concerned with the more general problem of a tubular antenna driven on the outside surface by a generator that is not necessarily rotationally symmetrical. A practical example is the thick monopole driven from a coaxial line excited in the the TE_{11} mode. For the rotationally symmetrical excitation in the TEM mode, which is included as a special case, the integral equation derived by King and Wu (1967) and again by Otto (1968) is, of course, obtained. Since the Seshadri-Wu method introduces neither a discontinuous potential nor a fictitious magnetic current source, it is no doubt the least "artificial."

Perhaps, now that three different methods for deriving the same integral equation have been published, the matter can rest and work can proceed with its solution.

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A Review of Scattering From Surfaces With Different Roughness Scales

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An examination and review of various theories for scattering from surfaces with different roughness scales has appeared in a recent report by Barrick and Peake. This note briefly summarizes the report by presenting some of the more important results contained therein.

1. Introduction

A recent report (Barrick and Peake, 1967) examines and reviews various theories for scattering of electromagnetic waves from surfaces with different roughness scales. The first two sections of the report deal with slightly rough and very rough surface scales; they review and compare various analyses already available and explicitly show the approximations under which the theories are valid. The third section, treating composite surfaces (i.e., surfaces having slightly rough and very rough scales simultaneously), presents a new approach as far as the Western literature is concerned. The fourth section applies the preceding theories to roughened spherical surfaces, such as those of a planet, immersed in the illuminating field. The emphasis is on the physical process or mechanism behind the analysis, rather than on the mathematical detail. The various theories are compared with measured data. Closed-form results are presented, where available, and polarization dependence is retained for both bistatic and backscatter. Finitely conducting homogeneous surfaces, as well as the perfectly conducting case, are treated.[1]

[1] The referenced report is too lengthy to be published without serious reductions. Hence, this note is to provide a digest or summary of some of the significant points of the report. Where mathematical results and curves are repeated here, the results will be restricted to backscatter.

2. Slightly Rough Surface

A slightly rough surface is defined as one having $k_o\zeta(x, y) \cos\theta_i < 1.0$ (for backscatter), where $k_o = 2\pi/\lambda$ is the free-space wavenumber, $\zeta(x, y)$ is the height of the random surface above a mean plane, and θ_i is the angle of incidence from the mean normal to the surface. While two theories have appeared in the literature for this type of surface, viz., physical optics (Eckart, 1953; Davies, 1954; Beckmann and Spizzichino, 1963) and a boundary perturbation approach, only the perturbation technique is strictly valid in the low-frequency limit ($k_o \to 0$). A requirement on the applicability of physical optics, or alternately the tangent plane approximation, is that $k_o\rho > 1$, where ρ is the local radius of curvature of the surface. In the low-frequency limit, this requirement fails at all surface points. An important difference between the two approaches is the fact that only the results from the perturbation shows any polarization dependency for backscatter, a fact which has been observed experimentally. The effects of multiple scattering and shadowing, important near grazing, are included in the perturbation approach, in contrast with the physical optics technique.

The boundary perturbation approach (Rice, 1951; Peake, 1959) is based upon an expansion of both the surface and the scattered fields into Fourier series involving the same eigenvalues. The perturbation, or smallness parameters, are $k_o\zeta \cos\theta_i$, $\partial\zeta/\partial x$, and $\partial\zeta/\partial y$ (the latter are the surface slopes). The results are valid at arbitrary scattering angles, but near the specular direction a strong coherent reflected field term must be added also. For backscatter, the average incoherent scattering cross section per unit area derived in this manner is

$$\sigma^0_{\eta\xi} = 4\pi k_0^4 \cos^4\theta_i |\alpha_{\eta\xi}|^2 W[-2k_0 \sin\theta_i, 0], \tag{1}$$

where η and ξ refer to the scattered and incident polarization states, respectively, and $\alpha_{\eta\xi}$ is proportional to a scattering matrix element; $W(p, q)$ is the roughness height spectral density of the surface, and is the Fourier transform of the surface height correlation function. For the linear polarization states,

$$\alpha_{hh} = \frac{(\mu_r - 1)[(\mu_r - 1)\sin^2\theta_i + \epsilon_r\mu_r] - \mu_r^2(\epsilon_r - 1)}{[\mu_r \cos\theta_i + \sqrt{\epsilon_r\mu_r - \sin^2\theta_i}]^2},$$

$$\alpha_{vv} = \frac{(\epsilon_r - 1)[(\epsilon_r - 1)\sin^2\theta_i + \epsilon_r\mu_r] - \epsilon_r^2(\mu_r - 1)}{[\epsilon_r \cos\theta_i + \sqrt{\epsilon_r\mu_r - \sin^2\theta_i}]^2}, \tag{2}$$

with $\alpha_{hv} = \alpha_{vh} = 0$. These solutions are valid only to the first order in the perturbation parameters; the second-order term has been derived by Valenzuela (1967), but is complicated and not readily interpreted. It can be seen that it is the presence of surface roughness frequencies, p, near $2k_o \sin\theta_i$ which produce backscatter. The highest roughness frequencies which can enter the scattering process occur at grazing incidence, where $p \to -2k_o$. Also, depolarization is absent to the first order, but the returned power is polarization dependent. Equation (1) represents a true low frequency solution, as evidenced by the k_0^4 dependence. For a perfectly conducting surface, one merely permits $\epsilon_r \to \infty$ in (2) to give $\alpha_{hh} = 1$, $\alpha_{vv} = (1 + \sin^2\theta_i)/\cos^2\theta_i$. The report also expresses the solutions for the circular polarization states, as well as for arbitrarily directed linear polarization states. A comparison of calculated solutions with measurements (Wright, 1966) is shown in figure 1, and the agreement is in evidence. Further curves for this model are found in Radar Cross Section Handbook (Ruck, Barrick, and Stuart, 1968).

3. Very Rough Surface

The very rough surface is defined as having $k_o\zeta(x, y) \cos\theta_i > 1.0$. In this limit, the physical optics, or tangent plane, approach is the applicable technique for solution. This demands, further, that $k_o\rho > 1.0$, i.e., surface radius of curvature nearly everywhere be larger than wavelength. There have been several vector formulations of the physical optics integral proceeding from the exact Chu-Stratton integral equation (Barrick, 1965; Semenov, 1965; Hagfors, 1966; Stogryn, 1967; Kodis, 1966). The last reference evaluates the integral at the stationary phase points before squaring and averaging, while all of the preceding references invert this order. Barrick (1968b), who employs Kodis' solutions, shows that the results are the same regardless of the order of evaluating the integral and average. In addition, a third heuristic approach (Muhleman, 1964) gives the same solution. Hence, one can be reasonably assured that, despite the seeming disparity of results reported in the Western literature in recent years, all valid solutions can and do agree with each other in the high-frequency limit.

The average backscattering cross section per unit area derived from any of these approaches can be expressed in the following manner:

$$\sigma^o = \pi \sec^4\theta_i |R(0)|^2 p(\tan\theta_i, 0), \tag{3}$$

where $p(\zeta_x, \zeta_y)$ is the joint probability density function for the surface slopes in the x- and y-directions. This slope probability density is related to the joint probability density for the surface height at different points by Barrick (1968a). (It is assumed here that the mean plane of incidence contains the x axis.) The Fresnel reflection coefficient for normal incidence is $R(0)$.

The various approaches to the above solution show that backscattering from a very rough surface in the high-frequency limit arises from areas whose slopes are oriented normal to the line of sight. This fact was first pointed out in the Western literature on radar scattering by Hagfors (1964) and Muhleman (1964), but was recognized at an earlier date in the area of optics (Cox and Munk, 1954). This mechanism also predicts no depolarization for backscatter, since the reflecting surface facets normal to the line of sight are insensitive to polarization direction.

Shadowing and multiple scattering between different surface points are neglected in these theories, as contrasted with the slightly rough surface solution. Shadowing, however, does not seriously degrade the solution for mean-square surface slopes less than 25° until one is within 10° of grazing ($\theta_i = 80°$), at which point (3) is too large by 3dB (Wagner, 1967). No estimate is presently available for the effect of multiple scattering, but it is not expected to be a serious factor as long as roughness slopes are not too precipitous and near-grazing angles are avoided.

4. Composite Surfaces

The composite surface under consideration has two general classes of roughness: a very rough scale and a slightly rough scale. Such surfaces are commonly created by natural forces, for one seldom finds a very rough surface which does not have some slight roughness superposed. The report shows in a heuristic manner that to a first order, one may simply add the average incoherent scattering cross section per unit area for the slightly rough surface to that for the very rough surface. Scattered power from a very rough surface is shown to arise from facets oriented so they specularly reflect into the desired direction according to optics principles. The slightly rough surface produces scattered power proportional to the roughness spectrum, and the mechanism, which is analogous to Brillouin scattering in the Bragg limit, is entirely different from that of the very rough scale. Hence, when the two scales are superposed, each scatters a component of power independent and incoherent of the other, and they may be added (to a first order). This approximation holds for surfaces whose slopes are not too precipitous. While this demonstration is based more upon physical intuition, a more exact—although mathematically involved—proof can be found in two recent Soviet articles (Semenov, 1966; Fuks, 1966).

Previous results have shown that very rough surfaces alone produce backscattered power which is very strong near the vertical direction, but which falls off rapidly as one approaches grazing. The slight roughness, however, scatters a much lower intensity near vertical, but this level does not fall off nearly as rapidly toward grazing. Hence, for scattering from composite surfaces, the region near vertical incidence is accounted for by the dominance of the specular, or very rough scale scatter; this is in agreement with previous suggestions in the literature, which termed this the "quasi-specular" region. The scatter due to the slight roughness scale predominates near grazing, however; the scattered power in this region is commonly termed the "diffuse" component. Since experimental data has shown that this diffuse component appears to follow a $\cos^n \theta_i$ law where n often ranges between 1 and 2, there has been no general agreement on the source of this component. This study suggests that the diffuse component, which dominates near grazing, is produced by the scale of roughness falling in the slightly rough category.

To demonstrate the behavior of backscatter curves obtained by adding the two roughness scale power components, figure 2 shows one of a family of such curves taken from the report for a dielectric surface having $\epsilon_r = 5$. Figure 3 is a measured curve for a relatively calm sea surface at X band. There appears to be a general agreement in the nature of the curves, and especially in the polarization dependence. Similar curves are shown in the report comparing cir-

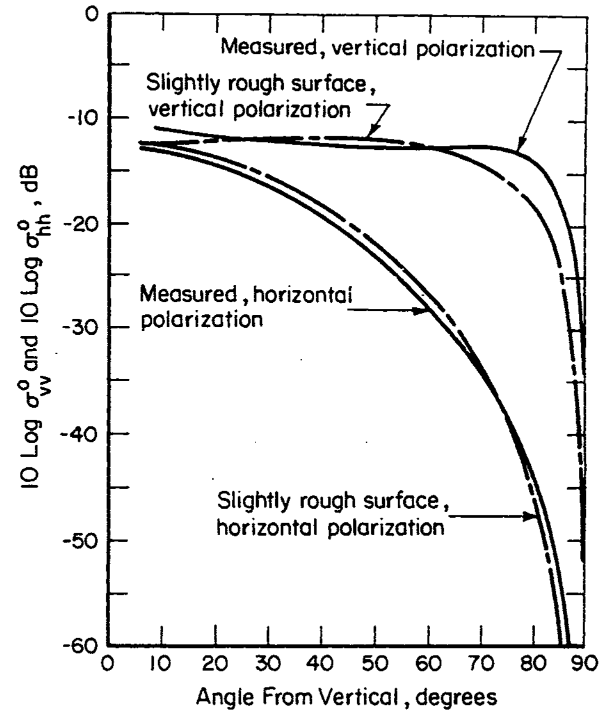

FIGURE 1. *Comparison of measured average backscattering cross section per unit area (solid curves) for slightly rough, fresh water surface at X band with theoretical results (dashed curves) for the vertical and horizontal polarization states.*

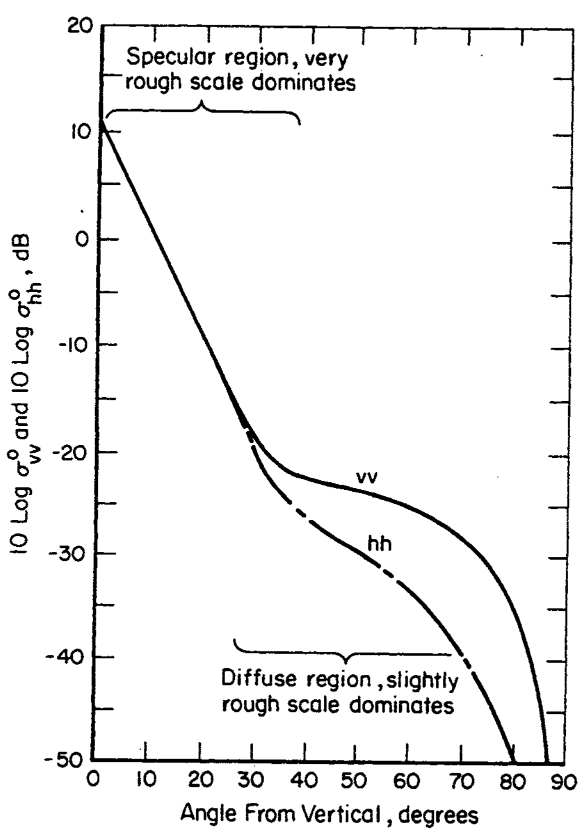

FIGURE 2. *Theoretical curves for backscattering cross section per unit area of composite surface having a dielectric constant $\epsilon_r = 5.0$ for vertical and horizontal polarization states.*

The very rough scale has an exponential slope distribution with rms slope, s_1, defined by $\tan^{-1} s_1 = 10°$, while the slightly rough scale has rms height, h, and rms slope, s_2, defined by $k_0 h = 0.1$ and $\tan^{-1} s_2 = 11.3°$.

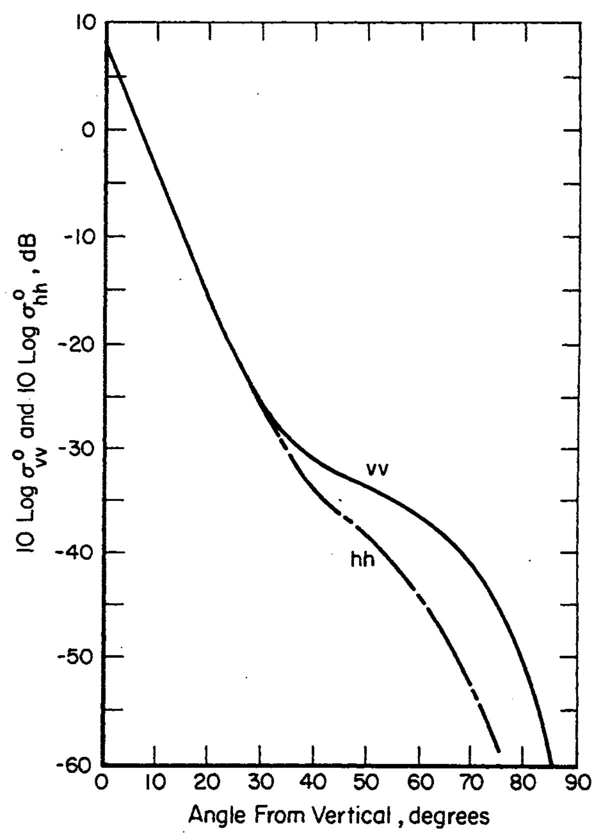

FIGURE 3. *Measured average backscattering cross section per unit area of relatively calm sea surface at X band for vertical and horizontal polarization states.*

cularly polarized returns from the moon with calculated curves.

There are, of course, surfaces for which the above composite model is not appropriate. For example, for surfaces having deeply reentrant fissures, or a cover of vegetation, the scattering may be entirely diffuse, i.e., there is no observable quasi-specular component. This scattering arises from quite different mechanisms, in which multiple scattering and reflection, or scattering by many individual objects, is important. This class of surface is usually discussed (Cosgriff et al., 1960) in terms of empirical scattering laws, of which the best known are the Lambert law ($\sigma' \propto \cos^2 \theta_i$; large reentrant fissures) and the Lommel-Seeliger law ($\sigma^\circ \propto \cos \theta_i$; many layers of independent small scatterers) and its generalizations.

5. Roughened Spherical Surfaces

The report employs the results of the preceding models and calculates the average backscatter cross section for a roughened sphere immersed simultaneously in a CW incident field. Computer-generated curves show that when slight roughness alone is present, its effect is usually small compared to the strong coherent specular return from the front cap, i.e., $\pi A_r^2 |R(0)|^2$, where A_r is the radius of the sphere. On the other hand, when a very rough scale is present, the average incoherent backscatter cross section will increase by a few percent over the result for a smooth sphere. For all practical purposes, however, the average cross section of a very rough sphere can be taken to be the same as that for a smooth sphere.

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