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Cross-shore hydrodynamics within an unsaturated surf zone

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Abstract

This paper concerns the hydrodynamics induced by random waves incident on a steep beach. New experimental results are presented on surface elevation and kinematic probability density functions, cross-shore variation in wave heights, the fraction of broken waves and velocity moments. The surf zone is found to be unsaturated at incident wave frequencies, with a significant proportion of the incident wave energy remaining at the shoreline in the form of bores. Wave heights in both the outer and inner surf zones are best described by a full Rayleigh distribution [Thornton, E.B., Guza, R.T., 1983. Transformation of wave height distribution. J. Geophys. Res. 88, 5925–5938], rather than a truncated Rayleigh distribution as used by Battjes and Janssen (1978) [Battjes, J.A, Janssen, J.P., 1978. Energy loss and setup due to breaking of random waves. Proc. 16th Int. Conf. Coastal Eng. ASCE, New York, pp. 569–588]. A new parametric wave transformation model is outlined which provides explicit expressions for the fraction of broken waves and the energy dissipation rate within the surf zone. On steep beaches, the model appears to offer improved predictive capabilities over the original Battjes and Janssen model. Cross-shore variations in the velocity variance and velocity moments are best described using Linear Gaussian wave theory, with less than 20% of the velocity variance in the inner surf zone due to low frequency energy. © 1998 Elsevier Science B.V. All rights reserved.

Keywords: Random waves; Unsaturated surf zone; Breaking waves; Kinematics; Beaches

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1. Introduction

The surf zone is the nearshore region in which the relatively well-ordered irrotational motion of deep water waves is transformed through wave breaking into a range of different fluid motions. These may include turbulent bores, large scale vortices, low-frequency motions and mean cross-shore and longshore flows (Battjes, 1988). These processes govern the nature and rate of sediment transport in the nearshore region and therefore determine the morphological behaviour of beaches. The inner surf zone is of further importance because it governs the shoreline boundary conditions and therefore the hydrodynamics and sediment dynamics of the swash zone.

However, the majority of probabilistic or parametric wave transformation models assume that the inner surf zone is saturated (wave height or wave energy in the inner surf zone independent of the incident wave height or energy) and that the incident short wave energy is completely dissipated at the shoreline (e.g., Battjes and Janssen, 1978; Thornton and Guza, 1983; Dally et al., 1985). In contrast, on steep beaches the surf zone is frequently very narrow and there is insufficient time for all the incident short wave energy to be dissipated (Wright and Short, 1984; List, 1991). The inner surf zone is therefore unsaturated and short wave bores may reach the shoreline, driving swash motions at short wave frequencies (Hibberd and Peregrine, 1979; Kobayashi et al., 1989; Hughes, 1992). Consequently, saturated and unsaturated surf zones induce different hydrodynamic conditions at the shoreline (Battjes, 1974) and this has significant implications for the modelling of sediment dynamics, littoral currents and forces on coastal structures.

The present paper addresses this point and presents new experimental data on the cross-shore variations in the hydrodynamics within unsaturated surf zones. Section 2 reviews existing wave transformation models and previous data. In Section 3 a parametric wave transformation model based on the Battjes and Janssen (1978) approach is reformulated in order to determine the height of short wave bores incident on the shoreline. Section 4 describes the experimental instrumentation and measurement techniques. Section 5 presents a discussion of the experimental data and comparisons with numerical model predictions and linear wave theory, with final conclusions drawn in Section 6.

2. Previous work

Surf zones may be broadly divided into two types, depending on the characteristics of the incident waves and the beach gradient seaward of the shoreline. On mildly sloping beaches and with steep incident waves the surf zone is wide and the wave height in the inner surf zone is largely controlled by the local water depth. Any increase in the incident wave energy is dissipated through an increase in wave breaking and therefore the wave heights in the inner surf zone remain constant, i.e., the surf zone is saturated. Consequently, it is generally assumed that there is no wave group structure in a saturated inner surf zone (Symonds et al., 1982).

However, on steeper beaches, or with low steepness incident waves, wave breaking may occur much closer to or even at the shoreline as ‘shore breaks’ (Guza and

Thornton, 1982; Raubenheimer et al., 1996). As a result, an increase in the offshore incident wave height will result in an increase in the wave height in the inner surf zone, i.e., the surf zone is unsaturated and wave groupiness may still be apparent in the inner surf zone (Foote et al., 1992; Holmes et al., 1996). Indeed, wave grouping may increase shorewards on steep beaches (Kobayashi et al., 1989). This difference between saturated and unsaturated surf zones may be commonly observed on both macro- and micro-tidal beaches if there is a distinct change in the beach gradient around the low water mark.

However, most wave transformation models assume that in the inner surf zone the incident short waves are always depth limited and therefore that the surf zone is saturated. For example, monochromatic wave models assume that the wave height, H, is a linear function of the water depth, h, with the constant of proportionality, γ, taking a value of about 0.8–1.2 (e.g., Battjes, 1974). Random wave transformation models for more realistic natural sea states may be generally divided into two classes; probabilistic models and parametric models (Roelvink, 1993).

Probabilistic models (Mizuguchi, 1982; Mase and Iwagaki, 1982; Dally and Dean, 1986; Dally, 1992) subdivide an incident distribution of wave heights (and wave periods) into a number of discrete classes. These classes are then propagated shorewards independently using an appropriate monochromatic wave model and a new probability density function (p.d.f.) constructed at the required location. These methods are particularly useful if a detailed wave height distribution is required in the inner surf zone. However, the models are limited by the fact that both wave–wave interactions in the nearshore and slowly varying water level fluctuations must be minimal (Hamm et al., 1993).

Parametric models are generally based on the work of Battjes and Janssen (1978) and seek to describe the characteristics of the sea state in terms of local and time-averaged parameters. These are typically the wave energy (E) or root-mean-square wave height (H_{rms}), the peak spectral frequency (f_p), the local water depth (h, including wave set-up/set-down) and the fraction of broken waves (Q_b). Wave directionality may be included, although in the present paper discussion is restricted to waves normally incident on a beach. This paper also focuses on parametric models as these appear to be the most widely used in cross-shore morphological models due to their computational efficiency.

Parametric models determine wave transformation by applying an energy flux balance across the surf zone. As a result:

$$\frac{\partial\left(EC_g\right)}{\partial x} = -\langle \varepsilon \rangle \tag{1}$$

where C_g is the group velocity of the peak spectral frequency, x is the horizontal coordinate, positive onshore and $\langle \varepsilon \rangle$ is the time-averaged energy dissipation due to broken waves and frictional losses at the bed.

Note that here Eq. (1) does not include reflected wave energy. However, although very recent work by Baquerizo et al. (1997) showed that this could be included in a parametric model of this form, the solution requires prior knowledge of the cross-shore variation in wave height. Consequently, since the intention of the present work is to formulate a cross-shore wave transformation model for general conditions, where this information will be typically unavailable, reflected wave energy is not considered. Furthermore, even with a reflection coefficient of 30%, the ratio of reflected to incident wave energy is less than 10%, and therefore the errors introduced by neglecting reflected energy in Eq. (1) should be relatively small.

Given initial conditions in deep water, Eq. (1) is then integrated across the surf zone to obtain the wave energy in the nearshore. In order to determine the energy dissipation rate, a bore model is generally applied (LeMehaute, 1962), and the time-averaged rate of energy dissipation is found following Stoker (1957) and Battjes and Janssen (1978):

$$\langle \varepsilon \rangle = \frac{1}{4} \rho g f_{\mathrm{p}} B \frac{H^3}{h} Q_{\mathrm{b}} \tag{2}$$

where B is a constant of order one if the model is accurate and H is the height of either an individual wave or a representative broken wave. Q_{b} is determined by assuming a wave height p.d.f., within which some fraction of waves above a certain height are breaking. Following Battjes (1972), Battjes and Janssen (1978) assumed that the wave height p.d.f. could be modelled with a Rayleigh distribution truncated at a maximum limiting height, H_{m}, such that all breaking waves have a height equal to $H_{\mathrm{m}} = H_{\mathrm{b}}$. Q_{b} was then found from the implicit equation:

$$\frac{1 - Q_{\mathrm{b}}}{\ln Q_{\mathrm{b}}} = -\frac{H_{\mathrm{rms}}^2}{H_{\mathrm{m}}^2} \tag{3}$$

However, Battjes and Janssen (1978) found that close to the shoreline the model appeared to underestimate the energy dissipation and consequently predicted $H_{\mathrm{rms}} > H_{\mathrm{m}}$. The solution was therefore forced such that $H_{\mathrm{rms}} = H_{\mathrm{b}}$, i.e., a saturated inner surf zone was assumed. Thornton and Guza (1983) adopted a very similar approach but modelled the proportion of breaking waves using an empirical function based on field data. More recently, following Svendsen (1984), Lippmann et al. (1996) extended the Battjes and Janssen (1978) approach by including energy dissipation due to surface wave rollers and the shear stress at the wave/roller interface. Thornton and Guza (1983) additionally showed that the wave height distribution in the surf zone was best described by a standard Rayleigh distribution, rather than the truncated distribution used by Battjes (1972). Similar findings were also reported by Whitford (1988), although Klopman and Stive (1989) proposed an alternative description of the saturated nearshore wave heights based on a Weibull distribution.

Although parametric models have proved remarkably accurate in describing wave height transformations across saturated surf zones, there appear to be few comparisons of such models against wave height data from unsaturated surf zones. Indeed, initial comparisons of a numerical model based on the Battjes and Janssen (1978) approach with laboratory data from unsaturated surf zones showed that the inner surf zone wave heights were underestimated by up to 50% (see Section 5). Similar results were found by Cox et al. (1994) for a steep concave beach. Consequently, the Battjes and Janssen (1978) model has been reformulated based on a standard Rayleigh p.d.f. and abandoning the depth limitations on nearshore wave heights (see Section 3).

Alternatives to modelling cross-shore variations in wave heights by either probabilistic or parametric techniques are provided by time-domain models based on either the nonlinear shallow water wave equations or Boussinesq equations. These derive from the work of Abbott et al. (1978), Hibberd and Peregrine (1979) and Packwood (1980) and have been extended by a number of authors (e.g., Kobayashi et al., 1989; Madsen et al., 1991; Watson and Peregrine, 1992). These models are capable of accurately describing the evolution of both unbroken and broken waves across the surf zone (Raubenheimer et al., 1995, 1996). However, at present they remain fairly computationally expensive and have yet to be generally incorporated into morphological beach models. Nevertheless, such models can provide valuable physical insights into the complex hydrodynamic processes within the surf zone (see Hamm et al., 1993 for a review).

3. Model formulation

The model outlined here is a parametric model, reformulated from the approach of Battjes and Janssen (1978). It incorporates the observation by Thornton and Guza (1983) and Whitford (1988), and also found in the present experimental study (see Section 5), that the wave heights in the surf zone are best described by a Rayleigh distribution. This is given by:

$$p\left(\frac{H}{H_{\mathrm{rms}}}\right) = 2\frac{H}{H_{\mathrm{rms}}}\exp\left[-\left(\frac{H}{H_{\mathrm{rms}}}\right)^2\right] \tag{4}$$

and is assumed to be valid for both broken and unbroken waves (Fig. 1). However, the empirical breaking wave distribution used by Thornton and Guza (1983), obtained from field data in a saturated surf zone, may not be constant for different surf zone conditions and beach slopes. In the present paper a simpler model for wave breaking is proposed, which although probably less realistic, is widely used and does not assume prior knowledge of the surf zone conditions. Therefore, adopting the usual criterion that a wave is breaking when its height, H_{b}, exceeds some fraction of the water depth, the proportion of breaking waves is obtained directly from the Rayleigh distribution. Fig. 1

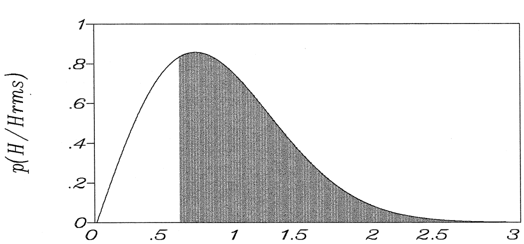


Fig. 1. Theoretical wave height and breaking wave height distributions. (————) Rayleigh distribution, (shaded area) $H/H_{\mathrm{rms}} \geq H_{\mathrm{b}}/H_{\mathrm{rms}}$ (breaking).

shows the resulting wave height distribution, including breaking waves, when $H/H_{rms} \geq H_b/H_{rms} = 0.6$.

The proportion of broken waves, Q_b, is found by integrating the Rayleigh distribution over all waves for which $H/H_{rms} \geq H_b/H_{rms}$:

$$Q_b = \int_{H*}^{\infty} p\left(\frac{H}{H_{rms}}\right) d\left(\frac{H}{H_{rms}}\right) \tag{5}$$

where $H* = H_b/H_{rms}$, resulting in an explicit expression for Q_b in terms of H_b and H_{rms}:

$$Q_b = \exp\left[-\left(\frac{H_b}{H_{rms}}\right)^2\right] \tag{6}$$

Therefore, if $H_b \gg H_{rms}$, $Q_b = 0$, and as $H_b/H_{rms} \to 0$, $Q_b \to 1$. Furthermore, if $H_b/H_{rms} = 1$, $Q_b \approx 0.4$, in contrast to the Battjes and Janssen (1978) model where all the waves are broken, i.e., $Q_b = 1$. However, if the wave heights in the surf zone are given by a Rayleigh distribution, then Eq. (6) is clearly more appropriate than Eq. (3). Indeed, the breaking wave distribution found by Thornton and Guza (1983) gives $Q_b \approx 0.5$ when $H_b = H_{rms}$, even for a saturated surf zone (their Fig. 11).

The total energy dissipation rate is found by multiplying the energy dissipation due to each broken wave, H, by the probability of that wave height occurring. Thus using Eq. (5), and following Battjes and Janssen (1978) by taking $H/h \approx 1$ in Eq. (2) gives:

$$\langle \varepsilon_b \rangle = A \int_{H*}^{\infty} H^2 p\left(\frac{H}{H_{rms}}\right) d\left(\frac{H}{H_{rms}}\right) \tag{7}$$

where A is given by:

$$A = \frac{1}{4} \rho g f_p B \tag{8}$$

Integrating Eq. (7) results in a simple explicit expression for $\langle \varepsilon_b \rangle$ in terms of H_b and H_{rms}:

$$\langle \varepsilon_b \rangle = A \exp\left[-\left(\frac{H_b}{H_{rms}}\right)^2\right]\left(H_b^2 + H_{rms}^2\right) \tag{9}$$

If $H_{rms} = H_b$, the usual assumption for a saturated surf zone, then:

$$\langle \varepsilon_b \rangle = 2A \frac{H_b^2}{e} \tag{10}$$

about 75% of the dissipation rate calculated by the Battjes and Janssen (1978) approach. Note that although bed friction is included in the model, which is based on the work of Nairn (1990) and Southgate and Nairn (1993), friction effects are minimal in the surf zone and have negligible influence on the model predictions. In very shallow water as:

$$H_b \to 0, \quad \langle \varepsilon_b \rangle \to A H_{rms}^2 \tag{11}$$

which gives the same energy dissipation rate as the Battjes and Janssen (1978) approach. The breaker height may be evaluated from any of the conventional expressions. Here, the expression developed by Battjes and Stive (1985) and modified by Nairn (1990) is used:

$$\frac{H_b}{h} = 0.39 + 0.56\tanh(33S_o) \tag{12}$$

where S_o is the offshore wave steepness. Note that this expression does not include the effect of the bed slope on wave breaking which was identified in a saturated surf zone by Raubenheimer et al. (1996). However, in an unsaturated surf zone, the offshore wave steepness is likely to be important and consequently Eq. (12) is chosen here. The wave height transformation across the surf zone is subsequently obtained from Eq. (1), including wave shoaling (Southgate and Nairn, 1993) and wave set-up/set-down (Longuet-Higgins and Stewart, 1964), and comparisons between the original Battjes and Janssen (1978) approach and the present formulation are presented in Section 5. The value of B in Eq. (8) is taken to be 1 throughout. In the present paper, the model is not calibrated for any particular beach slope and parameters previously found to apply over a range of beach slopes have generally been used, i.e., B in Eq. (8) and H_b in Eq. (12). However, alternative values for these two parameters may be appropriate if the surf zone conditions or breaker types differ very significantly from those for which they were originally proposed (e.g., Battjes and Janssen, 1978; Battjes and Stive, 1985; Nairn, 1990).

4. Experimental setup

4.1. Wave flume

The experiments were carried out in the large wave flume in the Civil Engineering Department at Imperial College. This flume is 50 m long, 3 m wide and was used with a working water depth of 0.9 m (Fig. 2). Waves were generated by a plane bottom-hinged hydraulically driven wave paddle which can generate both regular waves or random waves up to frequencies of 2 Hz, with simultaneous absorption of short waves reflected from the far end of the flume. Preliminary measurements showed that the wave motion generated by the paddle was highly repeatable, even in the inner surf zone, and no evidence of cross-tank oscillations or seiching was found. A slope (β) of gradient 0.05 starts 32 m from the paddle, rising to 0.1 in the surf and swash zones. The last 6 m of the flume are subdivided into three sections, providing two working sections of width 0.9 m and a central section for access to instrumentation. Perspex panels allow additional optical access within the swash zone.

4.2. Instrumentation

The vertical elevation of the water surface was measured using standard surface-piercing resistance-type wave gauges. The absolute accuracy of each gauge is of order

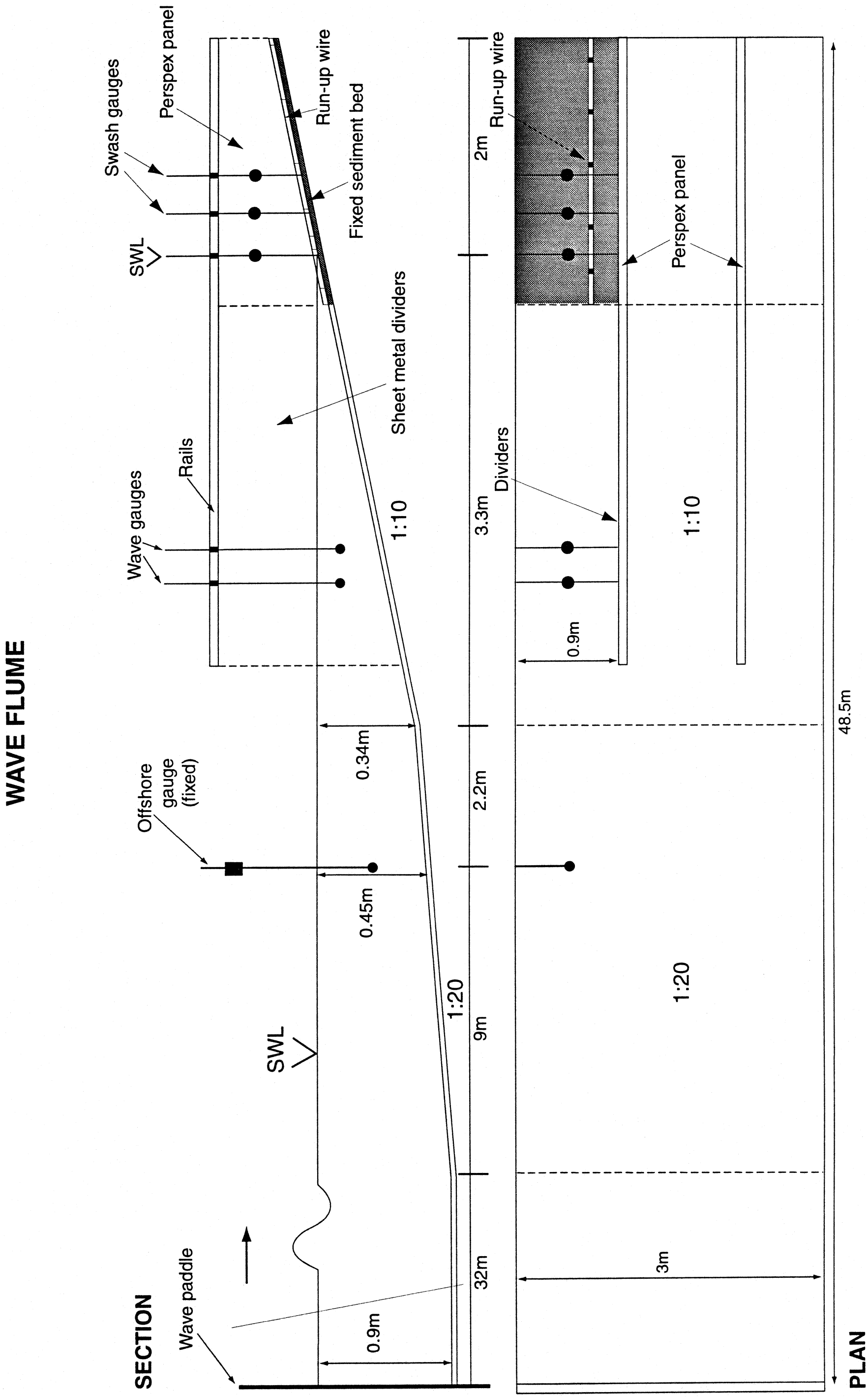


Fig. 2. Wave flume and instrumentation.

± 1 mm. For use within the inner surf zone and swash zone, the probes were modified to enable them to measure water depths ranging from zero to 150 mm. The output from the gauges was linear over their complete length, enabling accurate measurements of the water depth down to 2 mm. Five gauges were used to provide data across the surf zone, with a further gauge in a fixed offshore position. Taking the origin of the horizontal coordinate at the still water line (SWL), the offshore gauge was positioned at $x = 5.5$ m, in a water depth of 0.45 m.

Video analysis was used to determine the proportion of broken waves passing the wave gauges at different cross-shore locations. Water particle kinematics were measured with a 2-D sideways looking Acoustic Doppler Velocitimeter (ADV). Kinematic measurements were made 5 mm above the bed, outside the expected wave induced boundary layer flow region ($\delta \approx 1$–2 mm, Nielsen, 1992). Noise induced by bubbles from breaking waves was found to be small, even in very shallow water, allowing kinematic data to be obtained in water depths down to 5 mm. Wave reflections were estimated using linear long wave theory (Guza et al., 1984) with the ADV co-located with a wave gauge at $x = -1.8$ m. The reflection coefficient was estimated to be in the range 10–15%, although some uncertainties may be introduced due to the presence of wave grouping and nonlinear harmonics. Furthermore, very recent work has suggested that the ratio of reflected to incident wave energy may increase shoreward as the incident wave energy is dissipated through wave breaking (Baquerizo et al., 1997). The shoreline motion was measured with a run-up wire fixed 3 mm above the bed. These measurements have a relative accuracy of about 1 mm, but, due to the finite thickness of the component wires, the absolute accuracy was estimated to be of order 20 mm parallel to the bed. Further details may be found in Baldock et al. (1997).

4.3. *Wave generation*

The present investigation considers the hydrodynamics of three random wave simulations, based on Jonswap spectra with varying H_s and T_p, each record being 380 s in length. The wave heights shown in Table 1 are the measured root-mean-square wave heights at the offshore gauge, denoted by a subscript ‘o’, where H_{rmso} is taken as $\sqrt{(8m_o)}$, and will include any reflected wave energy. The surf similarity parameter (Iribarren and Nogales, 1949),

$$\zeta = \frac{\tan\beta}{\sqrt{H_{rmso}/L_o}} \tag{13}$$

where L_o is the wavelength at the offshore gauge, is typically in the range 0.5–0.7, suggesting a combination of spilling and plunging waves within the surf zone. These random wave simulations represent relatively steep wave conditions, $H_{rmso}/L_o \approx 0.03$, compared to $H_{rmso}/L_o \approx 0.01$ in typical field studies (e.g., Thornton and Guza, 1983; Lippmann et al., 1996). Consequently, locally forced low-frequency waves are expected to be more important than free incident long waves, which tend to be dominant in low steepness wave conditions (Huntley and Kim, 1984; Raubenheimer et al., 1995).

Table 1
Experimental wave conditions

Case	T_p (s)	H_{rmso}	H_{rmso}/L_o	Surf similarity, ζ
J1	1.5	90	0.033	0.55
J2	1.5	74	0.027	0.61
J3	1.0	46	0.031	0.57

Each random wave simulation was generated using a linear representation of the water surface at the wave paddle. Although this induces some spurious free long waves, previous work has shown that generating the waves in deep water reduces their magnitude to a minimum (Baldock et al., 1996). Furthermore, tests indicated that the phase relationship between the wave groups and the bound long waves was consistent with second-order wave theory (e.g., Longuet-Higgins and Stewart, 1964). This would not be the case if either shoreward or seaward propagating free long waves were present. Note that, because the same random wave simulations were run repeatedly to obtain the wave heights and kinematics at different cross-shore locations (each run uses the same control signal), each random wave spectrum may be considered deterministic, rather than a single representation of a random process. Consequently, the spectral plots presented later do not require the confidence limits associated with stochastic processes (Baldock et al., 1996). Spectra were computed via an FFT using 8192 data points sampled at 25 Hz, with smoothing over 20 adjacent frequency bins.

5. Discussion of results

5.1. Surface elevation and wave heights

Figs. 3 and 4 show the probability density functions (p.d.f.'s) of the water surface elevation at four locations across the surf zone for cases J1 and J3 (case J2 gives very similar results, not shown). In each case, the measured data are compared to normal (Gaussian) probability density functions with the same mean values and standard deviations. At all locations across the surf zone, the p.d.f.'s are close to Gaussian but, as expected, show a small positive skewness due to wave nonlinearity (Ochi and Wang, 1984). The skewness in the inner surf zone is typically in the range 0.8–1. However, it is interesting to note that at the SWL ($x = 0$), despite the cut-off in the data imposed by the bed, the data still fit reasonably well to the Gaussian distribution. The positive value of the skewness is therefore largely due to this physical constraint.

Wave heights were determined from the surface elevation data using the zero-up crossing method about the mean water surface elevation at each spatial location. Slightly different results might be obtained from a down-crossing analysis. Note that the data have not been high-pass filtered to exclude low frequency (surf beat) energy. This is because this energy is expected to be predominantly bound to the incident wave groups (there is little free incident long-wave energy in the random simulations) and therefore should be included in the wave-height analysis. Surf beat energy should only be

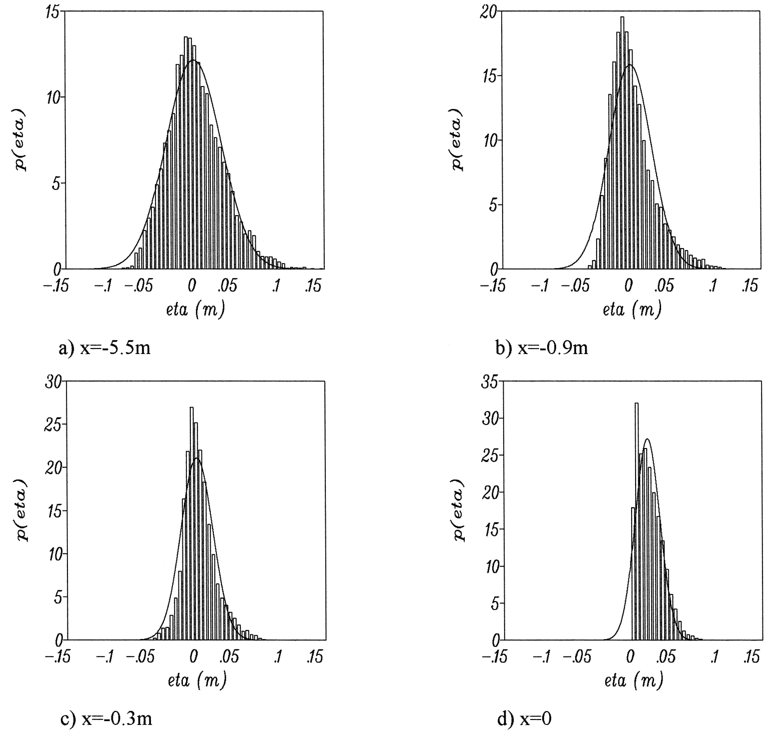


Fig. 3. P.d.f. of water surface elevation $(1/m)$, case J1. (Empty vertical box) Measured, (————) Gaussian distribution. (a) $x = -5.5$ m, (b) $x = -0.9$ m, (c) $x = -0.3$ m, (d) $x = 0$.

excluded if due to free long waves which change the effective water depth for the incident short waves (Thornton and Guza, 1983). The resulting measured wave-height probability density functions are compared to the Rayleigh p.d.f. (4) in Figs. 5 and 6. Also shown on these figures is the proportion of the p.d.fs. where $H/H_{rms} \geq H_b/H_{rms}$, with H_b calculated from Eq. (12), including wave set-up/set-down (Longuet-Higgins and Stewart, 1964), and using the H_{rms} obtained from the zero-crossing analysis. Wave set-up/set-down (not shown) was found to be very small at all locations across the surf zone, typically of order 2–3 mm, and only a small fraction of the wave height. The hypothesis of Rayleigh distributed wave heights is generally supported by the data for both the unbroken and the (assumed) broken waves, although discrepancies are apparent at smaller wave heights. Similar discrepancies appear in the data of Thornton and Guza (1983). However, the key feature in the data is the proportion of waves with height $H/H_{rms} \geq H_b/H_{rms}$, i.e., broken waves, that still fit the Rayleigh distribution. Indeed, broken waves of height $2H_{rms}$ and $4H_b$ are found in the inner surf zone. The surf zone is therefore unsaturated and a Rayleigh distribution truncated at $H/H_{rms} = H_b/H_{rms}$ is clearly inappropriate for these surf zone conditions.

Fig. 7a and b compare two different estimates of the cross-shore variation in the fraction of broken waves, Q_b, both using Eqs. (5) and (12). The first estimate assumes

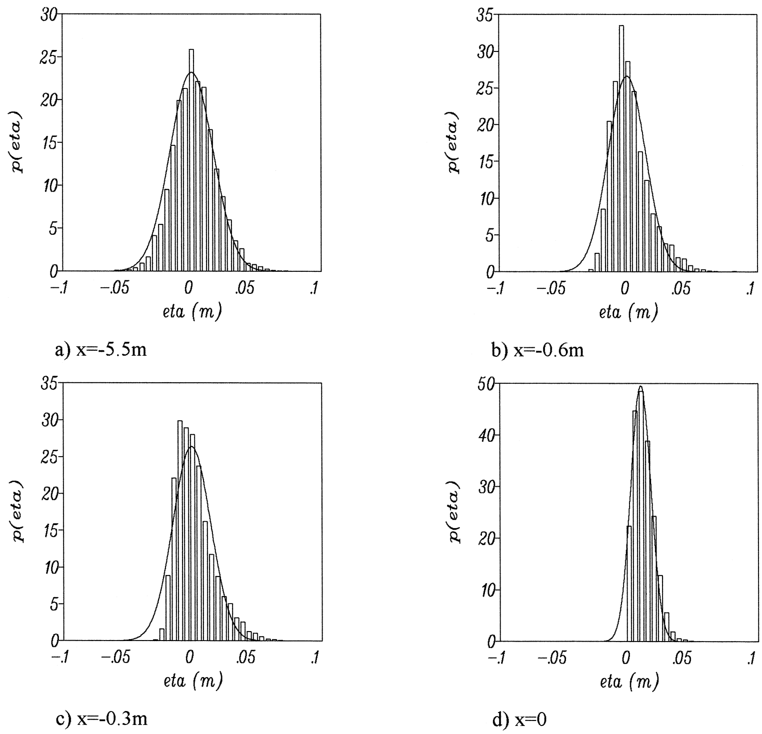


Fig. 4. P.d.f. of water surface elevation $(1/m)$, case J3. (Empty vertical box) Measured, (————) Gaussian distribution. (a) $x = -5.5$ m, (b) $x = -0.6$ m, (c) $x = -0.3$ m, (d) $x = 0$.

Rayleigh distributed wave heights, and hence Eq. (6), with H_{rms} taken as the observed local value. The second estimate uses Eq. (5) directly, i.e., integrating the observed local wave height distribution over the range $H/H_{rms} \geq H_b/H_{rms}$. Although there are some small differences, the measured data compare very well to the theoretical distribution. This shows that, despite the discrepancies between the measured and theoretical p.d.f.'s on Figs. 5 and 6, the total fraction of broken waves is very well described by a Rayleigh p.d.f. if a standard expression for the minimum breaker height (12) is used in both instances. These calculations therefore support some of the key assumptions in the model (Eqs. (5) and (12)).

Comparisons between the true proportion of broken waves obtained from the video analysis and that predicted using the Battjes and Janssen (1978) cross-shore model and the new Rayleigh model (Eqs. (1)–(12)) are shown on Fig. 8a–c. Note that the values of H_{rms} used in Eq. (6) are based on the model predictions, not the observed values. For all three random wave cases the Battjes and Janssen (1978) model significantly overestimates the proportion of breaking waves, and over a large part of the nearshore zone gives $Q_b = 1$, i.e., a saturated surf zone. The Rayleigh model gives a much better fit to the measured data, which show a significant proportion of the waves breaking close to the shoreline. Indeed, for case J3 approximately 40% of the waves are 'shore breaks'.

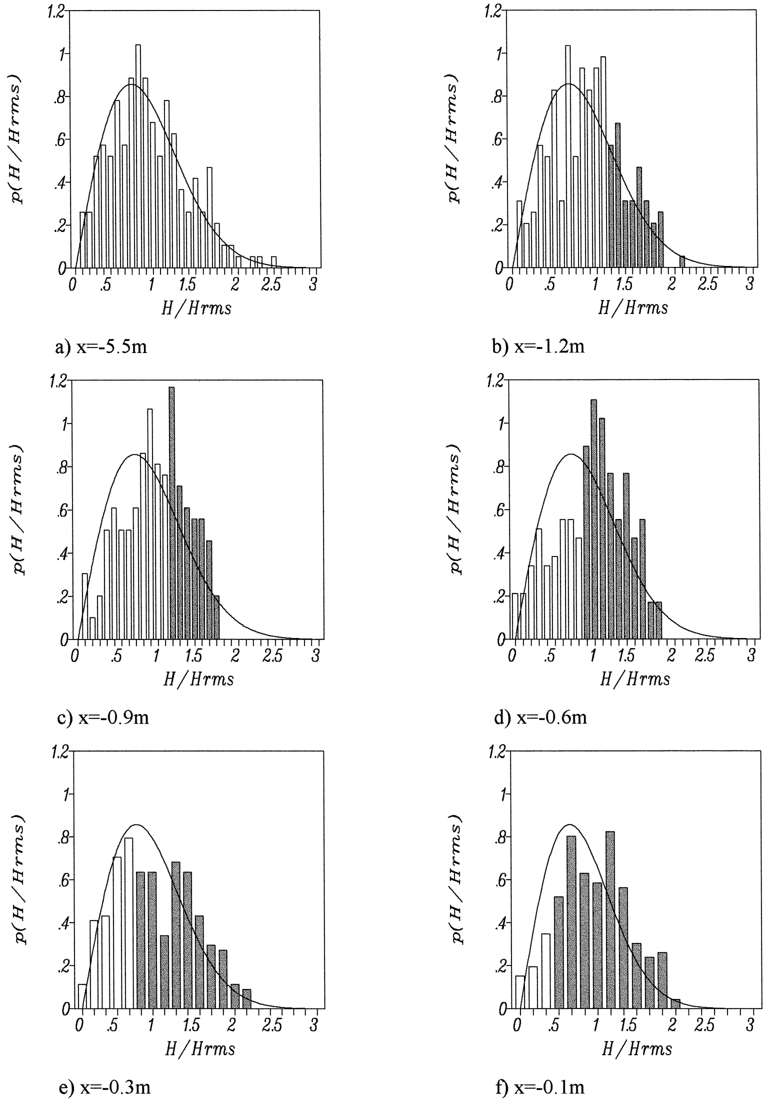


Fig. 5. Wave height probability density functions, case J1. (Empty vertical box) Measured, (shaded vertical box) $H/H_{rms} \geq H_b/H_{rms}$, (————) Rayleigh distribution. (a) $x = -5.5$ m, (b) $x = -1.2$ m, (c) $x = -0.9$ m, (d) $x = -0.6$ m, (e) $x = -0.3$ m, (f) $x = -0.1$ m.

This is consistent with a recent analysis of field data by Raubenheimer et al. (1996) which shows that γ increases with increasing beach slope and therefore more 'shore breaks' are expected. Consequently, on a steep beach, waves can propagate into very shallow water before fully breaking. It should be noted that Battjes and Janssen (1978) did not attempt to describe accurately the fraction of broken waves, but aimed to model the overall energy dissipation rate within the surf zone. However, their model has been

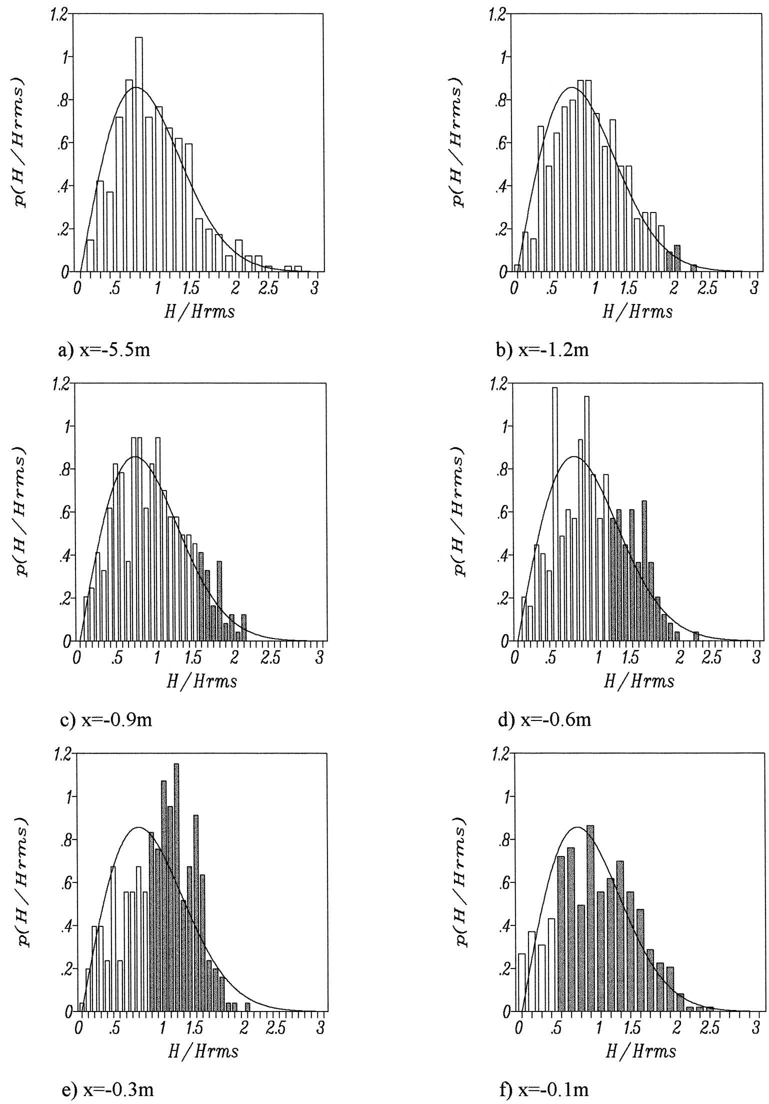


Fig. 6. Wave height probability density functions, case J3. (Empty vertical box) Measured, (shaded vertical box) $H/H_{rms} \geq H_b/H_{rms}$, (———) Rayleigh distribution. (a) $x = -5.5$ m, (b) $x = -1.2$ m, (c) $x = -0.9$ m, (d) $x = -0.6$ m, (e) $x = -0.3$ m, (f) $x = -0.1$ m.

widely used to estimate the fraction of broken waves for undertow calculations (e.g., Southgate and Nairn, 1993).

Comparisons between the measured wave heights from the zero-crossing analysis and the wave heights estimated from the variance of the water surface elevation (m_o) by assuming a Rayleigh distribution are shown on Fig. 9a and b. For a narrow banded Gaussian process and a Rayleigh wave height p.d.f., the variance or energy based wave

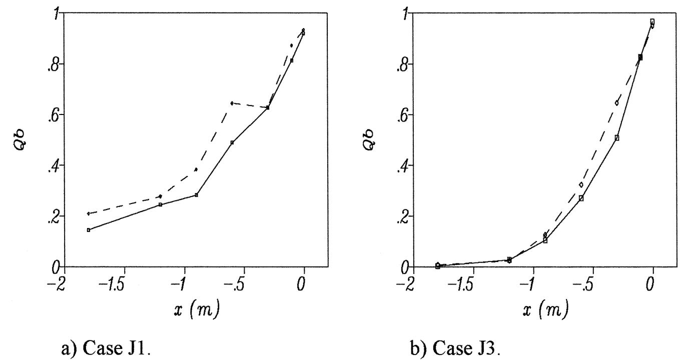


Fig. 7. Estimated cross-shore variation in the fraction of breaking waves. (————) Rayleigh p.d.f. using observed H_{rms}, (---) measured wave height p.d.f. (a) Case J1, (b) Case J3.

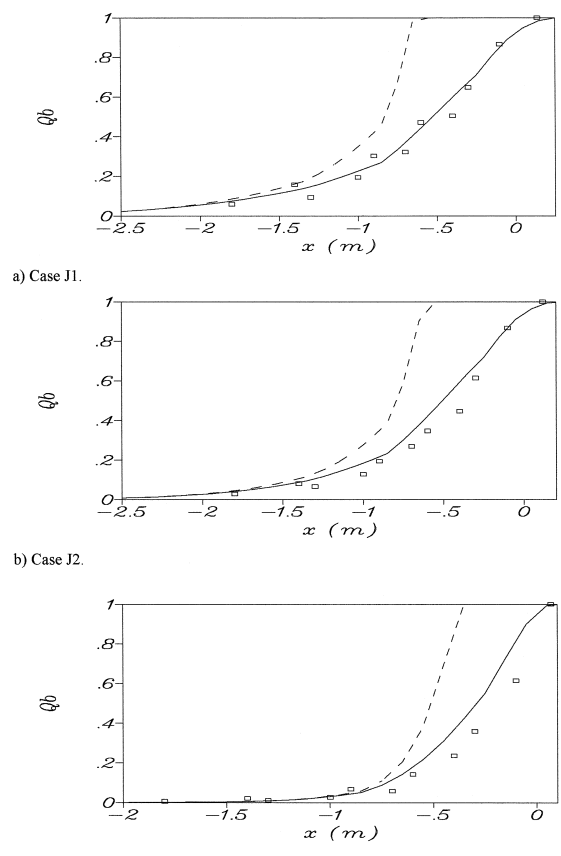
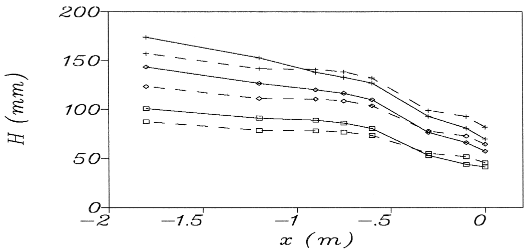
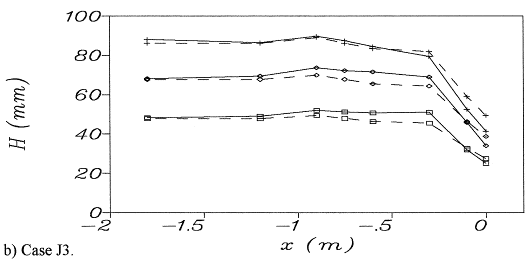


Fig. 8. Measured and predicted cross-shore variation in the fraction of breaking waves. (□ □ □ □) Data points (Video analysis), (————) Rayleigh model, (---) Battjes and Janssen (1978). (a) Case J1, (b) Case J2, (c) Case J3.



a) Case J1.



b) Case J3.

Fig. 9. Measured cross-shore variation in wave heights. (——— □ ———) H_{rms}, (--□--) H_e (Rayleigh), (———◇———) $H_{1/3}$, (--◇--) $H_{1/3}$ (Rayleigh), (——— + ———) $H_{1/10}$, (--+--) $H_{1/10}$ (Rayleigh). (a) Case J1, (b) Case J3.

height, $H_e = \sqrt{(8 m_o)}$, is equivalent to H_{rms}; the significant wave height, $H_{1/3}$, is equal to $1.416 H_e$; and $H_{1/10}$ is given by $1.8 H_e$. For case J1, the equivalent measures of wave height differ by about 10–15% in the outer surf zone but the difference decreases shorewards. Indeed, at the SWL position, H_{rms} and H_e are nearly identical. The difference in the outer surf zone is likely to arise from nonlinear profile deformation due to bound higher harmonics, the effects of which reduce as energy is dissipated shorewards. For case J3, the equivalent wave heights are very similar in the outer surf zone, with the difference increasing slightly up to the principal break point. However, at the SWL, the Rayleigh model again gives a very close approximation to the measured H_{rms}.

Comparisons between the measured cross-shore variation in wave heights (H_e), predictions from the Battjes and Janssen (1978) wave model and the new Rayleigh model (9) are shown on Fig. 10a–c. In the outer surf zone the Rayleigh model predicts slightly larger wave heights than the Battjes and Janssen (1978) model, since Eq. (9) gives a reduction in $\langle \varepsilon_b \rangle$ when $H_{rms} \leq H_b$. In the inner surf zone the Battjes and Janssen (1978) model forces $H_{rms} = H_b$, i.e., a saturated surf zone. However, the data clearly show that H_e (and H_{rms}, see Fig. 9), exceeds H_b in the inner surf zone and that a significant proportion of the initial incident wave energy remains at the start of the swash zone ($x \approx 0$). The Rayleigh model provides a much better estimation of the nearshore wave heights, particularly for cases J1 and J2, although the model is unable to predict the very rapid decrease in wave energy after the break point for case J3. This is because 'shore breaks' cannot be described realistically by the bore approximation used in the model. The agreement between the Rayleigh model and the measured data might

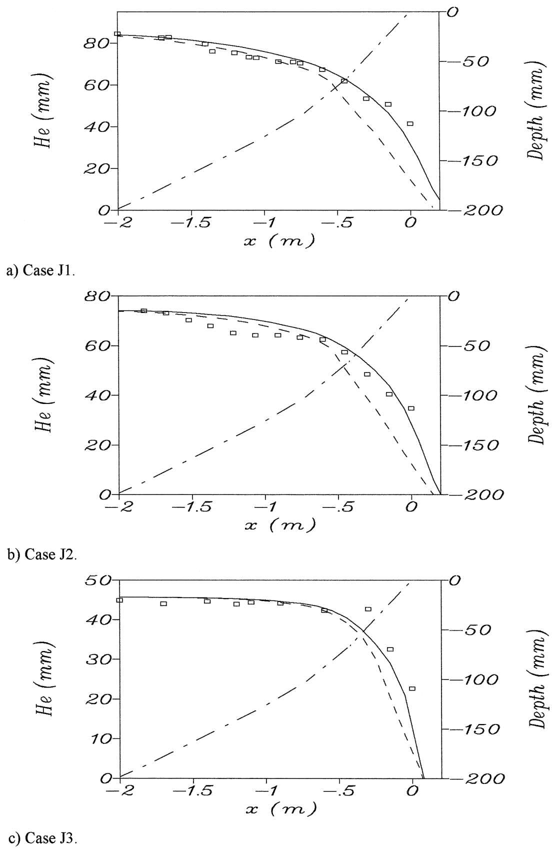
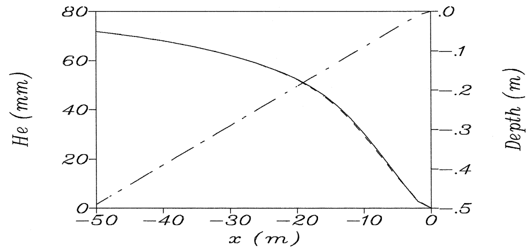


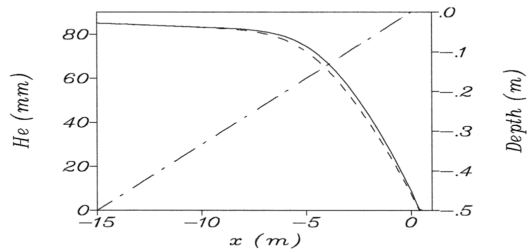
Fig. 10. Measured and predicted cross-shore variation in wave height. (□ □ □ □) Data points (H_e), (————) Rayleigh model, (- - -) Battjes and Janssen (1978), (— · —) beach profile (rhs). (a) Case J1, (b) Case J2, (c) Case J3.

also be further improved if wave reflections could be accounted for. Comparing the data for case J1 and J2 also confirms that the nearshore wave height increases with an increase in offshore wave height, i.e., the inner surf zone is unsaturated.

The Rayleigh model must, however, also be capable of predicting the wave height transformation across mildly sloping beaches, where it is well known that $H_{rms} \approx \gamma h$ (i.e., the surf zone is saturated (Thornton and Guza, 1983; Raubenheimer et al., 1996)). Fig. 11a and b compare the results predicted by the Battjes and Janssen (1978) model with those predicted by the Rayleigh model. For both a $1/100$ and $1/30$ surf zone slope, the two models give nearly identical results for the same incident wave conditions (case J1). For mildly sloping beaches, both models therefore give the same mean energy dissipation rate, despite the different predicted cross-shore variation in the fraction of broken waves (Fig. 12). The reason for this is that although the fraction of broken waves is smaller using the Rayleigh model, the Rayleigh model includes a greater rate of energy dissipation from the largest waves in the p.d.f. The energy dissipation is therefore



a) β=1/100.



b) β=1/30.

Fig. 11. Numerical predictions of cross-shore variation in wave height. (————) Rayleigh model, (- - -) Battjes and Janssen (1978), (— · —) beach profile (rhs). (a) $\beta=1/100$, (b) $\beta=1/30$.

more realistic than that in the Battjes and Janssen (1978) approach, since large broken waves will clearly dissipate more energy than small broken waves. The Rayleigh model also gives very similar results to the original Battjes and Janssen (1978) model for wave transformation over a bar on a 1/20 slope (Battjes and Janssen, 1978, case 15). The Rayleigh model therefore appears suitable for a wider range of surf zone conditions than the original Battjes and Janssen (1978) model.

5.2. Spectral characteristics and kinematic moments

Fig. 13a and b show the spectra of the run-up and water surface elevation at $x=-5.5$ m and $x=0$ for cases J1 and J3. In each case there is little low-frequency energy (<0.2 Hz) at the offshore location, but the low-frequency energy increases

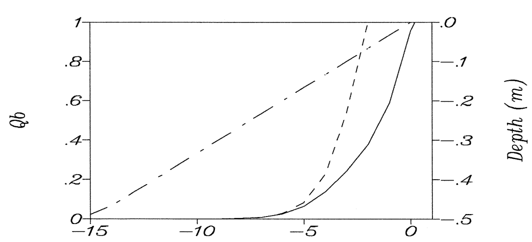


Fig. 12. Numerical predictions of cross-shore variation in the fraction of breaking waves. (————) Rayleigh model, (- - -) Battjes and Janssen (1978), (— · —) beach profile (rhs). $\beta=1/30$.

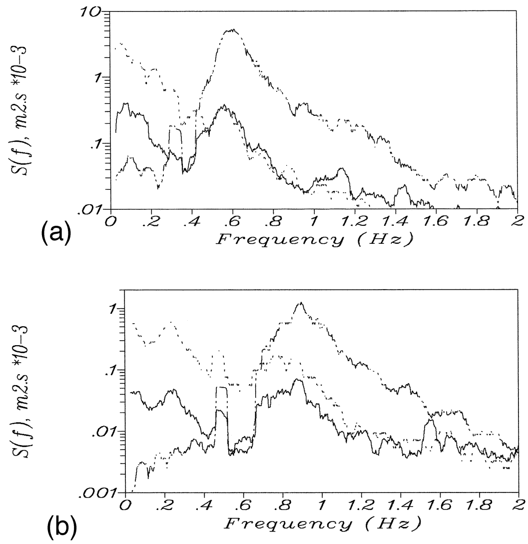
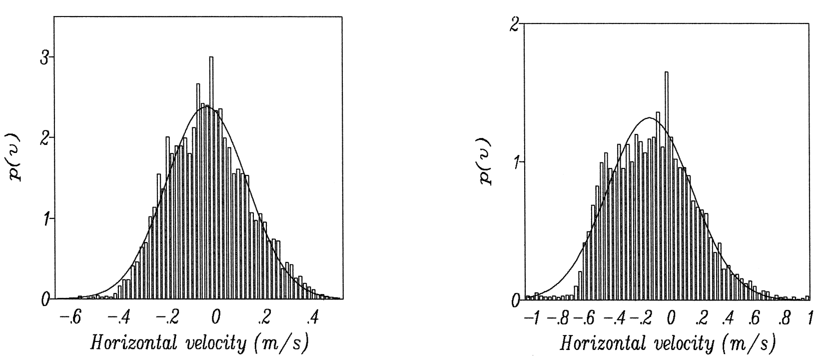


Fig. 13. (a) Power spectra of run-up and water surface elevation, case J1. (····) Run-up wire, (————) $x=0$, (—··—) $x=-5.5$ m. (b) Power spectra of run-up and water surface elevation, case J3. (····) Run-up wire, (————) $x=0$, (—··—) $x=-5.5$ m.

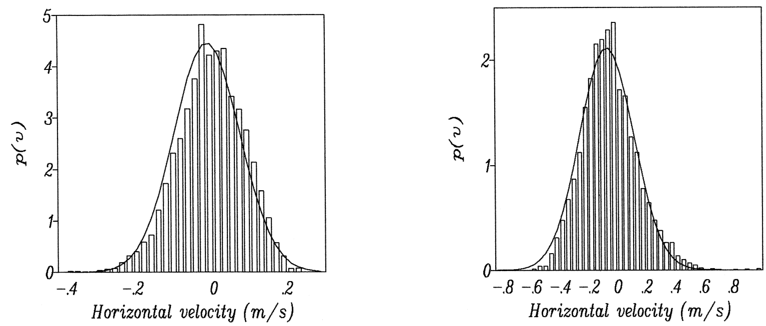
shorewards, reaching a maximum in the swash zone. Much of the initial incident short wave energy (0.5–1.5 Hz) also remains in the inner surf zone and swash zone, consistent with the previous data. The order of magnitude difference between the low-frequency energy at $x=0$ and in the swash is largely due to the effects of wave grouping in the run-up (Baldock et al., 1997).

Probability density functions of the wave-induced horizontal velocity 5 mm above the bed in the outer and inner surf zone are shown on Figs. 14 and 15 for cases J1 and J3 (positive velocity onshore). In each case, the kinematics are reasonably well described



a) x=-1.8m b) x=-0.3m

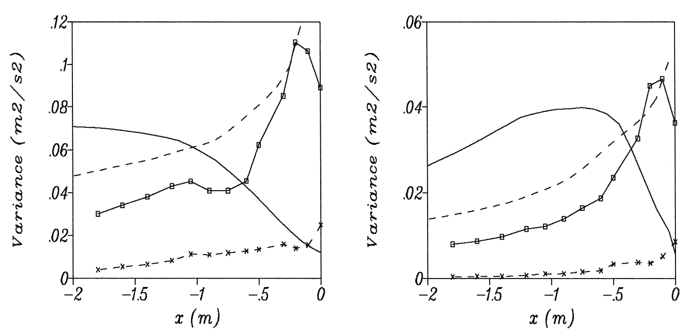
Fig. 14. P.d.f. of horizontal velocity 5 mm above bed (s/m), case J1. (Empty vertical box) Measured, (————) Gaussian distribution. (a) $x=-1.8$ m, (b) $x=-0.3$ m.



a: x=-1.8m b) x=-0.3m

Fig. 15. P.d.f. of horizontal velocity 5 mm above bed (s/m), case J3. (Empty vertical box) Measured, (————) Gaussian distribution. (a) $x = -1.8$ m, (b) $x = -0.3$ m.

by Gaussian probability density functions with the same mean and standard deviation as the measured data. Similar results are found at other cross-shore positions, with the greatest discrepancy between the Gaussian distribution and the measured data closest to the shoreline (Fig. 14b and Fig. 15b). The cross-shore variation of the velocity variance (m_0), with in this instance the measured data initially low-pass filtered at 2 Hz to remove noise due to bubbles from broken waves, is shown on Fig. 16. The velocity variance increases shorewards almost up to the SWL, with less than 20% of the velocity variance at frequencies below 0.2 Hz. For waves breaking close to the shoreline, numerical computations by Kobayashi and Karjadi (1996) showed similar results. This is again in contrast to the velocity variance found in saturated surf zones, where low frequency energy is frequently dominant (Guza and Thornton, 1985; Battjes, 1988). Predicted values of the velocity variance using Vocoidal wave theory (Swart, 1978; see also Nairn, 1990) are generally poor, with the predicted variance decreasing to zero at the shoreline. In contrast, linear shallow water wave theory based on $H_e = H_{rms}$



a) Case J1. b) Case J3.

Fig. 16. Measured and predicted cross-shore variation in horizontal velocity variance. (——— □ ———) Total, (- - × - -) low frequency (< 0.2 Hz), (————) predicted (Southgate and Nairn, 1993), (- - -) predicted (linear theory). (a) Case J1, (b) Case J3.

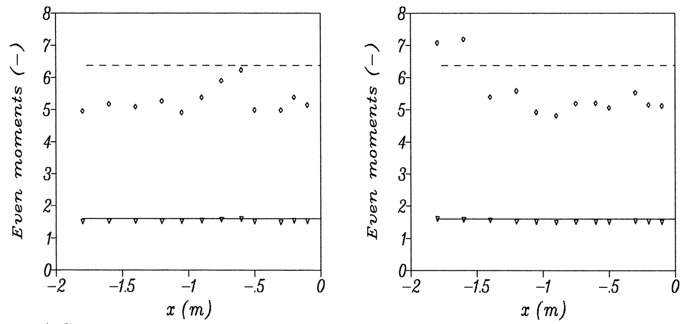


Fig. 17. Measured and estimated nondimensional cross-shore variation in even velocity moments. ($\nabla\nabla\nabla\nabla$) u^{3*}, ($\diamond\diamond\diamond\diamond$) u^{5*}, (————) u^3* Gaussian (linear), (---) u^{5*} Gaussian (linear). (a) Case J1, (b) Case J3.

($u_{\max} = (H_e/2)\sqrt{(g/h)}$, $m_o = 1/8 H_e^2 g/h$) provides a much better description of the velocity variance. This is consistent with the field data of Guza and Thornton (1980), who showed that linear theory was accurate to within about 20% in the surf zone.

In order to calculate the short wave velocity moments the data were first band passed filtered over the range 0.2–2 Hz. The third ($u^{3*} = \langle |u|^3 \rangle / (\langle u^2 \rangle)^{3/2}$) and fifth ($u^{5*} = \langle |u|^5 \rangle / (\langle u^2 \rangle)^{5/2}$) nondimensional even moments are reasonably well described by a linear Gaussian model (Guza and Thornton, 1985) and, although the fifth moment appears to be overestimated by about 20%, both show very little cross-shore variation (Fig. 17a and b). This is consistent with the surface elevation data and the Rayleigh distributed wave heights in the inner surf zone. The measured odd velocity moments ($I_1 = \langle u^3 \rangle, I_2 = \langle |u|^3 u \rangle$) are very small over much of the surf zone (Fig. 18a and b), also consistent with the linear Gaussian model (odd moments zero), although in the inner surf zone the calculated moments appear to be influenced to some degree by noise

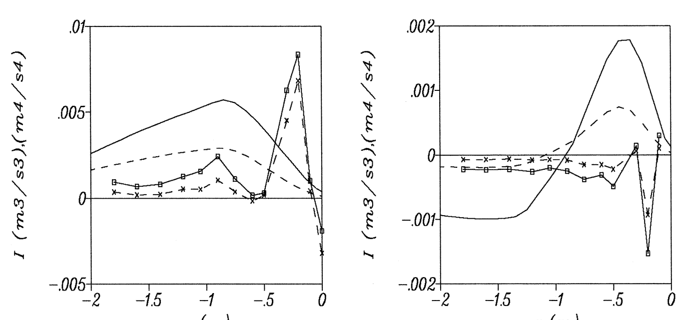


Fig. 18. Measured and predicted cross-shore variation in odd velocity moments. (——□——) I_1, (--×--) I_2, (————) I_1 predicted (Southgate and Nairn, 1993), (---) I_2 predicted (Southgate and Nairn, 1993). (a) Case J1, (b) Case J3.

induced by breaking waves. Note that the linear Gaussian model will always predict zero odd velocity moments, whereas in reality nonlinear waves will generally exhibit some shorewards skewness, particularly under broken waves (Guza and Thornton, 1985). However, the data trends are again poorly predicted by Vocoidal wave theory. All the kinematic data is therefore consistent with a Gaussian model for the water surface elevation and shallow water linear wave theory.

6. Conclusions

New experimental data on cross-shore variations in random wave hydrodynamics on a steep beach have been presented. The surf zone is found to be unsaturated at incident short wave frequencies and, consequently, significant wave grouping and short wave energy remain at the shoreline. Wave heights in both the outer and inner surf zones are reasonably well described by a normal Rayleigh p.d.f., rather than a truncated p.d.f. as used by Battjes and Janssen (1978).

A new parametric wave transformation model is presented, based on a normal Rayleigh p.d.f. and a standard expression for the minimum breaker height. This allows explicit expressions for both the fraction of broken waves and the energy dissipation rate to derived. The model proves to give good results for the cross-shore variation in both wave height and the fraction of broken waves on a steep laboratory beach and very similar results to the original Battjes and Janssen (1978) model on mildly sloping beaches.

Cross-shore variations in the velocity variance and higher order velocity moments are best described by linear Gaussian wave theory, rather than Vocoidal wave theory as proposed by Swart (1978). This is a result of the narrow surf zone, where there is insufficient time for the classical vertical and horizontal asymmetry of shallow water waves to develop fully and is consistent with the relatively large fraction of waves that break very close to the shoreline.

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