Doppler Frequency Estimation and the Cramér-Rao Bound

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Abstract—This paper addresses the problem of Doppler frequency estimation in the presence of speckle and receiver noise. An ultimate accuracy bound for Doppler frequency estimation is derived from the Cramér-Rao inequality. It is shown that estimates based on the correlation of the signal power spectra with an arbitrary weighting function are approximately Gaussian distributed. Their variance is derived in terms of the weighting function. It is shown that a special case of a correlation-based estimator is a maximum-likelihood estimator that reaches the Cramér-Rao bound.

These general results are applied to the problem of Doppler centroid estimation from SAR data. Different estimators known from the literature are investigated with respect to their accuracy. Numerical examples are presented and compared with experimental results.

Keywords—Doppler frequency estimation, Cramér-Rao bound, Synthetic Aperture Radar, Doppler centroid estimation.

I. INTRODUCTION

ESTIMATION of Doppler frequency shifts is a fundamental operation in radar data processing: Weather radar and moving-target indication radar exploit the Doppler shift of each radar return to measure the velocity of scatterers. In Synthetic Aperture Radar (SAR) the Doppler history of a target, while traversing the beam, is used to focus the data in the azimuth direction. Here, the mean Doppler frequency, the *Doppler centroid*, is a measure of the effective antenna squint angle. It is used to adjust the bandpass characteristic of the azimuth compression filter to the location of the signal spectrum. An inaccurate Doppler centroid not only affects resolution and the signal-to-noise ratio, but also allows aliased azimuth frequency components to fall within the passband of the compression filter, and thus reduces the signal-to-ambiguity ratio [1], [2].

If only isolated point scatterers are considered, Doppler estimation means finding the frequency shift of an *a priori* known signal corrupted by additive receiver noise. This leads to the concept of matched filtering and ambiguity function. Ultimate bounds of the accuracy of such estimators have been derived in [3], [4].

For weather radar and SAR, the assumption of point targets is not applicable. An extended target, e.g., a surface much larger than a resolution cell with a high roughness compared to the wavelength, is a more appropriate model. The radar data from such a Rayleigh scattering target suffer from speckle noise, which must be considered when designing an optimum estimator.

This paper addresses the problem of Doppler estimation under the assumption of both additive (thermal) and multiplicative (speckle) noises. The organization is as follows: In Section II the assumed statistical properties of the signals under consideration are given. In Section III the ultimate accuracy bound of Doppler frequency estimation based on the Cramér-Rao inequality is presented. In Section IV the accuracy of Doppler frequency estimators using correlation is derived. Section V presents the optimum estimator; i.e., one that meets the Cramér-Rao bound. In Section VI the general mathematical results of the preceding sections are applied to Doppler centroid estimation from SAR data. Several known estimators are compared with respect to the Cramér-Rao bound. Numerical examples are given. In Section VII the theoretical findings are compared with experimental results, and Section VIII supplies concluding remarks.

II. MATHEMATICAL PRELIMINARIES

Assume that the radar data are coherently demodulated and sampled at a period of Δt to form the complex vector:

$$u = u_s + u_n = (u[1], \cdots, u[k], \cdots, u[N]) \quad (1)$$

where u_s represents the radar signal; and u_n, the receiver noise. In the cases of weather radar and moving target indication, u may be the data from one return. For SAR, u represents data in the azimuth direction.

Throughout the paper, extended targets of uniform backscatter are assumed. Thus signal and noise are assumed to be complex, Gaussian, zero-mean, and stationary processes orthogonal to each other. The assumption of uniform backscatter does not mean any loss in generality, since it can be shown [5], [6] that all the results of this paper can be easily adapted to the case of nonhomogeneous targets if the number of samples N is multiplied by a contrast dependent factor:

$$\frac{\langle I \rangle^2}{\langle I^2 \rangle} \le 1 \quad (2)$$

where I is the radar image pixel intensity; and "$\langle \ \rangle$" means the spatial or temporal average.

The information contained in U is fully present in its discrete Fourier transform:

$$U = U_s + U_n = (U[1], \cdots, U[i], \cdots, U[N]). \quad (3)$$

Since Gaussian processes are invariant against Fourier transform, the signal spectrum U_s and the noise spectrum U_n are, again, complex Gaussian, zero-mean, and orthogonal processes. Due to the assumed stationarity of U, the spectral samples $U[i]$ are mutually uncorrelated.

It is important to understand that the *phase* of U carries no information about the Doppler frequency, since U has been modeled as a stochastic process with the aforementioned properties. Hence it is sufficient to consider the power spectrum:

$$S = (S[1], \cdots, S[i], \cdots, S[N])$$
$$= (|U[1]|^2, \cdots, |U[i]|^2, \cdots, |U[N]|^2) \quad (4)$$

as a starting point for Doppler frequency estimation. S has the following properties:

$$E\{S[i]\} \equiv E\{|U_s[i] + U_n[i]|^2\}$$
$$= E\{|U_s[i]|^2\} + E\{|U_n[i]|^2\} \quad (5)$$

$$E\{|U_s[i]|^2\} = A_s(i \cdot \Delta f - f_D) \quad (6)$$

$$E\{|U_n[i]|^2\} = A_n = \text{const} \quad (7)$$

with $E\{\cdot\}$ denoting the expectation value.

Equation (7) means that the thermal noise has been considered to be white with a power spectral density proportional to A_n. $A_s(f)$ in (6) is the *a priori* known power spectral density of the signal; Δf is the frequency sampling interval; and f_D is the Doppler frequency shift to be estimated. Combining (5) to (7) leads to:

$$E\{S[i]\} = A(i \cdot \Delta f - f_D) \quad (8)$$

with

$$A(f) = A_s(f) + A_n. \quad (9)$$

$A(f)$ will be referred to as the *nominal power spectrum*; it includes both signal and noise and is assumed to be periodic with period $1/\Delta t$.

Since U is a complex Gaussian process, the probability density function of each sample $S[i]$ under the condition of a particular Doppler frequency f_D is given by the well-known exponential distribution (χ^2 distribution with two degrees of freedom) [7]:

$$p(S[i]; f_D) = \frac{1}{A(i \cdot \Delta f - f_D)}$$
$$\cdot \exp\left\{-\frac{S[i]}{A(i \cdot \Delta f - f_D)}\right\}. \quad (10)$$

Obviously, the power spectrum S exhibits the typical speckle statistics. This implies that the variance of $S[i]$ is:

$$\text{var}\{S[i]\} = E\{(A(i \cdot \Delta f - f_D) - S[i])^2\}$$
$$= A(i \cdot \Delta f - f_D)^2. \quad (11)$$

Note that the contributions $A_s(f)$ and A_n from the signal and noise do not show up separately.

III. THE CRAMÉR–RAO BOUND FOR DOPPLER FREQUENCY ESTIMATION

Suppose that an estimator operates on the power spectrum S of the data as defined in (4) and shall exploit all the *a priori* knowledge about the nominal power spectrum $A(f)$ and speckle statistics given by (10). Then the fundamental Cramér–Rao inequality from estimation theory [8]–[10] states that the variance of any (unbiased) Doppler frequency estimate ϕ will be lower bounded by:

$$\text{var}\{\phi\} \geq \frac{1}{E\left\{\left(\frac{\partial \ln p(S; f_D)}{\partial f_D}\right)^2\right\}}. \quad (12)$$

Here, $p(S; f_D)$ is the joint probability for all values $S[i]$ under the condition of f_D. Several authors have interpreted this fundamental limit in the context of Doppler or time delay estimation [3], [11], [12]. Translated to the notations of this paper, their results say that the Cramér–Rao bound for a Doppler frequency estimate ϕ from N data samples is:

$$\text{var}\{\phi\} \geq \frac{\Delta f}{\int \left[\frac{A'(f)}{A(f)}\right]^2 df} = \frac{\Delta f}{\int \left[\frac{A'(f)}{A_s(f) + A_n}\right]^2 df}. \quad (13)$$

(Unless otherwise stated, all integrations extend over one spectral period $1/\Delta t$.)

Trivially, var $\{\phi\}$ highly depends on the shape of $A(f)$. Equation (13) gives a design criterion for $A(f)$. All the spectral regions where the derivative $A'(f)$ is high and $A(f)$, which is proportional to the speckle noise, is low contribute most to the accuracy of the estimator.

IV. CORRELATION-BASED DOPPLER FREQUENCY ESTIMATORS

It is a common strategy of Doppler centroid estimators for SAR data [2], [5], [6], [13]–[15] to correlate the power spectrum S with some weighting function $B(f)$ centred at a trial Doppler centroid value ϕ:

$$D(\phi) = \sum_{i=1}^{N} S[i] \cdot B(i \cdot \Delta f - \phi). \quad (14)$$

The value of ϕ with:

$$D(\phi) = 0 \quad (15)$$

is taken as the Doppler centroid estimate. Such an estimator may be implemented either directly in the frequency domain or by comparing the energies of different looks in the time domain. Both approaches are equivalent to each other from the point of signal theory (Parseval's theorem). Equation (14) implies that the design of such a correlation based Doppler frequency estimator is fully determined by the choice of the weighting function $B(f)$.

In this section a mathematical relationship between the shape of $B(f)$ and the estimation accuracy is derived. This result is compared with the Cramér–Rao bound from (13).

The function $D(\phi)$ of (14) is the weighted sum of N uncorrelated data $S[i]$. Hence $D(\phi)$ is a stochastic process with approximate Gaussian distribution, no matter how the samples $S[i]$ were originally distributed (central limit theorem). Its expectation is:

$$E\{D(\phi)\} = \sum_{i=1}^{N} A(i \cdot \Delta f - f_D) \cdot B(i \cdot \Delta f - \phi)$$

$$\simeq \frac{1}{\Delta f} \cdot \int A(f - f_D) \cdot B(f - \phi) \, df. \quad (16)$$

For an unbiased estimator, $B(f)$ must be chosen so that $E\{D(f_D)\} = 0$.

Near $\phi = f_D$ the function $E\{D(\phi)\}$ can be expanded into a Taylor series:

$$E\{D(\phi)\} = k \cdot (\phi - f_D) + \text{higher order terms} \quad (17)$$

with

$$k = \frac{dE\{D(\phi)\}}{d\phi}\bigg|_{\phi = f_D} \simeq \frac{-1}{\Delta f} \cdot \int A(f) \cdot B'(f) \, df$$

$$\simeq \frac{1}{\Delta f} \cdot \int A'(f) \cdot B(f) \, df. \quad (18)$$

The last expression of (18) holds, since both $A(f)$ and $B(f)$ are periodic with a period of $1/\Delta t$; i.e., $A(-1/(2 \cdot \Delta t)) = A(1/(2 \cdot \Delta t))$ and $B(-1/(2 \cdot \Delta t)) = B(1/(2 \cdot \Delta t))$.

With (11) the variance of $D(\phi)$ becomes:

$$\text{var}\{D(\phi)\} = \sum_{i=1}^{N} A(i \cdot \Delta f - f_D)^2 \cdot B(i \cdot \Delta f - \phi)^2$$

$$\simeq \frac{1}{\Delta f} \cdot \int [A(f - f_D) \cdot B(f - \phi)]^2 \, df. \quad (19)$$

Then

$$\text{var}\{\phi\} = \frac{\text{var}\{D(f_D)\}}{k^2} \quad (20)$$

i.e., the variance of the estimate ϕ is finally found to be (see also [16]):

$$\text{var}\{\phi\} = \Delta f \cdot \frac{\int [A(f) \cdot B(f)]^2 \, df}{\left[\int A(f) \cdot B'(f) \, df\right]^2} \quad (21)$$

or

$$\text{var}\{\phi\} = \Delta f \cdot \frac{\int [A(f) \cdot B(f)]^2 \, df}{\left[\int A'(f) \cdot B(f) \, df\right]^2}.$$

The approximation of (20) is sufficient if the higher order terms in (17) can be neglected for $|\phi - f_D| < 3 \cdot$

$\sqrt{\text{var}\{\phi\}}$. For the number N approaching infinity this is always true. Then ϕ is also Gaussian distributed.

Note that only the variance of the power spectrum as given by (11) (and not its particular probability density function) has been used to derive (21).

Equation (21) is an objective criterion for the optimization of the correlation kernel $B(f)$, provided that the nominal spectrum $A(f)$, i.e., both its signal part and noise part, is known. A "good" weighting function $B(f)$ is obviously one that averages over many samples of $S[i]$ to keep the variance of $D(\phi)$ low, and, on the other hand, allows for a high discrimination power of $D(\phi)$ by generating a large slope k of the correlation result.

V. The Optimum Estimator

It is known that maximum-likelihood estimators reach the Cramér–Rao bound; i.e., the optimum Doppler frequency estimator is one that maximizes the *a posteriori* likelihood:

$$p(S; \phi) = \prod_{i=1}^{N} \frac{1}{A(i \cdot \Delta f - \phi)} \cdot \exp\left\{-\frac{S[i]}{A(i \cdot \Delta f - \phi)}\right\}. \quad (22)$$

It can be easily shown [11] that the (optimum) maximum-likelihood Doppler frequency estimator is a correlation-based estimator as defined by (14) using as a weighting function:

$$B(f) = -\frac{d}{df}\left(\frac{1}{A(f)}\right) = \frac{A'(f)}{A(f)^2} = \frac{A_s'(f)}{[A_s(f) + A_n]^2}. \quad (23)$$

Inserting this particular function $B(f)$ into (21) indeed leads to the Cramér–Rao bound of (13).

The optimum weighting function of (23) has been used in [6], [15] for Doppler centroid estimation, without considering the additive noise term A_n, however.

A closer look at (23) might be of interest. The maximum-likelihood estimator obviously searches the zero of the correlation between the measured spectrum and the derivative of $1/A(f)$. This is equivalent to the minimization of the correlation of the measured spectrum with the *reciprocal* of the nominal spectrum. Correlating with $1/A(f)$ means that spectral regions with low energy, and thus with low speckle noise, contribute more than the areas of S with high energy. This is an intuitive explanation for the superior performance of this estimator in the presence of *multiplicative* noise.

Often, Doppler frequency estimation is performed on power spectra obtained by incoherently averaging the spectra of several uncorrelated signals. The results derived so far are also applicable for this case if the number N is replaced by the *overall number* of samples contributing to the estimate. Denoting the number of averaged spectra by M, this means that either N is substituted by $N \cdot M$ or Δf is replaced by $\Delta f/M$.

VI. Doppler Centroid Estimation from SAR Data

The results of the preceding sections will now be applied to the problem of estimating the Doppler centroid

from SAR data. It is assumed that a data block of N_r azimuth lines, of N_a complex samples each, is used for estimation. In this context, the applied mathematical notations obtain the following physical meanings: f_D = Doppler centroid; $\Delta t = 1/\text{PRF}$ (with PRF being the pulse-repetition frequency); $\Delta f = \text{PRF}/N$; $N = N_a \cdot N_r \cdot B_s/f_s$ (with f_s/B_s being the range oversampling factor); B_s = range bandwidth; f_s = range-sampling frequency; S = azimuth power spectrum of the SAR data; and $A_s(f)$ = nominal azimuth signal power spectral density, reflecting the aliased azimuth two-way antenna gain pattern.

Assuming an antenna of length L without beam shaping on a platform with a velocity v, the nominal azimuth signal power spectrum can be modeled as:

$$A_s(f) \sim \sum_{n=-\infty}^{+\infty} \text{sinc} \left((f - n \cdot \text{PRF})/f_0 \right)^4 \qquad (24)$$

with

$$f_0 = \frac{2 \cdot v}{L}. \qquad (25)$$

For sensors like Seasat, ERS-1, or SIR-C/X-SAR, the PRF's cover the range of:

$$0.9 \cdot f_0 < \text{PRF} < 1.5 \cdot f_0.$$

For these values, (23) can be well approximated by its first-order Fourier series [17]:

$$A_s(f) \sim a_0 + a_1 \cdot \cos (2\pi f/\text{PRF}). \qquad (26)$$

Any noise term A_n only contributes to a_0. Thus $A(f)$ can be assumed to be of the following form:

$$A(f) = 1 + m \cdot \cos (2\pi f/\text{PRF}). \qquad (27)$$

The factor m depends on both the additive noise level and degree of aliasing. In the numerical examples of this section,

$$m = 0.7 \qquad (28)$$

is assumed. Fig. 1 shows this particular nominal spectrum.

In the following, four different Doppler centroid estimators, i.e., different weighting functions $B(f)$, will be compared.

A. Energy Balancing [2], [14]

Here the spectrum S is "cut" into two parts at the Doppler centroid estimate ϕ. The estimate is accepted if the sums of the spectral values for $i < \phi/\Delta f$ and $i > \phi/\Delta f$ are equal; i.e., their difference is zero. Thus:

$$B(f) = \begin{cases} 1, & \text{for } - \text{PBW}/2 < f < 0 \\ -1, & \text{for } 0 < f < \text{PBW}/2 \\ 0, & \text{otherwise} \end{cases} \qquad (29)$$

and

$$B'(f) = \delta(f + \text{PBW}/2) - 2 \cdot \delta(f) + \delta(f - \text{PBW}/2) \qquad (30)$$

where $\text{PBW} \leq \text{PRF}$ is the processing bandwidth (see Fig. 2(a)).
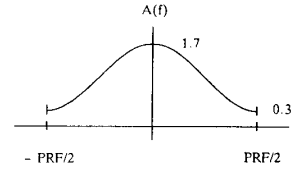

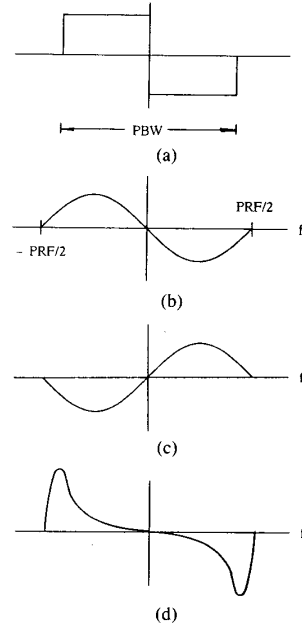Fig. 1. Nominal azimuth power spectrum assumed for numerical examples.

Fig. 2. Weighting functions $B(f)$ of different Doppler centroid estimators.

Application of (21) yields (see also [5], [6], [15]):

$$\text{var} \{\phi\} = \frac{\text{PRF}}{N} \cdot \frac{\int_{-\text{PBW}/2}^{+\text{PBW}/2} A(f)^2 \, df}{4 \cdot [A(0) - A(\text{PBW})/2]^2}. \qquad (31)$$

For the nominal spectrum of (27) the variance is lowest for $\text{PBW} = \text{PRF}$:

$$\text{var} \{\phi\} = \frac{\text{PRF}^2}{N} \cdot \frac{1}{16} \cdot \left(\frac{1}{m^2} + \frac{1}{2} \right). \qquad (32)$$

With $m = 0.7$, the standard deviation of the energy balancing estimator is:

$$\text{SD} \{\phi\} = 0.3985 \cdot \frac{\text{PRF}}{\sqrt{N}}. \qquad (33)$$

B. Correlation with the Nominal Spectrum

Energy balancing does not incorporate any information about $A(f)$ in $B(f)$ and thus might be intuitively considered to be only suboptimum.

From detection theory it is known that correlation with the nominal function followed by a maximum detection is optimum for signals buried in additive white Gaussian noise. For computational reasons it is more convenient to

correlate with the first derivative of the nominal function and look for a zero. Applying this approach to Doppler centroid estimation means (Fig. 2(b)):

$$B(f) = A'(f). \tag{34}$$

Then (21) becomes:

$$\text{var } \{\phi\} = \frac{\text{PRF}}{N} \cdot \frac{\int [A(f) \cdot A'(f)]^2 \, df}{\left[\int A'(f)^2 \, df\right]^2}. \tag{35}$$

For the particular spectrum $A(f)$ of (27), correlation with $A'(f)$ yields:

$$\text{var } \{\phi\} = \frac{\text{PRF}^2}{N} \cdot \frac{1}{2 \cdot \pi^2} \cdot \left(\frac{1}{m^2} + \frac{1}{4}\right). \tag{36}$$

For $m = 0.7$ a standard deviation of:

$$\text{SD } \{\phi\} = 0.3407 \cdot \frac{\text{PRF}}{\sqrt{N}} \tag{37}$$

is expected, which is better than the one obtained with energy balancing. This result is still not optimum because of the multiplicative character of speckle noise.

C. Correlation Doppler Centroid Estimator [5], [12], [16], [18], [19]

This method uses the Fourier relationship between the power spectrum S and autocorrelation function s of the data. The phase gradient of s around zero lag is proportional to the frequency location of the centroid S. This gradient may be estimated by calculating the phases of $s[1], s[2], \cdots$. For most SAR data only the value $s[1]$, i.e., for lag $= 1/\text{PRF}$, is of a sufficiently high signal-to-noise ratio. From its phase, the Doppler centroid is estimated in the following way:

$$\phi = \frac{\arg \{s[1]\}}{2\pi} \cdot \text{PRF} \tag{38}$$

where

$$s[1] = E\{u^*[k] \cdot u[k + 1]\} \tag{39}$$

and $u[k]$ being the azimuth raw data.

From

$$s[1] = \sum_{i=1}^{N} S[i] \cdot \exp \left\{ j2\pi \left(\frac{i \cdot \Delta f - \phi}{\text{PRF}}\right)\right\}$$
$$\cdot \exp \{j2\pi\phi/\text{PRF}\} \tag{40}$$

and

$$\arg [s[1]] = \arctan \left[\frac{\sum S[i] \cdot \sin \left(2\pi \dfrac{i \cdot \Delta f - \phi}{\text{PRF}}\right)}{\sum S[i] \cdot \cos \left(2\pi \dfrac{i \cdot \Delta f - \phi}{\text{PRF}}\right)}\right]$$
$$+ \frac{2\pi\phi}{\text{PRF}} \tag{41}$$

it follows that this Doppler centroid estimator tries to find that value of ϕ, where the inner product of S and a sine function is zero. Hence this correlation Doppler centroid

estimator has the same performance as one with a weighting function of the form:

$$B(f) = \sin (2\pi f/\text{PRF}) \tag{42}$$

i.e., for nominal spectra like the one in (27), this estimator is identical to a correlation with $A'(f)$. Therefore:

$$\text{SD } \{\phi\} = 0.3407 \cdot \frac{\text{PRF}}{\sqrt{N}} \tag{43}$$

also for this estimator.

Note that the term "correlation Doppler centroid estimator" has been adopted from [5], [19] and should not be confused with the class of "correlation-based estimators" as defined in Section IV.

D. Optimum Estimator [6], [11], [15]

According to (23), the optimum weighting function is:

$$B(f) = \frac{A'(f)}{A(f)^2}. \tag{44}$$

With this kernel the Cramér-Rao bound for Doppler centroid estimation is reached:

$$\text{var } \{\phi\} = \frac{\text{PRF}}{N} \cdot \frac{1}{\int \left[\dfrac{A'(f)}{A(f)}\right]^2 \, df}. \tag{45}$$

For the particular spectrum of (27) and (28), the standard deviation of the optimum estimator can be found by numerical integration:

$$\text{SD } \{\phi\} = 0.2516 \cdot \frac{\text{PRF}}{\sqrt{N}}. \tag{46}$$

Besides the choice of the weighting function $B(f)$, several other points have to be considered when constructing an accurate Doppler centroid estimator:

• In (2), it has already been noted that scene contrast reduces the effective number of samples N by the factor $\langle I \rangle^2 / \langle I^2 \rangle$. This problem can be coped with to a certain extent by estimating f_D separately from samples of different intensities and appropriately averaging the results, as described in [6], [15].

• High scene contrast also causes another nuisance: The data block that is used for the estimation of f_D contains not only the whole azimuth chirps, but also the high-frequency or low-frequency sections of partially covered chirps. If these belong to strong scatterers, the shape of the azimuth spectrum may be cruelly distorted. An azimuth compression prior to the selection of the estimation area avoids this problem. This compression in turn presumes an accurate value for f_D in order not to bias the estimate. Therefore a Doppler centroid estimator using focused data must work iteratively.

• If the data are precompressed for estimation, a residual distortion of the azimuth spectrum may still be caused by aliased frequency components; i.e., by azimuth ambiguities of strong scatterers, located one synthetic aperture away from the estimation region.

VII. EXAMPLES

In this section the above theoretical findings shall be compared to experimental results obtained from a Seasat scene of a uniform ocean surface. The accuracy numbers of the energy-balancing estimator and maximum-likelihood estimator have been adopted from [6], [15], and a correlation Doppler centroid estimator has been implemented by the author. In both cases, estimates have been obtained based on data blocks of 64 complex range bins by 4096 azimuth samples each. These estimates have been fitted to a linear function in range direction. The deviation of each estimate from this function has been taken as the estimation error. Since Seasat data are oversampled by a factor of 1.198 in range, the effective number of samples contributing to each estimate is $N = 218818$. In Table I, the standard deviations predicted from (33), (43), and (46) are listed, together with the experimentally obtained results.

VIII. CONCLUDING REMARKS

In this paper the problem of estimating a frequency shift of power spectra of *a priori* known shape in the presence of speckle and additive noise has been addressed. Extended targets have been assumed. An ultimate accuracy bound has been given and an optimum estimator has been formulated. It should be noted that this optimality refers not only to the known correlation-based estimators, but to any other estimator without additional *a priori* knowledge.

By substituting frequency with time, the results apply also to the case of time delay estimation; e.g., range measurement of extended targets by altimeters.

The comparison of Doppler centroid estimators showed accuracies much higher than needed for normal SAR processing. Methods like attitude reconstruction or multiple PRF technique, however, benefit from an accuracy on the order of a few hertz.

The differences between the Doppler centroid estimators are not dramatic, however, in the case of the particularly assumed spectrum. Other practical considerations might influence the choice of a weighting function. For example, it may be considered inconvenient that the weighting function of the maximum-likelihood estimator depends on the signal-to-noise ratio (see (33)), which has to be estimated itself.

Another point of concern is that nominal spectra like the one in (27) contain aliasing, especially near the minimum. Thus high image contrast tends to distort the minimum, rather than the maximum. The maximum-likelihood estimator may no longer be optimum in such cases, because the spectral values near that minimum contribute strongly.

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TABLE I
COMPARISON OF PREDICTED AND MEASURED ACCURACIES OF DIFFERENT DOPPLER CENTROID ESTIMATORS

Estimator	Standard Deviations	
	Predicted (Hz)	Measured (Hz)
Energy balancing	1.40	1.5
Correlation Doppler centroid estimator	1.20	1.3
Maximum-likelihood estimator	0.89	1.0

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