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A CASE STUDY OF WAVE HEIGHT VARIATIONS DUE TO CURRENTS IN A TIDAL ENTRANCE

JURJEN A. BATTJES

Department of Civil Engineering, Delft University of Technology, Delft (The Netherlands)

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ABSTRACT

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A case study is presented of the effects of tidal currents on the wave heights in a tidal entrance, viz. the Oosterschelde estuary in the Netherlands. Observations of the variation of wave height with tidal elevation during a tidal cycle show a hysteresis. In order to investigate this phenomenon, calculations have been made of wave height changes due to refraction by non-uniform depths and currents, including the occurrence of wave breaking on the shoals in the estuary delta. The calculated wave height variation with tide elevation is found to have a current-induced hysteresis similar to the observed one.

INTRODUCTION

Wind waves and swell can be modified appreciably when they propagate in regions with non-uniform currents, such as boundaries of ocean currents, or near tidal entrances. Although this phenomenon is well known in a qualitative sense, quantitative information about it appears to be scarce. In this paper, a case study is described of effects of tidal currents on wave penetration into an estuary mouth. Some measured data are discussed, and compared to results of calculations which include wave—current interactions.

The present investigation was made as part of a more comprehensive wave climate study for the design of a movable storm surge barrier in the Oosterschelde estuary in the southwest of The Netherlands. Reference is made to Vrijling and Bruinsma (1980) for a description of that study, including the manner in which the effects of the tidal currents on the wind wave penetration were taken into account.

The contents of the paper are as follows. First, the area of the measurements is described and examples are given of observed wave height variation at a point during a tidal cycle. Subsequently, a resume is presented of the schematization of the conventional linear current—depth refraction model, adopted for the present problem. This is followed by numerical calculations, and a comparison of the results with the field observations.

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OBSERVATIONS

The Oosterschelde is a tidal branch which is part of the estuarine region along the southwest coast of The Netherlands. It is about 8 km wide at the mouth, where the major channels have maximum depths of about 25 m to 35 m. The shoals in the offshore delta extend to an elevation of a few meters below mean sea level (MSL). The tide in the area is semi-diurnal, with a mean range of approximately 3 m. The tidal volume passing the mouth during an average flood- or ebb-cycle is about 1.2×10^9 m^3, with maximum tidal velocities of about 1.5 ms^{-1} in the channels. The normal tidal regime can be significantly altered by storms over the North Sea. Northwest storms in particular can cause a considerable set-up.

A study of the tides, storm surges and wind-waves in the Oosterschelde delta was initiated by the Dutch Board of Maritime Works (Rijkswaterstaat), in preparation of the design, construction and operation of a movable storm surge barrier in the Oosterschelde mouth (see Fig. 1 for the projected barrier site). The estimation of the probabilities of occurrence of various extreme conditions necessitated a significant extrapolation of available field data. To aid in this, a mathematical model was developed (Vrijling and Bruinsma, 1980) incorporating the most important physical effects, so that the extrapolations could be based on more than purely statistical methods. An essential element in this model is the presence of extensive shoals in the offshore delta, which — because of wave breaking — effectively provide an upper limit

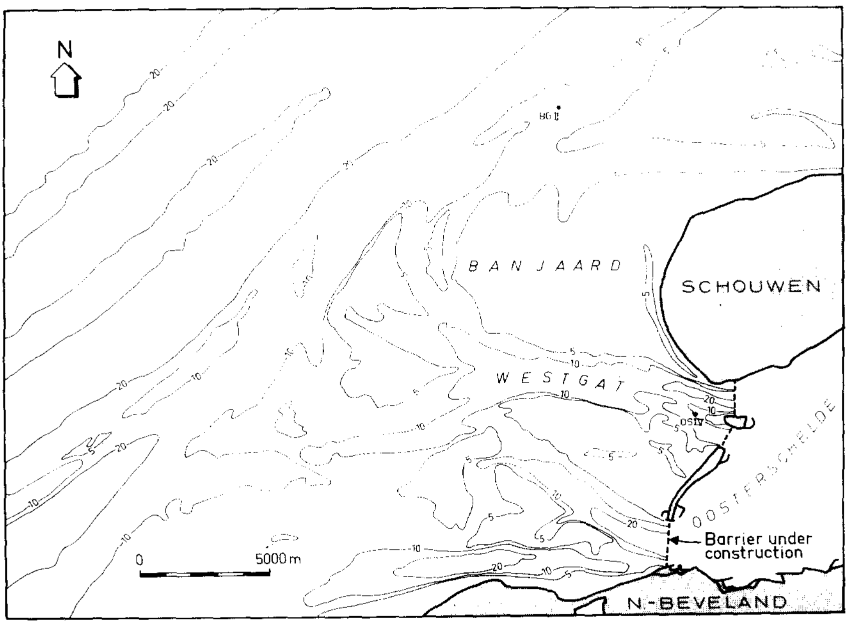

Fig. 1. Oosterschelde delta; contours of depth in m below LLWS.

to the level of wave energy transmission from the North Sea to the barrier site. Since this is a depth-controlled mechanism, a strong correlation was expected between the "instantaneous" tide level (η) above a reference plane (MSL), and the significant wave height inshore of the delta (H_s). Observations during rough weather, when the waves approaching the estuary from deep water could not pass unbroken over the shoals in the delta, did indeed show such correlation, but at the same time a significant deviation from a unique relation was consistently noted, in the sense that the variation of wave height with elevation during a tidal cycle was hysteretic.

Two examples of data showing the phenomenon mentioned above are given in Figs. 2 and 3, from observations at station "OS IV" in the Ooster-

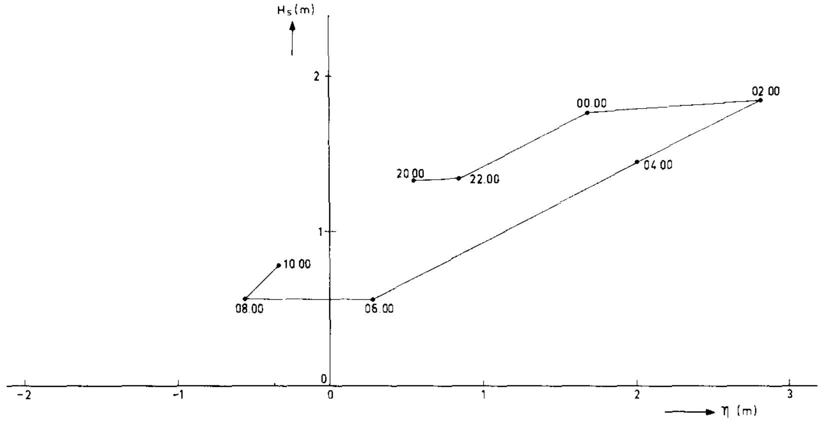

Fig. 2. Bihourly values of significant wave height vs tidal elevation, measured at station OS IV (see Fig. 1 for location) on April 2 and 3, 1973; the numbers indicate local time (20.00 h local = 19.00 h GMT).

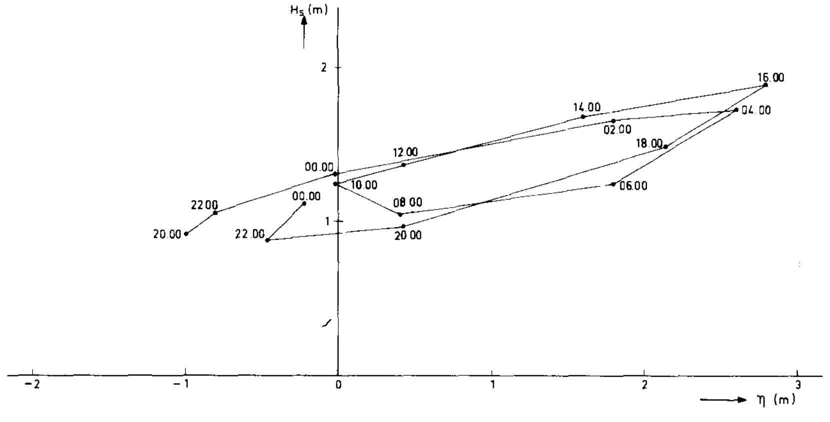

Fig. 3. Bihourly values of significant wave height vs tidal elevation, measured at station OS IV (see Fig. 1 for location) on November 12 and 13, 1973; the numbers indicate local time (20.00 h local = 19.00 h GMT).

schelde entrance (see Fig. 1 for location). The measurements were made during two storms, one in April 1973 and one in November 1973. These differed mainly in the wind direction. During the April storm (Fig. 2) the wind was from about 330° with respect to north, which is more or less across the major channels in the delta. In the November storm, the direction was about 290°, which is more nearly along the axes of the major channels (see Fig. 1). This difference is noted here in view of the calculations presented later.

Apart from the wind- and wave direction, the conditions in the two storms were rather much alike, at least in the time intervals shown in the figures. The average wind speed was about 18 m s^{-1}. The incident waves, measured at station "BG II" located at the outer rim of the delta (see Fig. 1), had a significant height of approximately 3.5 m and a mean zero-crossing period of about 6 s to 7 s.

Whereas the data shown in the Figs. 2 and 3 are only two examples, they are typical for all observations of $H_s(t)$ vs. $\eta(t)$ during rough weather. A notable feature is the hysteretic variation of wave height with tidal elevation. It seemed plausible that this would be due to the influence of tidal currents, but this was not certain. It was important to verify whether this was indeed the case, if alone because the outcome would influence the manner of extrapolation of the data to conditions after the construction of the storm surge barrier, since the latter — if closed — would greatly reduce the ingoing and outgoing tidal currents in the delta region.

REFRACTION MODEL

For the reason mentioned above, it was decided to look more closely at the possibility of current refraction as the cause of the observed hysteresis. To this end, calculations were carried out. In view of the restricted scope, a coarse schematization of the complex conditions in the area was deemed sufficient. Instead of a full two-dimensional model, requiring wave ray calculations in the inhomogeneous depth- and current-field, a quasi one-dimensional approximation was adopted. This requires straight and parallel depth contours, current velocities which are uniform along these contours, and waves propagating across them. Such conditions prevail more or less for waves incident from northerly to northwesterly directions, which propagate across the shoal "Banjaard" and the channel "Westgat" (see Fig. 1). This is the situation corresponding to the April 1973 storm discussed above. For waves from more westerly directions of incidence, i.e. more nearly along the channel axis, this approach is not feasible.

A monochromatic, long-crested wave model was applied, based on the linear, potential-flow theory, except for a semi-empirical limiting wave height due to breaking on the shoals. The model which was used is described in the following.

The propagation is considered of short waves through a region in which mean depth (h) and current velocity ($\vec{U}$) vary slowly with position ($\vec{x}$) and time (t). Effects of the waves on the mean depth and current are ignored in this paper.

In the refraction approximation, the waves are locally treated as uniform. Cumulative effects of variations in depth and current velocity are calculated on the basis of conservation laws. Reference is made to Whitham (1974) for derivations and for a presentation of the equations for the general case. In view of the schematization adopted here, we will state the reduced form of the general equations for a temporally periodic ("steady") situation, uniform in one horizontal coordinate (apart from phase differences).

Assuming a periodic incident wave field, the use of a steady model is permissible if the time scale of the variations of h and $\vec{U}$ is large compared to the travel time of wave modulations traversing the study area at the group velocity. In the applications given below, h and $\vec{U}$ vary with the semi-diurnal tide, with a period of about 12.4 hours, which is an order of magnitude more than a travel time as described here, which is less than one hour (see below). For time-harmonic motion, the surface elevation ζ can be written as:

$$\zeta(\vec{x}, t) = a(\vec{x}) \cos [\omega t - \Psi(\vec{x})] \tag{1}$$

The amplitude (a) and phase (Ψ) are slowly varying with $\vec{x}$. The frequency is denoted by ω, and the local wavenumber vector is:

$$\vec{k} = \nabla \Psi \tag{2}$$

Frequency and wavenumber are related through the dispersion equation:

$$\omega = \sigma + \vec{k} \cdot \vec{U} \tag{3}$$

in which:

$$\sigma \equiv (gk \tanh kh)^{1/2} \tag{4}$$

is the intrinsic wave frequency, i.e. the frequency observed in a frame of reference moving with the current.

The concept of wave action plays a central role in the dynamics of wave—current interactions. To leading order, the action density (A) equals the ratio of energy density (E) to the intrinsic frequency (σ):

$$A = E/\sigma = \tfrac{1}{2}\rho g a^2/\sigma \tag{5}$$

in which ρ is the mass density of the water and g is the gravitational acceleration. The flux of wave action ($\vec{F}$) is given by:

$$\vec{F} = (\vec{U} + \vec{c}_g)A \tag{6}$$

in which $\vec{c}_g$ is the group velocity of the waves, given by:

$$\vec{c}_g = \left(\frac{\vec{k}}{k}\right) \frac{\partial \sigma}{\partial k} \tag{7}$$

In the absence of production or dissipation of energy of the wave current system, wave action is conserved.

For temporally periodic motion, uniform in one direction (say the y-direction) the conservation laws for wave number and wave action, and the irrotationality condition of $\vec{k}$ (see eq. 2), reduce to:

$$\omega = \text{const} \tag{8}$$

$$F_x = (U_x + c_g \cos\theta)(\tfrac{1}{2}\rho g a^2/\sigma) = \text{const} \tag{9}$$

and

$$l \equiv k_y = k \sin\theta = \text{const} \tag{10}$$

in which the subscripts denote vector components, and θ is the direction of wave propagation with respect to the x-axis, which is normal to the depth contours. The values of the constants in eqs. 8 through 10 must be given, e.g. in a reference point in deep water.

Substitution of $k_x = (k^2 - l^2)^{1/2}$ in the dispersion equation (eq. 3) gives:

$$\omega = (gk \tanh kh)^{1/2} + (k^2 - l^2)^{1/2}\, U_x + lU_y \tag{11}$$

For given values of the constants ω, l and g, the local value of k can be calculated from eq. 11, in any point where the local values of h and $\vec{U}$ are given. Subsequently, θ follows from eq. 10, σ from eq. 4, and c_g from eq. 7. Finally, the amplitude (a) is calculated from eq. 9, assuming no production or dissipation of energy of the wave-current system.

As noted above, energy dissipation is built into the model via a breaking criterion. For periodic waves on a horizontal or gently sloping bottom, the criterion derived by Miche (1944) can be used, viz.:

$$(ak)_{max} = \alpha \tanh kh \tag{12}$$

in which $\alpha \simeq 0.14\pi \simeq 0.44$ according to Miche's theory; empirically, a somewhat lower value $\alpha \simeq 0.12\pi \simeq 0.38$ seems to be more appropriate (Danel, 1952).

In the applications to be given here, we deal with random waves in fairly shallow water, in which case eq. 12 is replaced by an analogous expression, viz.:

$$(H_s k)_{max} = \beta \tanh kh \tag{13}$$

in which H_s is the significant wave height and k is a characteristic wave number, e.g. at the spectral peak. The semi-empirical coefficient β can be adjusted in a calibration of the model. In the calculations presented below, a value of β was chosen a priori, based on observations reported by Svasek (1969) for the ratio $H_{s,max}/h$ in deltaic areas, which he found to be (0.40 ± 0.05). For this reason, $\beta = 0.4$ is used here.

NUMERICAL RESULTS

We now consider waves approaching the Oosterschelde delta from northerly to northwesterly directions, such that they propagate across the shoals and channels, as required in the one-dimensional theoretical model.

The study area extends from the northern rim of the delta (the 20 m contour, say) to the axis of the channel "Westgat", a distance of about 14 km (see Fig. 1). A typical value of the group velocity there is 6 m s^{-1} (see Table I), giving a travel time of about 40 min. It is perhaps more relevant to consider the location of minimum shoal depth as the outer boundary of the study area, since the depth there controls the wave heights to a large extent; the corresponding travel time would be about 20 min. In either case, the travel time is small compared to the tidal period, so that the assumption of quasi-steadiness appears reasonable.

Numerical results will be presented for two different cases. The first of these is given for purposes of illustration and for a preliminary assessment of the effects of tidal currents; the tide level is held constant in this case. In the second case, the tide level and the current velocities are given a sequence of different values, simulating a tidal cycle.

In the first schematization, in which the tide level is held constant in space and time, the depth contours are assumed to be straight and parallel in the entire region from deep water to the channel (see Fig. 4a). Only three steady current fields are considered: one which is typical for flood flow, one for ebb flow, and one for slack water (zero current).

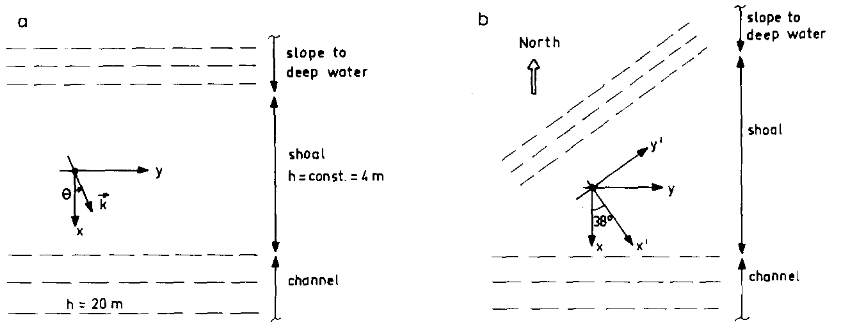

Fig. 4. a. Topography adopted in calculations for Table I. b. Schematization of part of the Oosterschelde delta, used in the calculations for Fig. 5.

Calculations have been made for an incident wave with a period $T = 8$ s and direction in deep water $\theta_0 = 45°$ (see Fig. 4a). The columns 2, 3 and 4 of Table I list the values of the other input parameters. The depth $h = 4$ m represents the shoal area, and $h = 20$ m the channel on its lee side. All values in the columns 2, 3 and 4 have been assumed, except U_x in the channel, which is determined from the other values and the equation of conservation of mass for the mean flow ($U_x h$ = const). Although the values of the independent parameters have been assumed, they are nevertheless realistic for the area concerned.

The results of the calculations are presented in the columns 5 through 10 of Table I. The numerical subscripts in the columns 8 through 10 refer to the depth (expressed in meters). The amplitude ratios a_{20}/a_4 (column 8) are based on the conservation of wave action, eq. 9. Assuming that the incident wave height is sufficiently large to cause substantial breaking on the shoals, the wave height at h = 4 m is set equal to its limiting value calculated from eq. 13, with β = 0.4 (column 9). Finally, the wave height in the channel (H_{s20}, column 10) is calculated as (a_{20}/a_4) times H_{s4}.

TABLE I

Calculated wave parameters

Tidal phase	h (m)	U_x (ms^{-1})	U_y (ms^{-1})	k (m^{-1})	θ (°)	c_g (ms^{-1})	a_{20}/a_4 (1)	H_{s4} (m)	H_{s20} (m)
—	∞	0	0	0.063	45	6.25			
Ebb	4	−0.7	−0.7	0.155	16.7	5.25		1.42	
	20	−0.14	−1.5	0.082	32.8	6.57	0.87		1.23
Slack	4	0	0	0.131	19.8	5.51		1.47	
	20	0	0	0.071	38.9	7.41	0.95		1.39
Flood	4	0.7	0.7	0.112	23.4	5.69		1.50	
	20	0.14	1.5	0.061	46.8	8.30	1.03		1.55

The following features are noted in the results:

(1) The wavelength over the shoals (column 5, h = 4 m) is affected considerably (± 17%) by the tidal currents, but the wave height (column 9) only moderately (± 3%). The relatively small influence on the wave height is due to the fact that kh is relatively small over the shoal, so that $H_{s,max}$ is determined almost exclusively by the depth (which is held constant in Table I).

(2) At the transition from shoal to channel, the wave height decreases during ebb and it increases during flood. This is due to the velocity component normal to the depth contours. (The component parallel to the contours would cause an opposite effect.)

(3) The influences of the currents on the wave height mentioned under (1) and (2) are reinforcing each other, with the result that the wave height in the channel during flood is about 11% greater than it is in the absence of currents, and during ebb it is about 11% smaller.

The result mentioned in (3) is qualitatively in agreement with the observations discussed above, since these showed a larger wave height during rising tide, which mainly coincides with flood flow, than during falling tide, which mainly coincides with ebb flow.

In the second set of calculations, an entire tidal cycle was simulated. Also, the bottom was schematized differently, by taking into account that the depth contours on the seaward side of the shoal "Banjaard", which are more or less parallel, run at an angle of about 38° with respect to those on the south side and along the channel, which also are more or less parallel. The depth between these two transition regions is approximately uniform. It is therefore possible to use a two-dimensional schematization in each of the two transition regions, and to apply a coordinate axes rotation in the uniform-depth region between them (see Fig. 4b).

Tidal information needed in the calculations was taken from a two-dimensional numerical tidal model, which has been extensively calibrated and verified (Langerak et al., 1978). (The use of calculated values, instead of measured data, had the advantage that the (future) situation of a closed storm surge barrier could also be dealt with.) From this numerical model, two-hourly values of tidal elevations (assumed to be uniform in the study area) and current velocity vectors were obtained for a typical spring tidal cycle (see Table II). A wind set-up of 1 m was added to these tidal elevations so as to approximate more closely a northwest storm situation. The shoal elevation was taken at 4 m below MSL, as in the calculations presented above. The incident wave period and direction were taken as 6 s, and 150° with respect to north (from 330°), which are representative values for the April 1973 event shown in Fig. 2. The calculated significant wave heights in the channel, at a mean depth of 20 m, are plotted against tidal elevation in Fig. 5.

TABLE II

Surface elevations (η) and velocities (V) during a typical spring tidal cycle; h_0 is the bottom elevation below MSL; t is time with respect to HW; η is tide level above MSL (including 1 m wind set-up); V is current velocity magnitude; δ is current velocity direction with respect to north

Time	—	Outer rim	(h_0 = 19 m)	Shoal	(h_0 = 4 m)	Channel	(h_0 = 19 m)
t (hr)	η (m)	V (ms^{-1})	δ (°)	V (ms^{-1})	δ (°)	V (ms^{-1})	δ (°)
−6	−0.40	0.9	225	0.6	173	0.2	90
−4	0.25	0.7	220	0.4	160	0.6	92
−2	1.85	0.6	72	0.4	86	1.2	97
0	2.70	0.8	44	0.4	19	0.4	92
2	1.85	0.4	17	1.0	321	1.2	281
4	−0.05	0.4	274	0.6	317	1.1	296
6	−0.75	0.9	225	0.6	165	0	—

DISCUSSION

The calculations presented above were aimed primarily at an assessment of the effects of the tidal currents on the wave heights inshore of the delta. A precise, quantitative prediction of the wave heights was not intended. For

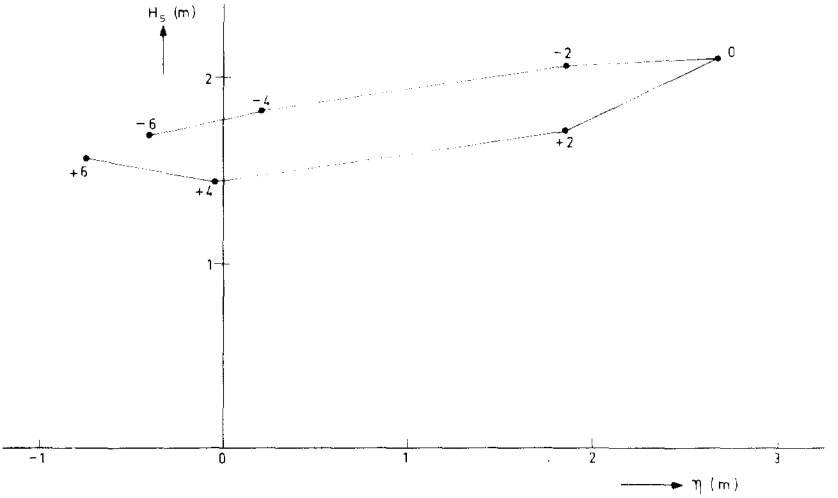

Fig. 5. Calculated bihourly values of significant wave height in the channel, vs tidal elevation; the numbers indicate the time in hours relative to the time of high water.

this reason numerous features and effects such as a two-dimensional bottom topography and current field, energy losses in boundary layers, diffraction, and a continuous frequency- and direction spectrum were not considered. The discussion of the calculated results should be consistent with this approach.

A comparison of Fig. 5 with Fig. 2 (the April 1973 event, simulated in the calculations) shows that the wave heights are overestimated. This is not considered further, in view of the preceding arguments.

The mean variation (trend) of wave height with tidal elevation appears to be underestimated in the calculations. An inspection of the intermediate results showed that the wave height amplification from shoal to channel was largest when the depth over the shoal — and the limiting wave height there as well — was minimal, because the refraction effects between shoal and channel are then maximal. This explains why the calculated wave height in the channel varies relatively weakly with tide level, compared with the wave height over the shoal, which is about 0.4 the mean depth. It is not clear, however, why the calculated trend is much less than the observed one for comparable incident wave conditions (Fig. 2). The trends in Fig. 5 and Fig. 3 are roughly the same, but that may be fortuitous, since Fig. 3 is representative for waves propagating more or less along the channel axis, contrary to the computed case.

The principal feature in the comparison of the figures 2 and 5 is that the observed hysteretic variation of wave height with tidal elevation during a tidal cycle appears to be well reproduced. In this respect, it is relevant to point out that in the calculations the tide elevation η was assumed uniform in the area, and that a quasi-steady approximation was employed. In other words, effects of derivatives of η were excluded, so that effectively the situations at two instants of equal η (one at rising tide, one at falling tide) differed only with respect to the tidal currents. This implies that the calculated hysteresis is entirely due to these currents. The latter conclusion in turn was taken as sufficient evidence that the hysteresis observed in the field data could also be ascribed to tidal currents. This interpretation was used in the estimation of extreme conditions for the Oosterschelde storm surge barrier, including the future situations with modified currents due to closure of the barrier (Vrijling and Bruinsma, 1980).

SUMMARY AND CONCLUSIONS

A case study has been presented of wave height variations in the tidal delta region of the Oosterschelde estuary in the Netherlands.

During rough conditions, when the incident waves were breaking on the shoals in the delta, a consistent and significant non-uniqueness was observed in the relation between the wave heights in the estuary entrance and the tide level, in the form of an hysteresis.

In a schematic theoretical model, which includes depth- and current-refraction and a semi-empirical shallow-water breaking criterion, the effects of the tidal currents on the wave heights have been isolated, and shown to give an hysteresis similar to the observed one.

Although in this paper only a single case study has been presented, the phenomenon considered can be and has been observed qualitatively in numerous locations. It is hoped that the quantitative empirical data and their interpretation, as presented here, will be of use elsewhere.

ACKNOWLEDGEMENTS

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REFERENCES

Danel, P., 1952. On the limiting clapotis in gravity waves. N.B.S. Circular 521, pp. 35—38.

Langerak, A., de Ras, M.A.M. and Leendertse, J.J., 1978. Adjustment and verification of the Randdelta II model. Proc. 16th. Int. Conf. on Coastal Eng., Hamburg, I, pp. 1049—1070.

Miche, R., 1944. Mouvements ondulatoires des mers en profondeur constante ou décroissante. Ann. des Ponts et Chaussées, pp. 25—78, 131—164, 270—292, 369—406.

Svasek, J.N., 1969. Statistical evaluations of wave conditions in a deltaic area. Proc. Symp. Research on Wave Action, Delft Hydraulics Laboratory, Vol. 1, Paper 1.

Vrijling, J.K. and Bruinsma, J., 1980. Hydraulic boundary conditions, hydraulic aspects of coastal structures. Delft University Press, Part 1, pp. 109—133.

Whitham, G.B., 1974. Linear and Nonlinear Waves. Wiley and Sons, New York, N.Y.