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The behaviour of waves on tidal streams

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The paper discusses the manner in which waves change their characteristics when they pass through regions where the water has a streaming motion. The treatment applies to tidal streams, the velocity of which depends both on time and position. Some experimental evidence is provided in support of the theory.

Introduction

Since early in 1945 wave records from the coast of Cornwall have been submitted to frequency analysis for the purpose of studying the generation and propagation of waves and swell. Some results have been reported by Barber & Ursell (1948). The evidence indicates that in a storm, trains of waves are generated of a variety of wave-lengths up to a maximum wave-length depending on the wind strength and that each wave train advances across the ocean with a speed approximately equal to $gT/4\pi$, which is the group velocity indicated by theory for waves whose period is T. The separation between the longest and shortest waves travelling in the same direction must therefore increase with the distance travelled from the generating area, and because of this dispersion the first wave trains to arrive at a distant recording station will exhibit a long natural period. Wave trains of shorter period arrive later, and it is to be expected that the swell arriving at the coast will show a period which decreases continuously with time.

The frequency spectra of waves recorded at Pendeen and Perranporth illustrate this behaviour, but they show that the period of the swell arriving at the coast does not vary exactly in accordance with the simple theory. The swell from a distant storm is observed to have a period which fluctuates by as much as ± 1 sec. in cycles of $12\frac{1}{2}$ hr. about the smooth curve drawn through the observations to represent the general trend towards shorter periods. This fluctuation is evident in the curves of figure 1, which are based on observations made about the time of spring tides.

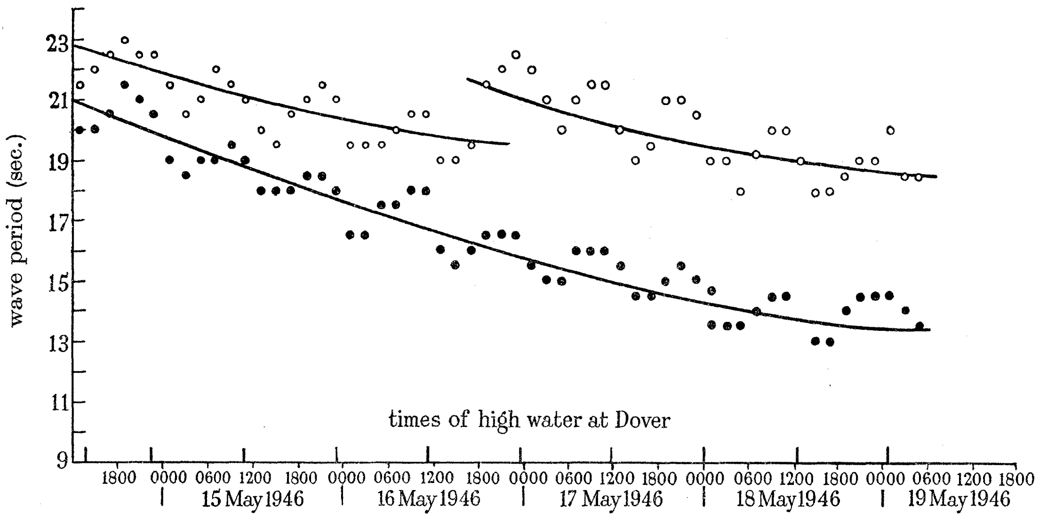


FIGURE 1. The maximum and minimum periods limiting the 24 to 14 sec. frequency band in the wave spectra of 14 to 19 May 1945 at Perranporth.

The fluctuation in period has been attributed to the effect of tidal streams through which the swell had to travel in the last 200 miles of its journey to the coast. The present aim is to discuss this effect in more detail.

THEORY OF THE STEADY STATE

Unna (1942) has discussed the steady state of a wave train entering streams whose velocity does not change with time. He shows that where the stream has a velocity u the waves may be expected to exhibit a new wave-length λ and velocity c given by the rule

$$\lambda_0/c_0 = \lambda/(c+u),$$

where the zero subscript denotes the values of these quantities for that part of the wave train which is on slack water. The equality is obtained by assuming that an equal number of wave crests per unit time must pass any fixed observer. The argument presumes that it is physically possible to have a train of waves, extending from slack water to moving water, whose waves retain their identity, no new ones appearing and none disappearing. No description of such a train has yet been given in terms of a velocity potential, and this may suggest that such a train cannot exist; there may, for instance, be some reflexion of the wave train. But if the existence of the wave train is assumed, and if the wave-length and wave velocity of any part are related by the usual equation, the changes in velocity or length may be predicted; in deep water the rule

$$\lambda = 2\pi c^2/g$$

enables the first equation to be written in the form

$$0 = (c/c_0)^2 - c/c_0 - u/c_0,$$

which is equivalent to formulae due to Unna. The equation describes two types of wave train:

(1) If c/c_0 is greater than $\frac{1}{2}$, the wave train is one which may extend into slack water; it cannot extend into an opposing stream whose velocity is greater than $\frac{1}{4}c_0$. Before this point the waves steepen and break.

(2) If c/c_0 is less than $\frac{1}{2}$, the wave train is one which rides on an opposing stream whose velocity is greater than the group velocity, $\frac{1}{2}c$, of the waves. The train cannot extend into slack water; before it arrives there the waves steepen and break. This wave train might be generated as the wake of a boat moving up stream.

The arguments have been extended by Johnson (1947) to the two-dimensional problem of waves crossing obliquely the boundary between two currents. He infers that when the waves approach the boundary obliquely they may fail to cross it, their energy being dissipated by reflexion or in breaking.

The non-steady problem

It will be appreciated that the tidal variations of period observed at Pendeen and Perranporth cannot appropriately be dealt with by steady-state theory. The velocities of the tidal streams change considerably during the time that the swell takes to cross the area of streams lying between the deep sea and the coast. It is desirable to develop some treatment suitable for waves on streams whose velocity changes with time.

Unna (1941), discussing the behaviour of short waves riding on swell, treats the short waves as if they expanded and contracted with the water mass on which they move. The short waves then attain their greatest length as the trough of swell passes them and attain their smallest wave-length when they are overtaken by the crest. This somewhat intuitive treatment can be justified by reducing the problem to the steady state. A uniform velocity impressed on the system will bring the swell to rest and it may be regarded as slight variations in space, but not in time, of a rapid stream on which the short waves move. This leads to the formulae given by Unna.

The idea that waves expand or contract with the water mass can be justified, however, by another means which is found to provide a solution to the general problem. This treatment is given below, and its conclusions will be seen to be in fairly good agreement with experimental observations.

Kinematics of waves on streams

The classical theory of water waves shows that if the height of the waves is sufficiently small the system can always be regarded as due to the superposition of a sufficient number of elementary wave trains, each consisting of long parallel crests equally spaced and moving with a speed appropriate to the wave-length. In the resulting interference pattern the elementary waves lose their identity, and the more complicated phenomena of group motion, in which waves appear and disappear as they travel in groups, take their place.

In discussing waves on streaming water two assumptions will be made. By analogy with the theory of waves on slack water it will be assumed that a complicated wave motion on streaming water can be looked upon as being the superposition of a sufficient number of elementary trains which could have an independent existence and in which the wave crests would retain their existence as they progressed through the streaming water. Whether such elementary wave trains are possible on streaming water is not known, but the analogy with classical wave theory makes it plausible.

It will also be assumed that in each of these elementary trains of waves the length of the wave and its velocity relative to the water are related by the equation of classical theory

$$c^2 = g\frac{\lambda}{2\pi}\tanh\frac{2\pi h}{\lambda}, \tag{1}$$

where c = wave velocity relative to the water mass, λ = wave-length, g = acceleration due to gravity, h = water depth.

This equation is strictly applicable only to the waves of an elementary train in slack water, but it seems likely that it may apply to an elementary train on streaming water provided:

(*a*) that the changes in stream velocity are small during a very large number of wave periods;

(*b*) that the stream velocity is very nearly constant over a large number of wave-lengths;

(*c*) that the streaming velocity is sensibly uniform over a depth equal to half a wave-length.

With these assumptions it is possible to consider the one-dimensional case illustrated in figure 2. The velocity u of the stream is prescribed at each point x as a function of the time t. An elementary train of waves is present on the surface, and the length λ and velocity c of the waves will in general be functions of both the position x and time t.

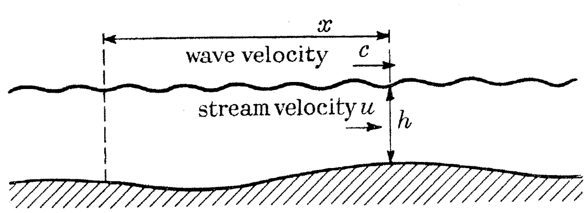


Figure 2. Kinematics of waves on streams (one-dimensional problem).

Considering a wave crest which is at position x at time t, its velocity relative to the water is c and relative to the fixed system of co-ordinates is $(c+u)$. The Nth wave in succession from this one will be at a distance $N\lambda$ from it, and the velocity of this wave will differ slightly from the velocity of the first one, the difference being

$$N\lambda\partial(c+u)/\partial x.$$

This relative velocity may also be expressed as the time rate of change of the distance between the two crests. The value of λ depends both upon x and t, and since the waves move forward a distance $(c+u)\,\delta t$ in a brief interval δt, the rate of change of $N\lambda$ with time is

$$N(\partial\lambda/\partial t+(c+u)\;\partial\lambda/\partial x).$$

The two expressions for relative velocity may be equated to give the relation

$$\frac{1}{\lambda}\left(\frac{\partial\lambda}{\partial t}+(c+u)\frac{\partial\lambda}{\partial x}\right)=\frac{\partial(c+u)}{\partial x}. \tag{2}$$

Waves on water which is deep or of constant depth

The formulae (1) and (2) may now be combined, but the precise form of equation (1) is not involved at this stage, and it is only necessary to observe that the velocity c has been assumed to be a function of λ, g and h and not a function, for instance, of the space and time derivatives of these quantities.

In the first instance it will be assumed that the waves are in water whose depth is uniform and constant, or else that the water in which the waves are travelling is deeper than a wave-length. The equation (1) then shows that we may write

$$\frac{\partial c}{\partial x}=\frac{dc}{d\lambda}\frac{\partial\lambda}{\partial x}.$$

This relation when substituted in equation (2) gives

$$\frac{1}{\lambda}\left[\frac{\partial\lambda}{\partial t}+\left(u+c-\lambda\frac{dc}{d\lambda}\right)\frac{\partial\lambda}{\partial x}\right]=\frac{\partial u}{\partial x}. \tag{3}$$

The right-hand side of this equation measures the rate of expansion of the water surface which is produced by the streaming motion. The left-hand side is the fractional rate of increase of wave-length with time; the expression does not refer to a given set of waves, but to those waves which happen to be in the vicinity of an observer who moves relatively to the water with a velocity

$$c-\lambda dc/d\lambda.$$

This expression is the usual definition of the group velocity of the waves in classical theory. In this problem, therefore, as in many others, the group velocity enters into the equation with a peculiar significance. It appears that if an observer follows a given wave group through moving water, *the average length of the waves in the group expands or contracts at the same rate as the general surface of the water upon which the group is moving.* It will be appreciated that this result determines the whole behaviour of the waves, for the velocity and period may be inferred when the wave-length has been found.

It is convenient to use a special symbol to denote the time rate of change of some characteristic of the waves in a particular group, and using the notation

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \left(u + c - \lambda \frac{\partial c}{\partial \lambda}\right) \frac{\partial}{\partial x}, \tag{4}$$

the equation (3) may be written as

$$\frac{1}{\lambda} \frac{D\lambda}{Dt} = \frac{\partial u}{\partial x}. \tag{5}$$

The general case

It is clear that in tidal regions the water depth is a function both of place and time, and the effective value of gravity to be used in equation (1) may also vary; the waves are riding on a mass of water whose vertical acceleration must modify the effective force of gravity on the waves. It can be shown that these two effects rarely appear simultaneously; the changes in gravity are of importance in discussing the behaviour of wind waves riding on long swell in water whose depth is comparable with the length of the swell, and the changes in depth are of importance in discussing the behaviour of waves on tides. For the sake of generality, changes in both h and g will be discussed.

The wave equation (1) is a relation between the quantities c, λ, g and h, and if the wave period is denoted by T, where

$$T = \lambda / c,$$

it may be shown that the following relations hold between the partial differential coefficients

$$\left.\begin{aligned} \left(\frac{\partial c}{\partial h}\right)_{\lambda g} &= \frac{1}{\lambda}\left(\frac{\partial \lambda}{\partial h}\right)_{Tg}\left(c - \lambda\left(\frac{\partial c}{\partial \lambda}\right)_{gh}\right), \\ \left(\frac{\partial c}{\partial g}\right)_{\lambda h} &= \frac{1}{\lambda}\left(\frac{\partial \lambda}{\partial g}\right)_{Th}\left(c - \lambda\left(\frac{\partial c}{\partial \lambda}\right)_{gh}\right), \end{aligned}\right\} \tag{6}$$

where the subscripts denote the quantities held constant during the differentiation. The quantity $c - \lambda(\partial c/\partial \lambda)_{gh}$ in these expressions is recognizable as the group velocity and will be denoted by G.

Substitution may now be made in the kinematic relation (2). The quantity $\partial c/\partial x$ on the right-hand side of this equation may be expanded as

$$\frac{\partial c}{\partial x} = \left(\frac{\partial c}{\partial \lambda}\right)_{gh} \frac{\partial \lambda}{\partial x} + \left(\frac{\partial c}{\partial h}\right)_{\lambda g} \frac{\partial h}{\partial x} + \left(\frac{\partial c}{\partial g}\right)_{\lambda h} \frac{\partial g}{\partial x},$$

and using the equations (6) the kinematic equation becomes

$$\frac{1}{\lambda} \frac{D\lambda}{Dt} = \frac{\partial u}{\partial x} + \frac{1}{\lambda}\left(\frac{\partial \lambda}{\partial h}\right)_{Tg} G \frac{\partial h}{\partial x} + \frac{1}{\lambda}\left(\frac{\partial \lambda}{\partial g}\right)_{Th} G \frac{\partial g}{\partial x}. \tag{7}$$

This expression for the fractional rate of increase of wave-length in a wave group is similar to the equation (5) derived previously, but it includes two terms which show the effects of changes in depth and in effective gravity. This may be regarded as the fundamental equation governing the change of character of the waves of a group as the group advances through streaming water. From the wave-length the period or velocity or group velocity may be inferred, but it is possible to obtain the changes in these quantities explicitly. Thus if M denotes one of the quantities c, T or G it can be shown that

$$\frac{DM}{Dt} = \left(\frac{\partial M}{\partial \lambda}\right)_{gh} \lambda \frac{\partial u}{\partial x} + \left(\frac{\partial M}{\partial h}\right)_{g\lambda} \left(\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x}\right) + \left(\frac{\partial M}{\partial h}\right)_{Tg} G\frac{\partial h}{\partial x}$$
$$+ \left(\frac{\partial M}{\partial g}\right)_{h\lambda} \left(\frac{\partial g}{\partial t} + u\frac{\partial g}{\partial x}\right) + \left(\frac{\partial M}{\partial g}\right)_{Th} G\frac{\partial g}{\partial x}. \tag{8}$$

A reasonable physical interpretation may be given to equations (7) and (8). From equation (7) it will be seen that the time derivatives of g and h do not affect the wave-length, and so long as the waves remain on a mass of water which is all behaving in a very similar way the changes in wave-length are produced solely by the contraction or expansion of the water surface; changes in g or h only affect the wave-length in so far as the group experiences them as a result of moving with velocity G on to new masses of water where g or h are different. When moving on to new water, the changes in wave-length occasioned by changes in g or h take place as if the period of the waves remained constant. Thus if waves are present on the surface of water in a tank which suddenly begins to accelerate upwards or downwards the length of the waves will not change, but the period will alter appropriately to the new value of g; when waves pass into shoaling water which has no streaming motion the period remains constant, but the wave-length alters appropriately to the new depth of water. The other wave characters experience changes which may be inferred from these changes in wave-length or period.

Changes in the apparent period

A wave characteristic which is not included in the general treatment of the previous section is the apparent period of the waves as recorded by a stationary observer. This will be written as

$$T^* = \lambda/(c+u), \tag{9}$$

and it is this apparent period which is measured in the frequency spectra or waves recorded by an instrument fixed upon the sea bed.

The differentiation of equation (9) gives

$$\frac{1}{T^*}\frac{DT^*}{Dt} = \frac{1}{\lambda}\frac{D\lambda}{Dt} - \frac{1}{(c+u)}\left(\frac{Dc}{Dt} + \frac{Du}{Dt}\right),$$

where the operator D/Dt has the significance previously given to it in (4). The quantities $D\lambda/Dt$, Dc/Dt and Du/Dt may be obtained from equations (7), (8) and (4), and upon substitution of these quantities the equation governing the apparent period appears as

$$\frac{1}{T^*}\frac{DT^*}{Dt} = -\frac{1}{(u+c)}\left[\frac{\partial u}{\partial t} + \left(\frac{\partial c}{\partial h}\right)_{\lambda g}\frac{\partial h}{\partial t} + \left(\frac{\partial c}{\partial g}\right)_{\lambda h}\frac{\partial g}{\partial t}\right]. \tag{10}$$

This equation shows that the changes in the apparent period depend only upon the time derivatives of u, g or h; if, therefore, a wave group passes through an area of sea in which the streaming motion is everywhere constant with time, the apparent period of the waves in the group remains unchanged throughout its progress. The equation shows how the apparent period will change if the waves pass through a tidal area in which u, g and h vary with time.

When wave groups complete their passage through a tidal area in a small fraction of a tidal cycle the change in apparent period will be proportionately small. Comparing two wave-measuring stations screened from the ocean by different widths of continental shelf, it may be inferred that the corrections which must be made to the measured periods in order to obtain the true wave periods in the open sea will be smaller for the station with the narrower continental shelf, assuming that the stream velocities and tidal ranges are similar in the two cases. The proposed wave measuring stations at Casablanca and Wellington, N.Z. are likely therefore to provide a simpler picture of the state of the sea than do the stations in Cornwall.

A practical instance of the tidal variation in apparent period

(a) *Swell travelling from the Southern Ocean*

The equations (5), (7) and (8) derived in the previous sections make it possible to deduce what changes will take place in waves when they cross an area of sea in which the stream velocity and the depth of water are known as functions of time and position. In any practical case it is unlikely that the stream velocity and depth can be expressed as analytic functions of x and t, and a graphical method is more suitable for the integration of the equations. This method has been used to estimate the total variation to be expected in the apparent period of a wave train having a period of 18 sec. in the open sea, which arrives at Perranporth, Cornwall, after being generated in the region of the Falkland Islands. The fluctuations observed experimentally have been shown in figure 1.

Figure 3 is a refraction diagram for such waves as they cross the continental shelf, and the full line represents the path of a wave group arriving at Perranporth. Figure 4 shows the velocity of the tidal streams at various positions on this path throughout the tidal cycle, the value used being the component of the stream velocity parallel to the line of the path; these values have been estimated from the Admiralty Atlas of tidal streams. Most of the curves of figure 4 are approximately sinusoidal, and irregularities are probably due to errors in interpolating from the Atlas. They are more complicated in the region of Perranporth, for the tidal streams on the north Cornish coast appear to show a double maximum flood stream. Figure 5 shows the mean depth of water at points along the path, values of the tidal range in depth and the time interval by which high water at various points along the path precedes high water at Dover; this information is obtained from the Admiralty Atlas of tides, and high water at Dover is used as a reference time so as to conform with usual practice.

The data of figures 4 and 5 were used in the integration of equation (10) to obtain the total change in apparent period which the waves would show at Perranporth; only the first two terms on the right-hand side of this equation were considered. The values of the quantity $\partial u/dt$ were obtained for points at intervals of 10 miles along the path by taking differences between successive hourly values of the stream velocities in figure 4. These values were written in array upon a diagram whose axes were the distance from Perranporth and the time relative to high water at Dover. On this diagram straight lines were drawn to represent the

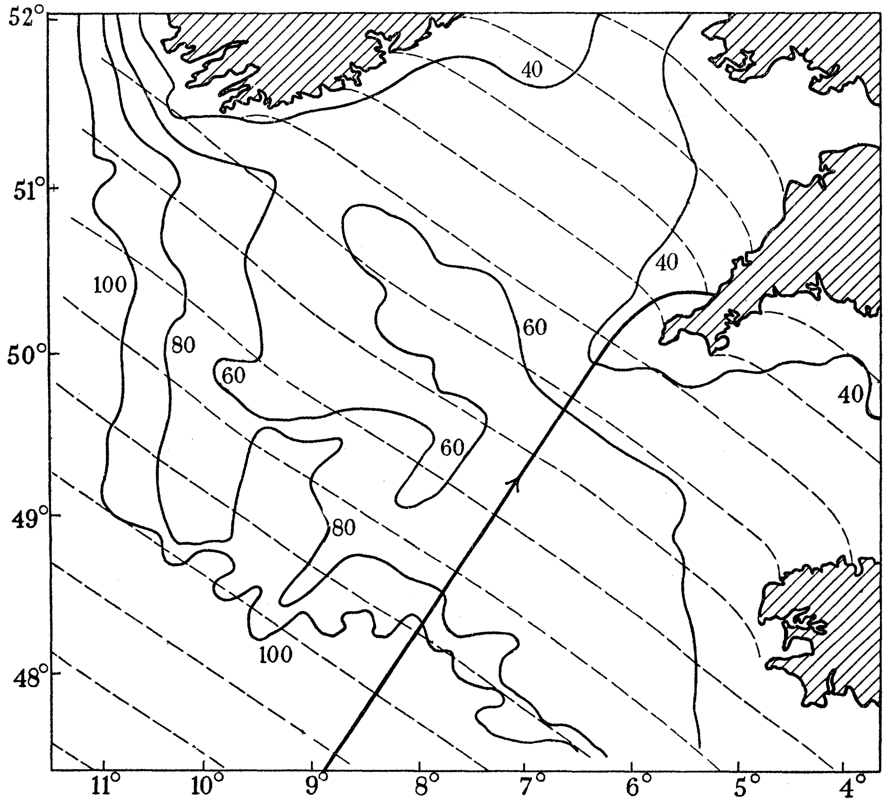
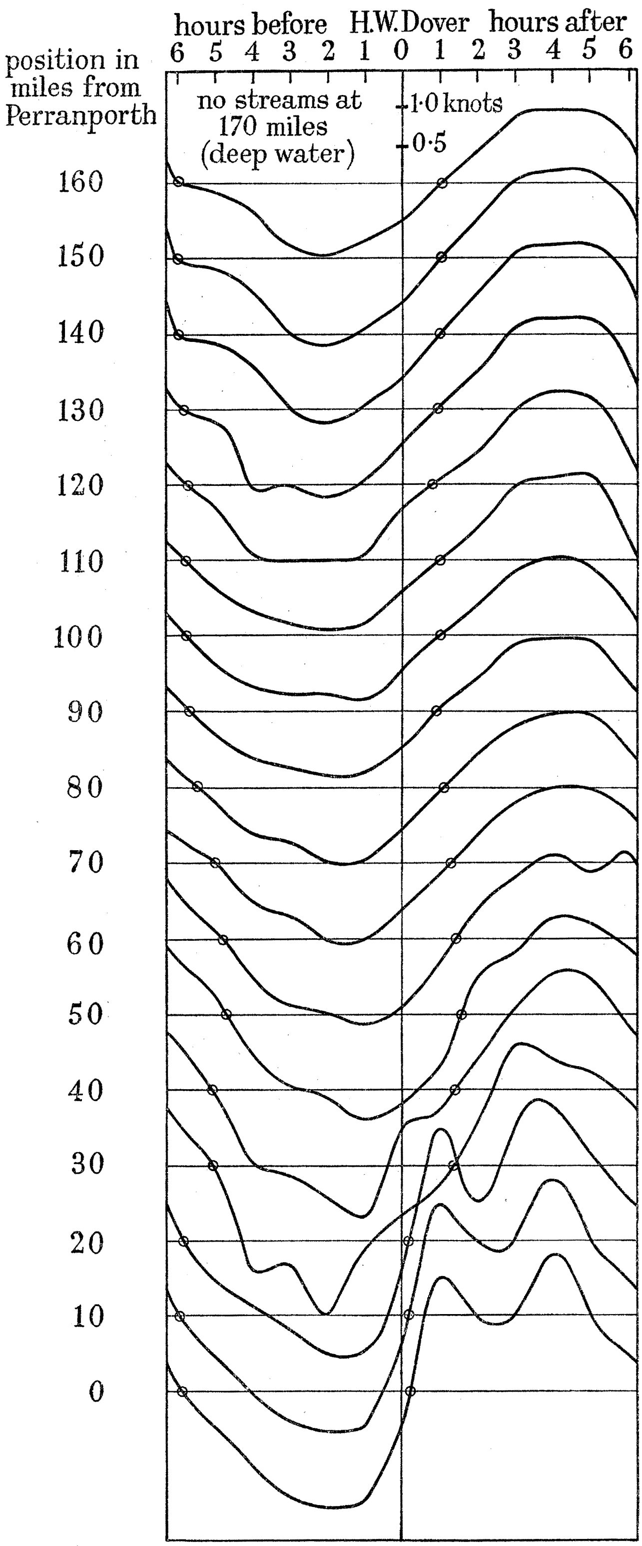


FIGURE 3. Refraction diagram for waves crossing the continental shelf from the direction 215°. Depth contours in fathoms. Broken lines show every 100th wave crest. Full line shows the path of a group arriving at Perranporth.

progress of a wave group arriving at Perranporth at various hours before and after the time of high water at Dover, the gradient of the lines being appropriate to a uniform velocity of the group equal to 32 knots. Some approximation is involved here, since the different depths of water along the path lead to theoretical values of the group velocity ranging from 33·5 to 30·0 knots, and the tidal streams themselves are of the order of a knot and affect the progress of the group over ground. These differences mean that the position of a wave group at hourly intervals would be somewhat different from the positions obtained by the assumption of a constant value for the group velocity, and the values of $\partial u/\partial t$ at these positions



Graphs against time, of the component of velocity of tidal stream (at springs) in the direction of wave progress from Falkland Islands, past Land's End to Perranporth. The curves are based on the Admiralty charts of tidal streams issued for hourly intervals relative to high water at Dover. The horizontal lines are at intervals of 1 knot on the vertical scale. The times at which the streams are zero are indicated by small circles.

FIGURE 4. Variations of the component of tidal stream at various points upon the path in figure 3. The velocity plotted is the component of stream velocity along the path.

would differ in consequence; it was considered, however, that assumption of a constant group velocity of 32 knots would not lead to any serious error. From the diagram it was possible to infer the values of $\partial u/\partial t$ which the groups would experience at equal intervals during their progress and to obtain, by addition, the

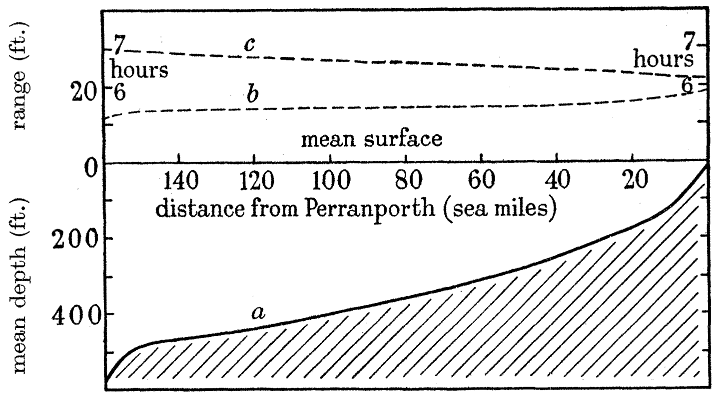


FIGURE 5. Mean depth of water at various points along the path shown in figure 3. *a*, mean depth of water; *b*, range of tide at springs; *c*, hours by which high tide precedes the time of high tide at Dover.

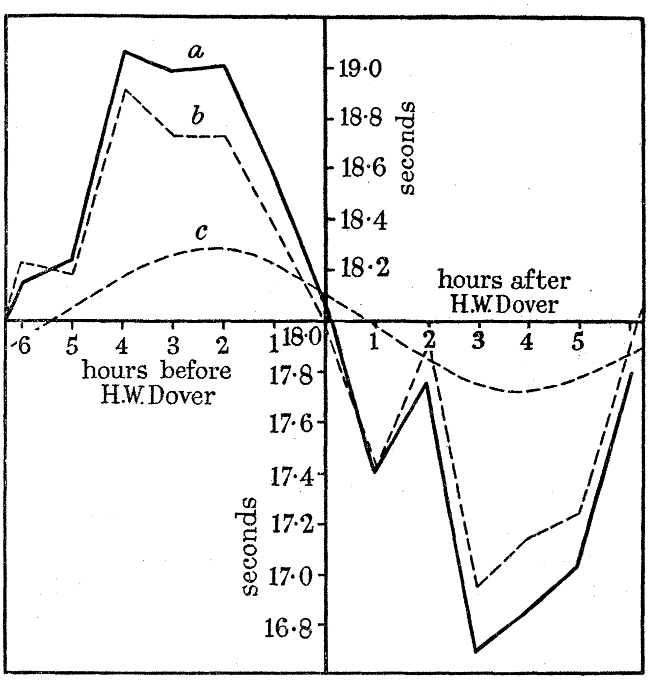


FIGURE 6. Estimated values of apparent period of waves originally of 18 sec. period which travel across the continental shelf in a direction 035° as in figure 3, and arrive at Perranporth at various tidal times. *a*, total value; *b*, effect of tidal stream velocities; *c*, effect of changing water depth in tides.

integrated value of the first term in equation (10). The changes in apparent period, due to the term $\partial u/\partial t$, which the various groups would exhibit on arrival at Perranporth are shown by the broken line b in figure 6. It seems likely that much of the erratic nature of this curve is due to errors in interpolation in reading from the Atlas. The large fluctuation at 2 hr. after high water at Dover may be real however, since it is associated with the double flood stream near the north Cornish coast.

The integral of the second term in equation (10) was obtained somewhat differently. The maximum value of the quantity $\partial h/\partial t$ at various positions on the path was obtained from the tidal ranges shown in figure 4 on the assumption that the rise and fall of water-level was sinusoidal with tidal period, and the values of the coefficient $\frac{1}{c}\left(\frac{\partial c}{\partial h}\right)_{\lambda g}$ were evaluated for the various positions on the path using the mean water depths shown in figure 5 and assuming a value of 18 sec. for the wave period. The quantities were then compounded in a vector diagram, the amplitudes of the vector elements being the maximum values of the quantity

$$\frac{1}{c}\left(\frac{\partial c}{\partial h}\right)_{\lambda g}\frac{\partial h}{\partial t}$$

obtaining at the various positions on the path, and the angles of the vector elements being the hour angle by which a wave group arriving at Perranporth at the time of high water at Dover would pass each of the positions prior to the time of maximum $\partial h/\partial t$ at that position. The summation of these elements in the vector diagram showed that the greatest increase in apparent period due to this cause would occur in waves arriving at Perranporth $2\frac{1}{4}$ hr. before high water at Dover, and it would amount to an increase of 0·28 sec.; for the remainder of the tidal cycle it is indicated by the sinusoidal broken curve c in figure 5.

The sum of the two calculated changes is shown as the full line a in figure 6. Disregarding certain irregularities in the curve it appears that the overall fluctuation in apparent period should be about 2·3 sec., and the greatest period should be shown by waves arriving at Perranporth about 3 hr. before the time of high water at Dover. This is in fair agreement with the fluctuations in the apparent period evident in figure 1.

(*b*) *Swell travelling from the west*

Calculations similar to those above have been made to find the change in apparent period that might be expected in swell arriving at Perranporth after having crossed the continental shelf from the west. The calculated curves are shown in figure 7; these suggest that the greatest increase in period would be about 0·6 sec. and would be shown by swell arriving at Perranporth about half an hour before the time of high water at Dover. The swell generated in storms in the North Atlantic does not usually show such regular and well-defined fluctuations in period as does swell coming from the Southern Ocean, and it can only be said that the calculated changes shown in figure 7 are not in violent disagreement with the observations.

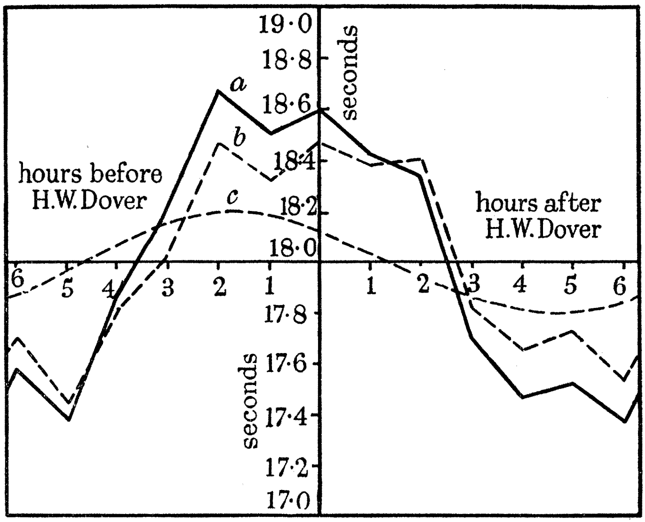


FIGURE 7. Estimated values of apparent period of waves originally of 18 sec. period which travel across the continental shelf in a direction 090° and arrive at Perranporth at various tidal times. *a*, total value; *b*, effect of tidal streams; *c*, effect of changing water depth in tides.

CONCLUSIONS

Fluctuations in tidal cycles of the period of waves recorded by a stationary instrument can satisfactorily be attributed to the action of tidal streams. In deep water the average length of waves appears to expand or contract at the same rate as the general surface of water on which they are moving. The change in period will be proportionately small when the tidal streams are weak or where the waves complete their passage through the tidal area in a small fraction of a tidal cycle.

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