A state-space model for ocean drifter motions dominated by inertial oscillations

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[1] Coincident ocean drifter position and surface wind time series observed on hourly timescales are used to estimate upper ocean dissipation and atmosphere-ocean coupling coefficients in the Labrador Sea. A discrete-process model based on finite differences is used to regress ocean accelerations on ocean velocity estimates but fails because errors in the discrete approximations for the ocean velocities are biased and accumulate over time. Model identification is achieved by fitting a stochastic differential equation model based on classical upper ocean physics to the drifter data via the Kalman filter. Ocean parameters are shown to be nonidentifiable in a direct application to the Labrador Sea data when the known Coriolis parameter is not identified by the model. To address this, the ocean parameters are estimated in an empirical sequence. Data from the Ocean Storms experiment are used to estimate ocean dissipation in isolation from complexities introduced by strong and variable winds. Given a realistic estimate of the ocean dissipation, a second application in the Labrador Sea successfully estimates atmosphereocean coupling coefficients and reproduces the Coriolis parameter. Model assessments demonstrate the robustness of the parameter estimates. The model parameter estimates are discussed in comparison with Ekman theory and results from analyses of the global ocean surface drifter data set.

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1. Introduction

[2] Data from modern upper ocean drifters provide unprecedented volumes of high-quality information that is well suited for analyses using advanced statistical methodology. This work exploits drifter observations of position (leading to upper ocean velocity) and surface vector winds on hourly timescales in the Labrador Sea to deduce classical parameters of upper ocean physical balances extending back to the work of *Ekman* [1906]. To reduce uncertainties in estimates for atmosphere-ocean coupling and upper ocean dissipation effects from momentum convergence, a statespace model is identified in an empirical sequence, adding physical and statistical sophistication only as necessary to fit the drifter data. Our work is primarily methodological and demonstrates application of well-developed statistical meth-

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ods to a newly abundant data set in an ocean setting that is realistically complex due in part to vigorous and highly variable wind forcing. An important oceanographic aspect of our work derives from the simultaneous observations of ocean currents and surface vector winds by the ocean drifters. With a precision not accessible in the absence of these upper ocean drifter data, the estimated parameters validate a classical, wind-driven upper ocean model.

[3] Starting with the Ocean Storms experiment in 1987, oceanographers have deployed large arrays of satellite located surface drifters to measure the circulation of the global upper ocean. These modern drifters are carefully constructed to be "Lagrangian"; that is, to follow horizontal water motion at some preselected shallow depth. A drifter typically consists of a small surface float that contains electronics, power and a satellite transmitter. The surface floatation is tethered to a large subsurface drogue, typically at 15 m depth [*Niiler*, 2001]. A variety of environmental sensors have been attached to upper ocean drifters, including instruments from which surface wind speed and wind direction can be determined. In the fall and winter of 1996-1997, time series of upper ocean drift and surface vector winds were obtained in the Labrador Sea [Milliff et al., 2003] from a deployment of 21 upper ocean drifters with a variety of sensor systems. Here we use a subset of six buoys, requiring that each buoy provides simultaneous position and wind measurements. These unique data present

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an opportunity to study, in a detail previously not attainable, the processes by which local winds drive near surface ocean currents in a wintry and wave-tossed sea.

[4] In his theory of local wind-driven ocean currents, Ekman [1906] postulates that, under steady winds, the principal horizontal momentum balance in the open ocean is between the Coriolis acceleration and the force due to the vertical convergence of the horizontal turbulent stresses caused by the action of the wind and waves. Ekman's theory implies that in the Northern Hemisphere, the surface ocean current vector orients 45° to the right of the surface wind stress vector; and deeper currents spiral further to the right with increasing depth. Ekman interpreted several observations of his time within the context of his theory to show that the strength of the current was proportional to the wind speed and inversely proportional to the square root of the Coriolis parameter. Recently, Ekman's theory of the wind-driven ocean has been validated by velocity observations from moored buoys [Weller, 1981; Price et al., 1987] and his theory of upper ocean current dependence on wind speed and the Coriolis parameter has been validated from drifter observations [Ralph and Niiler, 1999].

[5] In this paper, we incorporate the upper ocean horizontal momentum balance from Ekman dynamics in a statespace model for drifter position given time-varying surface winds. From this model we deduce upper ocean parameters of the Ekman balance and provide confidence intervals for the parameters of interest. Following *Pollard and Millard* [1970] and *Ralph and Niiler* [1999], we begin formulation of the physical model by considering the momentum balance in complex form:

$$\frac{d\mathbf{U}}{dt} + \mathbf{F}\mathbf{U} = -\mathbf{P} + \mathbf{A}\mathbf{W}.$$
 (1)

In (1), d/dt is the material time derivative following the drifter motion, $\mathbf{U} = u + \mathbf{i}v$ is the complex velocity vector for eastward component u and northward component v, measured at 15 m depth, and $\mathbf{F} = \gamma + \mathbf{i} f$ combines a dissipation term γ , here modeled as a Rayleigh friction, and the effects of Earth rotation through the Coriolis term $f = 2\Omega$ $\sin(\phi)$, defined for the angular rotation rate Ω and local latitude ϕ . **P** is a measure of the horizontal pressure gradient that is not correlated with the local surface winds. This term is negligibly small in the Labrador Sea case of our interest. In general, one can separate pressure gradient driven flows (i.e., geostrophic dynamics) from the wind-driven ocean response (i.e., Ekman dynamics) which is the dominant component in the regimes of interest here. The surface wind effect, which we show to be crucially important in the Labrador Sea, is modeled after the scaling and rotation effects described by Ralph and Niiler [1999] (see also Appendix C) as

$$\mathbf{AW} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} u^w \\ v^w \end{bmatrix}.$$
(2)

Here, $\mathbf{W} = [u^w, v^w]'$ is the surface wind vector, and $\mathbf{A} = (a_{ij})$ represents the air-sea coupling coefficients.

[6] The underlying physics of the model are appropriate for upper ocean regimes dominated by inertial currents [e.g., *Pollard and Millard*, 1970] (or see the pedagogic description by *Gill* [1982]). The classical notion of an inertial current arises in considering the response of a uniformly mixed upper ocean to an impulse forcing from, e.g., the sudden onset of a strong and uniform (in direction and magnitude) surface wind. In an inertial current response, parcels of upper ocean fluid traverse circular trajectories with dimensions proportional to wind strength, the local Coriolis parameter, and a local decay rate [e.g., see *Gill*, 1982, pp. 322–326]. In component form, our physical model equations are

$$\frac{du}{dt} - fv + \gamma u = a_{11}u^w + a_{12}v^w \tag{3}$$

$$\frac{dv}{dt} + fu + \gamma v = a_{21}u^w + a_{22}v^w.$$
 (4)

[7] Formally, the undetermined model parameters are: f, γ and a_{ij} . These are to be determined by fitting drifter position and wind vector data to the model equations. Since the Coriolis parameter is also knowable outside the model (i.e., it is a fixed function of the local latitude and Earth rotation rate), it provides a means of validating model approximations a posteriori. When the correct Coriolis term cannot be reproduced with sufficient certainty from the Labrador Sea data, such a posteriori validation is used to reject a model based on ocean-only variables (i.e., excluding terms in **AW**). Apparently, the ever-present and abruptly changing wind forcing obscures and confounds the information contained in the drifter data about f and γ .

[8] To address the confounding effects of the vigorous and variable wind forcing we take two steps. First, the ocean-only model is adapted to the well-known Ocean Storms experiment data [D'Asaro et al., 1995] when inertial currents were measured by ocean drifters in response to a single, strong surface-forcing event followed by calm winds. For this setting, a robust estimate for γ is obtained in a regime dominated by inertial currents. Second, given an appropriate estimate for γ , we return to the Labrador Sea drifter data and include explicit terms to couple the momentum forcing due to the vigorous and changeable surface winds in (3) and (4). Constraining f and γ , we now obtain robust estimates for a_{ij} , while accounting for measurement error and model uncertainty (noise terms). A final model verification is achieved when we compare the average rotational offset of the estimated surface currents with respect to the surface wind with similar estimates obtained in separate analyzes from the global ocean drifter data set [e.g., Ralph and Niiler, 1999].

[9] We note that there exists a rich recent literature on the assimilation of Lagrangian drifter data [e.g., *Özgökmen et al.*, 2000; *Castellari et al.*, 2001; *Molcard et al.*, 2003]. Using statistical interpolation techniques, this literature is generally concerned with deriving information about Eulerian model variables from Lagrangian observations, as well as with providing statistical information regarding the predictability of such flows [*Griffa et al.*, 2004; *Paldor et al.*, 2004; *Özgökmen et al.*, 2001]. To complement this work, we demonstrate statistical procedures for identification of the parameters of Lagrangian models that are observed at irregular times, and further provide uncertainty measures of the estimated parameters. Our modeling approach is similar to that of *Özgökmen et al.* [2001, 2003]



Figure 1. Interpolated buoy path (degrees latitude/longitude). Location measurements $\{x_{obs}(t_i), y_{obs}(t_i)\}$, i = 1, ..., 147, are denoted by asterisks.

who propose a Kalman filter for the assimilation of Lagrangian data. Here however, the time-dependent solutions for the transition and covariance matrices are used in the Kalman updating step. Further, we employ the method of maximum likelihood [e.g., *Bickel and Doksum*, 2001] to estimate model parameters.

[10] In the next section, we introduce ocean drifter data by means of descriptive statistics for a typical drifter from the Ocean Storms experiment. The ocean-only model is stated in section 3, and the model parameters f and γ are estimated for the special case of inertial currents responding to an isolated surface wind event in the Ocean Storms experiment. Ocean-only statistical models are developed, first in a discrete-process model (not satisfactory), and then in a continuous-process model using maximum likelihood methods. The Labrador Sea case is addressed in section 4. Drifter wind data are reviewed and an explicit account of surface wind forcing is implemented in an air-sea model. Section 5 provides a discussion of the statistical model caveats and useful future extensions. A summary is provided in section 6. Three appendices elaborate details of: the Kalman filter recursions (Appendix A); the calculation of time-dependent transition and covariance matrices (Appendix B); and the derivation of the form of the air-sea coupling terms (Appendix C).

2. Data Description

2.1. Data Collection

[11] The upper ocean drifter position time series are irregularly spaced in time for both the Ocean Storms

Experiment in the North Pacific Ocean, and the Labrador Sea Minimet deployments. In both cases, position data are telemetered from the in situ observing systems to spaceborne platforms of the System ARGOS satellite remote communications resources. Irregular temporal sampling is due to System ARGOS coverage and to data drop outs that are inevitable for sophisticated in situ ocean observing systems. The Ocean Storms temporal coverage is reviewed by *Large and Crawford* [1995]. The Labrador Sea coverage is described by *Milliff et al.* [2003], where hourly position data were obtained, on average, for 14 hours of every 24 hour period in the data record. On the same hourly timescale, the analyzed Labrador Sea Minimet drifters also telemetered surface vector wind information in addition to drifter position data.

2.2. Ocean Storms Experiment: Descriptive Statistics

[12] Drifter latitude and longitude positions $\{x_{obs}(t_i), y_{obs}(t_i)\}\$ for a typical drifter trajectory from the Ocean Storms Experiment are plotted in Figure 1. For this drifter, velocity estimates of the eastward and northward current components are calculated by taking first differences of the location time series. Since our focus is on inertial currents, which can exist in isolation or embedded in a variety of background flows, the location measurements are first adjusted by subtraction of the southward and eastward background current flows (see Figure 1). Here, the eastward background flow is estimated by regressing longitude measurements on time using a cubic polynomial, $x_{obs}(t_i) \approx b_{0,x} + b_{1,x}t_i + b_{2,x}t_i^2 + b_{3,x}t_i^3$. All subsequent analyses are based on the residual observations $\tilde{x}_{obs}(t_i) = \{x_{obs}(t_i) - y_{abs}(t_i) - y_{abs}(t_i) = \{x_{obs}(t_i) - y_{abs}(t_i) - y_{abs}(t_i) - y_{abs}(t_i) = \{x_{obs}(t_i) - y_{abs}(t_i) - y_{abs}(t_i) - y_{abs}(t_i) - y_{abs}(t_i) - y_{abs}(t_i) = \{x_{obs}(t_i) - y_{abs}(t_i) - y_{ab$



Figure 2. (top) Time series of estimated eastward velocity component, $\hat{u}(t_i^*)$. (bottom) Histograms of $\hat{u}(t_i^*)$ and $\hat{v}(t_i^*)$.

 $\hat{x}_{obs}(t_i)$ }, where $\hat{x}_{obs}(t_i)$ is the predicted value of $x_{obs}(t_i)$ obtained by the estimated regression. It should be noted that the specified regression model is employed only for convenience and that the $b_{i,x}$ coefficients are defined empirically. The latitude measurements $y_{obs}(t_i)$ are similarly adjusted using a separate cubic regression. Subtraction of the estimated background flow results in an inward spiraling path for the adjusted locations { $\tilde{x}_{obs}(t_i)$, $\tilde{y}_{obs}(t_i)$ }, reflecting a damped exponential.

[13] Velocity estimates $\{\hat{u}(t_i^*), \hat{v}(t_i^*)\}$ are obtained by taking first differences of the adjusted locations: $\hat{u}(t_i^*) =$ $\{\tilde{x}_{obs}(t_{i+1}) - \tilde{x}_{obs}(t_i)\}/(t_{i+1} - t_i) \text{ and } \hat{v}(t_i^*) = \{\tilde{y}_{obs}(t_{i+1}) - t_i\}$ $\tilde{y}_{obs}(t_i)$ / $(t_{i+1} - t_i)$, where $t_i^* = t_i + 0.5(t_{i+1} + t_i)$. Thus $\hat{u}(t_i^*)$ and $\hat{v}(t_i^*)$ represent the average drifter velocities over the time period (t_i, t_{i+1}) , and t_i^* is the midpoint between t_i and t_{i+1} . Figure 2 shows a time series of the estimated eastward velocity component $\hat{u}(t_i^*)$, along with histograms of $\hat{u}(t_i^*)$ and $\hat{v}(t_i^*)$. As can be seen, the eastward velocity component essentially dissipates over the 16 day measurement period. The northward velocity component (not shown here) behaves similarly. The estimated velocities $\hat{u}(t_i^*)$ and $\hat{v}(t_i^*)$ are approximately normally distributed with standard deviations of 0.302 ms^{-1} and 0.296 ms^{-1} , respectively. The mean speed of the drifter over the time period is 0.371 ms^{-1} .

[14] An intuitive first modeling approach involves first differences of irregular position time series data. We demonstrate in section 3 that this approach, the so-called discrete-process model, fails due to systematic errors that will accumulate for any velocity field approximated by first differences of surface drifter positions in a flow that includes a significant inertial oscillation component.

3. Ocean Model Parameter Estimation

[15] Using discrete- and continuous-process models, we present two methods to estimate the unknown parameters f and γ of the ocean process. The discrete-process method is based on regressing acceleration estimates on velocity estimates, while the continuous-process method is based on fitting a stochastic differential equation model to the data using the Kalman filter. Because differencing is a common and intuitive approach for obtaining flow information from Lagrangian trajectories [*Hernandez et al.*, 1995; *Ishikawa et al.*, 1996], we review this method in the context of parameter estimation. As will be highlighted by our discussion, the difference-based method is inadequate for sparsely sampled data dominated by inertial motion and contaminated by measurement error.

3.1. Discrete-Process Parameter Estimation

[16] Parameters in systems of linear differential equations can be estimated by considering discrete versions of continuous-process data models. Here we use first and second differences of the flow-adjusted location measurements and the method of least squares to estimate parameters. Using previously defined estimates $\hat{u}(t_i^*)$ and $\hat{v}(t_i^*)$, we approximate acceleration as: $\hat{u}(t_i') = {\hat{u}(t_{i+1}^*) - \hat{u}(t_i^*)}/{t_{i+1}^* - t_i^*}$ and $\hat{v}(t_i') = {\hat{v}(t_{i+1}^*) - \hat{v}(t_i^*)}/{t_{i+1}^* - t_i^*}$, where t_i' is the midpoint between t_i^* and t_{i+1}^* , i.e., $t_i' = t_i^* + 0.5(t_{i+1}^* + t_i^*)$. Note that,



Figure 3. Scatterplot of (top) $\hat{u}(t'_i)$ versus $\hat{v}(t^*_i)$ and (bottom) $\hat{u}(t'_i)$ versus $\hat{u}(t^*_i)$.

because of irregular temporal sampling, in general, $t'_i \neq t^*_i$. As can be seen in Figure 3, the scatter plots of $\hat{u}(t'_i)$ versus $\hat{v}(t^*_i)$ and $\hat{u}(t'_i)$ versus $\hat{u}(t^*_i)$ both indicate linear relationships. The regression lines in Figure 3 correspond to the estimates \hat{f} and $\hat{\gamma}$. To obtain the regression slopes, we separately fit the two regression models

$$\begin{split} \dot{u}(t'_i) &= f\hat{v}(t^\star_i) - \gamma \hat{u}(t^\star_i) + \eta_u(t^\star_i) \\ \hat{v}(t'_i) &= -f\hat{u}(t^\star_i) - \gamma \hat{v}(t^\star_i) + \eta_v(t^\star_i) \end{split}$$

using least squares. Here, $\eta_u(t_i^*)$ and $\eta_u(t_i^*)$ are noise terms representing discretization errors and errors due to small-scale variability.

[17] Parameter estimates for the first equation are $\hat{f} = 7.97 \times 10^{-5} \text{ s}^{-1}$ and $\hat{\gamma} = 5.98 \times 10^{-5} \text{ s}^{-1}$, with approximate 95% confidence intervals (CI) of (6.64 × 10^{-5}, 9.31 × 10^{-5}) and (4.76 × 10^{-5}, 7.20 × 10^{-5}), respectively. The second equation produced estimates of $\hat{f} = 7.82 \times 10^{-5} \text{ s}^{-1}$ and $\hat{\gamma} = 6.00 \times 10^{-5} \text{ s}^{-1}$, with respective CIs of (6.69 × 10^{-5}, 8.96 × 10^{-5}) and (4.76 × 10^{-5}, 7.25 × 10^{-5}). Note that neither CI for \hat{f} contains the Coriolis parameter for the centroid of the Ocean Storms drifter data at 47.4°N, where $f = 1.07 \times 10^{-4}$. Further, the estimated damping parameter $\hat{\gamma}$ of approximately $6 \times 10^{-5} \text{ s}^{-1}$ corresponds to an *e*-folding time of approximately 5 hours, implying that the upper ocean damping is unrealistically strong.

[18] There are two reasons why the difference-based least squares approach taken here produces poor estimates of f and γ . First, the location measurements are corrupted by observation noise, affecting the accuracy of velocity and acceleration estimates used as "data" to obtain $\hat{\gamma}$ and \hat{f} . Secondly, as the residual drifter track traverses a decaying

spiral (Figure 1), all velocity estimates are consistently underestimated. Unbiased estimates of $u(t_i)$ and $v(t_i)$ can only be obtained through integration along the true drifter path connecting { $\tilde{x}_{obs}(t_i)$, $\tilde{y}_{obs}(t_i)$ } and { $\tilde{x}_{obs}(t_{i+1})$, $\tilde{y}_{obs}(t_{i+1})$ }. However, correctly tracing this path is difficult absent a physical model for the data.

[19] To illustrate the effects of the measurement noise on the accuracy of $\hat{u}(t_i^*)$ and $\hat{u}(t_i^*)$ consider the simple measurement model $\tilde{x}_{obs}(t_i) = \tilde{x}(t_i) + \varepsilon_x(t_i)$, where the x observation is the sum of a "true" location $\tilde{x}(t_i)$ and a white noise term $\varepsilon_x(t_i)$. With $\delta_i = (t_{i+1} - t_i)$, we get $\hat{u}(t_i^*) = {\tilde{x}(t_{i+1}) - \tilde{x}(t_i)}/\delta_i + {\varepsilon_x(t_{i+1}) + \varepsilon_x(t_i)}/\delta_i$, and with $\sigma_x^2 = var{\varepsilon_x(t_i)}$, we obtain $var{\hat{u}(t_i^*)} = 2\sigma_x^2/\delta_i^2$. Thus the estimate $\hat{u}(t_i^*)$ becomes highly unreliable with a decreasing sampling interval. The acceleration estimate $\hat{u}(t_i')$ is even more adversely affected by the observation noise with a variance of

$$\operatorname{var}\left\{\widehat{\hat{u}}(t_{i}')\right\} = \frac{8\sigma_{x}^{2}}{\left(\delta_{i+1}+\delta_{i}\right)^{2}} \times \left[\frac{1}{\delta_{i+1}^{2}}+\frac{1}{\delta_{i+1}\delta_{i}}+\frac{1}{\delta_{i}^{2}}\right].$$

Moreover, along with inducing a strong correlation between the velocity and acceleration estimates, the difference-based estimation also produces unwanted lag-one autocorrelations for $\hat{u}(t_i^*)$ and $\hat{u}(t_i')$.

[20] Since the error terms $\eta_u(t_i^*)$ and $\eta_v(t_i^*)$ are neither identically distributed (the sampling intervals are irregular), nor independent (both velocity and acceleration estimates are estimated serially from the data), any inferential statistics (e.g., CIs) for γ and f are likely to be inaccurate. By fitting a weighted least squares (LS) regression, it may be possible to adjust the LS estimates for inefficiencies. However, since damping and rotation terms (resulting in the decaying spiral) have to be considered simultaneously, specifying a bias-corrected model will probably prove too complicated for most realistic settings.

[21] An alternative approach to improving estimation of the ocean velocity vector is to control the sampling interval δ_i . We again assume the previously defined location measurement model. Then, taking into account both drifter acceleration and measurement error, an approximately optimal sampling interval for estimation of the velocity vector $\{u(t), v(t)\}$ can be obtained by minimizing the mean squared error of $\{\hat{u}(t), \hat{v}(t)\}$, denoted MSE $_{\hat{u},\hat{v}}(\delta)$. With σ_x^2 and σ_y^2 representing the variance of the location errors, an approximate expression for the mean squared error is given by

$$\begin{split} \text{MSE}_{\hat{u},\hat{v}}(\delta) \approx & \frac{\delta^2}{4} \left[(-\gamma u(t) + f v(t))^2 + (-f u(t) - \gamma v(t))^2 \right] \\ & + \frac{4}{\delta^2} \left(\sigma_x^2 + \sigma_y^2 \right), \end{split}$$

and this term is minimized for

$$\delta = 2 \left[\frac{\sigma_x^2 + \sigma_y^2}{(\gamma^2 + f^2)(u^2(t) + v^2(t))} \right]^{1/4}.$$

[22] As indicated, the optimal sampling interval is inversely proportional to the drifter speed. Although informative, the result is not helpful since the actual sampling interval is not influenced by the drifter velocities. In fact, based on the velocity estimates $\hat{u}(t_i^*)$ and $\hat{v}(t_i^*)$, along with parameter estimates obtained in section 4.2, the sample correlation between the optimal and actual sampling interval is a low 0.17.

[23] For the inertial ocean model, the difference-based velocity and acceleration estimates have poor covariance and MSE properties, and cannot be expected to produce accurate estimates of γ and f. The adverse effects of the location errors in this problem are similar to those delineated by *Kuznetsov et al.* [2003], who study assimilation of simulated tracer paths in a point-vortex system, and our results are in agreement with findings demonstrating that difference-based assimilation methods generally produce inferior flow estimates [*Griffa et al.*, 2004].

[24] Next we describe a statistical method that models continuous processes from discrete data with nonnegligible measurement noise.

3.2. Continuous-Process Parameter Estimation

[25] The state of a drifter $\mathbf{s}(t_i)$ at time t_i is defined by its position $\{x(t_i), y(t_i)\}$ and its velocity $\{u(t_i), v(t_i)\}$. The state equation in continuous time is

$$d\begin{bmatrix} x(t)\\ y(t)\\ u(t)\\ v(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & -\gamma & f\\ 0 & 0 & -f & -\gamma \end{bmatrix} \begin{bmatrix} x(t)\\ y(t)\\ u(t)\\ v(t) \end{bmatrix} dt + \begin{bmatrix} 0 & 0\\ 0 & 0\\ g_{31} & 0\\ g_{41} & g_{42} \end{bmatrix} \begin{bmatrix} d\xi_u(t)\\ d\xi_v(t) \end{bmatrix}.$$
(5)

Here, $d\xi_u(t)$ and $d\xi_v(t)$ denote independent Wiener processes (i.e., random processes) with unit variance per unit time and representing unresolved small-scale variability. With $\mathbf{s}(t) = [x(t)y(t)u(t)v(t)]'$ and $d\mathbf{Z}(t) = [d\xi_u(t)d\xi_v(t)]'$, the state equation expressed in matrix notation is

$$d\mathbf{s}(t) = \mathbf{M}\mathbf{s}(t)dt + \mathbf{G}d\mathbf{Z}(t), \tag{6}$$

where **M** represents the continuous-time state transition matrix defined in (5). Premultiplying $d\mathbf{Z}(t)$ by the matrix $\mathbf{G} = (g_{ij})$, also defined in (5), allows the random input to the velocity to have a general covariance matrix

$$\mathbf{Q} = \mathbf{G}\mathbf{G}' = \begin{bmatrix} g_{31}^2 & g_{31}g_{41} \\ g_{31}g_{41} & g_{41}^2 + g_{42}^2 \end{bmatrix}.$$

The covariance matrix $\mathbf{Q} = (q_{ij})$ represents the instantaneous random input considered to be the ocean velocity component variances and covariance per unit time.

[26] The observation equation is

$$\begin{bmatrix} \tilde{x}_{obs}(t_i)\\ \tilde{y}_{obs}(t_i) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{vmatrix} x(t_i)\\ y(t_i)\\ u(t_i)\\ v(t_i) \end{vmatrix} + \begin{bmatrix} \varepsilon_x(t_i)\\ \varepsilon_y(t_i) \end{bmatrix}, \quad (7)$$

where the observational noise terms $\varepsilon_x(t_i)$ and $\varepsilon_y(t_i)$ are assumed zero-mean normally distributed, with general 2×2 covariance matrix $\mathbf{R} = (r_{ij})$. The observation equation in matrix notation is $\mathbf{y}(t_i) = \mathbf{Hs}(t_i) + \boldsymbol{\epsilon}(t_i)$, where $\mathbf{y}(t_i) = [\tilde{x}_{obs}(t_i), \tilde{y}_{obs}(t_i)]', \boldsymbol{\epsilon}(t_i) = [\varepsilon_x(t_i), \varepsilon_y(t_i)]'$, and the observation operator \mathbf{H} is defined by (7). It should be noted that the parameters describing the background flow (i.e., $b_{0,x}, b_{1,x}, \dots, b_{3,y}$) could be incorporated in the stochastic model (5) by augmenting the state vector, allowing for simultaneous parameter estimation within the Kalman filter framework. However, to simplify here, we fit the data using the residual location observations $[\tilde{x}_{obs}(t), \tilde{y}_{obs}(t)]'$.

3.2.1. Maximum Likelihood Calculations

[27] For the Ocean Storms Experiment, our primary interest is estimating the parameters pertaining to the acceleration of, the damping of, and the subgrid-scale inputs to the velocity components, i.e., in $\{f, \gamma, g_{11}, g_{21}, g_{22}\}$ from (5). To estimate parameters we use the method of maximum likelihood (ML), where the likelihood is evaluated using the innovations [see *Schweppe*, 1965]. For completeness, the ML procedure is outlined here in the context of the upper ocean drifter application.

[28] Let $\mathbf{Y}^n = \{\mathbf{y}(t_1), \dots, \mathbf{y}(t_n)\}$ represent all data up to and including time t_n . With $\boldsymbol{\Psi}$ denoting a vector of unknown parameters, the likelihood is defined through the probability density function $p(\mathbf{Y}^n | \boldsymbol{\Psi})$, which is conveniently decomposed by the identity $p(\mathbf{Y}^n | \boldsymbol{\Psi}) = \prod_{i=1}^n p(\mathbf{y}(t_i) | \mathbf{Y}^{i-1}; \boldsymbol{\Psi})$. Evaluation of this decomposition is nontrivial for general distributions, but our model assumptions allow for recursive tracking of the data forecast density $p(\mathbf{y}(t_i) | \mathbf{Y}^{i-1}; \boldsymbol{\Psi})$ using the Kalman filter [Kalman, 1960; Jones, 1993].

[29] Let $\mathbf{s}^{f}(t_{i}) = E\{\mathbf{s}(t_{i})|\mathbf{Y}^{i-1}; \Psi\}$ represent the conditional mean of $\mathbf{s}(t_{i})$ given \mathbf{Y}^{i-1} , and $\mathbf{P}^{f}(t_{i}) = E\{[\mathbf{s}(t_{i}) - \mathbf{s}^{f}(t_{i})][\mathbf{s}(t_{i}) - \mathbf{s}^{f}(t_{i})]'|\mathbf{Y}^{i-1}; \Psi\}$ the forecast error

Table 1. ML Parameter Estimates for the Inertial Model Specified in Equation (5)^a

\hat{r}_{11}	\hat{r}_{22}	\hat{g}_{31}	\hat{g}_{41}	\hat{g}_{42}	Ŷ
1.876×10^{5}	1.298×10^{5}	3.088×10^{-4}	110×10^{-4}	5.200×10^{-4}	1.876×10^{-6}
^a Here \hat{r}_{\cdots} repre	esents the estimated e	lements of R (m ²) (\hat{r}_2	$a \approx 0$ and $\hat{a} = (s^{-1})$ rem	presents those of G V	alue of f is fixed at

^aHere, \hat{r}_{ij} represents the estimated elements of **R** (m²) ($\hat{r}_{21} \approx 0$), and \hat{g}_{ij} (s⁻¹) represents those of **G**. Value of *f* is fixed at 1.069 × 10⁻⁴ s⁻¹.

covariance matrix. Note that the matrix $\mathbf{P}^{f}(t_{i})$ represents the average squared error of the predictor $\mathbf{s}^{f}(t_{i})$. With $p_{N}(\mu, \Sigma)$ denoting the normal probability density function with mean μ and covariance Σ , it can shown that $p(\mathbf{y}(t_{i})|\mathbf{Y}^{i-1}; \Psi)$ follows the normal distribution with mean $\mathbf{Hs}^{f}(t_{i})$ and covariance $\mathbf{HP}^{f}(t_{i})\mathbf{H}' + \mathbf{R}$, here denoted $N(\mathbf{Hs}^{f}(t_{i}), \mathbf{HP}^{f}(t_{i})\mathbf{H}' + \mathbf{R})$. Consequently, we have

$$p(\mathbf{Y}^{n}|\boldsymbol{\Psi}) = \prod_{i=1}^{n} p_{N} \big(\mathbf{Hs}^{f}(t_{i}), \mathbf{HP}^{f}(t_{i})\mathbf{H}' + \mathbf{R} \big).$$
(8)

For given data \mathbf{Y}^n , the ML estimate $\hat{\Psi}$ is obtained by maximizing (8) over Ψ .

[30] ML estimation in continuous-time Kalman Filter applications requires calculation of the time-dependent transition matrix $\Phi(\delta_t)$ along with the covariance matrix for the state input noise $\mathbf{Q}(\delta_t)$. For an arbitrary time step δ_t , the transition matrix provides the solution to the system of linear first-order differential equations given by $d\mathbf{s}(t) = \mathbf{Ms}(t)dt$. With **M** defined as in (5), $\Phi(\delta_t)$ is given by Gaussian with mean $\mu_0 = [\tilde{x}_o(1), \tilde{y}_o(1), 0, 0.25]'$ and diagonal covariance matrix \mathbf{P}_0 with entries $[10^6, 10^6, 1, 1]'$, specifying an initial location uncertainty of approximately 2 km radius of the first observation and initial velocities in the range -2 to 2 ms⁻¹. Thus $\mathbf{s}(0) \sim N(\mu_0, \mathbf{P}_0)$. The Matlab function "fminsearch" was used to obtain ML estimates of $\Psi = \{r_{11}, r_{21}, r_{22}, g_{31}, g_{41}, g_{42}, \gamma, f\}$, where r_{ij} and g_{ij} are components of **R** and **G**, respectively, and where the Coriolis parameter f was included to validate the inertial model specified in (5).

[34] The method of profile-logliklihood produced an approximate 95% CI for *f* of $(1.064 \times 10^{-4} \text{ s}^{-1}, 1.111 \times 10^{-4} \text{ s}^{-1})$. This method is based on the asymptotic probability distribution of the log likelihood-ratio between two competing models, which follows a Chi-squared distribution under the null hypothesis of no model differences [e.g., *Pawitan*, 2001]. Since the CI includes the range of the Coriolis parameter between the northernmost and southernmost observed drifter locations of 47.14°N ($f = 1.066 \times 10^{-4} \text{ s}^{-1}$).

$$\boldsymbol{\theta}(\delta_{i}) = \begin{bmatrix} 1 & 0 & \frac{1}{\gamma^{2}+f^{2}} \left\{ \gamma + e^{-\gamma\delta_{t}} [-\gamma \cos(f\delta_{t}) + f\sin(f\delta_{t})] \right\} & \frac{1}{\gamma^{2}+f^{2}} \left\{ f + e^{-\gamma\delta_{t}} [-\gamma \sin(f\delta_{t}) - f\cos(f\delta_{t})] \right\} \\ 0 & 1 & \frac{1}{\gamma^{2}+f^{2}} \left\{ -f + e^{-\gamma\delta_{t}} [\gamma\sin(f\delta_{t}) + f\cos(f\delta_{t})] \right\} & \frac{1}{\gamma^{2}+f^{2}} \left\{ \gamma + e^{-\gamma\delta_{t}} [-\gamma\cos(f\delta_{t}) + f\sin(f\delta_{t})] \right\} \\ 0 & 0 & e^{-\gamma\delta_{t}}\cos(f\delta_{t}) & e^{-\gamma\delta_{t}}\sin(f\delta_{t}) \\ 0 & 0 & -e^{-\gamma\delta_{t}}\sin(f\delta_{t}) & e^{-\gamma\delta_{t}}\cos(f\delta_{t}) \end{bmatrix} .$$
(9)

[31] Even for fairly simple systems, the entries of the covariance matrix $\mathbf{Q}(\delta_t)$ are typically lengthy algebraic expressions in terms of the parameters. As an illustration, we give the solution for the upper left entry of $\mathbf{Q}(\delta_t)$ for the case $g_{31} = g_{42} = g$ and $g_{41} = 0$:

$$\begin{aligned} \operatorname{var}\{x(t+\delta_t)|\mathbf{s}(t)\} &= \frac{e^{-\gamma\delta_t}g^2}{\gamma(f^2+\gamma^2)^2} \times [\gamma\{2\gamma\cos(f\delta_t) \\ &+ ((f^2+\gamma^2)\delta_t - 2\gamma)\cosh(\gamma\delta_t) \\ &- 2f\sin(f\delta_t)\} + \{\gamma^2(\gamma\delta_t - 1) \\ &+ f^2(\gamma\delta_t + 1)\}\sinh(\gamma\delta_t)]. \end{aligned}$$

This expression is obtained through symbolic manipulation using Mathematica.

[32] Although the discrete time solution to the system of stochastic differential equations specified by (5) is complicated in terms of the parameters, calculation of $\Phi(\delta_t)$ and $\mathbf{Q}(\delta_t)$ depends only on the eigenvalues and eigenvectors of **M** [*Jones*, 1993]. Efficient (and exact) computational solution procedures for finding $\mathbf{s}^f(t_i)$ and $\mathbf{P}^f(t_i)$ and for calculating $\Phi(\delta_t)$ and $\mathbf{Q}(\delta_t)$ are given in Appendices A and B, respectively.

3.2.2. Parameter Estimates for the Ocean Storms Experiment

[33] On the basis of the descriptive statistics of section 2.2, the prior distribution for the state was taken to be

 10^{-4} s^{-1}) and 47.59°N ($f = 1.074 \times 10^{-4} \text{ s}^{-1}$), we conclude that the model adequately identifies the effects of earth rotation. In the subsequent estimation of parameters in this section, f will be fixed at $1.069 \times 10^{-4} \text{ s}^{-1}$, the Coriolis parameter of the mean observed latitude at 47.39°N. Maximizing (8) over the set $\Psi = \{r_{11}, r_{21}, r_{22}, g_{31}, g_{41}, g_{42}, \gamma\}$, produced the ML estimates presented in Table 1.

[35] To find a more parsimonious model we test the hypothesis of uncorrelated noise processes. Maximizing the likelihood over the restricted set of parameters specified by setting $r_{12} = g_{41} = 0$ produced a nonsignificant change (p < 0.78) in the log likelihood compared to the unconstrained model (Table 1). We also fail to reject a test of equal noise variances $r_{11} = r_{22} \equiv r$ and $g_{31} = g_{42} \equiv g$ (p < 0.12). Estimates for the final fitted model are given in Table 2.

[36] The estimated standard deviation of the measurement errors suggest a location accuracy of roughly 0.8 km radius $(\pm 2\sqrt{\hat{r}})$, and the estimated damping parameter corresponds to an *e*-folding time of approximately 7 days. Using the method of profile log likelihood an approximate

Table 2. Final ML Estimates for the Inertial Model Fitted to the

 Ocean Storms Data

\hat{r}, m^2	\hat{g}, s^{-1}	$\hat{\gamma}, s^{-1}$		
1.6410×10^{5}	4.151×10^{-4}	1.678×10^{-6}		

95% CI for γ was estimated as (.116 \times 10⁻⁶ s⁻¹, 3.558 \times 10⁻⁶ s⁻¹).

4. Wind Model Identification

4.1. Labrador Sea Experiment: Descriptive Statistics

[37] Location measurements and concurrent surface vector wind observations for Labrador Sea drifter 16891 are shown in Figure 4. The observed wind vector, $\mathbf{w}_{obs}(t_i) =$ $\{u_{obs}^{w}(t_i), v_{obs}^{w}(t_i)\}$, is superimposed on the measured locations $\{x_{obs}(t_i), y_{obs}(t_i)\}$. Inertial oscillations can be seen in the beginning of the drifter path (see magnified inset, Figure 4), but note that no persistent background current exists. With velocity estimates calculated as in section 2.2, Figure 5 shows a time series of the estimated eastward ocean velocity component for drifter 16891. As can be seen, the eastward velocity component is dominated by inertial oscillations over approximately the first 10 days, and is continuously affected by the wind over the entire time period. The estimated northward ocean velocity component (not shown) behaves similarly. Figure 5 also shows histograms of $\hat{u}(t_i^*)$ and $\hat{v}(t_i^*)$, which like the velocity components for the Ocean Storms example (see Figure 2), are approximately normally distributed. The mean estimated velocities of $\hat{u}(t_i^*)$ and $\hat{v}(t_i^*)$ were calculated as 0.019 ms^{-1} and 0.026 ms^{-1} . The ocean velocity component standard deviations are 0.169 ms⁻¹ (zonal) and 0.148 ms^{-1} , respectively. The mean speed of the drifter over the time period is 0.190 ms^{-1} .

[38] Time series of the eastward and northward surface wind components for drifter 16891 are shown in Figure 6. Although the wind component processes are dominated by slowly varying modes (e.g., synoptic scales of several days), the presence of small-scale variability is evident [see also *Milliff et al.*, 2003]. The mean eastward and northward winds for drifter 16891 over the time period were 1.39 ms⁻¹ and -1.17 ms^{-1} , respectively. The wind component standard deviations were, 7.68 ms⁻¹ and 6.86 ms⁻¹, respectively. The sample correlation between $u_{obs}^{w}(t_i)$ and $v_{obs}^{w}(t_i)$ was 0.147. Descriptive statistics for all six drifters to be analyzed here are provided in Table 3.

4.2. Wind Model

[39] To estimate the wind-ocean coupling we require a model describing the wind process. Because the available wind data is not amenable to a Lagrangian interpretation, a statistical model is fit to the time evolution of $\mathbf{w}_{obs}(t_i)$. On the basis of the findings of *Milliff et al.* [2003], who emphasize mesoscale spatial and temporal variability of the wind forcing in the Labrador Sea regions, we represent the true wind innovations, i.e., $u^w(t_i) - u^w(t_{i-1})$ and $v^w(t_i) - v^w(t_{i-1})$, by a continuous time vector autoregressive process of order one:

$$d\begin{bmatrix} u^{w}(t)\\v^{w}(t)\end{bmatrix} = \begin{bmatrix} -\varphi_{u}^{w} & 0\\ 0 & -\varphi_{v}^{w}\end{bmatrix} \begin{bmatrix} u^{w}(t)\\v^{w}(t)\end{bmatrix} dt + \begin{bmatrix} g^{w} & 0\\ 0 & g^{w}\end{bmatrix} \begin{bmatrix} d\xi_{u}^{w}(t)\\d\xi_{v}^{w}(t)\end{bmatrix}.$$
(10)

In (10), φ_u^w and φ_v^w represent damping coefficients for the eastward and northward wind components, and $d\xi_u^w(t)$ and $d\xi_v^w(t)$ are independent Wiener processes with unit variance per unit time.

[40] The measurement equations for $u_{obs}^{w}(t_i)$ and $v_{obs}^{w}(t_i)$ are

$$u_{\text{obs}}^{w}(t_i) = u^{w}(t_i) + \varepsilon_u^{w}(t_i) \qquad v_{\text{obs}}^{w}(t_i) = v^{w}(t_i) + \varepsilon_v^{w}(t_i), \quad (11)$$

where the observation noise terms $\varepsilon_u^w(t_i)$ and $\varepsilon_v^w(t_i)$ are taken as independent Gaussians, with common standard deviation r^w .

[41] Since the sample correlation coefficients between $u_{obs}^w(t)$ and $v_{obs}^w(t)$ for the six drifters are negligible (see Table 3), we treat the wind components as uncorrelated in the estimation of parameters. To further simplify, we have constrained the standard deviation of the wind state input noise, represented by g^w in (10), to have the same value. ML parameter estimates for the wind data are given in Table 4.

[42] The mean damping coefficients of $\hat{\varphi}_u^w = 6.745 \times 10^{-6} \text{ s}^{-1}$ and $\hat{\varphi}_v^w = 7.751 \times 10^{-6} \text{ s}^{-1}$ represent day-to-day wind correlations of approximately 0.56 and 0.52, and are roughly consistent with timescales associated with polar-low propagation across the Labrador Sea region in winter [*Milliff et al.*, 2003; *Renfrew and Moore*, 1999; *Renfrew et al.*, 1999].

4.3. Wind-Forced Ocean Model

[43] To estimate the wind-ocean coupling coefficients described by the matrix **A** defined in (2) we extend the definition of the state $\mathbf{s}(t)$ of a drifter at time *t* to include surface wind terms in addition to position and velocity. With $\mathbf{s}(t) = [x(t), y(t), u(t), v(t), u^w(t), v^w(t)]'$, the state equation in continuous time is

$$d\begin{bmatrix} x(t)\\ y(t)\\ u(t)\\ v(t)\\ u^{w}(t)\\ v^{w}(t)\end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & -\gamma & f & a_{11} & a_{12}\\ 0 & 0 & -f & -\gamma & a_{21} & a_{22}\\ 0 & 0 & 0 & 0 & -\varphi_{u}^{w} & 0\\ 0 & 0 & 0 & 0 & 0 & -\varphi_{v}^{w} \end{bmatrix} \begin{bmatrix} x(t)\\ y(t)\\ u(t)\\ v(t)\\ u^{w}(t)\\ v^{w}(t)\end{bmatrix} dt + \begin{bmatrix} 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ g_{31} & 0 & 0 & 0\\ g_{31} & 0 & 0 & 0\\ g_{41} & g_{42} & 0 & 0\\ 0 & 0 & g^{w} & 0\\ 0 & 0 & 0 & g^{w} \end{bmatrix} \begin{bmatrix} d\xi_{u}(t)\\ d\xi_{v}(t)\\ d\xi_{v}^{w}(t)\end{bmatrix}.$$

$$(12)$$

[44] In (12), *f* represents the Coriolis parameter, γ Rayleigh friction, and $\{a_{11}, a_{12}, a_{21}, a_{22}\}$ denote the air-sea coupling coefficients. Further, $\{g_{31}, g_{41}, g_{42}\}$ weight the Wiener process terms $\{d\xi_{u}(t), d\xi_{v}(t)\}$ used to model, as random inputs, the effects of the physical processes that are not resolved by the air-sea model. As before, the Weiner processes are taken as independent with unit variances per unit time. The parameters modeling the wind processes are as described in section 4.2. The random input has general covariance matrix $\mathbf{Q} = \mathbf{GG}'$, but note that \mathbf{Q} has many structural zeros.



Figure 4. Path of buoy 16891 for the period 22 October 1996 to 11 December 1996. Superimposed on each of the plotted buoy locations $\{x_{obs}(t_i), y_{obs}(t_i): i = 1, ..., 740\}$ are the measured wind vectors $\mathbf{w}_{obs}(t_i)$. To facilitate visualization, the first 50 observations are magnified.



Figure 5. (top) Time series of estimated eastward velocity component $\hat{u}(t_i^*)$ for drifter 16891. (bottom) Histograms of $\hat{u}(t_i^*)$ and $\hat{v}(t_i^*)$.



Figure 6. Time series of eastward and northward wind components for drifter 16891, (top) $u_{obs}^{w}(t_i)$ and (middle) $v_{obs}^{w}(t_i)$. (bottom) Histograms of $u_{obs}^{w}(t_i)$ and $v_{obs}^{w}(t_i)$.

[45] The observation equation is given by

$$\begin{bmatrix} x_{obs}(t_i) \\ y_{obs}(t_i) \\ u_{obs}^w(t_i) \\ v_{obs}^w(t_i) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \\ u(t) \\ v(t) \\ u^w(t) \\ v^w(t) \end{bmatrix} + \begin{bmatrix} \varepsilon_x(t_i) \\ \varepsilon_y(t_i) \\ \varepsilon_u^w(t_i) \\ \varepsilon_v^w(t_i) \end{bmatrix}.$$
(13)

The observational error terms are taken as zero-mean Gaussian, and the location noise terms $\{\varepsilon_x(t_i), \varepsilon_y(t_i)\}\$ are assumed independent of the wind noise terms $\{\varepsilon_u^w(t_i), \varepsilon_v^w(t_i)\}\$. The observational noise covariance matrix is thus a block-diagonal matrix with blocks corresponding to the 2 × 2 matrices $\operatorname{cov}[\varepsilon_x(t_i), \varepsilon_v(t_i)]'$ and $\operatorname{cov}[\varepsilon_u^w(t_i), \varepsilon_v^w(t_i)]'$.

4.4. Parameter Estimates and Model Assessment

[46] We first fit the model specified in (12) and (13) to each of the 6 Labrador Sea drifters (Table 3), and in a second estimation step, all available data (n = 6116) are used to simultaneously estimate parameters. To obtain stable estimates for the wind-ocean coupling coefficients and the random input terms, the parameters related to the wind process are fixed at the values attained in section 4.2; specifically, for each drifter, we use the corresponding

 Table 3. Descriptive Statistics for Labrador Sea Data^a

The of Deben prive Standards for Data									
ID	Observation Period	п	S	\bar{u}^{w}	\overline{v}^w	S_u^w	S_v^w	$C_{u,v}^{W}$	
16887	25 Oct-2 Dec 1996	601	0.158	1.96	-1.45	7.11	5.86	0.141	
16891	22 Oct-11 Dec 1996	740	0.190	1.39	-1.17	7.67	6.86	0.147	
16892	30 Oct 1996-13 Feb 1997	1421	0.185	3.24	-0.72	9.02	7.64	0.228	
16896	25 Oct 1996-29 Jan 1997	1399	0.192	1.81	0.09	8.77	8.34	0.068	
16899	24 Oct 1996-3 Feb 1997	1302	0.162	2.54	-0.43	8.08	8.86	0.119	
16905	23 Oct-8 Dec 1996	653	0.183	4.00	-1.06	5.71	6.98	0.103	
Mean		1019	0.180	2.50	-0.635	8.06	7.72	0.137	

^aThe dates represent the first and last days of measurements, *n* represents the number of observations, *S* denotes mean drifter speed (ms⁻¹), \bar{u}^w and \bar{v}^w denote mean observed eastward and northward winds, respectively (ms⁻¹), s_u^w and s_v^w sample standard deviations of the observed winds (ms⁻¹), and $c_{u,v}^w$ denotes the correlation between u_{obs}^w and v_{obs}^w .

Table 4. ML Estimates for the Wind Model in Equation $(10)^{a}$

ID	$\hat{\phi}^w_u imes 10^{-6}$	$\hat{\phi}^{\scriptscriptstyle\! W}_{\scriptscriptstyle\! V} imes 10^{-6}$	\hat{g}^{w}	\hat{r}^{w}
16887	5.193	8.284	0.0227	2.534
16891	4.346	5.465	0.0231	1.804
16892	5.594	8.305	0.0313	2.350
16896	7.478	7.990	0.0343	2.175
16899	8.943	7.732	0.0347	2.366
16905	7.765	8.103	0.0279	1.729
mean	6.745	7.751	0.0304	2.196

^aHere, $\hat{\varphi}_{u}^{w}$ and $\hat{\varphi}_{v}^{w}$ denote estimated damping coefficients (s⁻¹), \hat{g} the standard deviation of the state input noise (s⁻¹), and \hat{r} is the observation noise standard deviation (ms⁻¹). Means are obtained by weighting according to sample size.

estimates from Table 4. As previously discussed, the upper ocean dissipation γ is similarly fixed at the estimate obtained in section 3.2.2, i.e., $\hat{\gamma} = 1.678 \times 10^{-6} \text{ s}^{-1}$, and the Coriolis parameter $f = 1.188 \times 10^{-4} \text{ s}^{-1}$ is set at its known middomain value. ML estimates for the parameter vector $\Psi = \{g_{31}, g_{41}, g_{42}, a_{11}, a_{12}, a_{21}, a_{22}, r_{11}, r_{12}, r_{22}\}$ are provided in Table 5.

[47] To assess the model fit, standardized residuals are calculated for each drifter using the parameter estimates of Table 5. Figure 7 depicts histograms and normal probability plots for the standardized longitude $(x_{obs}(t_i) - E\{x_{obs}(t_i)|\mathbf{Y}_{t-1}\})$ and latitude $(y_{obs}(t_i) - E\{y_{obs}(t_i)|\mathbf{Y}_{t-1}\})$ innovations. The observation innovations are standardized using the error standard deviations obtained through the first two diagonal elements of $\mathbf{P}^{f(t_i)}$. The residuals for the other drifters exhibit the same patterns shown for drifter 16891, with an overall mean skewness and kurtosis for the longitude observations of 0.07 and 5.51, respectively (-0.04 and 5.49 for latitude). Although the calculated kurtoses indicate slightly peaked residuals, the histograms and normal probability plots demonstrate that deviations from normality are not severe.

[48] To obtain a simplified wind-ocean model, two nested sets of hypotheses tests based on the likelihood ratio test are performed for each of the six drifters. By restricting $a_{11} = a_{22}$ and $a_{12} = -a_{21}$, the first test evaluates the evidence for a wind model reflective of Ekman dynamics only. The Ekman model can be expressed using an amplitude coefficient \tilde{A} , and the Ekman rotation angle, θ (see Appendix C). This simplification is rejected for drifters 16891 (p < 0.003) and 16896 (p < 0.008), but the remaining drifters produce pvalues greater than 0.1, supporting the model. A second hypothesis test evaluates the homogeneity of the ocean-state error process by setting $g_{31} = g_{42}$ and $g_{41} = 0$. With an overall Type I error rate of 0.05, the second hypothesis test is rejected for drifters 16896 (p < 0.002), and 16895 (p < 0.001). However, as indicated by the estimates \hat{g}_{31} , \hat{g}_{41} , \hat{g}_{42} in Table 5, the evidence against a model with simplified ocean-state noise structure is weak.

[49] The air-sea model defined in (12) and (13) can conveniently be extended for simultaneous parameter estimation using the data from all six drifters. Let $s_i(t)$ represent the state of drifter *i* at time *t*, and define $S(t) = [\mathbf{s}_1(t), \mathbf{s}_2(t),$ $\mathbf{s}_3(t)$, $\mathbf{s}_4(t)$, $\mathbf{s}_5(t)$, $\mathbf{s}_6(t)$]'. (Here, drifter 1 is synonymous with drifter 16887, drifter 2 with 16891, etc.; see column 1 of Table 5.) With I_6 representing the 6 \times 6 identity matrix and \otimes the Kronecker matrix product, we define the timedependent transition matrix $\Omega(\delta_t) = \mathbf{I}_6 \otimes \Phi(\delta_t)$. Similarly define the covariance matrix $\Sigma(\delta_t) = \mathbf{I}_6 \otimes \mathbf{Q}(\delta_t)$, and the observation operator $\mathbf{B} = \mathbf{I}_6 \otimes \mathbf{H}$, where **H** is the observation matrix from (13). Further let $\mathbf{y}_i(t) = [x_{obs}(t), y_{obs}(t), t]$ $u_{obs}^{w}(t), v_{obs}^{w}(t)$ be the observation for drifter *i* at time *t*. Then, employing the transition and covariance matrices along with an observation operator given by rows (i * 4)through (3 + i * 4) of **B**, we update the state vector S(t)using the data $y_i(t)$ via recursions specified by (A3), Appendix A. At each measurement time this model affects only the part of the state vector for which data is available.

[50] ML parameter estimates based on all n = 6116 observation vectors are presented in Table 6. Only the estimates for the air-sea coupling coefficients are provided since the estimates of the model and observation noise parameters are not meaningfully different from the mean values presented in Table 5. A 95% CI for the Ekman rotation angle $\hat{\theta}$ is given by (44.10°, 54.25°).

5. Discussion

5.1. Statistical Issues

[51] The analyses presented here motivate the need for advances in statistical research in a number of areas. First, the noisy and rapidly changing wind process is particularly challenging to model. It should be noted that data with similar properties are often encountered in financial time series, and are generally treated by specifying models with time-dependent covariance structures [e.g., *Tsay*, 2001]. However, such models have highly nonlinear transition structures and solution procedures, and are not practical in our setting which requires instantaneous wind forcing. One possibility for obtaining more accurate wind predictions is to fit a fixed-lag smoother by augmenting the state and observation vector with wind observations from one time step ahead. This method would produce a filtered, rather

 Table 5. ML Estimates for the Coupled Air-Sea Model^a

			-							
ID	\hat{g}_{31}	\hat{g}_{43}	\hat{g}_{42}	\hat{a}_{11}	\hat{a}_{12}	\hat{a}_{21}	â ₂₂	\hat{r}_{11}	\hat{r}_{22}	\hat{r}_{21}
16887	0.1657	0.1837	0.0317	0.1583	0.0042	-0.0281	0.0375	1.757	1.414	0.8698
16891	0.1401	0.1468	-0.0168	0.2041	-0.1519	0.4039	0.4410	3.233	1.345	0.5164
16892	0.2416	0.2338	0.0056	0.2983	-0.4456	0.4265	0.5882	2.181	0.8473	0.0787
16896	0.2012	0.1794	0.0093	0.3054	-0.6258	0.8132	0.5368	2.604	1.539	0.2175
16899	0.1683	0.1585	0.0010	0.5301	-0.5100	0.4582	0.4745	2.625	1.251	0.7397
16895	0.1867	0.1802	0.0229	0.0128	0.2281	0.3148	0.5741	2.744	1.719	-1.066
mean	0.1911	0.1841	0.0072	0.2936	-0.3488	0.4623	0.4788	2.518	1.301	0.2596

^aVariables $\hat{g}_{ij} \times 10^{-2} \text{ s}^{-1}$ are the entries of the random input term amplitude matrix, **G**, $\hat{a}_{ij} \times 10^{-6} \text{ s}^{-1}$ are the air-sea coupling coefficients, and $\hat{r}_{ij} \times 10^2$ m are the elements of the observation noise matrix **R**. Means are obtained by weighting according to sample size.



Figure 7. Histograms and normal probability plots for the standardized (top) longitude and (bottom) latitude residuals for drifter 16891. The plots indicate symmetric but slightly peaked residuals.

than predicted, wind vector as ocean forcing, allowing for more accurate modeling of rapid wind shifts, especially during periods of high variability. Another approach to modeling the abrupt level and variance shifts of the wind process is to represent the forecast density at each measurement time by a finite mixture distribution. For Gaussian mixture components the Kalman filter described in this paper can be applied to each component [e.g., *Chen and Liu*, 2000]. Note that such mixtures can be applied both at the level of the state, to address abrupt shifts, and at the level of the observations, to address heavy-tailed observation distributions. A non-Gaussian real-time Kalman filter specification is possible using the skewed-normal distribution of *Naveau et al.* [2004].

[52] Original attempts at estimating the inertial parameters using the Labrador Sea data failed, suggesting that the likelihood surface is extremely flat in the neighborhood of its maximum. We speculate that disproportionate variance in the wind process renders the simultaneous estimation of γ even more difficult. We therefore turned to data from the Ocean Storms Experiment to gain a physically realistic

estimate of γ , and in subsequent estimation procedures the dissipation was treated as fixed at $\hat{\gamma} = 1.678 \times 10^{-6} \text{ s}^{-1}$ (see Table 2). Since the covariance properties of $\hat{\gamma}$ are $\mathcal{O}(n^{-1/2})$, the asymptotic effects of treating $\hat{\gamma}$ as fixed on other parameter estimates are negligible. Yet, for moderate or small samples, the impact of replacing γ by $\hat{\gamma}$ is to increase uncertainty in the state and parameter estimates. To explicitly account for this uncertainty we could assign a prior distribution for γ based on the results of the Ocean Storms Experiment, and fit the Labrador Sea data using fully Bayesian techniques, e.g., the Dynamical Linear Model [West and Harrison, 1989], or Bayesian Hierarchical Modeling [Berliner et al., 2003; Royle et al., 1998]. However, due to the strongly nonlinear dependence of the transition and covariance matrices on all involved parameters, no closed form expressions exist with which to produce posterior state and parameter estimates. In particular, neither the state nor the parameters would be Gaussian. Thus computational integration techniques would have to be employed to obtain posterior distributions, but chain-linked samplebased integration methods (e.g., Gibbs sampling) require the

Table 6. ML Estimates of the Air-Sea Coupling Coefficients $\hat{a}_{ij} \times 10^{-6} \text{ s}^{-1}$, Wind Model Amplitude Coefficient $\hat{\mathbf{A}} \times 10^{-6} \text{ s}^{-1}$, and Ekman Rotation Angle $\hat{\theta}$

â ₁₁	â ₁₂	â ₂₁	â ₂₂	Â	$\hat{\boldsymbol{\theta}}$	Null Hypothesis
0.3170	-0.4230	0.4985	0.5086	NA	NA	$g_{ij}, a_{ij} \text{ free} g_{ij} \text{ free}; a_{11} = a_{22}, a_{12} = a_{21} g_{31} = g_{42}, g_{32} = 0; a_{11} = a_{22}, a_{12} = a_{21}$
0.4062	-0.4738	0.4738	0.4062	0.6241	49.39	
0.4047	-0.4687	0.4687	0.4047	0.6193	49.19	

chain to be restarted at each of the n = 6116 observation points and would clearly prove inefficient. Moreover, our initial attempts at estimating the inertial parameters from the Labrador Sea data suggest a complicated likelihood surface, and such a likelihood function would further compound computational inefficiencies. One approach may be to use a hybrid filter which treats the unknown parameters using sequential importance sampling methods [*Doucet et al.*, 2001], while the state is propagated using Kalman filter techniques.

[53] The possibility of using Kalman filter techniques for assimilation of Lagrangian data and Eulerian model variables is successfully demonstrated by various simulation studies [Kuznetsov et al., 2003; Molcard et al., 2003; Griffa et al., 2004] as well as by studies based on real data sets [Castellari et al., 2001; Özgökmen et al., 2001]. As demonstrated in section 4.4, the specified wind model scales easily to include a large number of drifting and moored buoys. It is straightforward to extend the model to include correlated wind and ocean state noise processes. Further, the possibility of assimilating data from different measurement platforms (e.g., satellite measurements) and across various spatial scales exists. To address the computational requirements of on-line assimilation in highdimensional systems we would explore the use of Monte Carlo based Kalman filter variants [Evensen, 1994; Anderson, 2001], or their deterministic counterparts [Tippett et al., 2003; Whitaker and Hamill, 2002]. Exact evaluation of time-dependent covariance structures may not be feasible in such systems. However, as demonstrated by our work, model identification and parameter estimation are attainable goals.

5.2. Oceanographic Validation

[54] While a central purpose of this paper has been to demonstrate statistical estimation methods for ocean velocity and surface wind data sets obtained from ocean drifters in a harsh environment, the success of the model in its present form yields parameter estimates that can be validated with prior work. Recall that the Labrador Sea data is unique in that ocean current and surface wind are observed simultaneously by the drifters at fine spatial and temporal scales. For these data there are no published studies with which direct comparisons can be made. In particular, the high-frequency, high-wave number [e.g., see Milliff et al., 2003] properties of the wind data, and its precise collocation with the current observations, are unique to the Labrador Sea drifter data set. However, the estimates of air-sea coupling a_{ij} , and Ekman rotation angle θ , from the aggregated Labrador Sea drifter data (Table 6) can be compared with similar parameters deduced by Ralph and Niiler [1999]. Their study assumes a steady Ekman balance over large spatial scales and for long timescales in the tropical Pacific.

[55] In one of the models evaluated by *Ralph and Niiler* [1999], they derive an amplitude coefficient relative to the friction velocity $u_* = [|\tau|/\rho_a]^{1/2}$, for surface wind stress τ and atmospheric density ρ_a . The model is $u = \beta u_* f^{-1/2}$, where the estimate $\hat{\beta} = 0.065$ is obtained by a least squares fit [see *Ralph and Niiler*, 1999, Table 1]. An estimate of their coupling coefficient relative to u_* , using *f* for the Labrador Sea, is 5.96. To compare with \hat{A} from Table 6

here, we divide by f and relate to u_* instead of u^w , to yield 4.18. The air-sea coupling coefficient derived here compares well with the average value estimated over a data set from 1503 drifters in the tropical Pacific [*Ralph and Niiler*, 1999].

[56] Similarly, our estimate of the Ekman rotation angle $\theta = 49.2^{\circ}$ (Table 6) is consistent with the Ekman rotation angle (48.7°) for the depth bin nearest the surface of *Ralph and Niiler* [1999]. Because their estimates are based on wind and ocean current data taken over a vast region and many years of drifter records, the ocean velocity estimates at the drifter drogue depth (15 m) occur over a wide range of depths relative to the time- (wind-) dependent Ekman depth H_* . The agreement here with the near-surface rotation angle is consistent with an H_* for the Labrador Sea in winter that is much deeper than 15 m.

[57] The results here indicate that $\gamma \ll f$. The magnitude of γ relative to f simplifies the form of the wind-ocean coupling derivations in Appendix C; that is, we have neglected γ dependence in **A**. By inclusion of nonlinear terms, this approximation can be explored more carefully within the context of a modified state-space model.

[58] Further ocean effects to be explored include the unmodeled influence of the ocean mesoscale [e.g., *van Muers*, 1998] on the inertial current damping time, and time-dependent models for γ and a_{ij} . These issues have not posed serious difficulties in our work because (1) the ocean mesoscale eddy amplitudes in the fraction of the Ocean Storms data we analyzed (and in the middle of the Labrador Sea basin) are relatively small and (2) the drifter time series for surface vector winds and ocean currents used here span only a part of a single winter season.

6. Summary

[59] A stochastic model for wind-driven upper ocean currents, treated as a continuous process, is evaluated in light of an upper ocean drifter data set in the Labrador Sea. Estimates are obtained for upper ocean dissipation due to momentum convergences, γ , and air-sea coupling coefficients, a_{ij} , as well as the Coriolis parameter, *f*. Second-order properties of model error terms are also given.

[60] The drifter data consist of coincident observations of drifter position and surface vector wind, reported on hourly timescales from 6 in situ platforms (Minimet drifters) for periods of about a month. The stochastic model framework also includes terms to quantify uncertainties in both the observations and the a priori assumptions regarding the important physical balances. Drifter observations impact the stochastic model via the Kalman filter, and parameter estimates are obtained using ML procedures.

[61] A simple (perhaps intuitive) model treats the ocean process as discrete, where the discretization is dictated by temporal increments in the drifter position time series. In the discrete-process model the ocean velocities and accelerations are computed using first and second difference approximations, but this model fails to reproduce the correct Coriolis parameter for the data set, which is knowable outside the model framework. The failure is attributed to error accumulations in the approximations for velocity and acceleration in flows characterized by significant contributions from ocean inertial oscillations, and the model framework yields biased and unreliable parameter estimates.

[62] The first application of the stochastic, continuousprocess model to the Labrador Sea data fails in a similar sense; that is, the known Coriolis parameter cannot be reproduced. The initial failure of the continuous-process model is attributed to the disproportionate variance contribution of the vigorous wind process, versus the contribution due to the ocean parameters. To address this, the estimation of γ is separated from the wind process by resorting to the drifter data from the Ocean Storms experiment. The Ocean Storms data are unique in that ocean inertial oscillations occurred for several days in the absence of significant wind, following the impulse forcing of a single initial storm event. A physically realistic and robust estimate for γ is obtained.

[63] The estimate for γ from the Ocean Storms setting is then held fixed in a second application to the Labrador Sea data. Robust estimates for $a_{i,j}$ and the observational and model error terms are obtained. Model simplifications are evaluated and quantified by likelihood methods. Model and observational error terms are small compared to the model parameter estimates. The average 15 m current rotation angle θ with respect to the surface vector wind is consistent with the upper ocean model based on Ekman dynamics.

Appendix A: Kalman Filter Recursions

[64] Let \mathbf{s}_t represent the unobserved state vector of the system at time *t* and let \mathbf{y}_t denote a new vector of observations. The data and the state are related by the observation equation

$$\mathbf{y}_t = \mathbf{H}_t \mathbf{s}_t + \mathbf{e}_t, \tag{A1}$$

where \mathbf{H}_t represents a linear observation operator and $\mathbf{e}_t \sim N(\mathbf{0}, \mathbf{R})$.

[65] We wish to update our knowledge of the unobserved state \mathbf{s}_t in light of the new data \mathbf{y}_t . Assuming initial knowledge of the system is given by the conditional forecast distribution $p(\mathbf{s}_t | \mathbf{Y}_{t-1})$, where \mathbf{Y}_{t-1} denotes all past data up to and including time t - 1, the update step combines the forecast distribution and the new vector of observations, yielding the posterior distribution $p(\mathbf{s}_t | \mathbf{Y}_t)$. The standard Kalman filter assumes that $p(\mathbf{s}_t | \mathbf{Y}_{t-1}) \sim$ $N(\mathbf{s}_t^t, \mathbf{P}_t^t)$, and a straightforward application of Bayes theorem yields

$$p(\mathbf{s}_t | \mathbf{Y}_t) = N(\mathbf{s}_t^u, \mathbf{P}_t^u), \qquad (A2)$$

where

$$\mathbf{s}_{t}^{u} = \mathbf{s}_{t}^{f} + \mathbf{K}_{t} \left(\mathbf{y}_{t} - \mathbf{H}_{t} \mathbf{s}_{t}^{f} \right)$$

$$\mathbf{P}_{t}^{u} = \left(\mathbf{I} - \mathbf{K}_{t} \mathbf{H}_{t} \right) \mathbf{P}_{t}^{f}.$$
(A3)

In (A3), \mathbf{K}_t denotes the Kalman gain matrix and is given by

$$\mathbf{K}_t = \mathbf{P}_t^f \mathbf{H}_t' \Big(\mathbf{H}_t \mathbf{P}_t^f \mathbf{H}_t' + \mathbf{R} \Big)^{-1},$$

where a prime superscript represents matrix transpose. To obtain the forecast distribution $p(\mathbf{s}_{t+1}|\mathbf{Y}_t)$ the update

distribution (A2) is propagated using the system dynamics represented in the state equation

$$\mathbf{s}_{t+1} = \mathbf{\Phi}(\delta_t)\mathbf{s}_t + \mathbf{v}_t, \tag{A4}$$

where $\mathbf{v}_t \sim N(\mathbf{0}, \mathbf{Q}_t)$. Since the system dynamics are linear, the forecast distribution will again be multivariate normal with closed forms for the mean and covariance:

$$\mathbf{s}_{t+1}^{\prime} = \mathbf{\Phi}(\delta_t) \mathbf{s}_t^u$$

$$\mathbf{P}_{t+1}^{\prime} = \mathbf{\Phi}(\delta_t) \mathbf{P}_t^a \mathbf{\Phi}(\delta_t)^{\prime} + \mathbf{Q}(\delta_t),$$
(A5)

where $\mathbf{\Phi}(\delta_t)$ and $\mathbf{Q}(\delta_t)$ are as described in section 3.2 and Appendix B.

[66] The form of the Kalman filter recursions (A3) and (A5) are the same for data sampled at regular and irregular time intervals. In the case of irregularly sampled data, both the transition matrix and state noise covariance matrix depend on δ_t , the time step between observations.

Appendix B

B1. Calculating $\Phi(\delta_t)$

[67] Without the white noise in (5), the system of firstorder linear differential equations can be written as

$$\frac{d}{dt}\mathbf{s}(t) = \mathbf{M}\mathbf{s}(t). \tag{B1}$$

The solution to this system of equations is

$$\mathbf{s}(t) = e^{\mathbf{M}t}\mathbf{s}(0),$$

where $e^{\mathbf{M}t}$ is defined as

$$e^{\mathbf{M}t} = \mathbf{I} + \mathbf{M}t + \frac{(\mathbf{M}t)^2}{2!} + \frac{(\mathbf{M}t)^3}{3!} + \dots$$
 (B2)

There can be numerical instabilities when evaluating matrix exponentials [*Moler and van Loan*, 1978], but one method that usually works is based on a diagonal representation of the matrix **M** using eigenvalues and eigenvectors. The right eigenvectors satisfy the right eigenvector equation

$$\mathbf{M}\mathbf{e}_i^{(r)} = \lambda_i \mathbf{e}_i^{(r)},$$

where $\mathbf{e}_i^{(r)}$ is the *i*th nonzero right eigenvector corresponding to the eigenvalue λ_i . If there are *n* linearly independent eigenvectors, they can be arranged as the columns of the matrix **E**, and we have

$$\mathbf{ME}_r = \mathbf{E}_r \mathbf{\Lambda},$$

where Λ is a diagonal matrix with eigenvalues on the diagonal. M can then be written in diagonal form as

$$\mathbf{M} = \mathbf{E}_r \mathbf{\Lambda} \mathbf{E}_r^{-1},$$

where the rows of \mathbf{E}_r^{-1} are the left eigenvectors of **M**. Since $\mathbf{E}_r^{-1}\mathbf{E}_r = \mathbf{I}$, we have $\mathbf{M}^k = \mathbf{E}_r \mathbf{\Lambda}^k \mathbf{E}_r^{-1}$ and the solution in (B2) can be written as

$$e^{\mathbf{M}t} = \mathbf{E}_r \left\{ \mathbf{I} + \mathbf{\Lambda}t + \frac{(\mathbf{\Lambda}t)^2}{2!} + \frac{(\mathbf{\Lambda}t)^3}{3!} + \dots \right\} \mathbf{E}_r^{-1} = \mathbf{E}_r e^{\mathbf{\Lambda}t} \mathbf{E}_r^{-1},$$
(B3)

where $e^{\Lambda t}$ is a diagonal matrix with entries $e^{\lambda i t}$.

[68] To find the transition matrix in our problem we note that M in (5) has two zero eigenvalues and a complex conjugate pair: $\lambda_1 = 0$, $\lambda_2 = 0$, $\lambda_3 - \gamma + if$, and $\lambda_4 = -\gamma - if$, where $i = \sqrt{-1}$. The four right eigenvectors $\mathbf{e}_i^{(r)}$ corresponding to the eigenvalues λ_i are given column-wise in the matrix \mathbf{E}_r :

$$\mathbf{E}_{r} = \begin{bmatrix} 1 & 0 & 1 & i \\ 0 & 1 & i & 1 \\ 0 & 0 & (-\gamma + if) & i(-\gamma - if) \\ 0 & 0 & i(-\gamma + if) & (-\gamma - if) \end{bmatrix},$$
(B4)

and the four left eigenvectors $\mathbf{e}_{i}^{(l)}$ are given row-wise in the matrix \mathbf{E}_r^{-1} :

$$\mathbf{E}_{r}^{-1} = \begin{bmatrix} 1 & 0 & \frac{\gamma}{\gamma^{2} + f^{2}} & \frac{f}{\gamma^{2} + f^{2}} \\ 0 & 1 & \frac{-f}{\gamma^{2} + f^{2}} & \frac{\gamma}{\gamma^{2} + f^{2}} \\ 0 & 0 & \frac{1}{2(-\gamma + if)} & \frac{-i}{2(-\gamma + if)} \\ 0 & 0 & \frac{-i}{2(-\gamma - if)} & \frac{1}{2(-\gamma - if)} \end{bmatrix}.$$
 (B5)

With Λ a diagonal matrix defined by the eigenvalues of M. the discrete time transition matrix $\Phi(\delta_t)$ in (9) is then obtained by (B3) with \mathbf{E}_r and \mathbf{E}_r^{-1} as defined in (B4) and (B5), respectively.

[69] The following matrices containing the right and left eigenvectors are used for the model in (12):

$$\begin{split} \mathbf{E}_{r,LS} &= \\ \mathbf{E}_{r,LS} &= \\ & \begin{bmatrix} 1 & 0 & 1 & i & -\frac{a_{11}(\gamma-\varphi_1)+a_{21}f}{\varphi_1(f^2+\gamma^2-2\gamma\varphi_1+\varphi_1^2)} & -\frac{a_{22}f+a_{12}(\gamma-\varphi_2)}{\varphi_2(f^2+\gamma^2-2\gamma\varphi_2+\varphi_2^2)} \\ 0 & 1 & i & 1 & \frac{a_{11}f-a_{21}(\gamma-\varphi_1)}{\varphi_1(f^2+\gamma^2-2\gamma\varphi_1+\varphi_1^2)} & \frac{a_{12}f-a_{12}(\gamma-\varphi_2)}{\varphi_2(f^2+\gamma^2-2\gamma\varphi_2+\varphi_2^2)} \\ 0 & 0 & (-\gamma+if) & i(-\gamma-if) & \frac{a_{21}f+a_{11}(\gamma-\varphi_1)}{f^2+\gamma^2-2\gamma\varphi_1+\varphi_1^2} & \frac{a_{22}f+a_{12}(\gamma-\varphi_2)}{f^2+\gamma^2-2\gamma\varphi_2+\varphi_2^2} \\ 0 & 0 & i(-\gamma+if) & (-\gamma-if) & -\frac{a_{11}f-a_{21}(\gamma-\varphi_1)}{f^2+\gamma^2-2\gamma\varphi_1+\varphi_2^2} & -\frac{a_{12}f-a_{12}(\gamma-\varphi_2)}{f^2+\gamma^2-2\gamma\varphi_2+\varphi_2^2} \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \\ \mathbf{E}_{r,LS}^{-1} &= \\ & \cdot \begin{bmatrix} 1 & 0 & \frac{\gamma}{\gamma^2+f^2} & \frac{f}{\gamma^2+f^2} & \frac{a_{21}f+a_{11}\gamma}{\varphi_1(f^2+\gamma^2)} & \frac{a_{22}f+a_{12}\gamma}{\varphi_2(f^2+\gamma^2)} \\ 0 & 1 & \frac{-f}{\gamma^2+f^2} & \frac{\gamma}{\gamma^2+f^2} & \frac{-a_{11}f+a_{21}\gamma}{\varphi_1(f^2+\gamma^2)} & \frac{-a_{12}f+a_{22}\gamma}{\varphi_2(f^2+\gamma^2)} \\ 0 & 0 & \frac{1}{2(-\gamma+if)} & \frac{-a_{11}+ia_{21}}{2(-\gamma+if)} & \frac{-a_{11}+ia_{21}}{2(f-i\gamma)(-if-\gamma+\varphi_1)} \\ 0 & 0 & \frac{-i}{2(-\gamma-if)} & \frac{1}{2(-\gamma-if)} & \frac{a_{21}f-a_{21}+ia_{22}}{2(f-i\gamma)(-if-\gamma+\varphi_1)} \end{bmatrix} . \end{split}$$

0 [70] Another approach to finding $\Phi(\delta_t)$ is to perform a rotation of the state vector based on the eigenvectors of M. Note that (6) can be written as

 $2(f-i\gamma)(-if$

1

 $\gamma + \varphi_1$

 $\gamma - if$

0

0

0

0 0

0

 $2(f-i\gamma)(-if$

0

1

$$d\mathbf{s}(t) = \mathbf{E}_r \mathbf{\Lambda} \mathbf{E}_r^{-1} \mathbf{s}(t) dt + \mathbf{G} d\mathbf{w}(t).$$

Premultiplying this equation by \mathbf{E}_r^{-1} yields a complex rotated state equation

$$d\mathbf{s}_r(t) = \mathbf{\Lambda}\mathbf{s}_r(t)dt + \mathbf{G}_r d\mathbf{w}(t), \tag{B6}$$

where $\mathbf{G}_r = \mathbf{E}_r^{-1} \mathbf{G}$. The rotation uncouples the state vector, and each element can be integrated separately. To get back to the unrotated state, premultiply by \mathbf{E}_r . This approach requires complex arithmetic, but is readily implemented computationally [see also Jones, 1993].

B2. Calculating $Q(\delta_t)$

[71] The random input to the state equation is integrated over the time interval δ_t as

$$\eta(\delta_t) = \int_0^{\delta_t} \Phi(\delta_t - t) \mathbf{G} \mathbf{w}(t) dt, \qquad (B7)$$

and the covariance matrix of this integrated random noise is

$$\mathbf{Q}(\delta_t) = \int_0^{\delta_t} \mathbf{\Phi}(\delta_t - t) \mathbf{G} \mathbf{G}' \mathbf{\Phi}'(\delta_t - t) dt.$$
(B8)

The covariance matrix of the input noise to the rotated state equation is

$$\mathbf{Q}_{r}(\delta_{t}) = \mathbf{E}_{r}^{-1}\mathbf{Q}(\delta_{t})\mathbf{E}_{r} = \left[\int_{0}^{\delta_{t}} e^{\mathbf{\Lambda}(\delta_{t}-t)}\mathbf{E}_{r}^{-1}\mathbf{G}\mathbf{G}'\left(\mathbf{E}_{r}^{\star}\right)^{-1}e^{\mathbf{\Lambda}^{\star}(\delta_{t}-t)}dt\right],$$
(B9)

where * denotes the complex conjugate transpose matrix. This matrix integration can be evaluated element-wise in terms of the elements of $\mathbf{K} = \mathbf{E}_r \mathbf{G} \mathbf{G}' (\mathbf{E}_r^*)^{-1}$, giving for the elements of $\mathbf{Q}_r(\delta_t)$

$$Q_{jk}^{(r)}(\delta_t) = K_{jk} \frac{e^{\left(\lambda_j + \bar{\lambda}_k\right)} - 1}{\lambda_j + \bar{\lambda}_k}, \lambda_j + \bar{\lambda}_k \neq 0$$
(B10)

$$Q_{jk}^{(r)}(\delta_t) = K_{jk}\delta_t, \lambda_j + \bar{\lambda}_k = 0$$
(B11)

where $\bar{\lambda}_k$ denotes complex conjugate. To obtain the covariance matrix for the integrated random noise $\eta(\delta_t)$, $\mathbf{Q}_r(\delta_t)$ is rotated back using \mathbf{E}_r and its complex conjugate, i.e.,

$$\mathbf{Q}(\delta_t) = \mathbf{E}_r \mathbf{Q}_r(\delta_t) \mathbf{E}_r^{\star}.$$
 (B12)

Note that the method outlined to calculate $\Phi(\delta_t)$ and $Q(\delta_t)$ only depends on the eigenvalues and eigenvectors of M.

Appendix C: Wind-Ocean Coupling

[72] *Ekman* [1906] derived a model for the response of upper ocean currents (u_E, v_E) to surface wind forcing. In complex variable notation we have

$$if(u_E + iv_E) = \frac{1}{\rho_o} \frac{\partial}{\partial z} (\tau^x + i\tau^y), \tag{C1}$$

where f is the Coriolis term, $\vec{\tau} = \tau^x + i\tau^y$ is the stress vector, ρ_o is the upper ocean density, and $i = \sqrt{-1}$.

[73] Analytic solutions to the Ekman equations (C1) are obtainable given assumptions and parameterizations of $\vec{\tau}$ [e.g., see Pond and Pickard, 1983; Pedlosky, 1987]. Ralph and Niiler [1999] fit ocean drifter data to these equations to determine coefficients in the theoretical solutions and evaluate parameterizations of $\partial \vec{\tau} / \partial z$ at z = 15 m. A salient feature of the theory and the data analyses is the rotation (to the right in the Northern Hemisphere) of the Ekman surface current vector with respect to the surface wind vector. From this, the form of the surface wind coupling terms used in this paper can be derived [e.g., see also Pollard and Millard, 1970]: we have

$$u_E + iv_E = \tilde{\mathbf{A}}e^{\mathbf{i}\theta}(u^w + iv^w), \tag{C2}$$

where A is an amplitude coefficient that depends on upper ocean stratification, a vertical length scale, and the wind stress. The angle θ is the rotation angle of the current vector (u_E, v_E) with respect to the surface wind vector (u^w, v^w) . We determine θ from the Labrador Sea drifter data in section 4.2 of the paper, and compare it with the results of Ralph and Niiler [1999].

[74] Matching real and imaginary coefficients in (C2), we have

$$u_E = \tilde{\mathbf{A}} u^w \cos(\theta) - \tilde{\mathbf{A}} v^w \sin(\theta), \qquad (C3)$$

$$= a_{11}u^w - a_{12}v^w, (C4)$$

$$v_E = \tilde{\mathbf{A}} u^w \sin(\theta) + \tilde{\mathbf{A}} v^w \cos(\theta), \tag{C5}$$

$$= a_{21}u^w + a_{22}v^w, (C6)$$

where the equations (C4) and (C6) lead to (2).

[75] We determine the a_{ij} from the Labrador Sea drifter data as well and note that these coefficients, determined independently, yield approximately the physically correct relations; $a_{11} = a_{22}$, and $a_{12} = -a_{21}$ (section 4.2).

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