac{\partial u_\alpha}{\partial t} + g\frac{\partial \eta}{\partial x} + u_\alpha\frac{\partial u_\alpha}{\partial x} + \frac{z_\alpha^2}{2}\frac{\partial^3 u_\alpha}{\partial x^2 \partial t} + z_\alpha\frac{\partial^2}{\partial x^2}\left(d\frac{\partial u_\alpha}{\partial t}\right) - D_b = 0 \qquad (5.b)$$

where the dissipative term due to breaking has the following expression :

$$D_b = \frac{1}{d+\eta}\frac{\partial}{\partial x}\left[\nu\frac{\partial}{\partial x}\left((d+\eta)u_\alpha\right)\right] \qquad (6)$$

The model and formulations used to compute the eddy viscosity ν, incipient breaking, etc. are implemented following the approach of Kennedy *et al.* (2000). The 1D code BSQ uses the finite difference technique and the high order numerical scheme proposed by Wei *et al.* (1995). First-order spatial derivatives are computed by a 4th order scheme while the time-integration is performed by a 5th order predictor and 6th order corrector method. The horizontal velocity as a function of the elevation z reads :

$$u(z) = u_\alpha + \left(\frac{z_\alpha^2}{2} - \frac{z^2}{2}\right)\frac{\partial^2 u_\alpha}{\partial x^2} + (z_\alpha - z)\frac{\partial^2(du_\alpha)}{\partial x^2} \qquad (7)$$

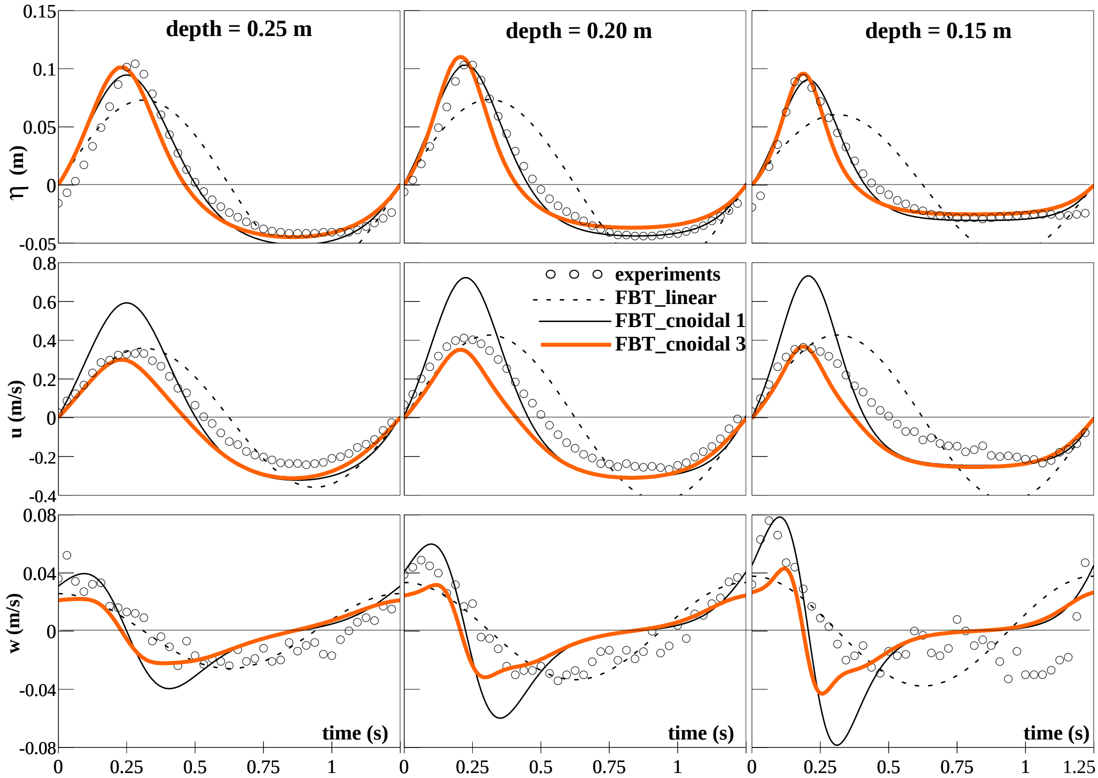

Figure 4 : Results of FBT methods for case 1 (spilling breaker)

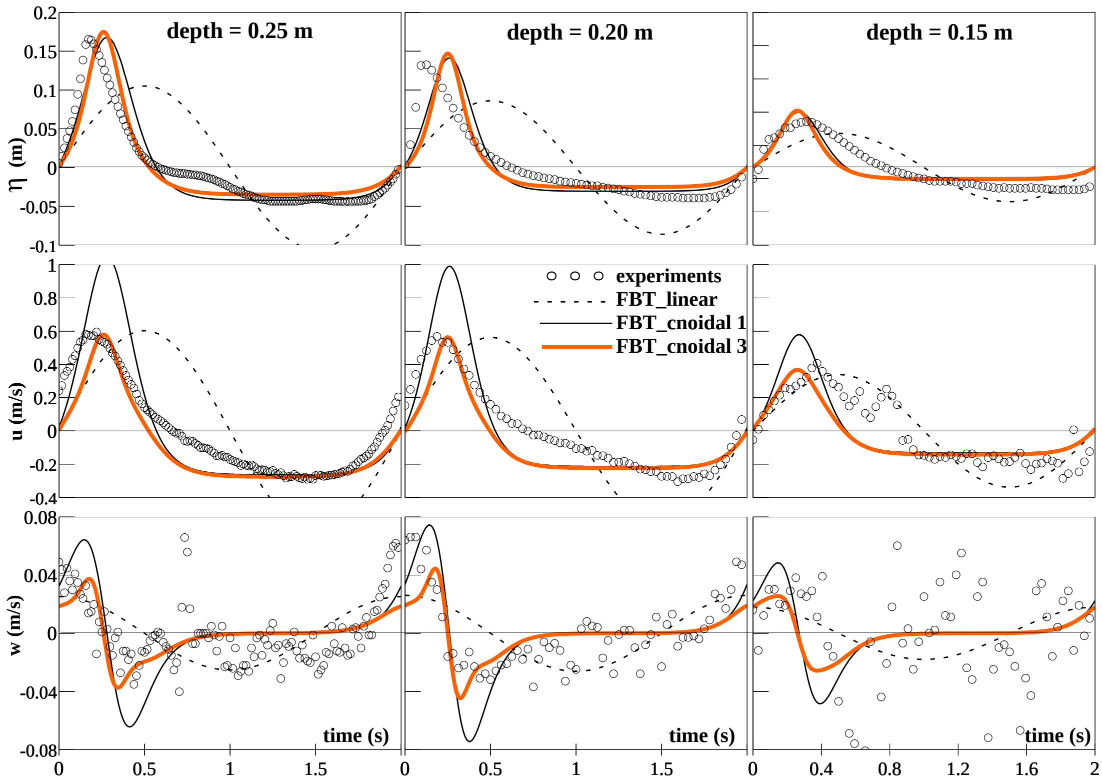

Figure 5 : Results of FBT methods for case 2 (plunging breaker)

4. ANALYSIS OF NUMERICAL PREDICTIONS FOR THE TWO TEST-CASES

4.1 Content and layout of figures of results

Results of the application of representative methods of the five categories of modeling techniques presented in Section 3 are plotted on figures 4 to 9. Figures 4, 6 and 8 correspond to test-case 1 (spilling breaker – T= 1.25 s), while figures 5, 7 and 9 correspond to test-case 2 (plunging breaker – T= 2 s). On each of these figures, nine graphs are plotted : each column corresponds to a value of the still water-depth (25 cm, 20 cm and 15 cm from the left to the right). On the first row, free surface elevations are plotted, while horizontal and vertical velocities at the elevation 2 cm above the bottom are plotted on the second and third rows respectively.

4.2 Flat-Bottom Theories (FBT)

Results are plotted on figure 4 for case 1 (spilling breaker) and figure 5 for case 2 (plunging breaker). It is clear from these figures that the linear theory fails to predict the profiles of both surface elevations and velocities. As the water depth decreases, the agreement is getting worse, as this method is unable to model both the vertical and horizontal asymmetries of the wave. This is very clear after wave breaking, in particular for the plunging case (figure 5) and confirms that the small amplitude theory should not be used in such near-breaking and breaking conditions. As they are able to represent the asymmetry of the waves with respect to a horizontal plane, cnoidal theories of order 1 and 3 bring some improvements. For the spilling case (figure 4), predictions of free-surface elevations are quite correct, in particular for the third order theory, even after breaking. The magnitudes of velocities are however overestimated by the first order cnoidal theory and this trend increases after breaking. Even if the shape of the horizontal velocity profile is not perfectly matched by the third order theory method, the magnitudes of the extrema are well predicted for the spilling case.

For the plunging case (figure 5), cnoidal theories are less successful in modeling the profile of free-surface elevation, because the asymmetry with respect to a vertical plane is also important in that case. Some comments arise for the predictions of kinematics : the first order cnoidal method significantly overestimates the maximum value of the horizontal velocity. Results of third-order theory are in better agreement with the measurements, but still suffer from the inability of the method to model such a skewed profile after breaking.

Another limitation related to FBT (including the stream function method) lies in the fact that the maximum height of a stable wave over a flat bottom is about 0.78 times the water depth. Under shoaling conditions however, it is well known that the ratio of wave height over water depth may exceed 1 and reach 1.3 or 1.4 for mild slopes (e.g. Goda 2000). Under such circumstances, it is expected that FBT methods will fail or diverge just prior to breaking, when the above ratio exceed 0.8.

4.3 Sloping-Bottom Theories (SBT)

From this class of methods, we only consider the method from Isobe and Horikawa (1982), hereafter referred as IH82. From that method, only the horizontal velocity can be computed. Results presented on figure 6 for case 1 (spilling breaker) show that a very

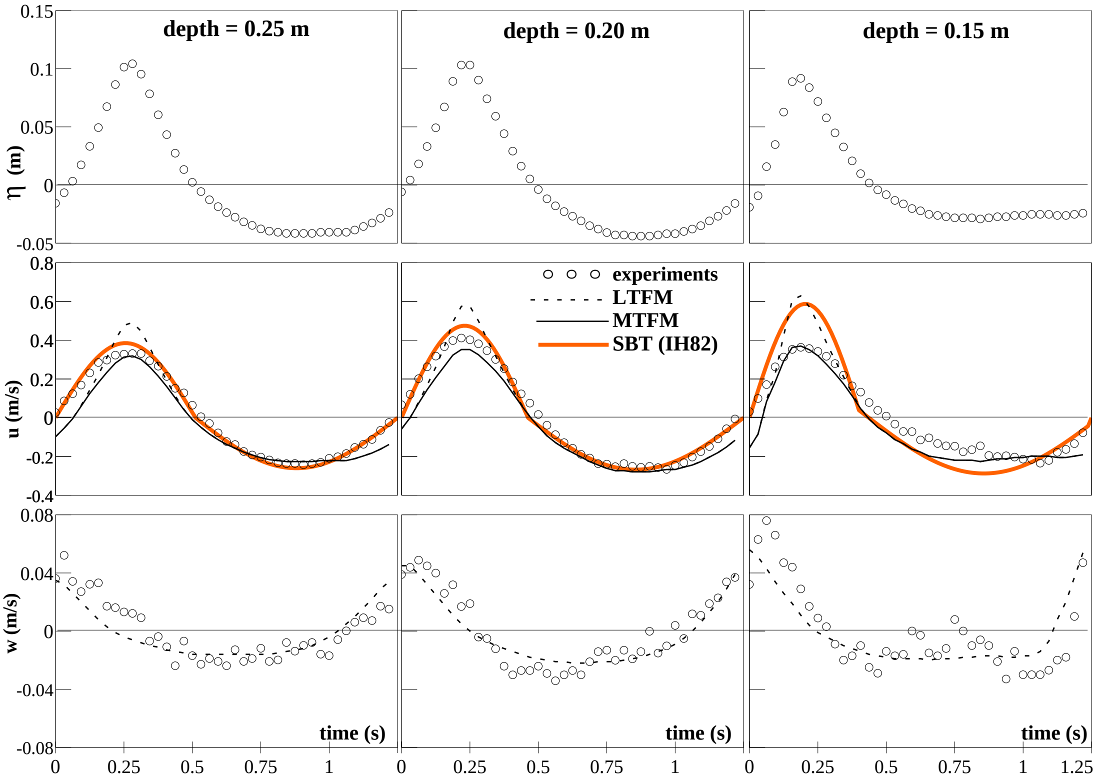

Figure 6 : Results of TFM and SBT(IH82) methods for case 1 (spilling breaker)

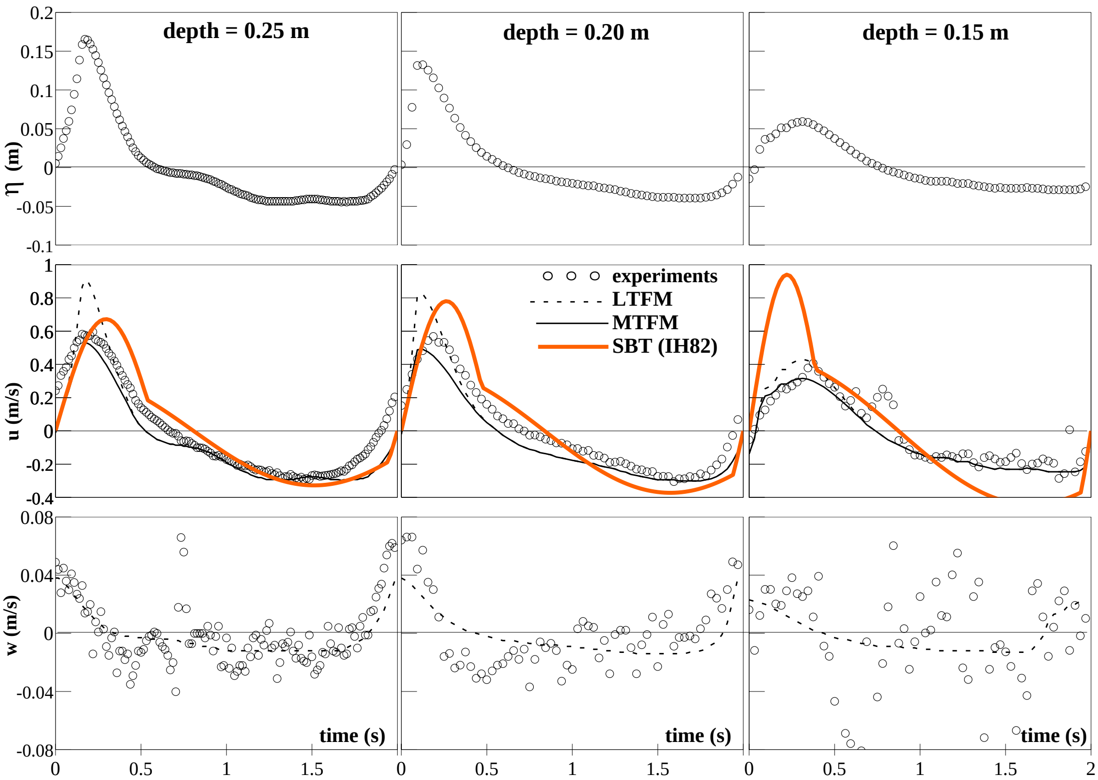

Figure 7 : Results of TFM and SBT(IH82) methods for case 2 (plunging breaker)

good agreement is obtained before breaking (depth = 25 cm), but as the water depth decreases the method tends to overestimate the peak value of the horizontal velocity. This is also observed for case 2 (plunging breaker) for which waves break offshore of the three measurement locations. This application of SBT_IH82 method clearly demonstrates that this technique should not be used in the breaking zone, although it may give accurate predictions of horizontal velocity before breaking. This limitation was already observed by several authors (e.g. Hamm, 1996).

4.4 Transfer Function Methods (TFM)

Results are plotted on figure 6 for case 1 (spilling breaker) and figure 7 for case 2 (plunging breaker). For these methods, the measured free-surface elevation signal is used on input. Thanks to this, it is observed for both cases that the predictions of kinematics from the (linear) LTFM method are significantly improved in comparison to the linear FBT predictions (figures 4 and 5) where only the wave height was used. However, there is a systematic trend of the method to overestimate the maximum value of the horizontal velocity under the crest. The MTFM method (Koyama and Iwata, 1986) partly overcomes this shortcoming and produces good estimations of the horizontal velocity before and after breaking, both for the shape and the extrema of the profile. Chevalier *et al.* (2000) came the same conclusion and recommended this method. The vertical velocity is well predicted by the LTFM method, but this component is significantly lower than the horizontal one.

4.5 Energy Flux Models (EFM)

Results are plotted on figure 8 for case 1 (spilling breaker) and figure 9 for case 2 (plunging breaker). For the spilling breaker case, EFM method gives very good estimation of the free-surface profile (as it was already observed in section 4.2 for the first order cnoidal FBT theory) because the asymmetry with respect to the vertical plane is low in the spilling breaking case. It must be emphasised that, for the EFM approach, the local wave height is computed from (4). For this case, this means that the combination of the first-order cnoidal theory and an efficient dissipation model allows to reach acceptable predictions for the free surface profile. For the kinematics however, we again notice the trend of first-order cnoidal method to overestimate the velocities under the crest, as was observed in section 4.2. Same comments are drawn from the plunging breaker case, for which, in addition (and as expected due to the strong skewed shape of the wave), the predictions of the free surface elevation are worse after breaking. For this case, at the water depth of 15 cm, the local wave height predicted by the EFM model is higher than the measured one, which means that the surf-breaking dissipation term D_b in (4) is not strong enough and should be re-calibrated for this type of breaking.

3.6 Nonlinear Deterministic Models (NDM)

Results are plotted on figure 8 for case 1 (spilling breaker) and figure 9 for case 2 (plunging breaker). The curves labeled BSQ_V1P1 refer to the Boussinesq equations of Nwogu (1993), whereas the curves labeled BSQ_V2P0 refer to the equations of Wei *et al.* (1995). For the spilling breaker case (figure 8), good predictions of the free-surface profile are obtained in particular from the fully nonlinear (to the order of the dispersive term) model BSQ_V2P0. Predictions of kinematics are also in close agreement with the

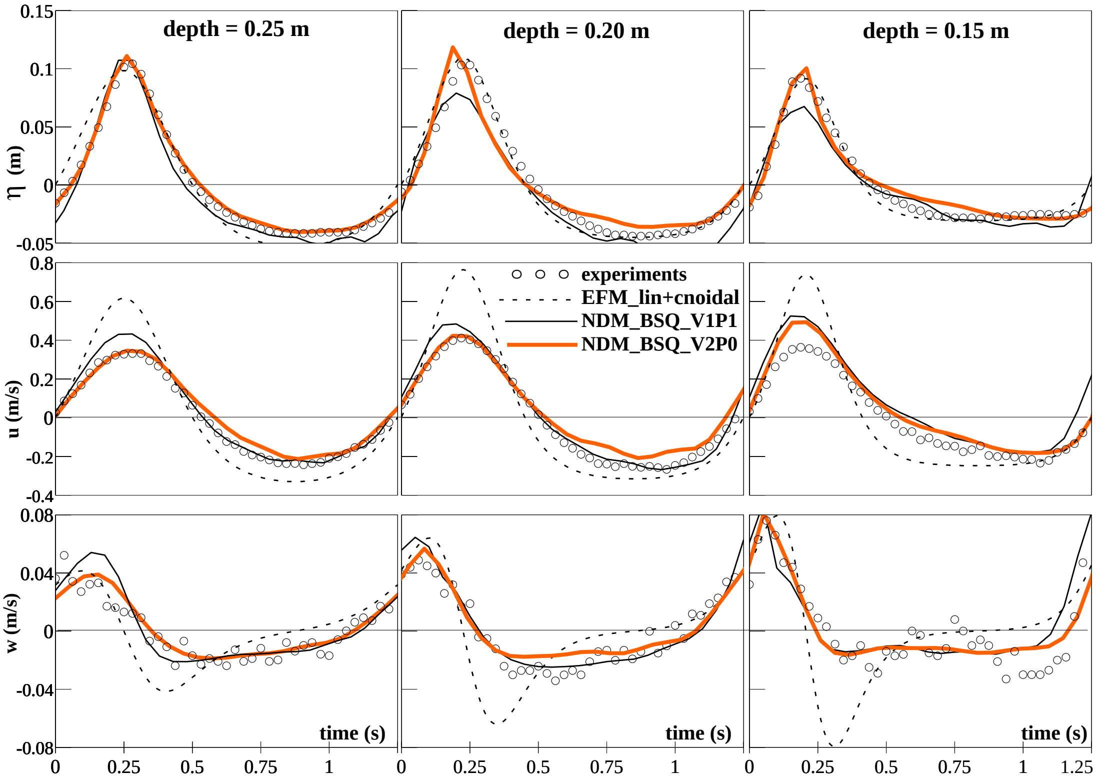

Figure 8 : Results of EFM and NDM methods for case 1 (spilling breaker)

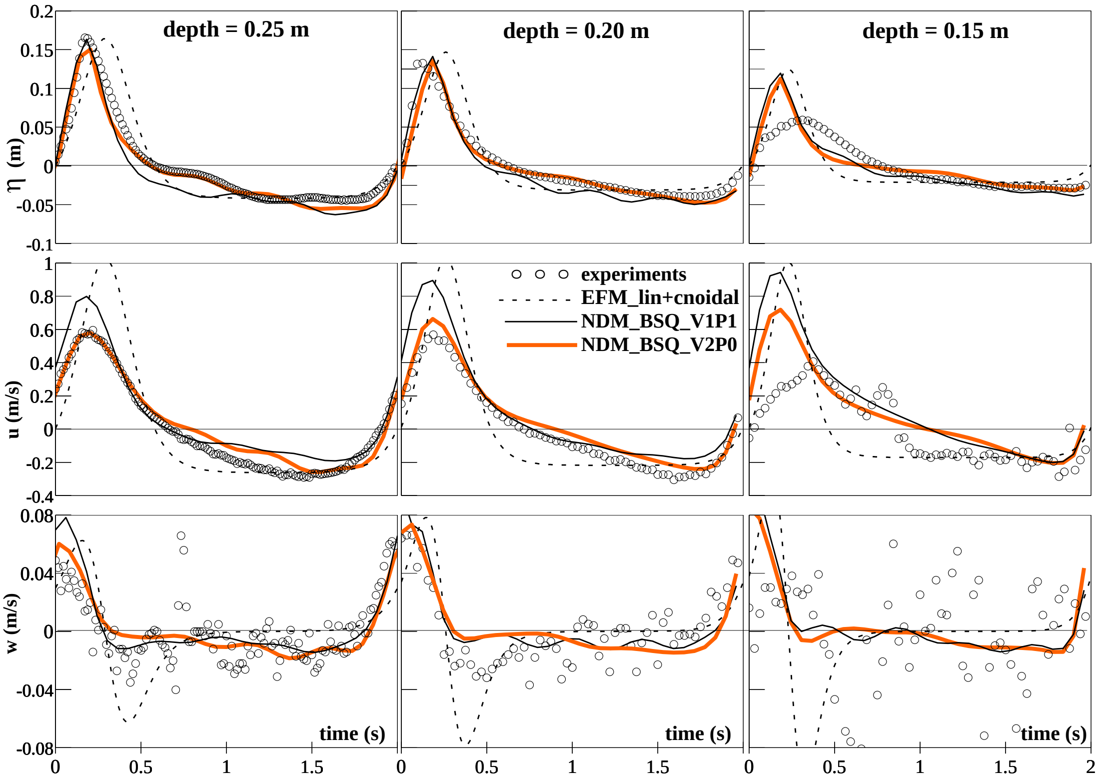

Figure 9 : Results of EFM and NDM methods for case 2 (plunging breaker)

measurements, and again BSQ_V2P0 performs slightly better than BSQ_V1P1. The peak horizontal velocity at the shallower water depth (d = 15 cm) is however a bit overestimated. Both models correctly reproduce the vertical velocity profiles.

For the plunging breaker case (figure 9), very good agreement with the measurements is observed after breaking (depth = 25 and 20 cm) for BSQ_V2P0. At the third station (depth = 15 cm), the shapes of free-surface elevation and velocities are not well resolved by the models. On one hand, the predicted wave height is higher than the measured one and on the other hand the measured profile is less skewed than the predicted one. This indicates that the model adopted for the breaking term in (5) is not fully adapted for this case. This is not surprising as the eddy viscosity formulation (6) is mostly suitable for moderate breaking conditions, such as spilling breakers.

5. CONCLUSIONS AND PERSPECTIVES

In this paper, through comparisons with laboratory experiments (regular waves over a smooth and plane slope of 1:20), we addressed the problem of accurate prediction of near-bottom kinematics under shoaling and breaking waves, both for spilling and plunging breaker conditions. The following conclusions can be drawn from the study :

1. <u>Flat Bottom Theories (FBT)</u> are not suitable for modeling waves in pre-breaking and breaking conditions. In particular, the linear (small amplitude) wave theory should not be used due to its inability to model the asymmetry of waves. For moderate breaking conditions (spilling breaker), cnoidal theories may be used as a first guess, but acceptable results were obtained only for the third-order theory (while the first order theory overestimates the horizontal velocity).
2. <u>Sloping Bottom Theories (SBT)</u> : the semi-empirical method of Isobe and Horikawa (1982) gives good predictions of horizontal velocities up to the breaking point, but should not be used after breaking as it then overestimates the horizontal velocity. Other approaches from this family are presently under test.
3. <u>Transfer Functions Methods (TFM)</u> : the LTFM (linear) method is not recommended, but the MTFM method (Koyama and Iwata, 1986) exhibited good overall performances, prior and after breaking. It is a recommended technique for use when the free surface signal is available.
4. <u>Energy Flux Models (EFM) :</u> Such models may be tuned for modeling wave height evolution, but kinematics remain poorly predicted, because these models invoke locally a Flat Bottom Theory (FBT) for the kinematics.
5. <u>Nonlinear Deterministic Models (NDM) :</u> Numerical models based on extended Boussinesq equations look promising, both for the shoaling and breaking zones. The model by Wei *et al.* (1995) gave better predictions than the model of Nwogu (1993), which is of lower order in nonlinearity. Better agreement was obtained for spilling breakers and improvements of the modeling of the breaking mechanism are needed for plunging breakers.

Ongoing and future work will consider the presence of a pipeline laid on the bottom and the determination of efforts due to near-breaking and breaking waves. Again, experimental tests will be performed and numerical prediction methods (based on the results of the present study for the kinematics) will be evaluated and improved.

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