REMOTE SENSING OF SEA STATE BY RADAR

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Abstract

In recent years several radar techniques have evolved which allow the remote measurement of certain parameters important in the description of sea state. At MF and HF, monostatic and bistatic configurations employing satellites, ships, islands, and/or land based stations can measure the ocean waveheight spectrum with several frequencies via first-order Bragg scatter. At high HF and VHF, the ocean waveheight spectrum can be estimated at a single carrier frequency via secord-order mechanisms; this technique is especially suited to remote sensing via long distance ionospheric propagation. At UHF, it is possible to measure the slope spectrum of the longer ocean waves via cross-correlation of simultaneous Bragg-effect returns at two frequencies. The short-pulse microwave satellite altimeter permits a direct measurement of the significant waveheight of the sea at the suborbital point via the specular point mechanism. Such techniques will be important both for detailed oceanographic study of ocean wave characteristics and for routine monitoring of sea state for maritime/meteorological purposes.

Introduction

This paper examines several radar concepts for remotely sensing sea state. "Remote" as applied to the concepts can mean as close as 10-20 nmi; other concepts--if implemented--could measure sea state from a land-based site as far away as 2000 nmi. Satellite and airborne sensor techniques are also included. Rather than presenting detailed theoretical derivations of electromagnetic wave scatter from the rough sea, an attempt is made here to discuss only those remote sensing concepts for which a clear physical understanding is available. Only when the interaction mechanism is understood to the point that an important characteristic of sea state relates clearly and directly to a simple radar observable can one reasonably expect success as a remote sensing concept. Rarely do mathematical manipulations lead to any significant remote sensing discovery. After a concept is uncovered and the expected physical mechanism identified, mathematical analysis can provide a valuable quantitative basis for further development and design of the technique. Experimental verification is of course ultimately necessary to demonstrate feasibility and accuracy.

Four interaction concepts for remotely sensing sea state are suggested here:
(1) First-order Bragg scatter at MF/HF;
(2) Second-order Bragg scatter at HF/VHF;
(3) Two-frequency correlation at UHF;
(4) Short-pulse altimetry at X-band. Of the four concepts, (1) and (4) have been analyzed in detail theoretically and verified in several configurations experimentally. Concept (2) has been partially analyzed theoretically, and tests are just being initiated to provide experimental verification. Only partial theoretical analysis is presently available to support concept (3). All four will be briefly discussed in the following sections. Rather than detailed mathematical derivations, the principal equation which quantitatively describes the scatter concept will be given and explained. When experimental verification is available, it will be presented.

The "radar range equation" describes the power received in terms of transmitted power, P_T, wavelength, λ, ranges from transmitter to target (R_T) and target to receiver (R_R), and antenna gains. When the target is a patch of sea of area dA, the average radar cross section describing scatter is $\sigma = \sigma^o dA$, where σ^o is the average radar cross section per unit area (dimensionless) for the sea. This equation then becomes

$$P_R = P_T \cdot \frac{G_T G_R F_L^2 \lambda^2}{(4\pi)^3 R_T^2 R_R^2} \cdot \sigma^o \, dA, \qquad (1)$$

where F_L^2 represents all losses greater than the free-space spreading loss (e.g., surface-wave losses, ionospheric absorption, attenuation through rain, etc.). For lossless transmission, $F_L^2 \to 1$. For surface-wave or line-of-sight radars, G_T and G_R are defined

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here to be the equivalent free-space gains of the antennas in the direction of the scattering patch. For ionospheric over-the-horizon propagation, these gains are as measured in the presence of the ground (i.e., about 6 dB higher than free-space gains.)

It is essential that the term "sea state" be defined at this point and the significant observable parameters describing it be discussed. Sea state in a practical sense refers to the height of the waves or roughness present on the surface of the ocean. Significant waveheight ($H_{1/3}$) is the maritime descriptor giving the height (peak-to-trough) of the highest 1/3 of the waves; it is roughly related to the rms waveheight, h, by $H_{1/3} \simeq 2.83\,h$. As winds drive the seas higher, they in essence increase the heights of longer, faster moving waves; the shorter waves are fully developed to their maximum heights. A deep-water wave of length L travels at velocity $v = \sqrt{gL/2\pi}$, where $g = 9.81\ ms^{-2}$ is the acceleration of gravity. These wind-driven waves will move predominantly with the winds. The length of the longest ocean wave which the wind can excite is one whose phase velocity, v, just matches the wind speed, u. Thus the crudest--but perhaps most important--descriptor of sea state is waveheight ($H_{1/3}$ or h). When the sea is fully developed by the winds, a rough estimate of h in terms of wind speed, u, is $h = .016u^2$ m, where u is in m/s. A more quantitative measure is the ocean waveheight spectrum, $S(\kappa)$, where $\kappa = 2\pi/L$ is the spatial wavenumber. The most detailed function describing the strengths of ocean waves moving in any direction, θ, is the directional spectrum $S(\kappa_x, \kappa_y) = S(\kappa \cos\theta, \kappa \sin\theta)$. See Kinsman [1] or Barrick [2] for a discussion of ocean wave characteristics.

MF/HF Bragg Scatter

Nearly two decades ago, Crombie [3] experimentally discovered the mechanism giving rise to radar sea scatter at HF by observing the Doppler spectrum of his received signal. It consisted almost entirely of two discrete lines shifted above and below the carrier ($f_0 = c/\lambda$) by an amount $\sqrt{g/(\pi\lambda)}$ Hz. For his ground-wave backscatter configuration, these Doppler shifts were seen to be produced by ocean waves whose lengths were one-half the radio wavelength ($L = \lambda/2$) moving toward and away from the radar. Thus the mechanism he deduced is Bragg scatter. Of all the ocean waves present, the only ones seen by the radar are those forming a diffraction grating with half-wavelength spacing, because in this case the returns from each wave crest will reinforce coherently in the backscatter direction. Later theoretical analyses [4,5] confirmed the correctness of Crombie's deductions; Barrick [5] showed that the magnitude of the average scattered signal spectrum, $\sigma(\omega)$, and normalized cross section σ^{o} ($\sigma^{o} = \frac{1}{2}\int_{\infty}^{\infty}\sigma(\omega)d\omega$) for ground-wave backscatter with vertical polarization is*

$$\left.\begin{array}{l}\sigma(\omega)\\ \sigma^{o}\end{array}\right\} = 2^{6}\pi k_0^4 \times \left\{\begin{array}{l} 2S(-2k_0,\ 0,\ \omega - \omega_0) \\ S(-2k_0,\ 0) \end{array}\right. , \quad (2)$$

where $k_0 = 2\pi/\lambda$ and $S(\kappa_x, \kappa_y, \omega)/S(\kappa_x, \kappa_y)$ are the directional spatial-temporal/spatial spectra of the ocean waveheight, normalized such that,

$$h^2 = \iiint S(\kappa_x, \kappa_y, \omega)\,d\kappa_x d\kappa_y\,d\omega.$$

The above equation shows Bragg scatter to explain the interaction mechanism, since the ocean wavenumber, κ, which is being observed is $2k_0$ (i.e., $L = \lambda/2$). The strength of the signal depends upon the height of the waves in the spectrum at this wavenumber. For a Phillips spectrum model (i.e., $S(\kappa) = .005/(2\pi\kappa^4)$ for fully developed waves), this gives rise to $\sigma^{o} = 0.02$ and $\sigma(\omega) = 0.04\ \delta(\omega - \omega_0 \pm 2\pi\sqrt{g/\pi\lambda})$ for first-order waves. The impulse functions equally spaced about the carrier explain the discrete Doppler shifts observed experimentally. The value $\sigma^{o} = .02 = -17$ dB is in quantitative agreement with experimental observations also [2].

Ongoing experimental confirmation and development of sensing techniques based upon this concept have been conducted by Crombie and are discussed in his paper at this meeting [6]. Rather than repeating his experimental observations here, the reader is referred to his paper. Instead, we will briefly describe several remote sensing configurations here which are based upon this principle. More detailed analyses of these are available elsewhere [2,6].

A stationary ground-based backscatter radar which can employ several carrier frequencies in the lower HF and upper MF region can provide estimates of the nondirectional waveheight spectrum, as shown in Crombie's Fig. 2 [6]. By locating this backscatter radar on a moving ship, the ship velocity imparts a Doppler bias to the signal spectrum which can permit measurement of the directional waveheight spectrum [2]. By placing a transmitter on a buoy (or ship) and a receiver bistatically on a non-synchronons satellite, the directional waveheight spectrum near the buoy can be obtained [2]. The directional waveheight spectrum ca: lso be obtained from bistatically separated ground-based transmitter and receiver configurations [2]; this has been verified experimentally and reported in Peterson et. al. [7].

HF/VHF Second-Order Bragg Scatter

The first-order theory discussed above shows that the received radar Doppler spectrum from the sea should consist of two narrow spikes (impulse functions), and nothing else. The dominance of the two spikes observed in experi-

* For other polarizations and bistatic directions, see Barrick [2].

mental records below about 5 MHz is striking; note Fig. 1 of Crombie [6], made at 2.9 MHz. In the higher HF and VHF region, however, two effects become evident upon examination of measured Doppler spectra: (1) the first-order "spikes" becomes broader, and (2) a larger, continuous "floor" between and near the spikes is seen to exist, often consisting of peaks near the first-order spikes. It was suggested by Hasselmann [8] and Barrick [9,2] that these second-order peaks near the first-order spikes should in themselves be measures of the ocean waveheight spectrum.

A recent example of such a signal spectrum is shown in Fig. 1; this was measured at 30 MHz with a ground-wave backscatter radar (vertically polarized) by Tyler et. al. [10]. Spectral

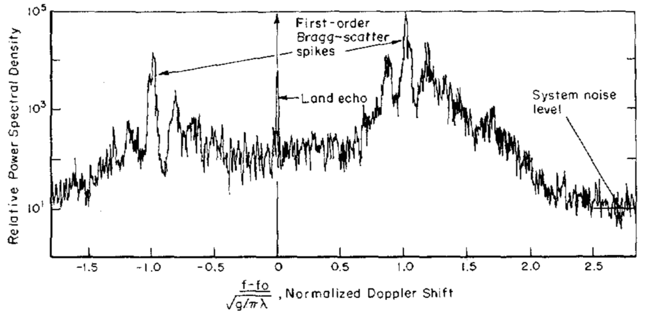

Fig. 1. Sea echo Doppler spectrum at 30 MHz for near-grazing backscatter, vertical polarization: January 25, 1972 [After Tyler, et. al., 10].

processing better than 0.001 Hz was obtained, and some spatial/spectral averaging was done to smooth the record. While quantitative details of the actual ocean waveheights in the measurement area were not available, the winds were observed to be fairly high and toward the radar, resulting in fairly rough seas. The first-order Bragg lines are evident--very near their predicted positions of 0.56 Hz; land echo appears as a narrow spike at zero Doppler. The presense of two peaks near the first-order echoes is quite evident; these peaks merge into a continuum, which rolls off to the system noise level above and below the Bragg spikes. Tyler, et. al, observe that for this record, about one-half the total sea echo power appears in these second-order sidebands.

A further example of such a sea-scatter spectrum made via ionospheric propagation is shown in Fig. 2. This was measured by Barnum (see Barrick [2]) at 25.75 MHz at a distance of 2700 km from the radar in southern California. Barnum's equipment permitted a spectral resolution of 0.04 Hz, and some spatial averaging was done to smooth the record. In the illuminated area of the Pacific, winds (and waves) were predominantly toward the west (away from the radar), producing higher negative Dopplers. The broadened first-order Doppler spikes, predicted to be ±0.518 Hz from the returning carrier, are clearly evident. Second-order spikes are again present. Some spectral smearing due to ionospheric motions is inevitable with such sky-wave sea-scatter records; there is also a small percentage of the time when disturbed ionospheric conditions preclude such measurements entirely.

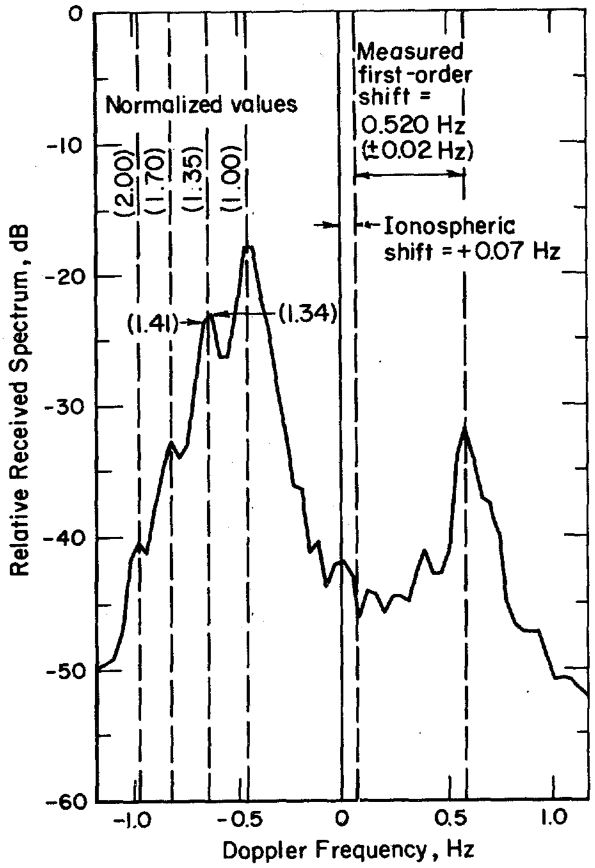

Fig. 2. Sea echo Doppler spectrum at 25.75 MHz for ionospherically propagated backscatter: March 30, 1971 [After Barnum, see [2]].

Barrick [2] has shown theoretically that both hydrodynamic and electromagnetic second-order effects produce the peaks and continuum near the first-order spectral spikes. An integral representation for the average backscattered spectrum, $\sigma(\eta)$, ($\eta = \omega - \omega_0$) can be written in terms of the first-order directional waveheight spectrum $S(\bar{\kappa}) = S(\kappa_x, \kappa_y)$ as follows:

$$\sigma(\eta) = 2^7 \pi k_0^4 \int\int_{-\infty}^{\infty} |\Gamma(\bar{\kappa}_1, \bar{\kappa}_2)|^2 \delta(\eta \pm \omega_1 \pm \omega_2) \times S(\bar{\kappa}_1) S(\bar{\kappa}_2) dp\, dq \quad , \qquad (3)$$

where $\bar{\kappa}_1 = (p - k_0)\hat{x} + q\hat{y}$; $\omega_1 = (g|\bar{\kappa}_1|)^{\frac{1}{2}}$; $\bar{\kappa}_2 = -(p - k_0)\hat{x} - q\hat{y}$; $\omega_2 = (g|\bar{\kappa}_2|)^{\frac{1}{2}}$; and δ = Dirac delta function.

A ground-wave geometry is assumed, with propagation along the x-axis. The kernal of the integral, $\Gamma(\bar{\kappa}_1, \bar{\kappa}_2)$, accounts for both electromagnetic and hydrodynamic effects. The former are obtained from the second-order terms of the Rice boundary perturbation theory [2], and indicate a double-bounce Bragg-reflection mechanism. The hydrodynamic contribution to Γ comes from the second-order terms in the perturbational expansion of the nonlinear boundary conditions at the free water surface; they indicate a continuum of trapped second-order ocean waves, from which first-order Bragg scatter takes place. Details of Γ for the two cases are found in Barrick [2]. As an example, Barrick numerically evaluated the integral for the cases of a semi-isotropic Phillips first-order spectrum with radar propagation in the upwind (or downwind) and cross-wind directions. The predicted results for different sea states are shown in Figs. 3 and 4. These numerical calculations show that the hydrodynamic second-order effects dominate the electromagnetic contributions.

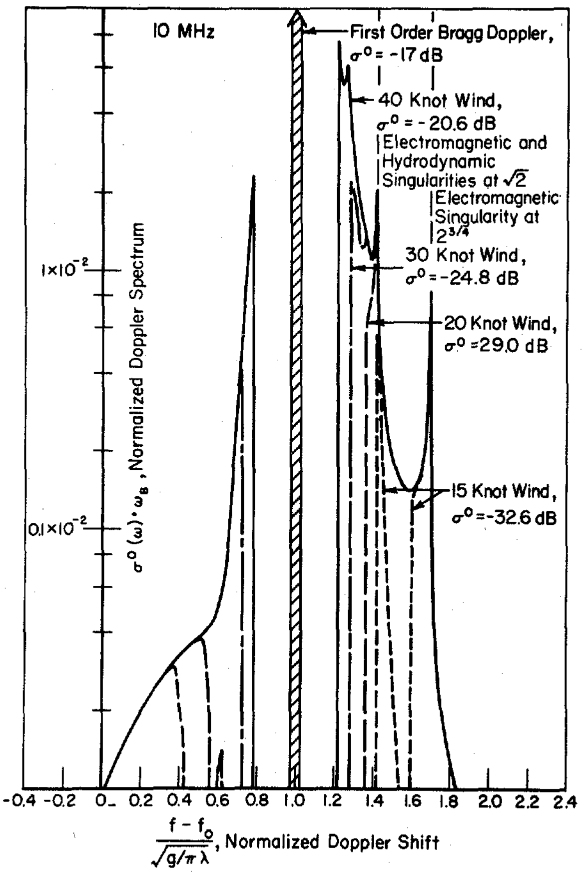

Fig. 3. Predicted Doppler spectrum of first and second-order near-grazing sea backscatter at 10 MHz for propagation in upwind direction (Phillips semi-isotropic waveheight spectrum assumed).

Since the sea will rarely be describable by a semi-isotropic Phillips spectrum, one would seldom observe second-order Doppler spectra which coincide with the above predictions. The significant effects, evident in both the predicted and measured spectra, are second-order peaks whose heights and proximity to the first-order spike increases with sea state. This, along with a comparison of the power in the first-order and second-order peaks, can provide a measure of the significant waveheight and the position of the lower-end cutoff for the waveheight spectrum. The practicality of such a sensing technique at

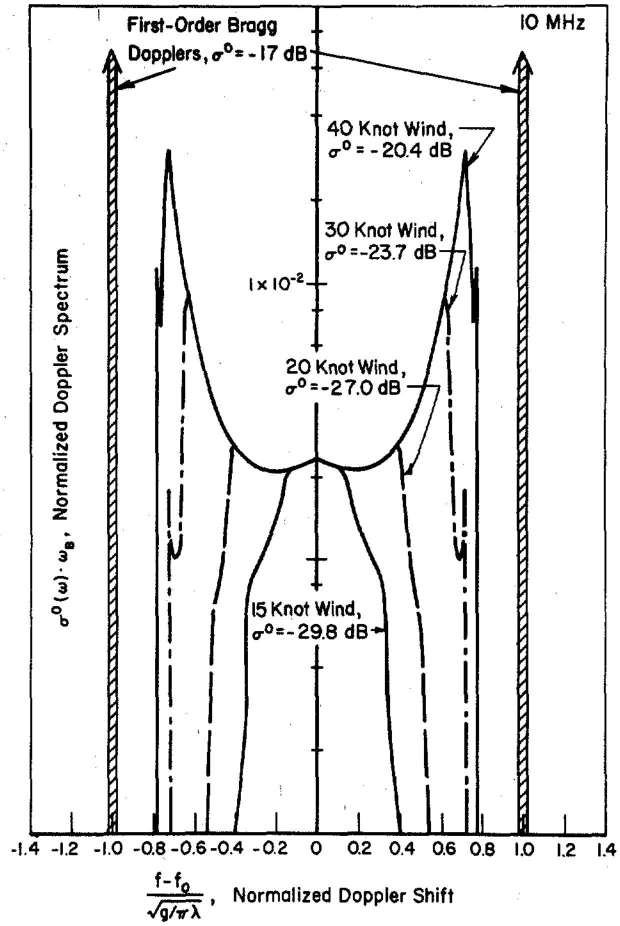

Fig. 4. Predicted Doppler spectrum of first and second-order near-grazing sea backscatter at 10 MHz for propagation in crosswind direction (Phillips semi-isotropic waveheight spectrum assumed).

high HF lies in the fact that this measurement is made at a single carrier frequency, rather than requiring a multitude of carriers from 1.5 to 15 MHz, as called for in the techniques discussed in the preceding section. This is especially suited to long range remote sensing via ionospheric propagation beyond the horizon, where ionospheric conditions at a given time of day normally restrict operation to a single frequency in the upper HF region.

UHF Two-Frequency Correlation Technique

The concept to be suggested here is based upon the fact that Bragg scatter--observed experimentally to explain the off-specular sea echo even at microwave frequencies--depends in intensity upon the slope of the surface at the local scattering patch. Thus, at UHF where the lengths of the Bragg-scattering ocean waves are about 15 cm, the local surface slope at a given scattering patch is determined by the longer and higher gravity waves upon which the 15 cm wavelets are riding. Thus if one employs a narrow-pulse UHF radar permitting, say, a fixed range gate about two meters long to be sampled, he would observe a fluctuating Bragg-scattered signal whose amplitude varied with the slope of the longer gravity waves passing through the range gate. Spectrum analysis of this amplitude will indirectly yield the slope spectrum of the longer "sea state" waves. This slope spectrum is of course directly related to the waveheight spectrum by a factor consisting of the square of the wavenumber. The short-pulse technique mentioned here was examined experimentally and theoretically by Soviet investigators [11].

An alternative to the short-pulse experiment discussed above can provide the same information--based upon the same mechanism--but eliminates the need for a Fourier transform of the received signal envelope. In this experiment, two frequencies are simultaneously transmitted. This can be accomplished by a balanced modulator at the transmitter output. The frequency separation, $\Delta f = f_2 - f_1$, results in a "separation wavenumber," $\Delta k = 2\pi\Delta f/c$. While scatter at the two UHF frequencies, f_1 and f_2, takes place via the familiar Bragg mechanism, the slopes of the longer, underlying gravity waves cause the scattered power at the two frequencies to become less correlated as the frequencies are more widely separated. Results of an analysis of this technique [2, 12] show that the covariance of the received power at the two frequencies for a one-dimensionally rough surface model at a given backscatter angle, θ, from the vertical is expressible as

$$\mathrm{Var}\,[P(\Delta k)] = K_1 + K_2\, W_{SL}\,(2\Delta k \sin\theta) \quad , \quad (4)$$

where K_1 and K_2 are constants (functions of the geometry and the pulse illumination pattern). $W_{SL}(\kappa)$ is the one-dimensional slope spectrum of the longer gravity waves. Equation (4) shows a surprising result; the slopes of the longer gravity waves appear to be measured by a Bragg-scatter process occurring at carrier frequency Δf, producing a sampling spatial wavenumber $2\Delta k \sin\theta$. Thus the frequency separation, Δf, should be varied between about 1.5 and 15 MHz to measure the wave spectrum of the longer "sea state" waves.

This technique, while as yet untested experimentally, has many potential advantages, primarily in equipment simplicities. Two frequencies with less than 2 percent maximum separation in terms of the carrier are easier to generate in a quasi-CW manner than implementation of a short-pulse system with a spectrum analyzer at the receiver output. This system could ultimately find application in airborne or satellite platforms, either separately or in bistatic conjunction with shipboard receivers.

Short-Pulse Radar Altimetry

Considerable interest in the past two years has turned to another remote-sensing technique for ocean profile observations: the short-pulse satellite-borne microwave radar altimeter. Initially such experiments were conceived for geodetic purposes, which could measure the instantaneous mean sea level to a precision of about 10 cm, thus obtaining maps of the geoid and slopes of gross oceanic trenches. It was recognized, however, that waveheight of the order of twenty feet or more would stretch a short pulse (e.g., one whose effective length is of the order of a foot), necessitating careful analysis and interpretation to extract the mean surface position at the suborbital point from the sea-distorted signal. A byproduct of this processing--which to some presently appears to be a more significant use of the altimeter than maps of the mean surface profile--is the significant waveheight of the ocean waves below the satellite.

The interaction mechanism between the pulse of microwave energy and the rough sea in this case is <u>not</u> Bragg scatter. A radar signal whose wavelength is short compared to the larger dimensions of the ocean waves is backscattered from the region near the vertical (i.e., the suborbital region) via the specular point mechanism. This means that only those wave facets whose normals point toward the radar produce scatter. This is the same mechanism as the dancing glitter points one observes visually due to the bistatic scatter of the sun or moon off a rough lake in the region near the specular direction. The reflection from each specular (or glitter) point is proportional in strength to the surface radii of curvature at that point, and the signals from various points combine incoherently due to random phase differences between them.

The analysis of the sea-scattered signal observed by a short-pulse altimeter via the specular point theory (i.e., either a geometrical or physical optics formulation) has been performed elsewhere. Miller and Hayne [13] used a simple model for specular facet scatter; the reader is referred to Barrick [2] for a straight-forward derivation in terms of the complete specular point theory and simple Gaussian models for the radar patterns. This leads to the following closed-form solution for the product of the radar cross section times the antenna gains--as a function of time--which appears in Equation (1) for the received altimeter power:

$$G^2\sigma(t) = (\pi^{3/2}\, H x_W/s^2)\, \exp\,[(t_p/t_s)^2 - 2t/t_s] \times [1 - \Phi(t_p/t_s - t/t_p)] \; , \quad (5)$$

where H = altimeter height ; s = rms surface slope ; τ = half-power pulse width ; $s \approx .074\sqrt{u}$ in terms of wind speed, m/s

ψ_B = one-way half-power antenna beamwidth ;
$x_W = c\tau/\,(4\sqrt{\ln 2}\,)$; $t_p = 2\sqrt{x_W^2 + 2h^2}/c$;
$t_s = 2H\psi_e^2/c$; $1/\psi_e^2 = (8\ln 2)/\psi_B^2 + 1/s^2$,

and h is the rms waveheight. The function Φ is the error function. It is assumed here that the altimeter beam is pointed directly toward the vertical and that an unskewed Gaussian function represents the height distribution of the ocean waves.

The constants t_p and t_s have interesting physical interpretations: t_p is the effective length of the stretched pulse after scatter from the waves distributed over a plane surface, while t_s is the (temporal) depth of the effective scattering region on the spherical earth surface. If $t_s \gg t_p$, the altimeter is said to be pulse-limited in its operation; the opposite extreme is called beam-limited operation (see Barrick [2] for more details). Satellite altimeters are nearly always pulse-limited because of their altitude and antenna sizes. In such operation, the slope of the leading edge of the return is influenced by sea state. This is illustrated by predictions from the model in Fig. 5 for a satelat 435 km, with a beamwidth of 3°, and whose effective pulse width is less than 15 ns.

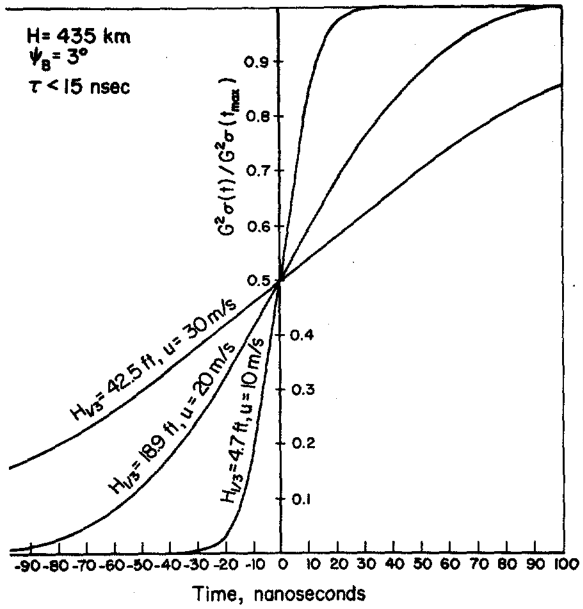

Fig. 5. Leading edge of averaged altimeter output versus time predicted for pulse-limited operation.

Limited experimental confirmation is available from aircraft quasi-pulse limited altimeter operation. Two such records made by Raytheon are shown here in Fig. 6 (see Barrick [2] for details). Surface winds in each case were measured at ~12 and 22 knots. From the rise times of the signals, the wind speeds inferred from Equation (5), along with the dependence of waveheight upon wind speed given in the Introduction, are 14.1 and 21.2 knots. Thus preliminary verification of the model is encouraging.

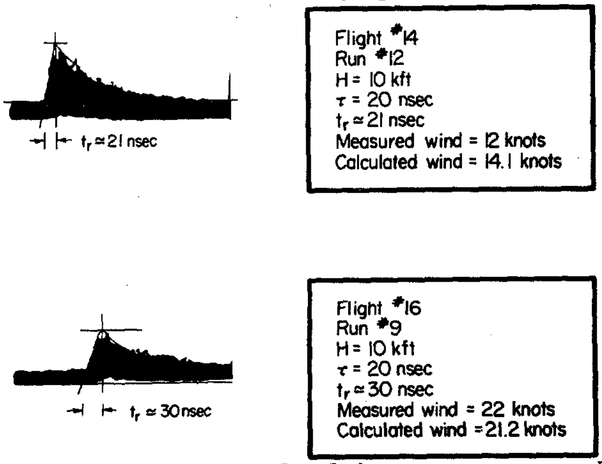

Fig. 6. Measured aircraft altimeter responses. Wind speeds inferred from rise times are compared to observed wind speeds.

Summary

Of the concepts presented here, the MF/HF first-order Bragg scatter techniques are the most advanced, both analytically and experimentally; they are perhaps the most limited in long-term potential because of the required wide frequency band of operation and limited remote-sensing coverage area per station. The second-order HF/VHF concept shows great potential for land-based distant sensing of large ocean areas via ionospheric propagation. The UHF two-frequency technique must be developed further to definitely establish its feasibility; in airborne or satellite-ship applications, it could provide several important pieces of data about sea state. Short-pulse satellite altimetry is proven experimentally; while providing only the significant waveheight along the orbital plane, it could perhaps be the first technique to reach the operational stage, providing needed sea-state information over large ocean areas on a daily basis.

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