On the turbulence generated by the potential surface waves

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[1] The turbulence (the random vortex motions) of the upper ocean is nourished by the energy and momentum of the surface waves (the potential motion). The statistical characteristics of the turbulence (turbulent kinetic energy, dissipation rate, and Reynolds stresses) depend on the state of the ocean surface waves. This paper discusses the possibilities of generating this turbulence using the vortex instability of the potential surface waves. The vortex component of fluctuations of velocity field and possibly the interaction between both the vortex and potential motions cause the vertical transport of the momentum. The Reynolds tensor is a linear function of the correlation tensor of vortex field. The initial small vortex perturbations always exist in the upper ocean because of the molecular viscosity influences, especially near the free surface, and the fluctuations of the seawater density. The horizontal inhomogeneities of the seawater density produce the vortex field even if the initial vorticity was zero and the initial flow was the potential flow. The evolution of the small initial vortex disturbances in the velocity field of potential linear surface waves is reduced to a coupled set of linear ordinary differential equations of the first order with periodic coefficients. The solution of this problem shows that the small initial vortex perturbations of potential linear surface waves always grow. The initial small vortex perturbations interacting with the potential surface wave produce the small-scale turbulence (Novikov's turbulence) that finally causes the viscous dissipation of the potential surface wave. The wave-induced turbulence can be considered as developed turbulence with a well distinguishable range of the turbulent wave numbers k where turbulence obeys the Kolmogorov's self-similarity law.

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1. Introduction

[2] The turbulent motion in the upper ocean is a highly specific example of turbulence in a liquid whose free surface is subject to wind friction. The ultimate result of this action is the formation of waves, pure drift currents, and turbulence, leading to strong vertical mixing of the upper ocean layer. In contrast to boundary layers at a solid wall, where mean velocity shear is the main source of turbulent energy, the turbulence in the upper ocean is governed in many respects by the wave motions. The energy transferred per unit time from the wind to the oscillations of water surface $\eta(\mathbf{r}, t)$ is an order of pure drift currents [*Kitaygorodskiy and Miropolskiy*, 1968; *Kitaygorodskiy*, 1970; *Benilov and Ly*, 2002], and it follows that the turbulence of the upper ocean is nourished by the energy accumulated in the waves. Consequently the turbulence characteristics should be depended on the state of the ocean surface.

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[3] The first attempt to take into account the wave influence on the turbulence was done in 1947 [*Dobroklonskiy*, 1947], there was obtained an equation for the eddy viscosity in a trochoidal wave. In 1950 [*Bowden*, 1950] it was proposed that the turbulence in a large- amplitude wave could be described by averaged fluid mechanics equations in which the mean velocity is taken to be the wave velocity field.

[4] In 1962, *Stewart and Grant* [1962] measured the turbulent kinetic energy spectra in the presence of waves were measured and estimated the turbulent kinetic energy dissipation rate, which had close connection to the wave state. Field measurements done in 1964–1966 [*Shonting*, 1966] showed considerable dawnward turbulent momentum fluxes, which depended on the wave motions. The more detailed measurements performed later by *Efimov and Hristoforov* [1971], *Kitaygorodskiy et al.* [1983], and *Datla and Benilov* [1997] confirmed conclusions by *Shonting* [1966]. Quantitative characteristics of the wave effects on the turbulence were described in early surveys [*Benilov*, 1969a; *Kitaygorodskiy*, 1970; *Gargett*, 1989; *Geernaert and Plant*, 1990] and in recent monographs by *Soloviev and Lukas* [2006] and *Babanin* [2011].

[5] In 1968 the turbulent kinetic energy equation was applied to study the wave turbulence [*Kitaygorodskiy and Miropolskiy*, 1968]. They took into account the effect of

the wave spectrum and calculated the vertical profile of the turbulent energy dissipation, explaining the field measurement in the upper ocean given in *Phillips* [1977]. By generalizing the results of *Stewart and Grant* [1962], *Kitaygorodskiy and Miropolskiy* [1968], *Phillips* [1977], and *Benilov* [1973a] proposed the equation of the turbulent energy balance, which takes into account the energy influx from the wave motions to the turbulence [see also *Benilov and Lozovatskiy*, 1977; *Kitaygorodskiy and Lumley*, 1983; *Benilov and Ly*, 2002].

[6] In 1970, *Benilov and Filyushkin* [1970] proposed a procedure of linear filtration to separate the turbulent and wave fluctuations in real recordings, obtained from measurements. Later, *Benilov* [1973b] developed a procedure based on the nonlinear analysis. The linear approach was used by *Matushevskiy* [1975], *Hristoforov and Zapevalov* [1979], *Kitaygorodskiy et al.* [1983], and *Cheung and Street* [1988] to study separately the turbulence and wave fluctuations. The measurements of the small-scale turbulence [*Jones*, 1985; *Solov'ev*, 1986; *Matusov et al.*, 1989; *Jiang et al.*, 1990] demonstrated a strong dependence of the turbulence intensity on the wave state.

[7] Hence, surface waves, considered as potential waves, have a strong influence on the turbulent motion, which is a vortex motion. Therefore, we shall discuss, following *Benilov* [1969b] and *Benilov et al.* [1993], the vortex instability of linear potential surface wave or, in other words, the possibility of generating turbulence by the vortex instability of linear surface waves.

2. Vortex Instability of Linear Surface Wave

2.1. Hydrodynamic Consequences

[8] This section considers three questions which can be answered from the hydrodynamic equations.

[9] 1. What is the contribution of potential surface waves into the spectra of the Reynolds stresses which are responsible for the vertical transport of the momentum? They play significant role on formation of the ocean upper layer vertical dynamic structure. It is well known that the wave Reynolds stresses induced by the potential surface waves do not contribute into the vertical transport of the momentum because the corresponding stresses equal to zero. To demonstrate this, first, we assume the field of surface waves as a random statistically homogeneous field. The field of the surface gravity wind waves satisfies well the conditions of statistical horizontal homogeneity in the range of the spatial scales at least up to tens of kilometers. The definitions and the mathematical technique of the homogeneous random fields can be found in *Batchelor* [1953], *Kinsman* [1965], and *Monin and Yaglom* [1987]. This theoretical approach is employed to study the ocean wind waves throughout of about 5 decades [*Kinsman*, 1965]. Let us put for the random wavefield of velocity the ordinary classic form of the potential approximation [*Kinsman*, 1965; *Phillips*, 1977; *Batchelor*, 1967; *Landau and Lifshitz*, 1987]

$$\mathbf{v} = \nabla\varphi, \quad \nabla\cdot\nabla\varphi = 0, \quad \mathbf{v}(\mathbf{r}, x_3, t) = [v_\alpha(\mathbf{r}, x_3, t), v_3(\mathbf{r}, x_3, t)],$$
$$\alpha = 1, 2, \tag{1}$$

$$\varphi(\mathbf{r}, x_3, t) = \int \exp(i\mathbf{pr} + px_3)dZ_\varphi(\mathbf{p}, 0, t), \mathbf{r} = (x_1, x_2),$$
$$\mathbf{p} = (p_1, p_2), \quad p = |\mathbf{p}|, \tag{2}$$

$$v_\alpha(\mathbf{r}, x_3, t) = i\int p_\alpha \exp(i\mathbf{pr} + px_3)dZ_\varphi(\mathbf{p}, 0, t),$$
$$v_3(\mathbf{r}, x_3, t) = -\int p \exp(i\mathbf{pr} + px_3)dZ_\varphi(\mathbf{p}, 0, t), \tag{3}$$

where φ is the potential of the wave velocity field, $\mathbf{r} = (x_1, x_2)$ is the Cartesian coordinate horizontal position vector, x_3 (≤ 0) is a vertical coordinate in upward direction, $\mathbf{p}$ is a wave vector, $dZ_\varphi(\mathbf{p}, 0, t)$ is Fourier-Stieltjes measure of potential $\varphi(\mathbf{r}, x_3, t)$ at $x_3 = 0$. Computing the wave Reynolds stresses $\overline{v_\alpha v_3}$ (α = 1, 2, and the bar at the top is the averaging) using (1)–(3) and taking into account that the wave Reynolds stresses are the real quantities, we find

$$\begin{gathered}\overline{v_\alpha v_3} = Reel\left[-i\int p_\alpha p \exp(2px_3)E_\varphi(\mathbf{p}, 0, t)\right]d\mathbf{p} = 0,\\ \overline{dZ_\varphi dZ_\varphi^*} = E_\varphi(\mathbf{p}, 0, t)d\mathbf{p},\end{gathered} \tag{4}$$

where $dZ_\varphi(\mathbf{p}, 0, t)^*$ is a complex conjugative quantity of the Fourier-Stieltjes measure of potential $\varphi(\mathbf{r}, x_3, t)$ at $x_3 = 0$, $E_\varphi(\mathbf{p}, 0, t) \geq 0$ is the spectral density of the wave potential $\varphi(\mathbf{r}, 0, t)$ and it is a real quantity. Therefore, the expression under the integral sign also is a real quantity, and then the wave Reynolds stresses $\overline{v_\alpha v_3}$ vanish and do not contribute into the vertical transport of the momentum. It means that the vortex component of fluctuations of velocity field and possibly the interaction between both the vortex and potential motions cause the vertical transport of the momentum.

[10] 2. What follows from the hydrodynamics equations about vortex disturbances in the potential flow? Let the velocity field $\boldsymbol{u}$ be the sum of the potential component $\boldsymbol{v}$ and the vortex component $\boldsymbol{w}$ that is

$$\begin{gathered}\boldsymbol{u} = \boldsymbol{v} + \boldsymbol{w}, \quad \boldsymbol{v} = \nabla\varphi, \quad \boldsymbol{w} = curl\boldsymbol{A}, \quad div\boldsymbol{A} = \mathbf{0},\\ (\nabla\cdot\nabla)\varphi = \mathbf{0}, \quad (\nabla\cdot\nabla)\boldsymbol{A} = -\omega, \quad \boldsymbol{\omega} = curl\mathbf{w},\end{gathered} \tag{5}$$

where φ is the scalar potential of the velocity field $\boldsymbol{v}$, and $\boldsymbol{A}$ is the vector potential of the velocity field $\boldsymbol{w}$. From equation set (5) the vortex velocity $\boldsymbol{w}$ can be expressed in terms of the vorticity either in the following differential form

$$\Delta\boldsymbol{w} = -curl\omega \tag{6}$$

or the integral form

$$\boldsymbol{w}(\boldsymbol{x}, t) = \int_{V_S} [curl\omega(\boldsymbol{x}', t)]G(\boldsymbol{x}', \boldsymbol{x})d\boldsymbol{x}' - \oint_S \boldsymbol{w}(\boldsymbol{x}_S, t)(\nabla G\cdot d\mathbf{S}), \tag{7}$$

where $G(\boldsymbol{x}', \boldsymbol{x})$ is Green's function of the Laplace's operator ($\Delta = (\nabla\cdot\nabla)$) for lower half-space ($-\infty < x_1,\ x_2 < \infty$; $-\infty < x_3 \leq \eta(\boldsymbol{r}, t)$) bounded by the random wave surface $\eta(r, t)$, the integration ($d\mathbf{x}' = dx'_1 dx'_2 dx'_3$) in the first integral of (7) is extended over the volume V_S of the entire region while the integration in the second integral is extended over the surface S of the region. The vector $d\mathbf{S} = \boldsymbol{n}\ dS$ represents the element dS of the surface S, the random wave surface

$\eta(\boldsymbol{r}_s, t)$, at $\boldsymbol{x} = \boldsymbol{x}_s = \{x_{1s}, x_{2s}, \eta(\boldsymbol{r}_s, t)\}$ with the outward normal $\boldsymbol{n}$ at this point. The first integral in (7) represents the contribution into the vortex velocity $\boldsymbol{w}(\boldsymbol{x}, t)$ vortices distributed over the entire volume V_S. The second integral in (7) represents the contribution into the vortex velocity $\boldsymbol{w}(\boldsymbol{x}, t)$ vortices distributed at the entire surface S or, in the case under examination, at the surface $x_3 = \eta(\boldsymbol{r}, t)$. Far from the boundary the Green's function is

$$G(\boldsymbol{x}', \boldsymbol{x}) = \frac{1}{4\pi|\boldsymbol{x}' - \boldsymbol{x}|} \tag{8}$$

so an approximation for $\boldsymbol{w}(\boldsymbol{x}, t)$ is

$$\boldsymbol{w}(\boldsymbol{x}, t) = \frac{1}{4\pi}\int_{V_S} \frac{curl\boldsymbol{\omega}(\boldsymbol{x}', t)}{|\boldsymbol{x}' - \boldsymbol{x}|} d\boldsymbol{x}'. \tag{9}$$

[11] The equations (7) and (9) determine the vorticity induced by the velocity component $\boldsymbol{w}(\boldsymbol{x}, t)$ of the velocity field $\boldsymbol{u}(\boldsymbol{x}, t)$ if the vorticity $\boldsymbol{\omega}(\boldsymbol{x}, t)$ is known. In the ocean, vorticity disturbances arise because there are always some velocity shear, viscosity influences (especially near the air-sea interface) and some density disturbances.

[12] When the density approximation $\rho = \rho_0 + \rho' \approx \rho_0 = const$ is sufficient, the inviscid hydrodynamics equations reduce to the Boussinesq approximation [*Phillips*, 1977; *Batchelor*, 1967; *Landau and Lifshitz*, 1987; *Saffman*, 1992] and yield the vorticity equation

$$\frac{\partial \boldsymbol{\omega}}{\partial t} + (\mathbf{u}\cdot\nabla)\boldsymbol{\omega} = (\boldsymbol{\omega}\cdot\nabla)\mathbf{u} + \frac{[\nabla\rho' \times \mathbf{g}]}{\rho_0}, \tag{10}$$

where $\mathbf{g} = (0, 0, -g)$ is the gravity acceleration vector. Equation (10) shows that the density horizontal inhomogeneities produce the vorticity even if the initial vorticity was zero and the initial flow was the potential flow. At the next stage the small vorticity disturbances interact with the wavefield of velocity $\mathbf{v}$.

[13] 3. What is a connection between the Reynolds stresses and the vorticity? To answer this question we shall present the fields $\boldsymbol{w}(\boldsymbol{x}, t)$ and $\boldsymbol{\omega}(\boldsymbol{x}, t)$ as random homogeneous fields. The numerous measurements of turbulence in the upper ocean [*Monin and Ozmidov*, 1981; *Soloviev and Lukas*, 2006] show that the ocean upper layer turbulence well satisfies the conditions of the random homogeneous fields in the range of spatial scales up to the wind waves variability. Assuming the random velocity and vorticity fields statistically homogeneous, let us consider the correlation tensors of the vortex velocity field

$$R_{mn}(\mathbf{l},\mathbf{t}) = \overline{w_m(\mathbf{x}_0 + \mathbf{l}, t) w_n(\mathbf{x}_0, t)} = \int_{-\infty}^{\infty} \exp(i\mathbf{k}\cdot\mathbf{l})\Phi_{mn}(\mathbf{k}, t) d\mathbf{k} \tag{11}$$

and the vorticity field

$$\Xi_{mn}(\mathbf{l},\mathbf{t}) = \overline{\omega_m(\mathbf{x}_0 + \mathbf{l}, t)\omega_n(\mathbf{x}_0, t)} = \int_{-\infty}^{\infty} \exp(i\mathbf{k}\cdot\mathbf{l})\Omega_{mn}(\mathbf{k}, t) d\mathbf{k}, \tag{12}$$

where m, n = 1, 2, 3, $\mathbf{k} = (k_1, k_2, k_3)$ is the 3-D wave vector of spectral components of the fields of the vortex velocity $\mathbf{w}(\mathbf{x}, t)$ and vorticity $\boldsymbol{\omega}(\mathbf{x}, t)$, $\Phi_{mn}(\mathbf{k}, t)$ is a spectral tensor of the Reynolds stress tensor of the vortex velocity field, and $\Omega_{mn}(\mathbf{k}, t)$ is a spectral tensor of the vorticity field. From the equation (6) the spectrum $\Phi_{mn}(\mathbf{k}, t)$ takes connection with the spectrum $\Omega_{mn}(\mathbf{k}, t)$ in the form

$$\Phi_{mn}(\mathbf{k}, t) = \frac{k_q k_g}{k^4}\Omega_{er}\varepsilon_{qem}\varepsilon_{grn} = \frac{1}{k^2}\left[\left(\delta_{mn} - \frac{k_m k_n}{k^2}\right)\Omega - \Omega_{mn}\right], \tag{13}$$

where $\Omega = \Omega_{\alpha\alpha}$ and $\varepsilon_{\alpha\beta\gamma}$ is the antisymmetrical unit third-order tensor. Last equations can be easy converted to find $\Omega_{mn}(\mathbf{k}, t)$ in respect on $\Phi_{mn}(\mathbf{k}, t)$, and, in result, we find the following expressions:

$$\Omega_{mn}(\mathbf{k}, t) = k^2\left[\left(\delta_{mn} - \frac{k_m k_n}{k^2}\right)E - \Phi_{mn}\right], \quad \Omega = k^2 E, \quad E = \Phi_{\alpha\alpha}, \tag{14}$$

which are well known from the theory of homogeneous turbulence [*Batchelor*, 1953]. Transforming (14) from the $\mathbf{k}$ space to $\mathbf{l}$ space, yields constraint equations for correlation tensors $R_{mn}(\mathbf{l},\mathbf{t})$ and $\Xi_{mn}(\mathbf{l},\mathbf{t})$ [*Batchelor*, 1953]

$$\Delta R_{mn} + \frac{\partial^2 R}{\partial l_m \partial l_n} = \Xi_{mn} - \delta_{mn}\Xi, \quad \Delta R = -\Xi, \quad R = R_{\alpha\alpha}, \quad \Xi = \Xi_{\alpha\alpha}. \tag{15}$$

[14] This determines the Reynolds tensor versus the correlation tensor of vorticity field as a solution of equation (15) at $\mathbf{l} = 0$.

[15] Hence, the initial small vortex perturbations always exist in the upper ocean, and might be enhanced by the wave motions. Then the Reynolds stresses, which provide the vertical transport of the momentum, are expected in accordance with the equations (13)–(15).

2.2. Vortex Instability

[16] We consider the simplest case of basic motion, the linear potential surface wave on the deep water [*Landau and Lifshitz*, 1987]

$$\begin{aligned}
&\varphi(x_1, x_3, t) = -\varphi_0 \exp(px_3)\,\mathrm{Cos}\vartheta, \quad p = (p_1, 0, 0), \quad p = p_1,\\
&v_1(x_1, x_3, t) = p\varphi_0 \exp(px_3)\,\mathrm{Sin}\vartheta,\\
&v_3(x_1, x_3, t) = -p\varphi_0 \exp(px_3)\,\mathrm{Cos}\vartheta,\\
&\eta(x_1, t) = a\,\mathrm{Sin}\vartheta, \quad \vartheta = p_1 x_1 - \sigma t, \quad a = \frac{\sigma\varphi_0}{g},
\end{aligned} \tag{16}$$

where $\varphi(x_1, x_3, t)$ is the wave potential, $\boldsymbol{p} = (p_1, 0, 0)$ is a wave number, η is a vertical displacement of the water surface, σ is frequency, a is the amplitude of the wave, $v_1(x_1, x_3, t)$ and $v_3(x_1, x_3, t)$ are correspondingly the horizontal and vertical components of the wavefield. We superimpose on (16) a small perturbation $\boldsymbol{w}$, which must be so small that the vorticity $\omega = curl\boldsymbol{u} = curl\boldsymbol{w}$ and the resulting velocity $\boldsymbol{u} = \boldsymbol{v} + \boldsymbol{w}$ do not violate equation (16). Using the equation (10) for $\rho' = 0$ and assumptions $|\mathbf{v}| \gg |\mathbf{w}|$, $|\partial_{x_m} v_n + \partial_{x_n} v_m| \gg |\partial_{x_m} w_n + \partial_{x_n} w_m|$, the last inequality is used here because the first term in the right hand side of

(10) is $\omega_m \partial_{x_m} u_n = 2^{-1}\omega_m(\partial_{x_m} u_n + \partial_{x_n} u_m)$), the equation for $\boldsymbol{\omega}$ takes the following form [*Batchelor*, 1967; *Landau and Lifshitz*, 1987; *Saffman*, 1992]:

$$\frac{d\boldsymbol{\omega}}{dt} = (\boldsymbol{\omega}\cdot\nabla)\mathbf{v}\,, \quad \boldsymbol{\omega}(x_0, t=0) = \boldsymbol{\omega}_0, \tag{17}$$

where $\boldsymbol{\omega}_0$ is the initial vorticity of the liquid particle that has the Lagrangian coordinate $\mathbf{x}_0(t=0)$. The vorticity $\boldsymbol{\omega}$ in (17) is associated with the fluid particle which moves with the wave velocity $\boldsymbol{v}$ (because $|\mathbf{u}|\approx|\mathbf{v}|\gg|\mathbf{w}|$) and therefore $\boldsymbol{\omega}$ is a function of the time t and the initial position $\boldsymbol{x}_0$. The right hand side of the equation (17) is also associated with the same fluid particle and therefore is a function of the time t and the initial position $\boldsymbol{x}_0$. Thus $\boldsymbol{\omega}$ is the vorticity of fluid particle moving on the trajectory of the potential motion. Under this assumption the equation (17) is the Lagrangian form and describes the evolution of small vortex disturbances in the coordinate system moving with the wave velocity $\mathbf{v}$. Expressions in (16), as it is well known, conform to that the fluid particles describe circles about the equilibrium point (x_{01}, x_{03}), and their motion can be written in the form [*Landau and Lifshitz*, 1987]

$$\begin{gathered}\ell_1 = x_1 - x_{01} = \ell_0\,\mathrm{Cos}\Theta, \quad \ell_3 = x_3 - x_{03} = \ell_0\,\mathrm{Sin}\Theta,\\ \ell_0 = a\exp(px_{03})\,, \quad \Theta = px_{01} - \sigma t\,.\end{gathered} \tag{18}$$

[17] From (16) the Euler's derivatives $\partial_{x_m} v_n$ are

$$\|\,\partial_{x_m} v_n\,\| = [\sigma ap\exp(px_3)]\left\|\begin{matrix}\mathrm{Cos}\theta & \mathrm{Sin}\theta\\ \mathrm{Sin}\theta & -\mathrm{Cos}\theta\end{matrix}\right\|. \tag{19}$$

[18] After substituting (18) in (19) the quantities $\partial_{x_m} v_n$ can be written in the form

$$\partial_{x_m} v_n = [\sigma ap\exp(px_{03} + p\ell_3)]\left\|\begin{matrix}\mathrm{Cos}(\Theta + p\ell_1) & \mathrm{Sin}(\Theta + p\ell_1)\\ \mathrm{Sin}(\Theta + p\ell_1) & -\mathrm{Cos}(\Theta + p\ell_1)\end{matrix}\right\|. \tag{20}$$

[19] The displacements $p\ell_1$ and $p\ell_3$ are small, $(p\ell_1, p\ell_3) \ll 1$. Expanding the functions $\exp(p\ell_3)$, $\mathrm{Cos}(p\ell_1)$ and $\mathrm{Sin}(p\ell_1)$ into power series and substituting into (17), (20), and keeping terms to second order in $k\ell_0$ reduces the equation (17) to the form

$$\begin{gathered}\frac{d\omega_1}{dt} = p\ell_0\sigma[\omega_1\,\mathrm{Cos}\Theta + \omega_3(p\ell_0 + \mathrm{Sin}\Theta)],\\ \frac{d\omega_2}{dt} = 0,\\ \frac{d\omega_3}{dt} = p\ell_0\sigma[\omega_1(p\ell_0 + \mathrm{Sin}\Theta) - \omega_3\,\mathrm{Cos}\Theta].\end{gathered} \tag{21}$$

[20] The equation set (21) is the basis for studying the vortex instability of the basic state (16). There is the trivial consequence: the simple harmonic plane surface wave does not interact with the vortex component that is perpendicular to the wave plane. In the wave case (21) it is the vortex component ω_2. It can be seen from (21) that the wave action on the vortex field vanishes in the large depth ($-x_{03} \gg k^{-1}$). Introduce a generalization of equation set (21), that is,

$$\begin{aligned}\frac{d\omega_1}{dt} &= \varepsilon_1\sigma[\omega_1\,\mathrm{Cos}\Theta + \omega_3(\varepsilon_2 + \mathrm{Sin}\Theta)],\\ \frac{d\omega_3}{dt} &= \varepsilon_1\sigma[\omega_1(\varepsilon_2 + \mathrm{Sin}\Theta) - \omega_3\,\mathrm{Cos}\Theta],\end{aligned} \tag{22}$$

where ε_1 and ε_2 are the dimensionless parameters. The condition $\varepsilon_1 = \varepsilon_2 = \varepsilon_0 = ap\exp(px_{03})$ reduces equation (22) to equation (21). Now the problem of vortex instability of the surface wave has been reduced to the problem of finding the stability region of the equation (22) with respect to the parameters ε_1 and ε_2. If a stable region exists and the line $\varepsilon_1 = \varepsilon_2$ intersects it, then the values of the parameter ε_0, which belong to the stable region in the (ε_1, ε_2) plane, give stable solutions of equation (21). In the opposite case, if the line $\varepsilon_1 = \varepsilon_2$ does never intersect the stable region, then solutions of equation (21) with any values of the parameter ε_0 are unstable.

[21] Introduce the complex vortex Ω and dimensionless time τ

$$\Omega = \omega_1 + i\omega_3\,, \quad \Omega^* = \omega_1 - i\omega_3\,, \quad \tau = \sigma t\,. \tag{23}$$

[22] After substituting (23) into equation set (22) we get

$$\begin{aligned}\frac{d\Omega}{d\tau} &= \varepsilon_1[i\varepsilon_2 + \exp(i\Theta)]\Omega^*,\\ \frac{d\Omega^*}{d\tau} &= \varepsilon_1[-i\varepsilon_2 + \exp(-i\Theta)]\Omega,\end{aligned} \tag{24}$$

for which initial conditions are $\Omega(0) = \omega_1(0) + i\omega_3(0)$ and $\Omega^*(0) = \omega_1(0) - i\omega_3(0)$. Eliminating Ω^* from (30) gives the second-order equation for Ω

$$\frac{d^2\Omega}{d\tau^2} + \frac{i\exp(i\Theta)}{i\varepsilon_2 + \exp(i\Theta)}\frac{d\Omega}{d\tau} - \varepsilon_1^2\left(1 + 2\varepsilon_2\,\mathrm{Sin}\Theta + \varepsilon_2^2\right)\Omega = 0, \tag{25}$$

with initial conditions

$$\Omega(0) = \Omega_0\,,\ \frac{d\Omega(0)}{d\tau} = \varepsilon_1[i\varepsilon_2 + \exp(i\Theta_0)]\Omega^*(0)\,,\ \Theta(0) = \Theta_0 = px_{01}\,. \tag{26}$$

[23] If the parameter ε_2 is zero, then equation (25) and the initial conditions in (26) yield to

$$\begin{gathered}\frac{d^2\Omega}{d\tau^2} + i\frac{d\Omega}{d\tau} - \varepsilon_1^2\Omega = 0,\\ \Omega(0) = \Omega_0\,, \quad \frac{d\Omega(0)}{d\tau} = \varepsilon_1\exp(i\Theta_0)\Omega^*(0)\,, \quad \Theta_0 = px_{01}\,.\end{gathered} \tag{27}$$

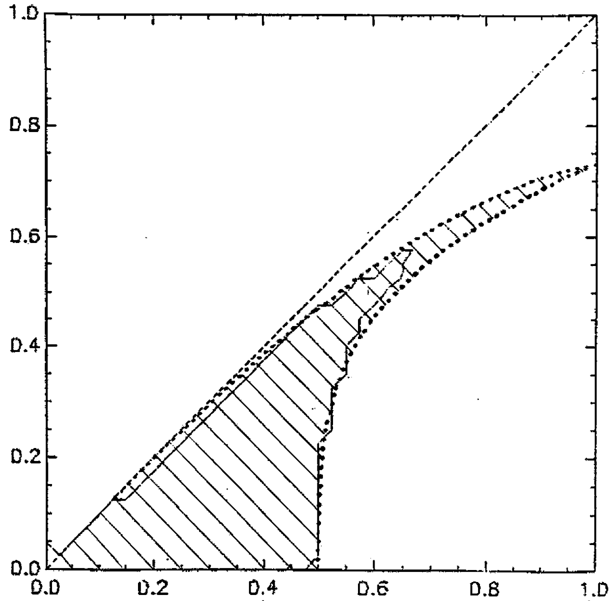


Figure 1. Diagram of stability of solutions of equation set (22) on the (ε_1, ε_2) plane. Here ε_1 is the horizontal axis and ε_2 is the vertical axis. F1 corresponds to $\varepsilon_2 = \varepsilon_1$; F2$_1$ corresponds to $\varepsilon_2 = \varepsilon_1 - 0.3\ \varepsilon_1^3 + 0.03\ \varepsilon_1^4$, $0 \le \varepsilon_1 \le 1$; F2$_2$ corresponds to $\varepsilon_2 = 0.9(\varepsilon_1 - 1/2)^{0.3}$, $1/2 \le \varepsilon_1 \le 1$; and F3, the dashed curve, corresponds to the numerical calculations using the Floquet theory [*Hale*, 1969].

[24] This equation is the first-order approximation with respect to the parameter $\varepsilon_0 = ap\ \exp(px_{03})$ of the basic problem (17). The solution of the problem (27) is

$$\begin{gathered}\Omega = \Omega_1 \exp(-i\nu_1\tau) + \Omega_2 \exp(i\nu_1\tau),\\ \nu_1 = 2^{-1}\left[1 + \left(1 - 4\varepsilon_1^2\right)^{1/2}\right], \quad \nu_2 = 2^{-1}\left[1 - \left(1 - 4\varepsilon_1^2\right)^{1/2}\right],\\ \Omega_1 = \frac{-\nu_2\Omega_0 + i\varepsilon_1 \exp(i\Theta_0)\Omega_0^*}{\sqrt{1 - 4\varepsilon_1^2}},\\ \Omega_2 = \frac{\nu_1\Omega_0 - i\varepsilon_1 \exp(i\Theta_0)\Omega_0^*}{\sqrt{1 - 4\varepsilon_1^2}},\\ \Omega_1^* = -i\exp(-i\Theta_0)\nu_2\Omega_2/\varepsilon_1, \quad \Omega_2^* = -i\exp(-i\Theta_0)\nu_1\Omega_1/\varepsilon_1.\end{gathered} \tag{28}$$

[25] Thus from this first approximate solution there are stable oscillations with two natural frequencies $\nu_1\sigma$ and $\nu_2\sigma$ if the parameter ε_1 is in the range $0 < \varepsilon_1 < 1/2$.

[26] For the limiting waves, $\varepsilon_1 = 1/2$ and $\nu_1 = \nu_2 = 1/2$, the initial vortex disturbance is linearly unstable. In this case the solution of equation (27) is

$$\Omega = \left\{\Omega_0 + 0.5\left[i\Omega_0 + \exp(i\Theta_0)\Omega_0^*\right]\tau\right\} \exp(-i\tau/2). \tag{29}$$

[27] The frequency of the vorticity oscillation is half of the wave frequency, and the amplitude of the oscillations is a linear function of the time τ. The solutions of equation (27) will be exponentially unstable for $\varepsilon_1 > 1/2$. Thus the first approximation already shows the wave effects on the vorticity field.

[28] Now we shall study the stability of the solutions of the system equation set (22). The analytical solution apparently cannot be found for arbitrary parameters ε_1 and ε_2. So, to solve this problem we have used a numerical calculation. The stability region has been calculated by two methods. The first one is a calculation of the functions $\omega_1(t)$ and $\omega_3(t)$ from (22) by the Runge-Kutta method using different values of the phase $\Theta_0(\Theta_0 = n\pi/6$, $n = 0, 1, \ldots, 6)$,different values of the parameters ε_1 and ε_2 of the domain $[0 < \varepsilon_1 \le 1, 0 < \varepsilon_2 \le 1]$, and for the initial conditions: $\omega_1(0) = \omega_3(0) = \omega_0$; $\omega_1(0) = \omega_0$, $\omega_3(0) = 0$; $\omega_1(0) = 0$, $\omega_3(0) = \omega_0$.

[29] The following quantities were taken into account:

$$\begin{gathered}\bar{f} = \frac{\varepsilon_1}{2n}\int_{-n/\varepsilon_1}^{n/\varepsilon_1} f(\tau + \tau')d\tau', \quad f_m = \max f(\tau + \tau'), |\tau'| \le n/\varepsilon_1, f\\ = \left\{|\omega_1|, \quad |\omega_3|, \quad |\omega|^2 = \omega_1^2 + \omega_3^2\right\}.\end{gathered} \tag{30}$$

[30] The quantities $\bar{f}$ and f_m were calculated at the time $\tau_k = (1 + 2k)n/\varepsilon_1$, for integer n and k. The solution was considered as stable if all the quantities $\bar{f}$ and f_m were such that the differences $\left|\bar{f}(\tau) - \bar{f}(\tau_k)\right|$ and $|f_m(\tau_{k+1}) - f_m(\tau_k)|$ were less than Δ while k increases. The quantities ε_1, ε_2, n, k, Δ have been taken as follows: $(\varepsilon_1, \varepsilon_2) = i\Delta\varepsilon$, $\Delta\varepsilon = 0.1$, $i = 1, \ldots, 10$, $k \ge 50$, $\Delta = 10^{-3} \div 10^{-4}$. The domain of the small values ε_1 and ε_2 (ε_1, $\varepsilon_2 \le 0.1$) was calculated with variable step $\Delta\varepsilon$ and such that min(ε_1, ε_2) was $\approx 10^{-4}$. From this calculation the stability region is

$$\begin{gathered}\varepsilon_1 = 0, 0 \le \varepsilon_2 \le 1,\\ 0 \le \varepsilon_1 \le 1/2, \quad 0 \le \varepsilon_2 \le \varepsilon_1 - 0.3\varepsilon_1^3 + 0.03\varepsilon_1^4,\\ 1/2 \le \varepsilon_1 \le 1, \quad 0.9(\varepsilon_1 - 1/2)^{0.3} \le \varepsilon_2 \le \varepsilon_1 - 0.3\varepsilon_1^3 + 0.03\varepsilon_1^4.\end{gathered} \tag{31}$$

[31] The stability domain is shown in Figure 1 by the shaded region bounded by Line F2$_1$ (F2$_1$ = $\varepsilon_2 = \varepsilon_1 - 0.3\ \varepsilon_1^3 + 0.03\ \varepsilon_1^4$, $0 \le \varepsilon_1 \le 1$) and Line F2$_2$ (F2$_2$ = $\varepsilon_2 = 0.9(\varepsilon_1 - 1/2)^{0.3}$, $1/2 \le \varepsilon_1 \le 1$). The function F1 = $\varepsilon_2 = \varepsilon_1 = \varepsilon_0$, Line F1, does not intersect the upper border of the stability region (31), Line F2$_1$.

[32] The other method of calculations the stability region bases on the numerical calculation of stability indexes using Floquet theory [*Hale*, 1969]. The Floquet theory is the theory of linear ordinary differential equations with periodic coefficients which allows presenting the solution of such linear ordinary differential equations in the form that characterizes stability its solutions by the characteristic exponent, called as stability indexes in some sources. The evolution equations for the vorticity field, (22), are a coupled set of linear ordinary differential equations with a periodic coefficients matrix. Using Floquet theory the stability of solutions as function of the embedded parameters has been determined efficiently from numerical solutions. The stability results are shown in Figure 1 by the dashed curve Line F3. The function F1 = $\varepsilon_2 = \varepsilon_1 = \varepsilon_0$, Line F1, does not intersect the stability region bounded by the dashed curve Line F3.

[33] Because the evolution of small vortical disturbances obeys the equation set (21) that corresponds F1 = $\varepsilon_2 = \varepsilon_1 =$

ε_0 and both methods of calculation the stability region of the solutions of the equation (22) agree very well, we conclude that the initial small disturbances of the vorticity field grows in time for all values of parameter ε_0 of the range $0 < \varepsilon_0 \le 1$.

[34] Considering our case of the system of two linear differential equation set (22), its phase portrait represents a set of its solutions, plotted as parametric curves (with t as the parameter) on the Cartesian plane tracing the path of each particular solution ($\omega_1(t)$, $\omega_3(t)$, $0 \le t < \infty$). A phase portrait is a graphical tool to visualize how the solutions of a given system of differential equations would behave in the long run. Figure 2 shows the examples of phase portraits for different values of parameters ε_1, ε_2 (ε_1 = 0.16, 0.34, 0.5; ε_2 = 0, 0.08, 0.16, 0.17, 0.25, 0.34, 0.5) and initial conditions ($\omega_1(0) / \omega_0 = 1$, $\omega_3(0) = 0$; $\omega_1(0) = 0$, $\omega_3(0) / \omega_0 = 1$). The transition from the stable oscillations (28) to the linear instability (29) is shown by the portraits (ε_1 = 0.16, 0.34, 0.5; ε_2 = 0). The portraits with (ε_1 = 0.16, 0.34, 0.5; ε_2 = 0.08, 0.17, 0.25) show the behavior of solutions of the system equation set (22) for the stable values of parameters ε_1, ε_2 of (31). The portraits ($\varepsilon_1 = \varepsilon_2 = \varepsilon_0$ = 0.16, 0.34, 0.5) show the evolution of the unstable solutions along the Line F1in Figure 1. Note that in the case of the system equation set (21), ($\varepsilon_1 = \varepsilon_2 = \varepsilon_0$), the vortex disturbances are analogous to Langmuir circulation, have the mean growth in the direction of wave propagation.

3. Turbulence of a Potential Surface Wave

3.1. Novikov's Turbulence by the Potential Wave

[35] Novikov's turbulence is the random vortex motions at very small spatial scales where the molecular viscosity strongly affects the motion and causes viscous dissipation [*Monin and Yaglom*, 1987]. If $|\mathbf{w}| \ll |\mathbf{v}|$ in (5) then the equation for the potential is solved independently of the vorticity equation because in this case the influence of the vortex component on the potential can be ignored. Then the velocity field $\boldsymbol{u}$ in (5) becomes $|\boldsymbol{u}| \approx |\boldsymbol{v}|$ and the vorticity field obeys equation (17) with the viscous term [*Saffman*, 1992]

$$\frac{d\boldsymbol{\omega}}{dt} = (\boldsymbol{\omega}\nabla)\mathbf{v} + \nu\Delta\boldsymbol{\omega}, \tag{32}$$

where ν is the kinematic molecular viscosity.

[36] Let us now consider a potential velocity field in which we are given vorticity perturbations, whose scales ℓ are small in comparison with the Kolmogorov scale$\eta = (\nu^3/\varepsilon)^{1/4}$, where ε is the dissipation rate of the turbulent kinetic energy. Within a spatial region of diameter ℓ the field $\mathbf{v}(\mathbf{x}, t)$ can be expanded into a Taylor series with respect to the coordinates and approximated by a linear vector functions corresponding to the sum of the terms of the zero and first order in that expansion. The coefficients of the linear vector function obtained derivatives $\partial v_i/\partial x_j$, will vary in time and space. At distances $\ell \ll \eta$, however, the instantaneous values of these derivatives will be practically constant, and the velocity field within the volume of linear dimensions $\ell < \eta$ can at first approximation be regarded as linear: that is, $v_{0i}(\mathbf{x}_0 + \mathbf{r}, t) = v_{0i} + a_{ik}r_k$, where $v_{0i} = v_{0i}(\mathbf{x}_0, t)$, $a_{ik} = \partial v_i(\mathbf{x}_0, t)/\partial x_{0k}$, while $\mathbf{x}_0$ is a fixed point within the isolated volume of liquid. Since the field $\mathbf{v}$ is potential, the quantities a_{ik} form a symmetrical tensor characterizing the deformation of the isolated liquid particle in the directions of the main axes of deformation. Converting to a system of coordinates connected with the main axes of deformation reduces equation (32) to the following form:

$$\left(\frac{\partial}{\partial t} - \nu\Delta\right)\omega_j + \sum_{\ell=1}^{3} a_\ell x_\ell \frac{\partial \omega_j}{\partial x_\ell} - a_j\omega_j = 0, \tag{33}$$

where a_ℓ are the principal values of the tensor $a_{k\ell}$. This equation is like the one analyzed in *Monin and Yaglom* [1987, p. 425] for weak eddy perturbations of regular sinusoidal form and for arbitrary tensor $a_{k\ell}$. In our case the components of the tensor can be found from the solution of the equation for potential. For gravitational waves of small amplitude the tensor $a_{k\ell}$will take form [*Landau and Lifshitz*, 1987]

$$\begin{pmatrix} -pv_3 & 0 & pv_1 \\ 0 & 0 & 0 \\ pv_1 & 0 & pv_3 \end{pmatrix}, \tag{34}$$

where p is the wave number of the potential component and $\ell \ll p^{-1}$. Reducing (34) to diagonal form, we obtain the matrix

$$\begin{Vmatrix} p\sqrt{T} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -p\sqrt{T} \end{Vmatrix}, \tag{35}$$

where $T = v_1^2 + v_3^2$.

[37] Substituting (35) in Novikov's formula [see *Monin and Yaglom*, 1987, p. 431] for the three-dimensional turbulent kinetic energy spectrum$E(k)$, ($p \ll \eta \ll k$), which is the consequence of the solution of equation (33)

$$E(k) = C\frac{\varepsilon^{2/3}}{k^{5/3}}(k\eta)^{2\sigma-4/3}\exp\left\{-\alpha(k\eta)^2\right\}, \tag{36}$$

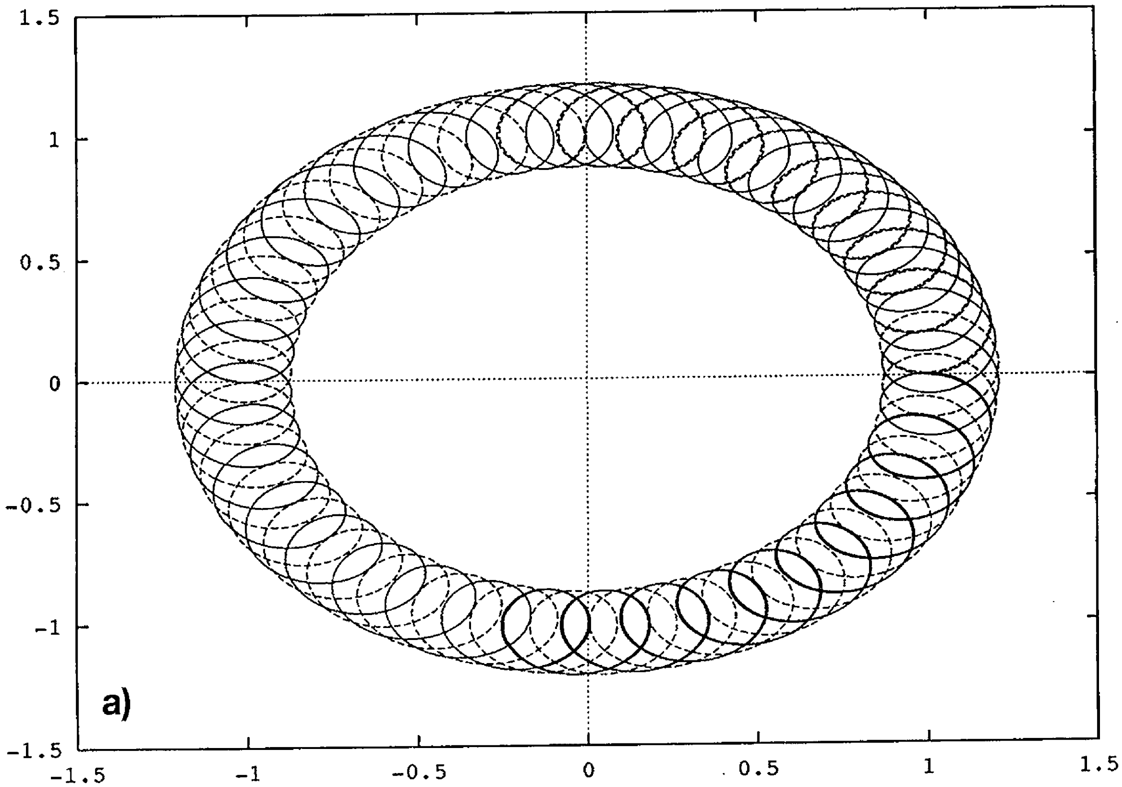
where $\sigma = -\frac{a_1}{a_3}$, $\alpha = -\frac{\sqrt{\varepsilon/\nu}}{a_3}$, we obtain $\sigma = 1$, $\alpha = -\frac{1}{p}\sqrt{\frac{\varepsilon}{\nu T}}$, and for dissipation due to the interaction of the vortex and potential components of the velocity field we shall have

$$\varepsilon = C^2\nu p^2 T, \quad \left(\int_0^\infty k^2 E(k)dk = \frac{\varepsilon}{2\nu}\right). \tag{37}$$

[38] Formulas (36) and (37) show that, first, short surface waves are better supporting eddy perturbations, since when $p \to 0$, $\alpha \to \infty$, and consequently the energy of eddy motion $E(k)$ will tend to zero; and, second, the small vortex perturbation causes the viscous dissipation of the potential wave.

3.2. Kolmogorov's Turbulence of the Potential Wave

[39] It is thus clear that the wave motion of a liquid can be a source of turbulence. As it has been discussed here, the wave-induced turbulence can be considered as developed turbulence with well distinguishable range of the turbulent wave numbers k where turbulence well follows to the Kolmogorov's self-similarity law which includes the Kolmogorov's



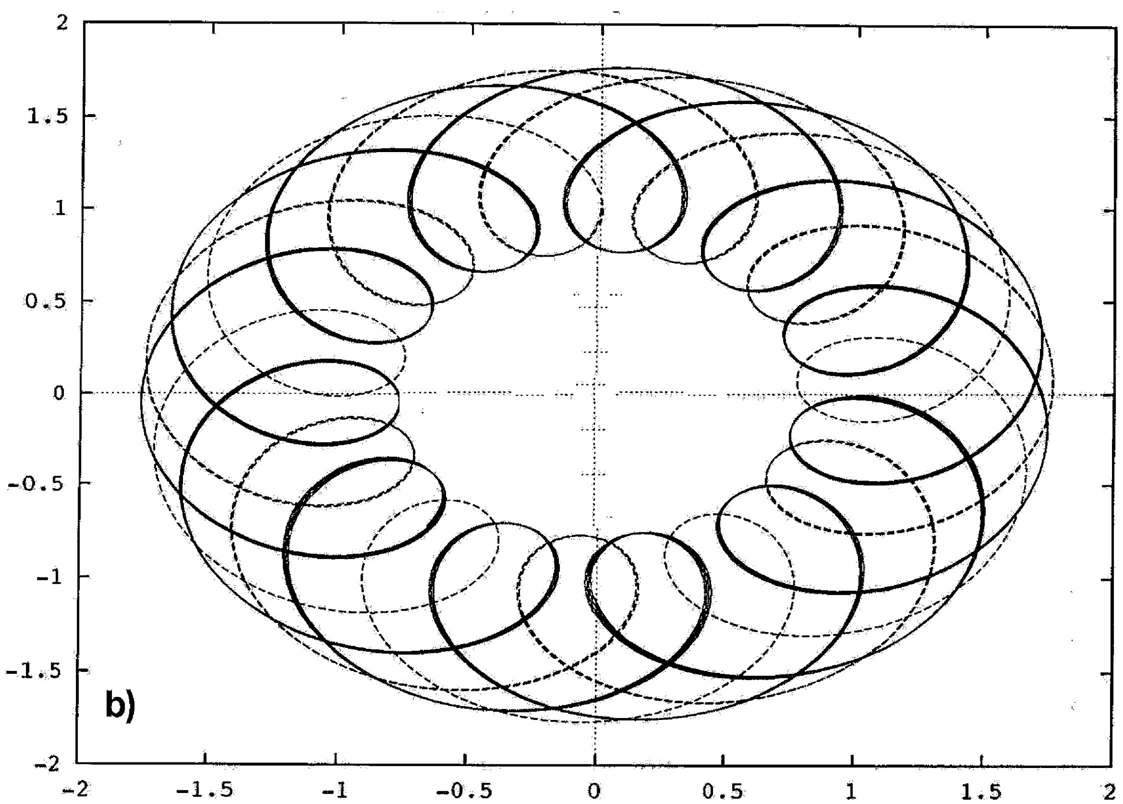
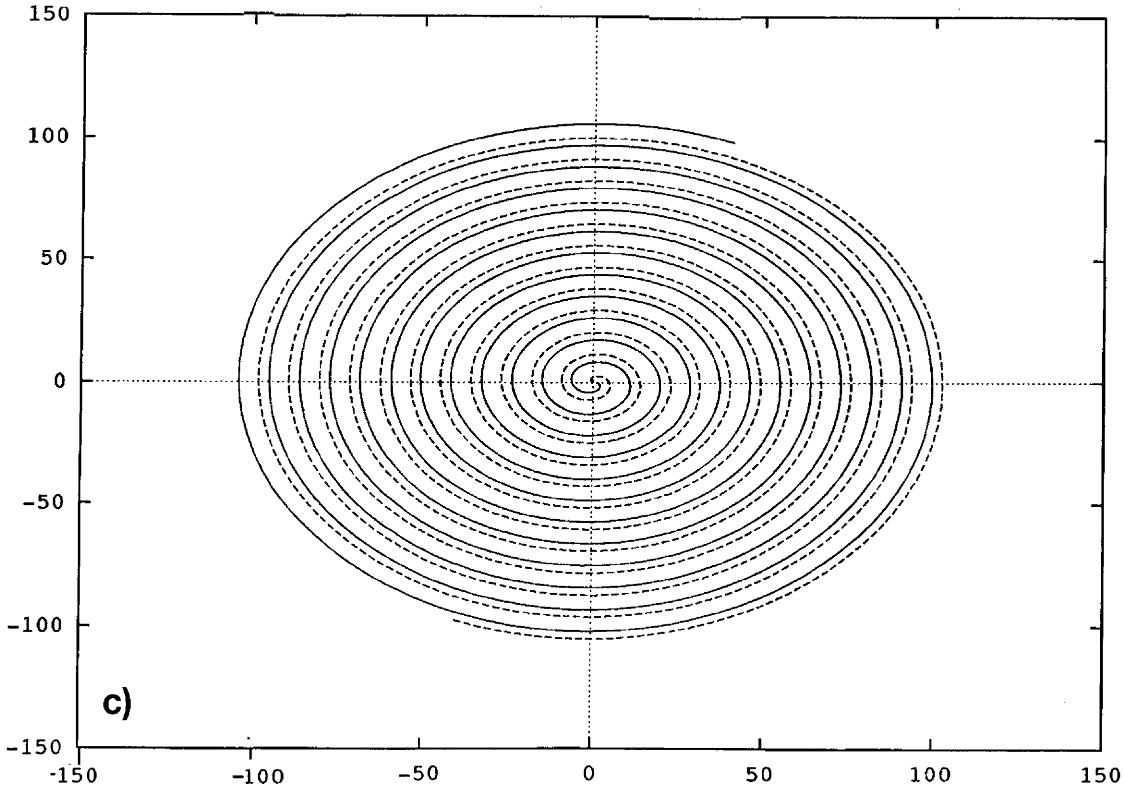
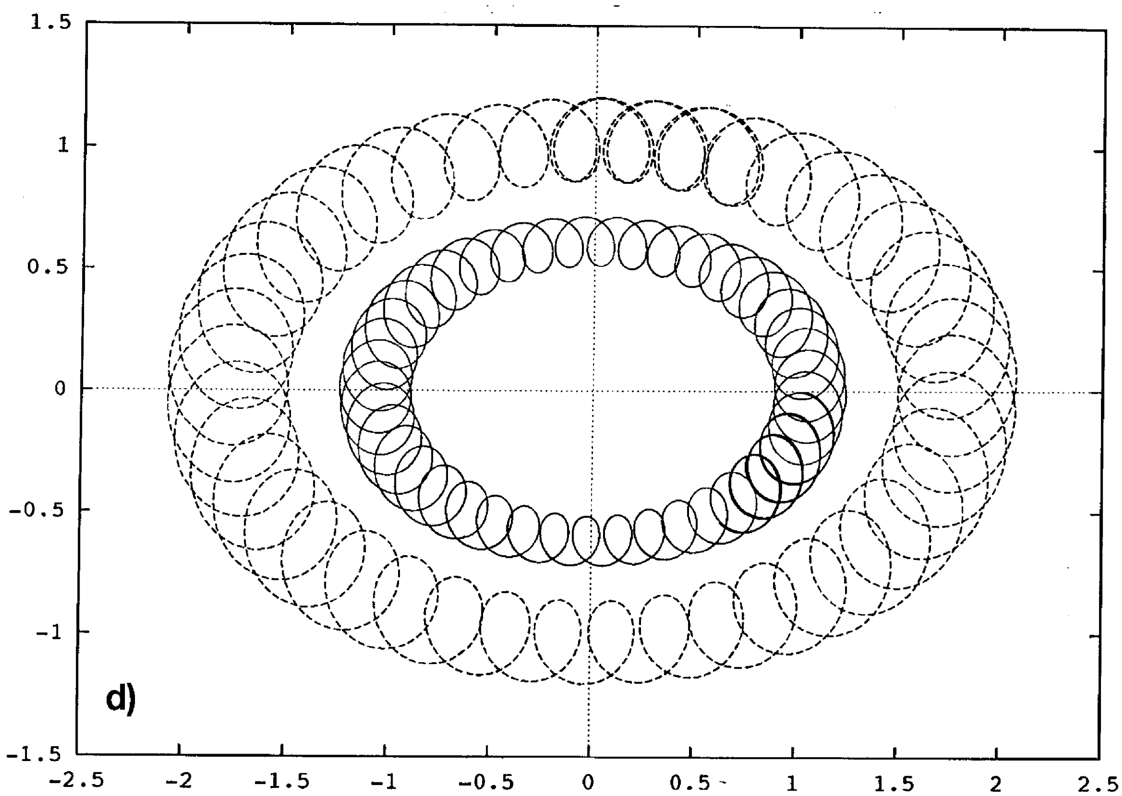


Figure 2. The examples of phase portraits of solutions of equation set (22) on the (ω_1, ω_2) plane for different values of the parameters ε_1 and ε_2: (a) $\varepsilon_1 = 0.16$, $\varepsilon_2 = 0$; (b) $\varepsilon_1 = 0.34$, $\varepsilon_2 = 0$; (c) $\varepsilon_1 = 0.5$, $\varepsilon_2 = 0$; (d) $\varepsilon_1 = 0.16$, $\varepsilon_2 = 0.08$; (e) $\varepsilon_1 = 0.34$, $\varepsilon_2 = 0.17$; (f) $\varepsilon_1 = 0.5$, $\varepsilon_2 = 0.25$; (g) $\varepsilon_1 = 0.16$, $\varepsilon_2 = 0.16$; (h) $\varepsilon_1 = 0.34$, $\varepsilon_2 = 0.34$; and (i) $\varepsilon_1 = 0.5$, $\varepsilon_2 = 0.5$. Here ω_1 is the horizontal axis and ω_2 is the vertical axis. The solid curve corresponds to $\omega_1(0) = 1$, $\omega_3(0) = 0$ and dashed curve corresponds to $\omega_1(0) = 0$, $\omega_3(0) = 1$.





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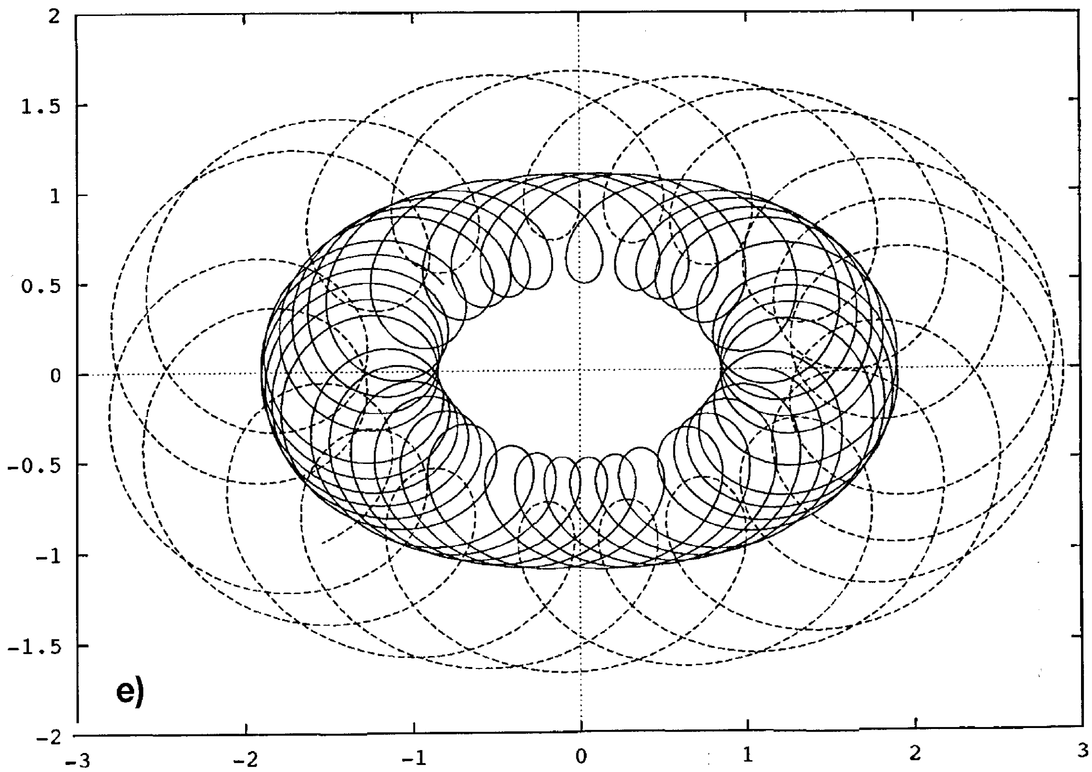


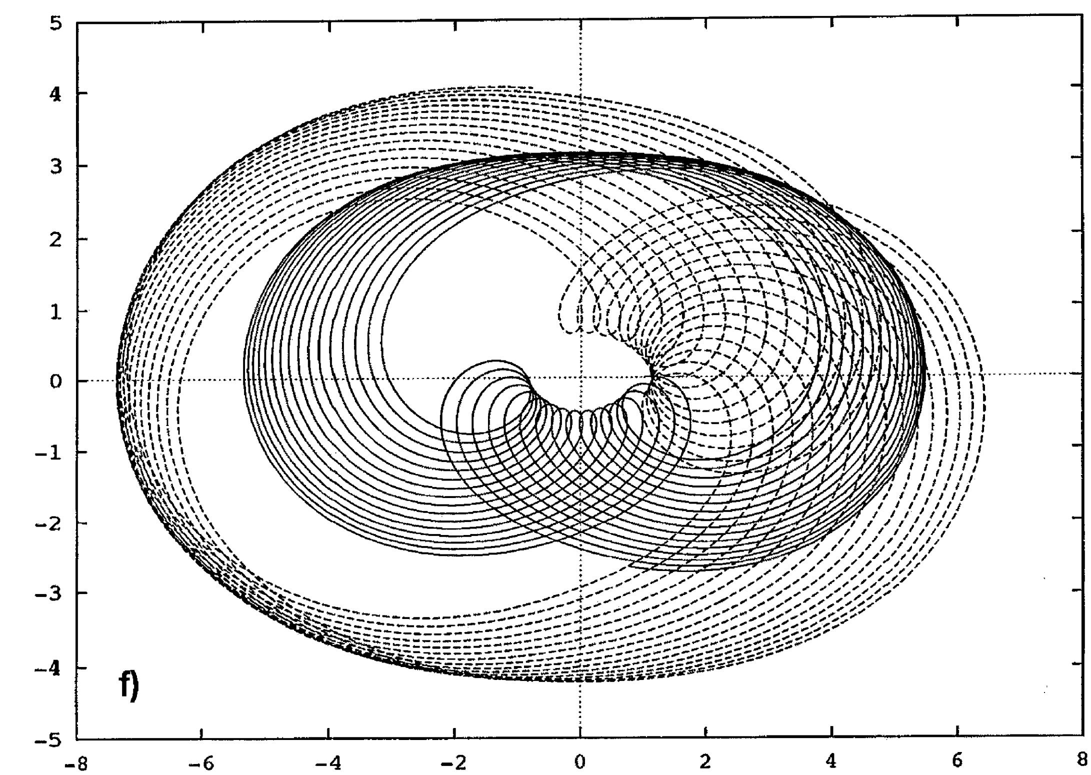
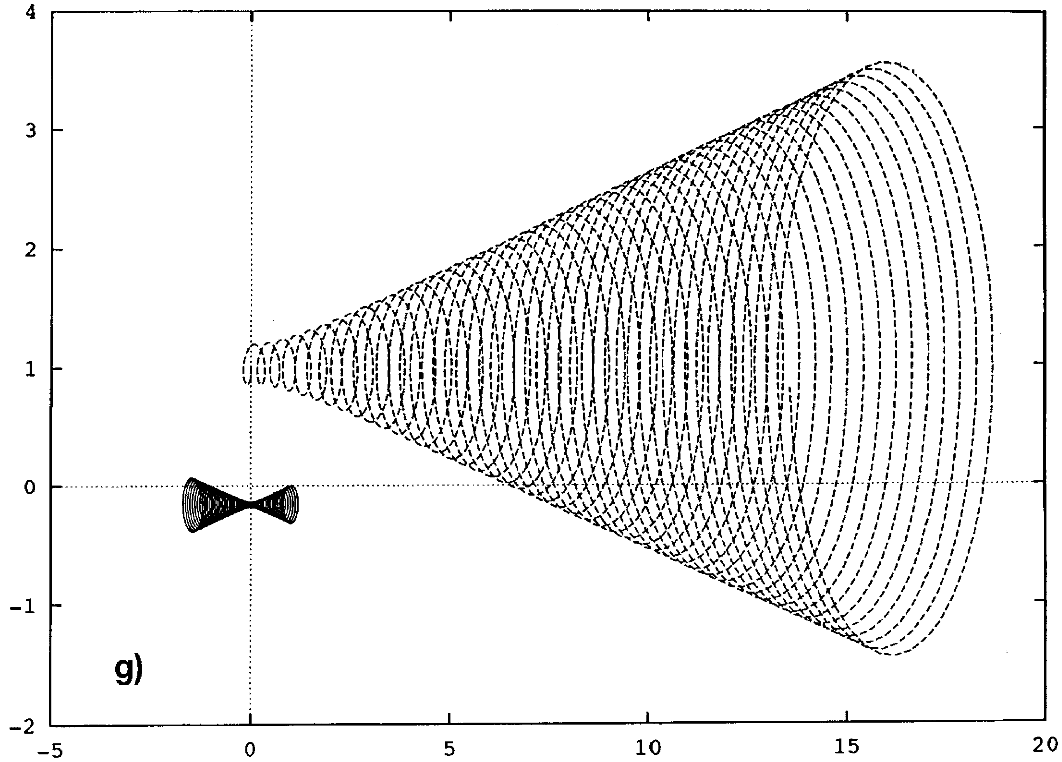


Figure 2. (continued)

law "–5/3." The experiments provide evidence of this. The presence of a locally isotropic region of turbulence ($k \geq 0.1$ cm^{-1}) allows certain conclusions relative to the dissipation of energy in waves [*Benilov*, 1969b; *Haskind*, 1973]. On the assumption of local isotropy of turbulence we can have the following estimate of the dissipation ε:

$$\varepsilon \sim \frac{\Delta u^3}{\ell}, \tag{38}$$



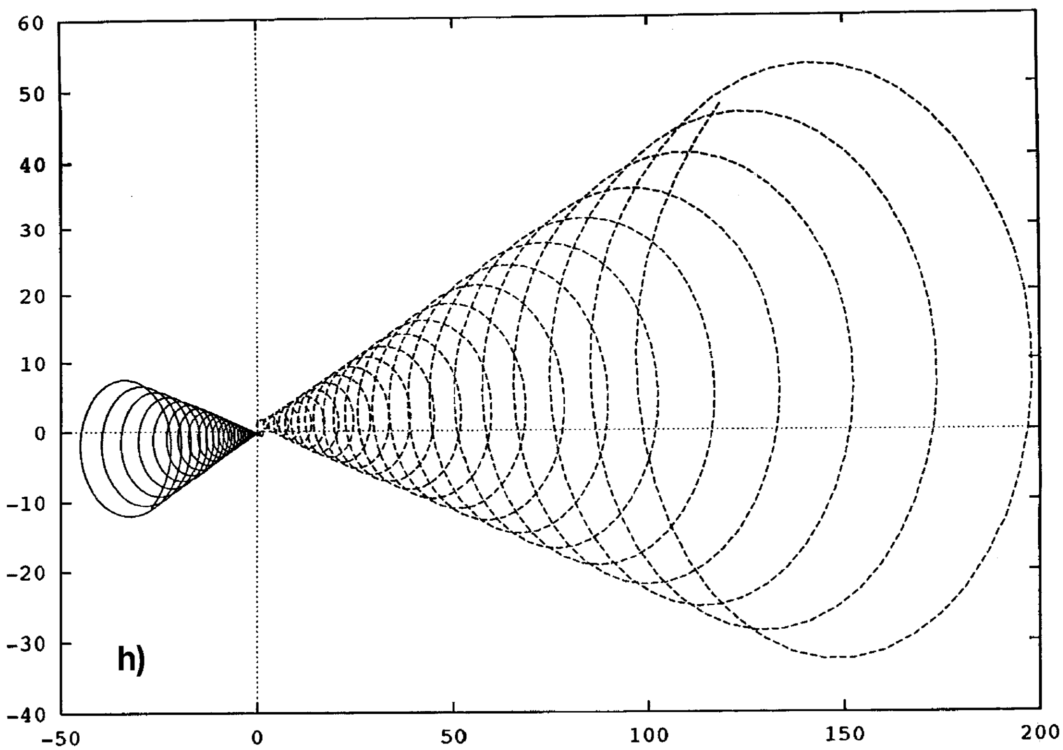


Figure 2. (continued)

where Δu is the characteristic scale of the velocity $\boldsymbol{u}$ variation throughout a distance equaled to the external scale of turbulence ℓ. Whereas the wave motion dominates in the ocean upper layer the velocity $\boldsymbol{u}$ has the same order of magnitude as the wave velocity $\boldsymbol{v}$, $|\boldsymbol{u}| \sim |\boldsymbol{v}|$. That leads to the scaling $\Delta \boldsymbol{u} \sim |\boldsymbol{v}| \sim \sqrt{T}$, where $T = v_i v_i$, and to the scaling of the external scale of turbulence ℓ which can be taken the order of magnitude

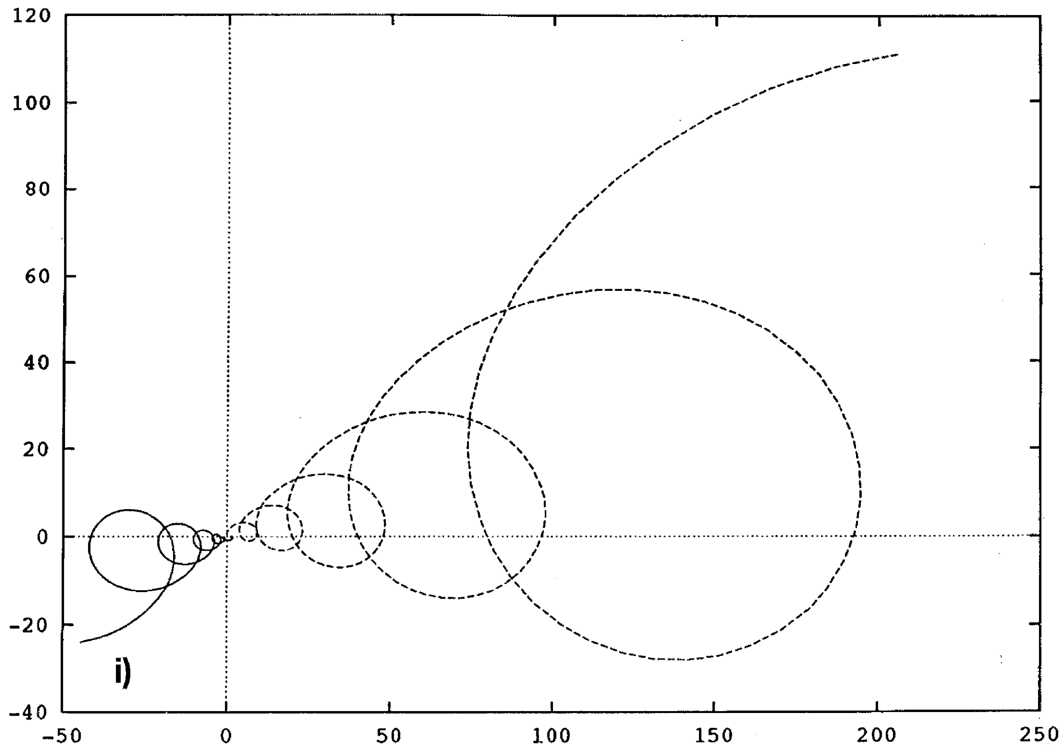


Figure 2. (continued)

of the wavelength $\lambda = 2\pi/p$. The dissipation rate (38) reduces to the following expression:

$$\varepsilon \sim \frac{T^{3/2}}{\lambda}. \tag{39}$$

[40] Also, the wave motion can be characterized by the wavelength $\lambda = 2\pi/p$, the wave height h and the group velocity U of the wavefield. Then the hypothesis of local self-similarity can be written in the form

$$\varepsilon = \varepsilon(z, h, \lambda, U) = \frac{U^3}{\lambda}\tilde{\varepsilon}\left(\frac{z}{\lambda}, \frac{h}{\lambda}\right), \tag{40}$$

where $\tilde{\varepsilon}$ is some dimensionless function of the arguments z/λ and h/λ. Let us expand $\tilde{\varepsilon}$ in the power series in h/λ

$$\varepsilon = \frac{U^3}{\lambda}\sum_{i=0}^{i=\infty}\tilde{\varepsilon}_i(z/\lambda)\left(\frac{h}{\lambda}\right)^{q+i}. \tag{41}$$

[41] When $h/\lambda \ll 1$ we can limit ourselves to the first term of the expansion (41) that corresponds to gravitational surface waves of small amplitude

$$\varepsilon = \frac{U^3}{\lambda}\left(\frac{h}{\lambda}\right)^q \tilde{\varepsilon}_0(z/\lambda). \tag{42}$$

[42] We can easily find the parameter q and the function $\tilde{\varepsilon}_0(z/\lambda)$ by using the known solution for such waves

$$v_x = -h\omega\sin(px - \omega t)e^{pz}, v_y = 0, v_x = h\omega\cos(px - \omega t)e^{pz}, z \le 0,$$
$$\omega = \sqrt{gp} = \sqrt{\frac{2\pi g}{\lambda}}, U = \frac{1}{2}\sqrt{\frac{g}{p}} = \frac{1}{2}\sqrt{\frac{g\lambda}{2\pi}}, p = \frac{2\pi}{\lambda}. \tag{43}$$

[43] Substituting (43) in (39) and taking (39) as equality, we obtain

$$\varepsilon = \gamma_1\frac{1}{\lambda}\left(\frac{g\lambda}{2\pi}\right)^{3/2}\left(\frac{h}{\lambda}\right)^3\exp\left(\frac{6\pi z}{\lambda}\right)$$
$$= \gamma\frac{U^3}{\lambda}\left(\frac{h}{\lambda}\right)^3\exp\left(\frac{6\pi z}{\lambda}\right). \tag{44}$$

[44] Hence $q = 3$ and $\tilde{\varepsilon}_0 = \gamma\exp(6\pi z/\lambda)$. The constant γ can be determined from measurements of the dissipation profile. The estimate in *Benilov* [1969b] gives $\gamma \approx 1$.

[45] The obtained formula, equation (44), indicates a dependence of ε on the parameters of wave motion and explains the rapid diminution of ε with depth (faster than $1/z$). Since the wave spectrum is fairly narrow, formula (44) can be applied for rough evaluation of ε using mean values of wave height and wavelength λ.

4. Conclusions

[46] 1. The turbulence (the vortex motion) of the upper ocean is nourished by the energy and momentum of the

surface waves (the potential motion). The statistical characteristics of the turbulence (turbulent kinetic energy, dissipation rate, Reynolds stresses) depend on the state of the ocean surface waves.

[47] 2. The vertical transport of momentum is induced by the vortex component of the fluctuations of the velocity field, and, possibly, by the interaction between both vortex and potential motions. The Reynolds tensor is a linear function of the correlation tensor of vortex field. The initial small vortex perturbations always exist in the upper ocean because the molecular viscosity influences, especially near the free surface, and the fluctuations of the seawater density. The horizontal inhomogeneities of the seawater density produce the vortex field even if the initial vorticity was zero and the initial flow was the potential flow. The linear problem of the stability of vortex disturbances in the velocity field of potential linear surface waves is reduced to a coupled set of linear ordinary differential equations of the first order with periodic coefficients. The analysis shows that the initial small disturbances of the vorticity field grow in time under the influences of the wave motions. The vortex disturbances are analogous to Langmuir circulation, and they have the growing in time the mean component in the direction of wave propagation.

[48] 3. The potential surface wave produces the small-scale turbulence, Novikov's turbulence, from the initial small vortex perturbations that finally causes the viscous dissipation of the potential wave. The wave-induced turbulence can be considered as developed turbulence with well distinguishable range of the turbulent wave numbers k where the turbulence obeys to the law of the Kolmogorov's self-similarity.

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