BY BURKARD BASCHEK AND DAVID M. FARMER

Kayaking with Bernoulli

INTRODUCTION

In the narrow channels along the coast of British Columbia, Canada, powerful tidal currents generate spectacular rapids sometimes exceeding 8.6 m s^{-1}. The Nakwakto tidal rapids are an especially impressive natural fluid mechanics laboratory and a beautiful example of Daniel Bernoulli's famous principle of energy conservation in a fluid. Boundary-layer separation from irregular shorelines generates intense shears with highly energetic whirlpools that draw gas bubbles to great depth, enhancing aeration of the water. Large standing waves form in the wake of an island. In these environments, fresh surface water from the extensive inlets of the mainland coast and saltier water from the Pacific Ocean are mixed thoroughly throughout the water column. In this article we describe unusual measurements acquired in Nakwakto Rapids, an outstanding educational laboratory with unique opportunities for studying hydrodynamics.

BERNOULLI

First, we offer a short discussion of Daniel Bernoulli, his interesting career path, and some of his work on fluid dynamics. He was born in Groningen, Netherlands, on February 8, 1700 (Guillen, 1995). After his family moved back to Basel, Switzerland, Daniel discovered his fascination for mathematics, which he studied from the age of 11. Although his father Johann and his uncles Nicolaus II and Jacob were renowned mathematicians, Daniel was pressured to pursue a career in business, which would allow him to earn his living. A compromise with his father allowed Daniel to study medicine in Basel, Strasbourg, and Heidelberg, and in his thesis, he used mathematics by applying his father's ideas of energy conservation to breathing. During his subsequent time in Venice, Italy, he published *Mathematical Exercises* and received the prestigious Paris Academy Award in 1725 for his invention of an hourglass that could be used for timekeeping at sea. In the same year, Daniel Bernoulli was offered a mathematics chair at the Russian Academy of Sciences in St. Petersburg. However, his brother Nicolaus III, who came with him to Russia, died within a year after their arrival. To cheer up Daniel, his father sent his best student, Leonard Euler, to St. Petersburg where he and the gifted mathematician soon established a fruitful scientific collaboration.

During this time, Daniel Bernoulli was looking for an improved method to measure pressure and realized that the height to which the fluid rose in a glass needle could be interpreted as a direct measure of pressure. More than two centuries earlier, Leonardo da Vinci had shown that a fluid accelerates when it flows through a contraction. When Bernoulli measured the pressure in a pipe with a contraction, he learned to his astonishment that the pressure in the fast-flowing water was less than in the slow moving water (Figure 1). It did not take him long to apply his discoveries to Leibniz's principle of "vis viva" conservation, which states that the sum of "vis viva" ("living energy") and "vis mortua" ("dead energy") of a solid object is

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constant. In the case of a fluid, however, it is the energy density, or pressure p, that is conserved in exchange with the kinetic energy ρu^2 (ρ is the density and u the velocity). In its modern form, Bernoulli's equation describes the movement of an inviscid and incompressible fluid along a streamline:

$$p + \frac{1}{2}\rho u^2 + \rho g h = const., \qquad (1)$$

where the potential energy $\rho g h$ has been added later, with h being the height and g the gravitational acceleration.

Daniel Bernoulli returned to Basel in 1732 to take up a chair in botany and astronomy. Soon after, in 1734, he and his father jointly received the Paris Academy Award for a second time. But Daniel's father Johann was so outraged by the fact that his son would now be considered his equal that he expelled him from home. Even worse, while Daniel was delighted to have published his extensive work in 1738 in his book *Hydrodynamica*, Johann claimed credit for the very same discoveries in the publication *Hydraulica* that he wrote shortly afterward, but predated to 1732.

In 1750, when Daniel succeeded his late father in the Basel physics chair, he could not suppress a feeling of late justice after all he had had to suffer since returning to Basel. By the time of his death on March 17, 1782, Daniel had won the Paris Academy Award 10 times and had left extensive contributions to fluid dynamics. Applied to the compressible "fluid" air, Bernoulli's equation (1) also provides the theoretical foundation for aerodynamics.

NAKWAKTO RAPIDS

In the natural environment, Bernoulli's principle is readily observed in rivers and in some narrow channels of the coastal ocean. The Nakwakto tidal rapids, located in the remote wilderness of the British Columbia coast (Figure 2), are a particularly striking example. A channel 400-m wide and 13-m deep connects the Pacific Ocean with an over 200-km-long fjord system forming Seymour and Belize inlets. The large tidal exchange through Nakwakto Rapids generates currents of over 8.6 m s^{-1}, making it one of the fastest navigable tidal channels in the world. In the middle of the rapids lies Turret Rock, locally known as Tremble Island because visitors have imagined feeling this 30-m-long island vibrate

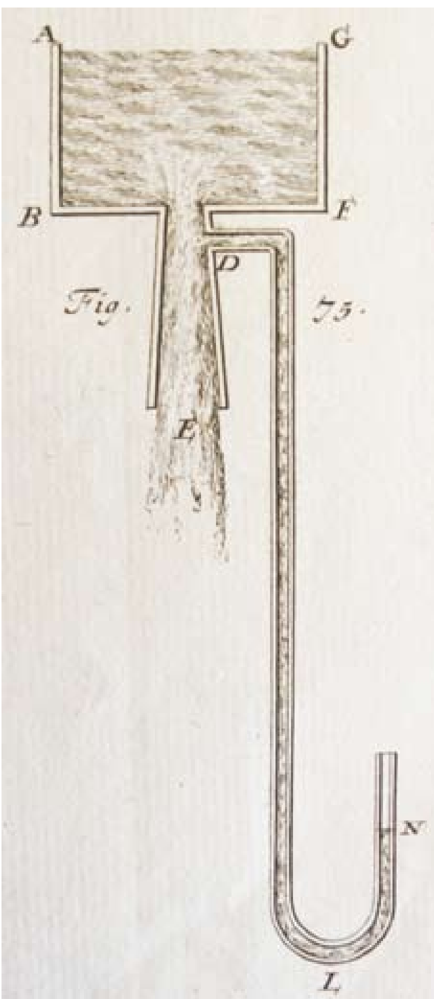

Figure 1. Daniel Bernoulli showed that the pressure in a moving fluid (E) is less than in a stationary one (L). *D. Bernoulli, 1738. Hydrodynamica, Fig. 75; Universitätsbibliothek Basel, Switzerland.*

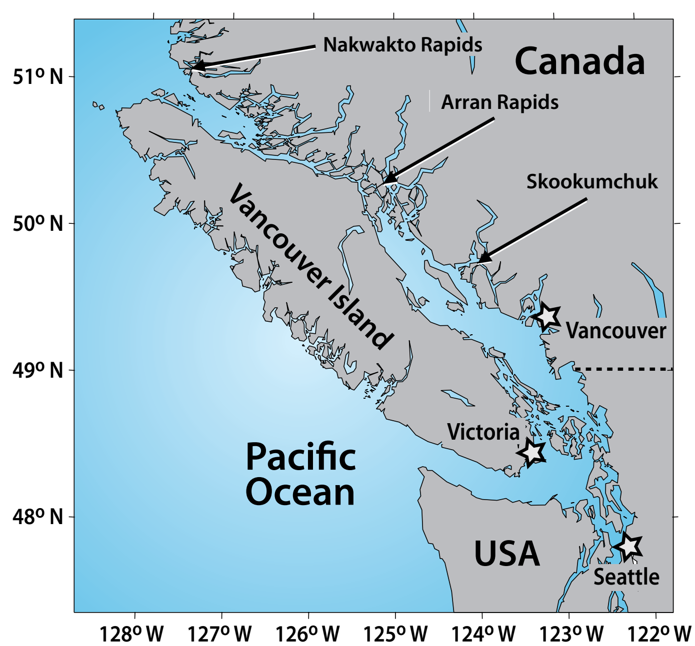

Figure 2. Map of southwestern British Columbia, Canada.

in the current. (Although, we did not visit it during the largest tide, we could not confirm any detectable trembling.) The island withstands the surging tidal current like a tugboat creating a prominent "bow wave" associated with a water-level drop of 2 m leading to a clearly visible slope in the sea surface (Figure 3). The water is constantly moving and even slack tide lasts only a few minutes.

In May 2000, our participation in a film shoot for *The Maelstrom*, a production by the British TV company Northlight Productions Ltd. (broadcast on Channel 4, UK, and Discovery Channel, USA/Canada), provided us with an opportunity to visit this natural fluid mechanics laboratory. With the support of Northlight Productions, we acquired brief but intriguing measurements with a global positioning system (GPS) unit, a sensitive Paroscientific atmospheric pressure sensor, and an internally recording Sea-Bird conductivity-temperature-depth (CTD) sensor deployed from a small boat. The CTD was used to verify the thoroughness of tidal mixing, while the boat with GPS served as a Lagrangian drifter to measure the surface current speed. The sensitivity of the atmospheric pressure sensor is equivalent to a vertical resolution of 0.2 m and was used to measure sea-surface elevation. Although noise is introduced to the measurements due to wave action and irregular motion of the boat in this turbulent environment, the magnitude of the current and the changes in surface elevation remain apparent in the data. Pressure measurements at a fixed shore location provided an atmospheric pressure reference. The measurements of surface elevation and flow speed thus serve as a basis for interpreting these extreme currents.

At ebb tide, the flow in the channel is three-dimensional and can be described as a "contracting jet" by the free streamline theory (Henderson, 1966). It separates from the western shore and is bounded by a relatively straight shear zone between the smooth flow in the center of the channel and the turbulence beyond (Figure 4). On the east side, the smooth flow is bounded by a turbulent wake shed by Turret Rock that is characterized by large standing waves (Figure 5). Both of these separated flows converge at location (C) in Figure 4a. Notwithstanding the complexities of these bounding flows, the central flow is free of significant turbulence, and it was in this flow that we carried out our measurements. Figure 4b shows the sea surface elevation and current speed along the measurement path marked by the solid black line in Figure 4a. This path represents the flow along a streamline as the boat was used as a passive drifter.

Figure 3. Turret Rock, locally known as Tremble Island, and the Nakwkato tidal rapids during ebb tide. The sea surface slope is clearly visible.

Although the flow reaches velocities of 6.1 m s^{-1}, it is subcritical in the middle of the channel as indicated by the Froude number $F^2 = u^2 / gH < 0.4$, where g is Earth's acceleration and H is water depth. Although we lack measurements in the area of the large standing waves (Figures 4 and 5), the water depth of 4 m and the flow speed in the deeper channel of 6.1 m s^{-1} suggest that the flow may be locally controlled ($F^2 = 1$).

The sea surface elevation drops from the upstream to the downstream side of the island by h = 2.5 m. The surface current reaches 2 m s^{-1} at the channel cross section (1) located right next to the island (see Figure 4a). With an estimated cross section of A_1 = 1250 m^2, this calculation results in a volume transport of Q_1 = 2500 $m^3 s^{-1}$. The current reaches maximal values of 6.1 m s^{-1} further downstream where the jet is contracted (cross section 2 in Figure 4a). Between (1) and (2) there will be an unknown

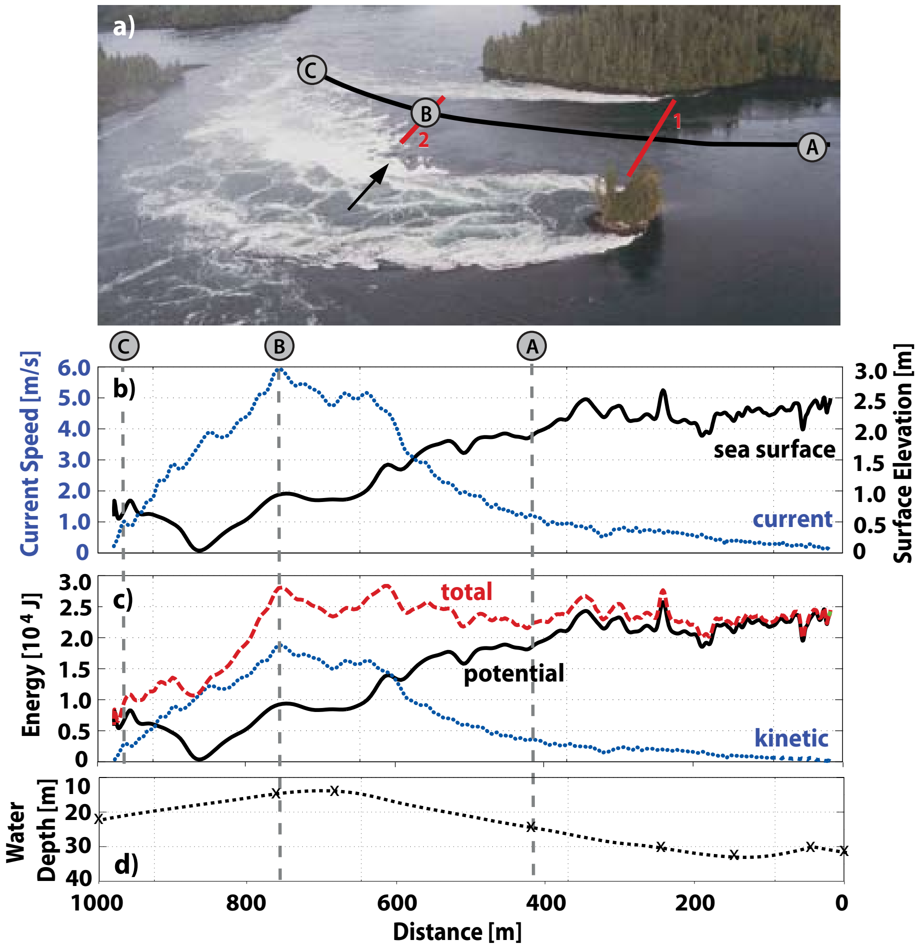

Figure 4. Energy dissipation. (a) Aerial view of the Nakwakto Rapids with transect and reference points A, B, and C for comparison with the lower panels; cross sections 1 and 2 are used to calculate flow volume. The arrow shows the location of the large standing waves shown in Figure 5. *Photograph from Hemmingway and Douglass (1999)* (b) Sea surface elevation (solid black) and surface currents (dotted blue). (c) Kinetic (dotted blue) and potential (solid black) energy of the flow in the Nakwakto Rapids. The sum of kinetic and potential energy is shown as a dashed red line. (d) Approximate water depth.

Figure 5. Standing wave in the wake of Turret Rock showing the transition to the dissipative region of flow separation.

loss of fluid through the separation boundary into the mixing layer beyond. However, we can estimate an upper bound to the resulting contraction coefficient under the assumption of zero loss (i.e., $Q_1 = Q_2$). This corresponds to a reduced cross sectional area of the strong current $A_2 = 0.33A_1$, which is similar to the expected coefficient of contraction $c = 0.29$ for flow through an orifice, derived from a modified form of the Bernoulli equation (1) $u^2 = \sqrt{2gH}$ (Toricelli's theorem) and the principle of flow continuity, yielding

$$c = \frac{Q_1}{A_1\sqrt{2gH}}. \qquad (2)$$

Actual values will be smaller owing to losses through the shear zones associated with flow separation.

The measurements can be used to calculate the kinetic and potential energy (Figure 4c). The potential energy drops with the sea surface elevation. At the same time, the flow accelerates as it descends to match the surface level of the channel to the south (the "downstream condition"). The kinetic energy is a quadratic function of the current speed and therefore has its maximal value at location (B) in Figure 4. The sum of kinetic and potential energy (Figure 4c in red) is constant within the uncertainty of measurement, as predicted by Bernoulli's equation (1) for flow along a streamline.

At point (B), the measurement path meets the eastern separation zone after which the flow speed declines, as does the total energy. At this point, the inviscid assumption invoked by the theorem breaks down, and the difference in total energy between upstream and downstream channels is dissipated by turbulence. The complex bathymetry is responsible for the three-dimensional distribution of the turbulent flow on both sides of the channel, which entrains water from the jet in the center of the channel. The turbulence includes standing waves downstream of the island (Figure 5), entrainment of slower fluid from the sides, and bubble injection by breaking waves. Flow separation at the bottom and entrainment of fluid from below is likely as indicated by the CTD profiles (see below). Between 850 m and 1000 m (Figure 4), the sea level rise contributes to the deceleration of the flow. Bottom friction within the jet does not seem to play a significant role, at least until point (B), as the total energy remains constant between (A) and (B).

It is this turbulent flow that is responsible for vertical mixing of the stratification. CTD profiles from both sides of the rapids (Figure 6b–d) show that the fresh surface layer on the upstream side and the water beneath it are mixed vigorously in the rapids. Also, a denser water mass that was not detected with the CTD on the upstream side is mixed into the water column. Less than 4% of the upstream energy is converted to increased potential energy of the water column via vertical mixing.

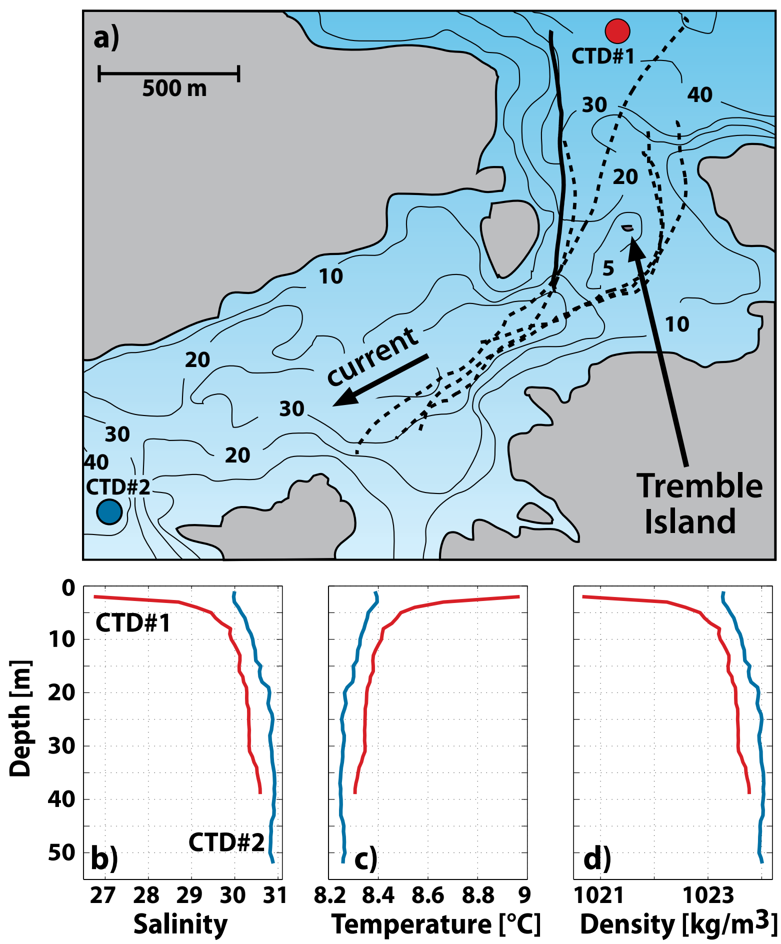

Figure 6. (a) Chart of Nakwakto Rapids. Dashed lines show successive downstream traverses; the solid line shows the traverse represented by the results in Figure 4. (b–d) Salinity, temperature, and density at CTD stations #1 (red lines) and #2 (blue lines).

SKOOKUMCHUK

Other tidal rapids along the coast of British Columbia also attract attention. This interest is especially true for the more accessible Skookumchuk or Sechelt Rapids. With its strong currents and large standing waves, it is a world-class surfing spot for whitewater kayakers. The waves that mark the transition to high dissipation reach a height of about 2 m at 5.7 m s^{-1} current speed. A white pillow of foam and gas bubbles forms on top of a glassy, blue water that invites the kayakers to ride the stationary wave (Figure 7). More advanced kayakers use the waves to perform artistic cartwheels or to spin around in their kayaks in any imaginable direction while remaining on the wave. When they are washed away by the currents, a large back eddy on one side of the channel allows them to return upstream against the current and back to the waves in order to practice more moves with colorful names such as “McTwist,” “Donkey-flip,” and “Air Screw.”

CONCLUSIONS

Our measurements of tidal currents and sea surface elevation show that Bernoulli’s principle applies to the flow in the Nakwakto Rapids within the observational error until air entrainment and turbulence cause energy dissipation. Measurements of surface current speed and sea surface slope, as well as density profiles, demonstrate the intense mixing occurring within the turbulent area of flow separation. About 4% of the

Figure 7. Kayakers surfing on a 2-m-high standing wave at the Skookumchuk tidal rapids, British Columbia, Canada.

dissipated energy was used for thorough mixing. In Nakwakto, the water retains some of its stratification after passing through the rapids; in other locations such as the Arran Rapids (Figure 2), mixing is complete.

The combination of narrow channels and strong currents causes large sea surface slopes, intense turbulence, and large standing waves in many tidal rapids. Kayakers exploit the large waves, where they perform their practical fluid dynamical experiments at the downstream limit where the flow becomes turbulent and this simple application of Bernoulli's principle breaks down.

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