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Separation of incident and reflected waves over sloping bathymetry

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Abstract

An existing 2D method for separating incident and reflected waves over a horizontal bed [Frigaard, P., Brorsen, M., 1995. A time domain method for separating incident and reflected irregular waves. Coastal Eng., 24, 205–215.] is modified to account for normally incident linear waves propagating over a bed with arbitrary 2D bathymetry. Linear shoaling is used to determine the amplitude and phase change between two measurement positions; thereafter the existing technique can be applied. Comparisons between the existing and modified methods are made using numerically simulated data. Errors in the reflection coefficient are found to be small for large reflection coefficients, but may become large if reflection is low. However, if an accurate assessment of the amplitude of the incident and reflected wave trains is required, the bathymetry must be accounted for in order to avoid significant errors (up to 90% for cases considered).

Keywords: Waves; Reflection; Sloping bathymetry; Beaches

1. Introduction

The separation of a wave train into the incident waves and the waves reflected from a beach or structure is a common requirement for both laboratory and field data. A large number of methods using spatially separated sensors are available for 2D waves propagating over a horizontal bed (Goda and Suzuki, 1976; Mansard and Funke, 1980; Frigaard and Brorsen, 1995; and others), which either provide incident or reflected wave spectra or the incident and reflected wave trains themselves.

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However, none of these methods strictly account for waves propagating over a sloping bed, although they are frequently applied in such circumstances. Errors in the resulting analysis are therefore likely, depending on the wave conditions and beach slope. Indeed, in order to determine if wave reflection is distributed across a sloping beach (Baquerizo et al., 1997), rather than occurring solely at the shoreline, then the effect of the bathymetry needs to be accounted for in the analysis. Alternative methods using co-located elevation and velocity sensors (e.g., Guza and Bowen, 1976; Hughes, 1993) avoid the variability in bathymetry, but noise in the data records tends to bias the reflection coefficient towards higher values (Huntley et al., 1999). Furthermore, if data are required at many cross-shore or longshore locations, the use of wave gauges or pressure transducers is much more economical (Hughes, 1993), and many laboratory facilities do not have access to multiple current meters.

This technical note presents a simple modification to an existing 2D wave separation technique using spatially separated sensors, in order to account for normally incident linear waves propagating over a bed with arbitrary 2D bathymetry. The method is principally applicable to laboratory conditions, but could be applied to field data if the waves were predominantly shore normal. The modification is an extension of that for a horizontal bed proposed by Frigaard and Brorsen (1995), which was chosen for the clarity of approach and which also readily allows separation of the incident and reflected time series, rather than just the incident and reflected wave spectra. However, the principle outlined below could equally be applied to the two wave gauge analysis technique of Goda and Suzuki (1976), and other 2D three wave gauge array methods. Section 2 reviews the relevant equations due to Frigaard and Brorsen (1995) and presents the modifications necessary for a sloping bathymetry. In Section 3, brief comparisons are made between the existing and modified method using simulated data and a plane bed. Final conclusions are presented in Section 4.

2. Theory

2.1. Original formulation

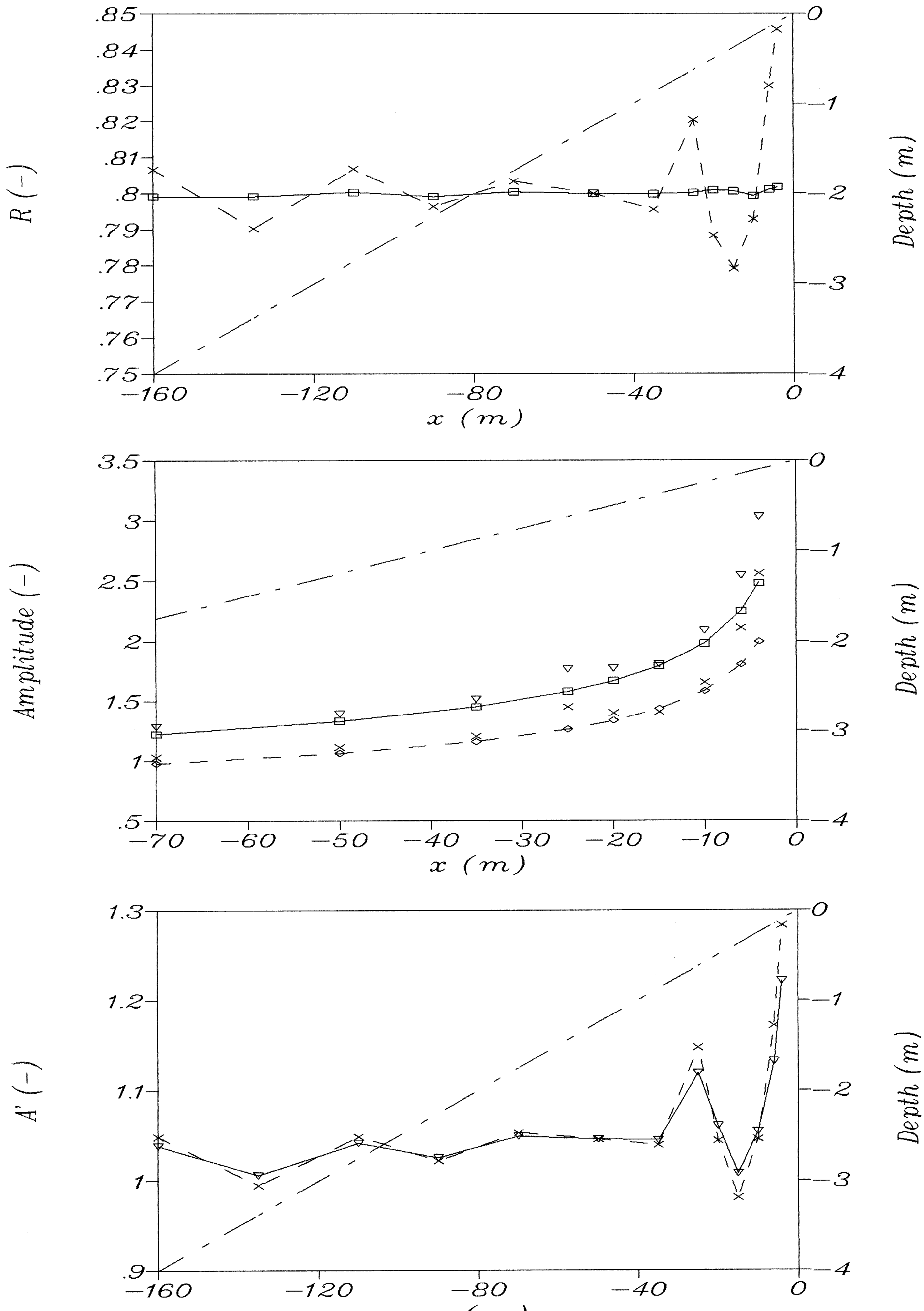
Linear theory gives the water surface elevation at two spatial locations, x_1 and x_2, Δx apart, with respective depths d_1 and d_2, as:

$$\eta(x_1,t) = a_\mathrm{i}\cos(\omega t - kx_1 + \phi_\mathrm{i}) + a_\mathrm{r}\cos(\omega t + kx_1 + \phi_\mathrm{r}) \tag{1}$$

$$\eta(x_2,t) = a_\mathrm{i}\cos(\omega t - kx_1 - k\Delta x + \phi_\mathrm{i}) + a_\mathrm{r}\cos(\omega t + kx_1 + k\Delta x + \phi_\mathrm{r}) \tag{2}$$

where t is time, ω is the wave angular frequency $(2\pi f)$, $a = a(f)$ is the wave amplitude, $k = k(f)$ is the wavenumber $(2\pi/L(f))$, $\phi = \phi(f)$ is phase and the indices

Fig. 1. (a) Calculated reflection coefficient, $\beta = 1/40$, $R = 0.8$, $f = 0.0488$ Hz. – · –, Depth (rhs); – □ – SB; – × – HB. (b) Calculated incident and reflected wave amplitudes, $\beta = 1/40$, $R = 0.8$, $f = 0.0488$ Hz. – · – Depth (rhs); □ a_i (SB); ▽ a_i (HB); ◇ a_r (SB); × a_r (HB); —— a_i (LS); --- a_r, (LS). (c) Error in incident and reflected wave amplitudes, $\beta = 1/40$, $R = 0.8$, $f = 0.0488$ Hz. – · – Depth (rhs); – ▽ – a_i (HB); – × – a_r (HB).



R (–)
Depth (m)
x (m)
.85
.84
.83
.82
.81
.8
.79
.78
.77
.76
.75
−160
−120
−80
−40
0
0
−1
−2
−3
−4
Amplitude (–)
Depth (m)
x (m)
3.5
3
2.5
2
1.5
1
.5
−70
−60
−50
−40
−30
−20
−10
0
A' (–)
Depth (m)
x (m)
1.3
1.2
1.1
1
.9

'i' and 'r' refer to incident and reflected components, respectively. x is taken as positive shorewards, in the direction of travel of the incident waves. Eqs. (1) and (2) show that between x_1 and x_2, the incident and reflected waves are simply physically phase shifted by $k\Delta x$ and $-k\Delta x$, respectively. Frigaard and Brorsen (1995) then applied theoretical phase shifts, $\varphi(f)$, and amplification factors, $C(f)$, to Eqs. (1) and (2), resulting in:

$$\eta^*(x_1,t) = Ca_\mathrm{i}\cos(\omega t - kx_1 + \phi_i + \varphi_1) + Ca_\mathrm{r}\cos(\omega t + kx_1 + \phi_\mathrm{r} + \varphi_1) \tag{3}$$

$$\eta^*(x_2,t) = Ca_\mathrm{i}\cos(\omega t - kx_1 - k\Delta x + \phi_\mathrm{i} + \varphi_2) + Ca_\mathrm{r}\cos(\omega t + kx_1 + k\Delta x + \phi_\mathrm{r} + \varphi_2) \tag{4}$$

and showed that $\eta^*(x_1,t) + \eta^*(x_2,t) \equiv a_\mathrm{i}\cos(\omega t - kx_1 + \phi_\mathrm{i}) \equiv \eta_\mathrm{i}(x_1,t)$ when:

$$\varphi_1 = k\Delta x + \pi/2 + m\pi + n2\pi \quad m,n = 0, \pm 1, \pm 2, \ldots \tag{5}$$

$$\varphi_2 = -\pi/2 - m\pi + n2\pi \tag{6}$$

$$C = \frac{1}{2\cos(-k\Delta x - \pi/2 - m\pi)} \tag{7}$$

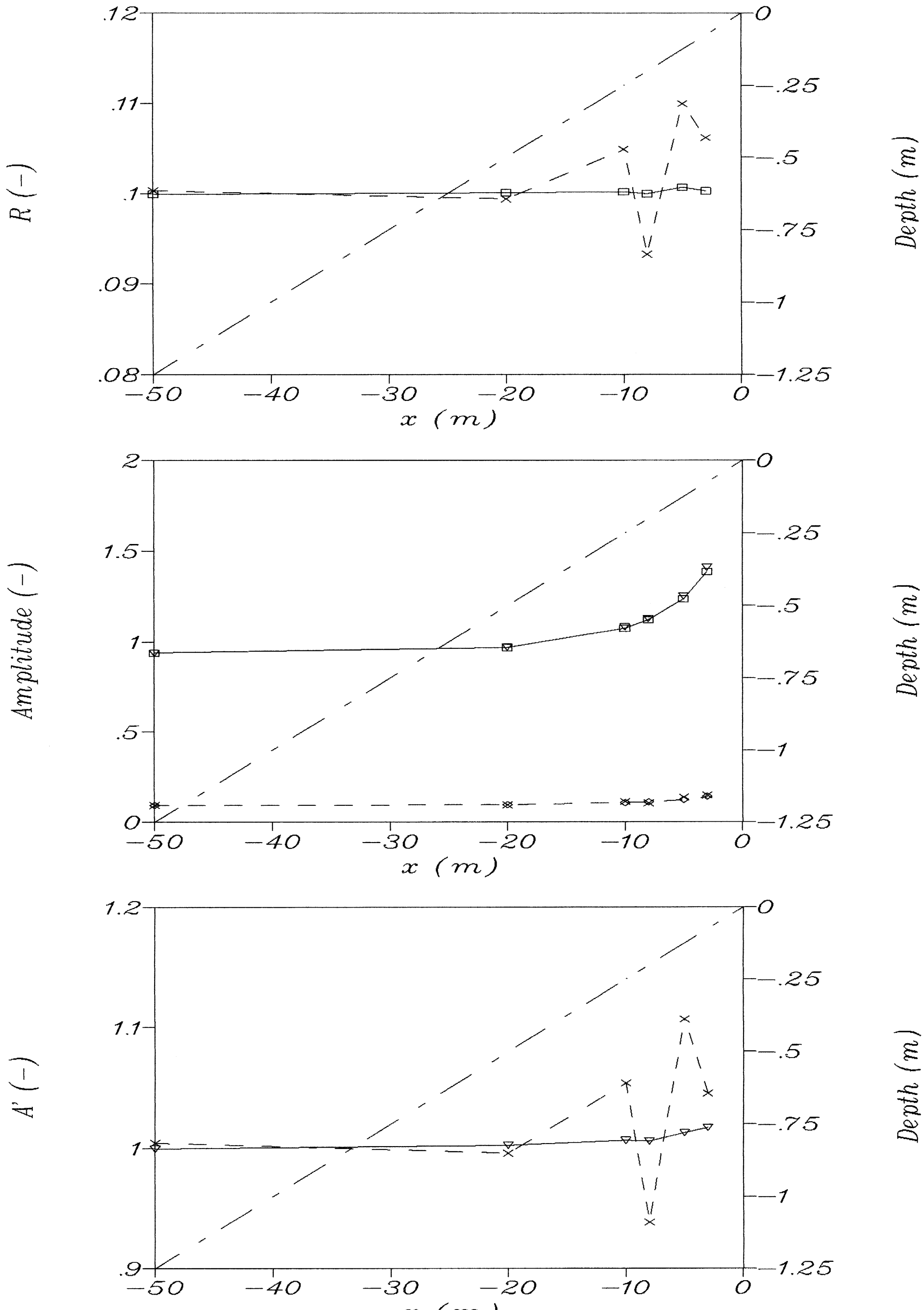
Therefore, transforming Eqs. (1) and (2) into the frequency domain, applying the phase shifts and amplification factors and inverse transforming back to the time domain allows recovery of the incident and reflected components of an irregular 2D wave train. Frigaard and Brorsen (1995) proceeded to design digital filters to accomplish this in real time for use in active wave absorption. However, we are not concerned here with real time output and therefore the phase shifting and amplification is carried out solely in the frequency domain.

2.2. Modification to account for bathymetry

The aim of this work is to modify Eqs. (3)–(7) to account for waves propagating over an arbitrary 2D bathymetry between locations x_1 and x_2. If it is assumed that the gradients in the bathymetry are mild, so that no reflection occurs between x_1 and x_2, then both a_i and a_r change between x_1 and x_2 by the same shoaling coefficient $K_\mathrm{s}(f)$, calculated using linear theory at the two water depths. Note that this does not preclude variations in reflection along a beach profile over larger spatial distances (Baquerizo et al., 1997). Similarly, the physical phase shifts $(\pm k\Delta x)$ change due to the bathymetry,

Fig. 2. (a) Calculated reflection coefficient, $\beta = 1/40$, $R = 0.1$, $f = 0.513$ Hz. – · – Depth (rhs); – □ – SB; – × – HB. (b) Calculated incident and reflected wave amplitudes, $\beta = 1/40$, $R = 0.1$, $f = 0.513$ Hz. – · – Depth (rhs); □ a_i (SB); ▽ a_i (HB); ◇ a_r (SB); × a_r (HB); —— a_i (LS); --- a_r, (LS). (c) Error in incident and reflected wave amplitudes, $\beta = 1/40$, $R = 0.1$, $f = 0.513$ Hz. – · – Depth (rhs); – ▽ – a_i (HB); – × – a_r (HB).

which may also be calculated from linear theory. Writing $k = \omega/c$, where c is the linear phase velocity at a depth d and equal to $\delta x/\delta t_p$, with t_p as the time taken for a wave to



propagate Δx over the arbitrary bathymetry, then the new physical phase shifts $(\pm k^{s}\Delta x)$ are simply given by:

$$k^{s}\Delta x = \frac{\omega\Delta x}{c} = \omega t_{p} = 2\pi f t_{p} \tag{8}$$

For arbitrary bathymetry, t_p may be calculated between d_1 and d_2 using linear theory and a simple time-stepping method. Hence $k^{s} = \omega t_{p}/\Delta x$. Therefore, the new theoretical phase shifts $\varphi_1^{s}(f)$, $\varphi_2^{s}(f)$ and amplification factors $C^{s}(f)$ may be obtained from Eqs. (5)–(7) by substituting $k^{s}(f)$ for $k(f)$. Finally,

$$\eta^{s^*}(x_1,t) = C^{s}\left[a_{i}\cos(\omega t - kx_1 + \phi_{i} + \varphi^{s_1}) + a_{r}\cos(\omega t + kx_1 + \phi_{r} + \varphi^{s_1})\right] \tag{9}$$

$$\eta^{s^*}(x_2,t) = \frac{C^{s}}{K_{s}}\left[a_{i}\cos(\omega t - kx_1 - k^{s}\Delta x + \phi_{i} + \varphi^{s_2}) + a_{r}\cos(\omega t + kx_1 + k^{s}\Delta x + \phi_{r} + \varphi^{s_2})\right] \tag{10}$$

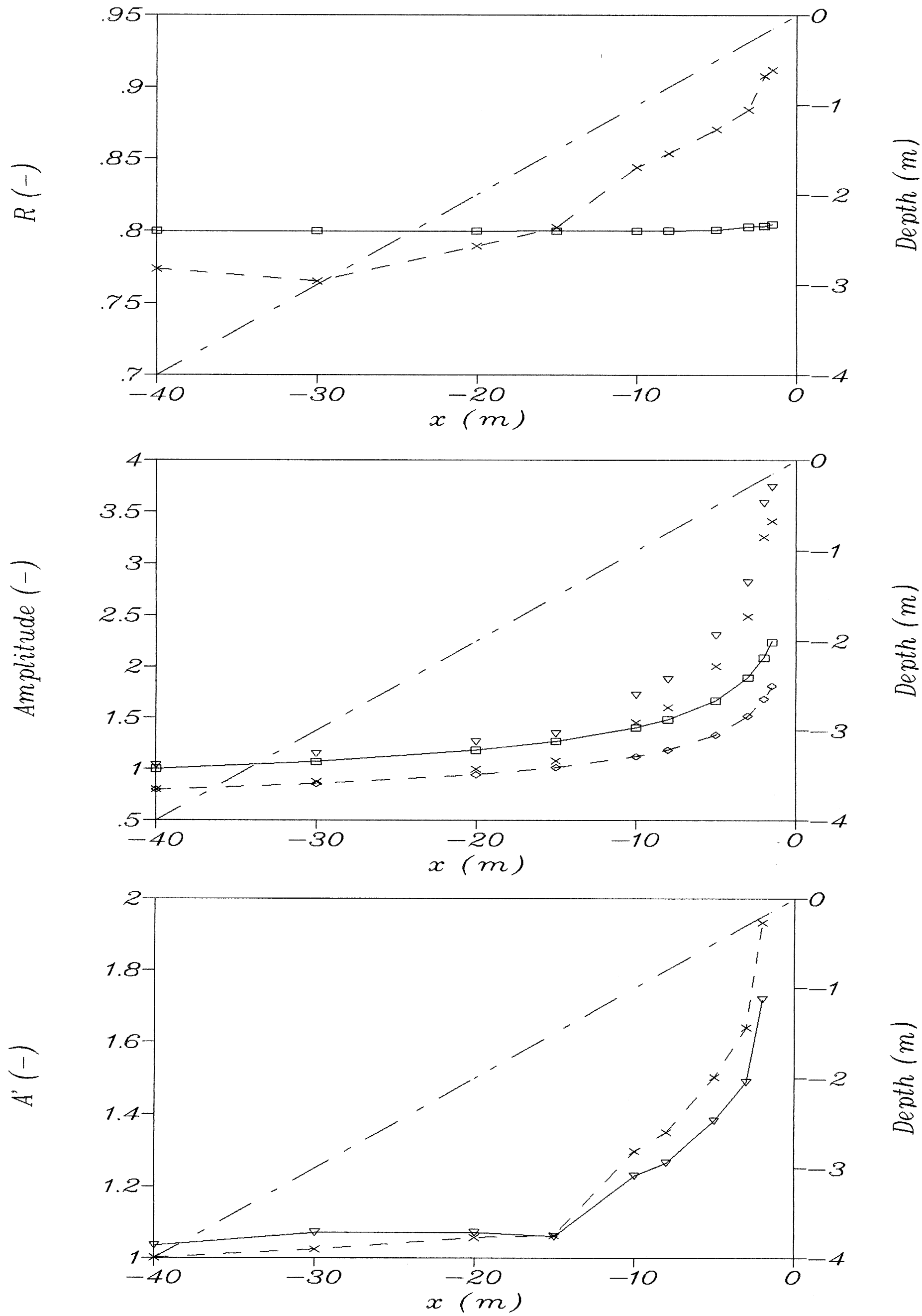
whereby $\eta^{s*}(x_1,t) + \eta^{s*}(x_2,t)$ is again equal to $a_{i}\cos(\omega t - kx_1 + \phi_{i})$ and $\eta_{i}(x_1,t)$. Note that the change in wave amplitude between x_1 and x_2 has been accounted for by dividing $\eta(x_2,t)$ by $K_{s}(f)$, again readily accomplished in the frequency domain, and both methods give the same results if the bed is horizontal. However, it is important to note that both the original and modified methods assume that linear theory is applicable and that there is no dissipation of wave energy (e.g., from wave breaking and bottom friction) between x_1 and x_2.

3. Sample results

Linear theory was used to calculate simultaneous time series of water surface elevation $[\eta(x_1,t),\eta(x_2,t)]$ at two spatial locations on a plane slope for a range of beach slopes, wave frequencies and reflection coefficient. The beach slopes and wave frequencies are typical of those that may occur in both large and small scale laboratory facilities. Although monochromatic waves and a plane beach have been used here for simplicity, the method works equally well for random waves and multiple component or arbitrary slopes. Comparisons are made between the modified method presented above and the original method due to Frigaard and Brorsen (1995) which assumes a horizontal bed. For the horizontal bed calculations the depth has been taken as the mean depth between x_1 and x_2. The spatial separation, $x_2 - x_1$, used for these calculations varied

Fig. 3. (a) Calculated reflection coefficient, $\beta = 1/10$, $R = 0.8$, $f = 0.0488$ Hz. – · – Depth (rhs); – □ – SB; – × – HB. (b) Calculated incident and reflected wave amplitudes, $\beta = 1/10$, $R = 0.8$, $f = 0.0488$ Hz. – · – Depth (rhs); □ a_i (SB); ▽ a_i (HB); ◇ a_r (SB); × a_r (HB); —— a_i (LS); --- a_r, (LS). (c) Error in incident and reflected wave amplitudes, $\beta = 1/10$, $R = 0.8$, $f = 0.0488$ Hz. – · – Depth (rhs); – ▽ – a_i (HB); – × – a_r (HB).

from $L/5$ offshore to 0.5 m inshore, where the proximity of the shoreline requires a closer spacing. Note that, for the horizontal bed method, greater errors will result with



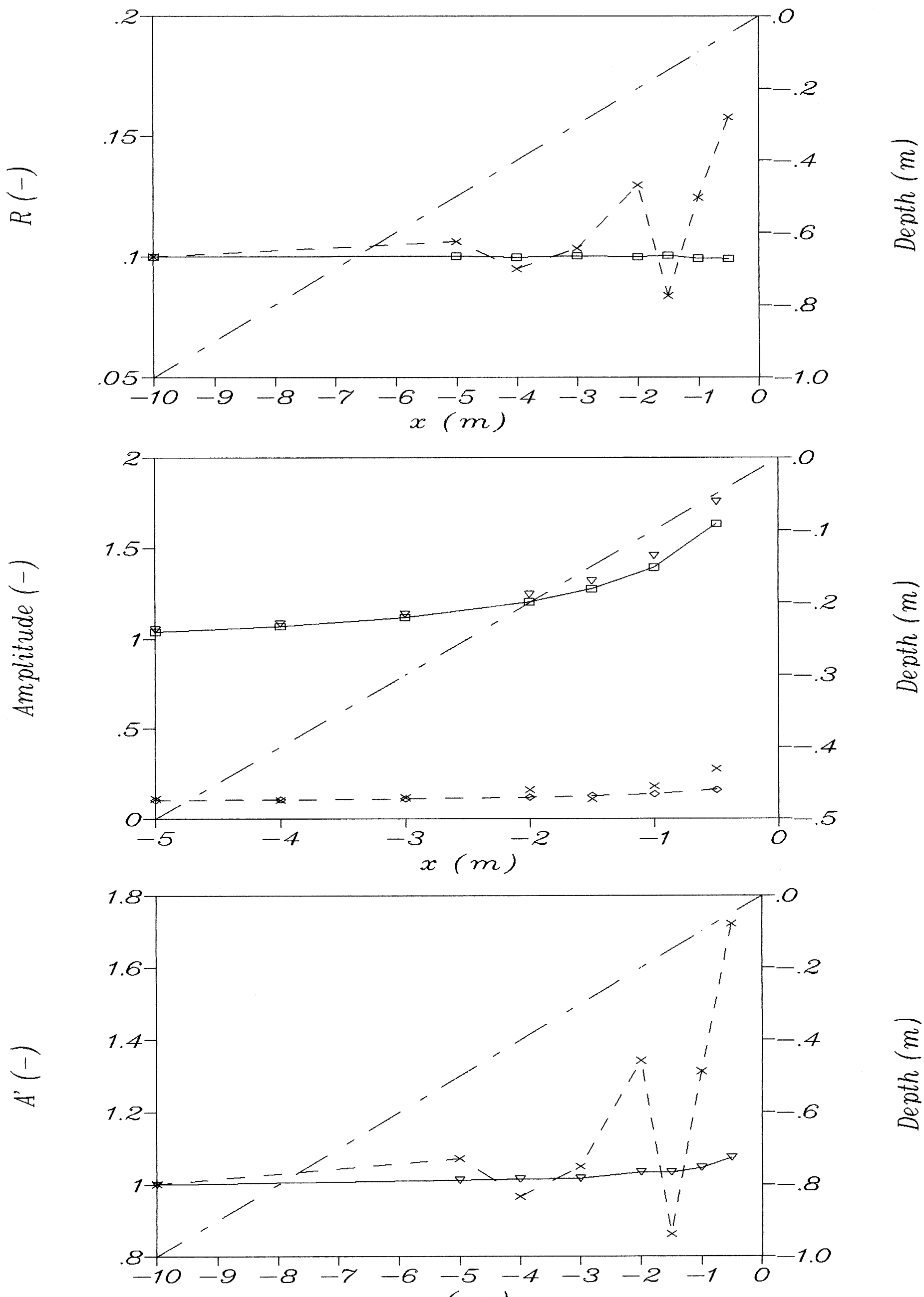
increased spatial separation. In the following figures, values calculated by the modified method for a sloping bed and the horizontal bed approach are denoted by SB and HB, respectively, with theoretical linear shoaling denoted by LS.

Fig. 1a shows the calculated reflection coefficient for a beach slope (β) of 1/40 and a unit amplitude wave with frequency 0.0488 Hz (a surf beat frequency at laboratory scales) and a reflection coefficient (R) of 0.8. The modified method gives reflection coefficients very close to 0.8 over the whole spatial domain (minor errors appear close to the shoreline when $x_2 - x_1$ becomes small compared to the wavelength). Neglecting wave shoaling, i.e., HB, clearly results in errors in the calculated reflection coefficient, particularly close to the shoreline, where long waves frequently dominate the hydrodynamics. However, the errors are relatively small, since the changes in wave amplitude and phase between $x_2 - x_1$ due to wave shoaling are small compared to the differences induced by high reflection. In contrast, for the same wave conditions and beach slope, but for $R = 0.2$ and $R = 0.1$, close to the shoreline the errors in the calculated reflection coefficient using the horizontal bed method reach 60% and 220%, respectively (not shown in figures). Similar errors arise in the calculated incident and reflected wave amplitudes. In this instance, the changes due to wave shoaling are similar in magnitude to those due to reflection. Fig. 1b shows the calculated incident and reflected wave amplitudes using the two different analysis techniques. As expected, the sloping bed method gives wave amplitudes that follow linear shoaling (with errors typically much less than 1%), whereas the horizontal bed method gives wave amplitudes that do not follow linear theory. Errors in the calculated incident and reflected wave amplitudes using the horizontal bed method, defined as the ratio of the calculated value divided by the value expected from linear shoaling (A'), approach 30% for this case (Fig. 1c).

Fig. 2a–c shows similar comparisons for a beach slope of 1/40, a wave frequency of 0.513 Hz and a reflection coefficient of 10%. For this short wave, errors in the horizontal bed method again approach 10%, but in absolute terms would probably be insignificant compared to noise and non-linear shoaling effects for real data sets. For a long wave propagating over a steep beach slope ($\beta = 1/10$, $f = 0.0488$, $R = 0.8$, Fig. 3a–c) the error in the reflection coefficient is again relatively small, although the results suggest neglecting shoaling biases the results towards higher reflection coefficients in the nearshore. However, the errors in wave amplitude using the horizontal bed method become very significant, reaching 90% near the shoreline for this case, and neither the incident or reflected wave amplitudes follow linear shoaling. Similarly, for a short wave propagating over a steep slope ($\beta = 1/10$, $f = 0.513$, $R = 0.1$, Fig. 4a–c) the error in the calculated reflection coefficient using the horizontal bed method is small in absolute terms. However, errors in the predicted wave amplitudes again reach 70%. Furthermore, neglecting linear shoaling introduces much greater errors for the reflected wave ampli-

Fig. 4. (a) Calculated reflection coefficient, $\beta = 1/10$, $R = 0.1$, $f = 0.513$ Hz. – · – Depth (rhs); – □ – SB; – × – HB. (b) Calculated incident and reflected wave amplitudes, $\beta = 1/10$, $R = 0.1$, $f = 0.513$ Hz. – · – Depth (rhs); □ a_i (SB); ▽ a_i (HB); ◇ a_r (SB); × a_r (HB); —— a_i (LS); --- a_r, (LS). (c) Error in incident and reflected wave amplitudes, $\beta = 1/10$, $R = 0.1$, $f = 0.513$ Hz. – · – Depth (rhs); – ▽ – a_i (HB); – × – a_r (HB).

tudes for low values of R (typically the case for short waves) than for high values of R, where the errors in the incident and reflected wave amplitudes are similar in magnitude.



Consequently, a conventional horizontal bed analysis is likely to show deviations from linear theory in the inverse shoaling of reflected waves on a slope.

4. Conclusions

An existing 2D method for separating incident and reflected wave trains propagating over a horizontal bed has been modified to account for 2D waves propagating over an arbitrary 2D bathymetry. Linear theory is used to account for the changes in wave amplitude and phase due to variations in the bathymetry between two spatially separated measurement locations; thereafter the existing method can be applied. The modified method is simple to apply in the frequency domain, with little additional computational effort, and is applicable to both monochromatic and random waves. In principle, it appears possible to extend the technique to 3D waves and 3D bathymetry by including refraction. Comparisons between the modified and original method for waves propagating over a plane slope show that neglecting shoaling effects has a relatively minor effect on calculated reflection coefficients for both long and short waves, although the errors in a conventional analysis increase in shallow water. However, neglect of linear shoaling may lead to very large errors (of order 90% for cases considered) in the calculated incident and reflected wave amplitudes, with the errors generally increasing with increasing beach slope and decreasing reflection. Consequently, if the amplitudes of incident and reflected wave trains propagating over an arbitrary bathymetry are required, then applying a conventional horizontal bed analysis may lead to significant errors.

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