AGU PUBLICATIONS

JGR

Journal of Geophysical Research: Oceans

RESEARCH ARTICLE

10.1002/2013JC009336

Key Points:

- A physical model for sea wave period from altimeter data is presented
- The resulting formula for wave period does not contain any empirical parameters
- Relevance of the new model is shown in a case study

Correspondence to:
S. I. Badulin,
badulin@ioran.ru

Citation:
Badulin, S. I. (2014), A physical model of sea wave period from altimeter data, *J. Geophys. Res. Oceans*, *119*, doi:10.1002/2013JC009336.

Received 6 AUG 2013
Accepted 21 JAN 2014
Accepted article online 28 JAN 2014

A physical model of sea wave period from altimeter data

S. I. Badulin[1,2,3]

[1]P. P. Shirshov Institute of Oceanology of the Russian Academy of Science, Moscow, Russia, [2]Novosibirsk State University, Nonlinear Wave Processes Laboratory, Novosibirsk, Russia, [3]Russian State Hydrometeorological University, Satellite Oceanography Laboratory, St. Petersburg, Russia.

Abstract A physical model for sea wave period from altimeter data is presented. Physical roots of the model are in recent advances of the theory of weak turbulence of wind-driven waves that predicts the link of instant wave energy to instant energy flux to/from waves. The model operates with wave height and its spatial derivative and does not refer to normalized radar cross-section σ_0 measured by the altimeter. Thus, the resulting formula for wave period does not contain any empirical parameters and does not require features of particular satellite altimeter or any calibration for specific region of measurements. A single case study illustrates consistency of the new approach with previously proposed empirical models in terms of estimates of wave periods and their statistical distributions. The paper brings attention to the possible corruption of dynamical parameters such as wave steepness or energy fluxes to/from waves when using the empirical approaches. Applications of the new model to the studies of sea wave dynamics are discussed.

1. Introduction

Altimeter data represent the most ample part of satellite records in number, space coverage and in duration of measurements. They are widely used in ocean studies, first and foremost, for monitoring sea level and estimating large-scale ocean currents based on the model of geostrophic balance.

Relatively coarse spatial resolution is generally considered as a key constraint of the spaceborne altimeters for wind wave studies. The footprint of a few kilometers does not allow to resolve wave profiles. On the other hand, it provides an estimate of sea state at scales of tens and hundreds lengths of wind-driven and swell waves. These scales are adequate to today needs of wave monitoring and forecasting. This is why these data continue to play an important role in wave studies from space being developed within international projects, e.g., ESA initiative Globwave (http://www.globwave.org).

Space altimeters give two parameters of sea state. First, significant wave height H_s is estimated from the front slope of electromagnetic pulse reflected by sea surface. Second, backscatter coefficient σ_0 characterizes sea roughness on the scales of the sounding electromagnetic pulse, i.e., on scales of capillary and gravity-capillary water waves. The latter issue is critical in the context of this work: while H_s can be related straightforwardly to sea waves of tens and hundreds meters length, it is not the case of the backscatter coefficient σ_0. The link of σ_0 to key parameters of sea waves is mostly empirical: it is hypothesized or postulated based on more or less stable correlation of collocated altimeter and in situ data. It hampers or, even, makes impossible further progress in understanding physics of wind-driven and swell waves [*Hwang et al.*, 2002].

Persistent correlation of σ_0 with in situ near-surface wind speed (U_{10} at standard height 10 m is implied below unless otherwise stated) allows to measure the latter with accuracy better than 1 m/s for winds below 20 m/s [e.g., *Brown et al.*, 1981; *Chelton and McCabe*, 1985; *Goldhirsh and Dobson*, 1985; *Abdalla*, 2007]. Even for higher winds up to 40 m/s reasonable estimates can be made by taking into account the effect of wind-wave age [e.g., *Zhao and Toba*, 2003], correct account of mechanisms of wave scattering [*Hwang et al.*, 2010] or purely empirically, again, assuming correlation of measured σ_0 and collocated in situ data [*Young*, 1993]. Within empirical approach, the deficiency of in situ measurements or indirect estimates of high wind speeds become a big problem even if theoretical and empirical results look consistent [cf. *Young*, 1993; *Hwang et al.*, 2010, Figures 1 and 6 correspondingly]. In absence of repeated verification, the parameterizations in terms of radar cross-section σ_0 look like an extrapolation to unexplored physical conditions rather than dependencies based on correct physics. Similar problems arise in inland basins in the whole range of

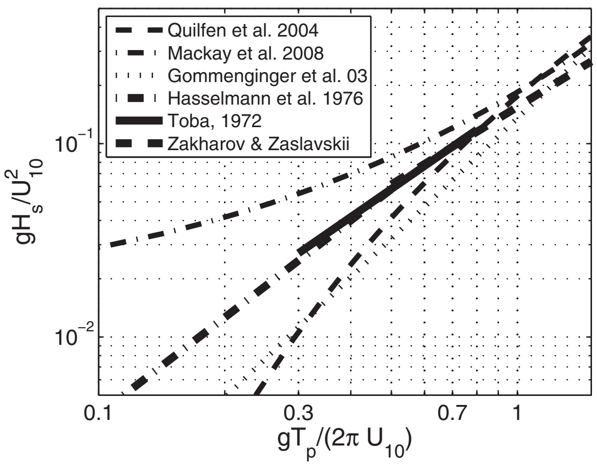

Figure 1. Dependence of nondimensional wave height $\tilde{H}$ on nondimensional wave period $\tilde{T}$ for reference regimes of wave growth (bold lines, in legend) and for parameterizations of altimeter measurements (thin lines, in legend). Wind speed $U_{10}=10$ m/s is fixed in the latter case.

wind speeds with in situ data shortage, specific features of water surface state (salinity, surface pollution, etc.), and peculiar wind-sea dynamics [e.g., *Vignudelli et al.*, 2011, sections 13–15].

Empirical parameterizations of sea wave periods rely upon both H_s and σ_0 from the very beginning in contrast to parameterizations of wind speed as function, as a rule, of the only variable σ_0. The resulting dependence $T(H_s, \sigma_0)$ being expressed in conventional form of dependence on wave height and wind speed $T(H_s, U_{10})$, evidently, does not reflect all the complexity of wind-sea interaction. At best, it can be considered as very preliminary approximation. A number of effects, like gustiness [*Abdalla and Cavaleri*, 2002], atmosphere stratification, etc., [*Donelan et al.*, 2005] can provide high variance of these dependencies. Nevertheless, the two-parametric dependencies $T(H_s, \sigma_0)$ are widely used for estimates of wave periods from altimeter data [e.g., *Gommenginger et al.*, 2003; *Quilfen et al.*, 2004; *Mackay et al.*, 2008].

One can specify two groups of models of sea wave period. The first one embraces the models derived from collocated data sets where formal fit procedures are used with no reference to physics of sea waves [e.g., *Gommenginger et al.*, 2003; *Quilfen et al.*, 2004; *Mackay et al.*, 2008]. These models show reasonable accuracy but cannot guarantee the relevance for arbitrary sea state: thorough repeated calibration is required.

Models of the second group [see *Hwang et al.*, 1998; *Zhao and Toba*, 2003] rely upon consistency of wind speed measurements and empirical laws of wind wave growth [e.g., *Toba*, 1972; *Hasselmann et al.*, 1976] written for nondimensional wave heights and periods

$$\tilde{H}_s = \frac{gH_s}{U_{10}^2}; \qquad \tilde{T} = \frac{gT}{U_{10}} \tag{1}$$

as simple power-like functions

$$\tilde{H}_s \sim \tilde{T}^R. \tag{2}$$

Exponent R in (2) varies in a range and can be used to discriminate different regimes of wave growth [*Gagnaire-Renou et al.*, 2011; *Badulin and Grigorieva*, 2012; *Grigorieva et al.*, 2012]: young waves grow faster (R is slightly higher) than older ones. Adequacy of wind speed scaling (1) and, to a lesser degree, proper choice of exponent R remain critical points of the approach.

Figure 1 represents motivation of this work in the context of previous models of wave periods from altimeter data. Empirico-theoretical dependencies of nondimensional significant wave height on nondimensional wave period [*Hasselmann et al.*, 1976; *Toba*, 1972; *Zakharov and Zaslavsky*, 1983] are shown by bold lines as the state-of-the-art of understanding laws of wind-wave growth. One of these dependencies proposed by *Hasselmann et al.* [1976] has been used by *Hwang et al.* [1998] and *Zhao and Toba* [2003] in their models. Pre-exponent of the authentic dependence by *Hasselmann et al.* [1976] has been considered as a tuning parameter in these papers for better fit of collocated altimeter and buoys data sets. Thin curves in Figure 1 represent the first group of wave period models [*Gommenginger et al.*, 2003; *Quilfen et al.*, 2004; *Mackay et al.*, 2008] where wave physics has not been regarded. One can see dramatic quantitative and, what is more important, qualitative deviations of the models from conventional empirical dependencies of wind wave growth. These deviations can only be partially explained by measurement peculiarities and data processing.

The primary goal of this paper is to focus on physical aspects of correct estimates of wave periods from altimeter data. The new physical model is based on recent advances of the theory of weak turbulence of wind-driven seas [*Zakharov*, 2010]. Within this model, we succeeded in avoiding any reference to backscatter coefficient σ_0 and to quite questionable hypotheses that link this characteristic of short capillary and gravity-capillary waves to much longer wind-driven waves. Our model operates with measured wave height and its spatial derivatives and does not contain any tuning parameters. Its validity is determined by the validity of wave turbulence theory itself and proper ranging of physical scales and scales of altimeter measurements (e.g., altimeter footprint as a scale of averaging and distance between successive data counts for estimates of wave field variations). Vice versa relevance of the wave period model can be considered as an independent support of the weak turbulence theory for sea waves.

We start with the theoretical background of the new model. The relevance of the new approach is discussed in terms of comparison with the results of previously proposed parametrical models [*Hwang et al*., 1998; *Gommenginger et al*., 2003; *Quilfen et al*., 2004; *Mackay et al*., 2008]. We find reasonable quantitative agreement with all these models. At the same time, we stress the problems of the empirical models in describing features of wave dynamics: they appear to be very restrictive in terms of the conventional dependencies $\tilde{H}_s(\tilde{T})$ (1). We discuss similar features of statistical distributions of wave steepness—the key parameter that characterizes nonlinear dynamics of sea waves. All the illustrations are given for one area only near the US Atlantic coast ($33^\circ-40^\circ$N and $71^\circ-78^\circ$W) and only for the ENVISAT mission. Default parameters of L2P data of the Globwave database have been accepted. In particular, 1 s averaged data have been used for this first step study.

Comparison with in situ data (e.g., NDBC buoys data base http://www.ndbc.noaa.gov) is avoided intentionally in this paper in order to focus on physical aspects of the new model. The model is presented as a physical concept rather than a tool for getting better quantitative agreement. Such advanced tool can be developed when we follow the actual wave physics in our choice of parameters of measurements and data processing. Possible ways to optimize the proposed algorithm are presented in the Discussion section. An important point of this discussion is an extension of the approach for estimating fluxes of energy and momentum related to wind-wave coupling from altimeter data.

2. Theory of Wave Turbulence as a Physical Background

Statistical description of wind-driven waves is based on the kinetic equation for weakly nonlinear water waves known as the *Hasselmann* [1962] equation. This equation written for spectral densities of wave action $N(\mathbf{k},\mathbf{x},t)$ or energy $E(\mathbf{k},\mathbf{x},t)$ is extremely inconvenient for theoretical study in general case when a number of physical processes should be incorporated into the model. Conventionally, three terms responsible for wave evolution are written in the right-hand side of the Hasselmann equation

$$\frac{dN}{dt}=S_{in}+S_{diss}+S_{nl}. \tag{3}$$

Term of wave input S_{in} describes processes of wave generation due to wind. Dissipation term S_{diss} includes a great number of physical effects: wave breaking, turbulent damping, bottom friction, etc. At the moment, there is no physical model that can describe all the complexity of wave generation and dissipation in a conceivable form. This is why different parameterizations of S_{in} and S_{diss} are used in wave modeling and forecasting [*Cavaleri et al*., 2007]. On the contrary, term of nonlinear interactions S_{nl} is derived from the first principles and given by explicit formulas [see *Badulin et al*., 2005, Appendices].

A conceptual breakthrough can be made within an asymptotic approach when the term of four-wave resonant interactions S_{nl} is assumed to be formally much larger than S_{in} and S_{diss} [*Zakharov*, 2005; *Badulin et al*., 2005, 2007, 2008; *Zakharov and Badulin*, 2011]. It gives two key advantages. First, one can get self-similar solutions for particular cases of wave development (duration-limited or fetch-limited). Second, poorly known terms S_{in}, S_{diss} enter this asymptotic model as integrals over all the wave scales and, thus, their peculiarities become unimportant. Based on these two points one can get a relationship between instant wave parameters and net input to waves [*Badulin et al*., 2007]:

$$\frac{E\omega_p^4}{g^2}=\alpha_{ss}\left(\frac{\langle S_{in}+S_{diss}\rangle\omega_p^3}{g^2}\right)^{1/3} \quad (4)$$

Here E is total energy (wave height variance) and ω_p is frequency of spectral peak. Self-similarity parameter α_{ss} depends on rate of wave growth and is estimated as $\alpha_{ss}=0.67$ for the case of constant energy flux $\langle S_{in}+S_{diss}\rangle=\mathrm{const}$ in extensive simulations of fetch-limited wave growth [*Gagnaire-Renou et al*., 2011]. This parameter varies slightly in a wide range of sea state conditions from young wind sea when wave age parameter $C_p/U_{10} \ll 1$ up to premature sea $C_p/U_{10} \approx 1$ (C_p is phase velocity of spectral peak component).

The asymptotic relationship (4) can be considered as a counterpart of the classic Kolmogorov-Zakharov relationships in the theory of wave turbulence [see *Zakharov*, 2010, and references herein]. Its relevance to observations of wind sea has been demonstrated for more than 20 experiments carried out for last 50 years [*Badulin et al*., 2007]. Being related with results by *Hasselmann et al*. [1976]; *Toba* [1972]; and *Zakharov and Zaslavsky* [1983], equation (4) gives consistent vision of wave growth as a continuity of stages of wave growth in terms of fluxes of wave momentum, energy and action [see *Gagnaire-Renou et al*., 2011, Table 1]. For power-like one-parametric dependencies (2), constant flux of wave momentum in (4) gives immediately $R=5/3$ by *Hasselmann et al*. [1976], constant energy flux—$R=3/2$ and law by *Toba* [1972], and constant flux of wave action provides slower dependence with $R=4/3$ for premature wind sea [*Zakharov and Zaslavsky*, 1983]. This is why all the mentioned dependencies have been put in Figure 1 as an empirico-theoretical background of wave period models.

Models by *Hwang et al*. [1998] and *Zhao and Toba* [2003] can be interpreted as ones relying upon a particular case by *Hasselmann et al*. [1976] of more general physical scheme (4). It is easy to see that this case corresponds to net input function

$$\langle S_{in}+S_{diss}\rangle \sim C_p U_{10}^2, \quad (5)$$

in (4), i.e., to the case of constant wave momentum flux or constant wind stress [*Gagnaire-Renou et al*., 2011, case 1 of Table 1].

Our explicit reference to wind speed scaling (1) in (5) is just a tribute to tradition of one-point measurements. Note, that net wave input in (4) can be estimated directly as variations of energy of wave field in space and time

$$\langle dE/dt\rangle=\langle S_{in}+S_{diss}\rangle$$

with no reference to wind or other mechanisms of wave evolution. This trivial note allows to propose a new method of estimating wave periods from altimeter data.

Assuming measurement setup to be stationary ($\partial E/\partial t \equiv 0$ as far as satellite travels at high speed and is able to fix variations in space only) and estimating $dE/dt=\mathbf{C_g}\nabla E$ ($\mathbf{C_g}$ is group velocity of spectral peak component, $|\mathbf{C_g}|=0.5C_p$ for deep water waves) one can convert the physical law (4) to a simple formula for wave period as function of wave height and projection of the height gradient onto the group velocity $\mathbf{C_g}$ of the spectral peak harmonics

$$T_p=2^{1/5}\pi\alpha_{ss}^{-3/10}\sqrt{H_s/g}|\nabla_{\mathbf{p}}H_s|^{-1/10} \quad (6)$$

where $\nabla_{\mathbf{p}}$ denotes directional derivative along the vector $\mathbf{C_g}$.

This is a basic formula of the new physical model of wave periods from altimeter data. This formula does not contain any empirical coefficients or tuning parameters. Self-similarity parameter α_{ss} is the only quantity that, formally, can require tuning as far as it depends on rate of wave growth. At the same time, α_{ss} enters (6) in power ($-3/10$) that reduces essentially the effect of its variations. As simulations show [see *Gagnaire-Renou et al*., 2011, Figures 8 and 9], α_{ss} varies within 15% of its magnitude in the range of wave steepness $0.04-0.12$, i.e., from the fully developed wind sea of *Pierson and Moskowitz* [1964] to very steep storm waves. Thus, generally, its tuning can be regarded as "an excessive accuracy" of the method.

In this work, we accept the following definition of wave steepness in terms of total energy E or significant wave height $H_s=4E^{1/2}$ and spectral peak wave number $\mathbf{k}_p$ (period T_p)

$$\mu=E^{1/2}|\mathbf{k}_p|=\frac{\pi^2 H_s}{gT_p^2}. \quad (7)$$

For this extremely important parameter of wave field, one has a remarkably concise reformulation of our key formula (6)

$$\mu=\frac{\alpha_{ss}^{3/5}}{2^{2/5}}|\nabla_{\mathbf{p}} H_s|^{1/5} \approx 0.596|\nabla_{\mathbf{p}} H_s|^{1/5} \quad (8)$$

In (8) and below, we fix $\alpha_{ss}=0.67$ following results by *Gagnaire-Renou et al.* [2011]. Thus, wave steepness depends on spatial gradient of wave height $\nabla_{\mathbf{p}} H_s$ only and can be estimated from altimeter measurements in two consecutive points. It resembles the widely used method of estimates of geostrophic currents from variations of sea surface height relative to the equipotential surface. In contrast to the inherently linear geostrophic relationship our dependence (8) is heavily nonlinear that implies specific limitations on accuracy of measurements and physical parameters of wave field.

Taking $\mu=0.04$ (close to the *Pierson and Moskowitz* [1964] case of fully developed sea) and along-track distance 6 km between two consecutive measurements of H_s, one has variation $\delta H_s \approx 0.008$ m, i.e., below the today accuracy of satellite altimeters. It does not mean that smooth (e.g., swell) sea is beyond the proposed method. Steepness (and periods) of such waves can be estimated for larger distances between consecutive measurements. Evidently, it implies additional physical conditions for validity of the asymptotic model (4). The ways to fit parameters of measurements and data processing for the physical criteria are presented in the Discussion section. Thanks to power 1/5 in (8), performance of the method grows up rapidly with wave steepness. For steep storm waves, $\mu=0.1$ (8) gives $\delta H_s \approx 0.8$ m that makes no problem for the today space-borne altimeters.

Extremely useful and prospective feature of (6) is low exponent (1/10) of dependence on spatial gradient of wave height. The gain of the fact is twofold. First, it reduces dramatically errors in estimating the derivative itself. It works like a smoothing filter for the inherently noisy significant wave height records (1 Hz data). We leave the discussion of this problem for future studies. In the present work, the effect of the noise leads to an additional scattering of estimates of T_p and μ but does not disaffirm relevance of the new method.

Second, the low exponent (1/10) is just to the point of satellite measurements where the derivative can be estimated along the track only. Uncertainty of the direction of wave propagation relatively to the satellite track appears to be not critical: even for the mismatch of the directions 80°, the resulting error of T_p will be less than 20% ($(\cos 80^\circ)^{1/10} \approx 0.84$)!!!

It should be noted that H_s and $\nabla_{\mathbf{p}} H_s$ can be considered as independent physical parameters. On the contrary, the pair (H_s–σ_0) of previously proposed models shows rather high correlation [see *Mackay et al.*, 2008, comments to Figure 4], which is a problem for the optimal choice of parameterizations $T(H_s, \sigma_0)$.

Finalizing this section, one should stress the most important benefit of formulas (6) and (8): they do not contain the backscatter coefficient σ_0. Weather conditions, water properties (e.g., salinity and surface pollution), and presence of broken ice can essentially affect σ_0 while H_s remains incomparably more robust. The proposed method is free of these problems as soon as it operates with measurements of the most reliable value—significant wave height H_s. Evidently, it cannot be considered as a perfect solution. The key question is validity of the asymptotic theory this method is based on. This question has many faces when choosing measurement setup, method, and parameters of data processing and a physical model for interpretation of results.

3. Verification of the New Method

3.1. New Method in a Case Study

Comparison with previously proposed methods is a straightforward way to validate our physical model. In this paper, we refer to four works. The first one by *Gommenginger et al.* [2003] has been proposed as an empirical model based on the geometric optics approximation. It predicts a power-law dependence on wave height close to $H_s^{1/2}$ quite similar to our model (6). Calibration for the collocated TOPEX altimeter and NDBC buoys data sets showed good performance of this model for zero-crossing T_z and mean T_m periods which are more stable as integral characteristics of wave field in contrast to peak period T_p derived from measured frequency spectra.

Everywhere in this paper, we refer to a characteristic period T that can be different in the discussed algorithms. Definitions of zero-upcrossing period T_z by *Gommenginger et al*. [2003] and *Mackay et al*. [2008] are equivalent to mean over spectrum period T_a by *Hwang et al*. [1998]. *Quilfen et al*. [2004] refer to the mean period T_m as defined by *Mackay et al*. [2008]. The new model operates with the spectral peak period T_p as the most convenient one in theoretical studies. This difference is not of great importance when we show our first results and ability itself of the new method to reproduce adequately the wave periods. Within the asymptotic approach presented above and the resulting self-similarity of wave spectra, all the periods differ from each other by fixed coefficients. *Hwang et al*. [1998] introduced the constant ratio $T_p/T_z=1.29\pm0.14$ that will be used below. In other works, the self-similarity of wave spectra is not assumed and the corresponding ratios depend essentially on wave scales. Sometimes, it leads to somewhat strange results when, for example, a mean period appears to be higher than the peak one [e.g., *Gommenginger et al*., 2003, Figure 1 and Table 1].

Quilfen et al. [2004] used neural network approach for their model of wave periods. H_s and σ_0 of altimeters were taken to fit wave heights and wave periods measured by a set of wave buoys. The most recent model of wave periods by *Mackay et al*. [2008] refers to the richest collocated data set of NDBC buoys and six satellite missions. Fitting coefficients in the resulting parametric formulas are different for these missions that reflects, in particular, the mentioned problem of "inconvenient" backscatter coefficient σ_0.

Work by *Hwang et al*. [1998] represents the second group of wave period models as specified in the previous section. It emphasizes the consistency of the models with empirico-theoretical dependencies $\tilde{H}(\tilde{T})$ in terms of the wind speed scaling (1) [*Toba*, 1972; *Hasselmann et al*., 1976; *Zakharov and Zaslavsky*, 1983]. As we show below, this emphasis, in fact, leads to severe restrictions of wave dynamics derived from the altimeter data.

Data of the European Space Agency (ESA) initiative Globwave have been used to compare our method with approaches mentioned above. A coordinate box $7^\circ\times7^\circ$ ($33^\circ-40^\circ$N, $71^\circ-78^\circ$W) near the US coast has been chosen as the study area. One of the argument for this choice is a great number of visual observations in this area. Voluntary observed ship (VOS) collection provides 600,939 telegrams for the period 2002–2012. A total of 246,129 cases have been classified as pure wind sea and 1348 as pure swell (V. Grigorieva, personal communication, 2013, see also data archive at http://www1.ncdc.noaa.gov/pub/data/icoads2.5). Almost 60% observations (351,939) with nonzero heights have been reported as crossing seas (both wind-driven waves and swell). Thus, the study area can be considered as a typical one with a wide ranging variety of parameters of wind and wave field. The ENVISAT mission (2002–2012) tracks in this area are shown in Figure 2.

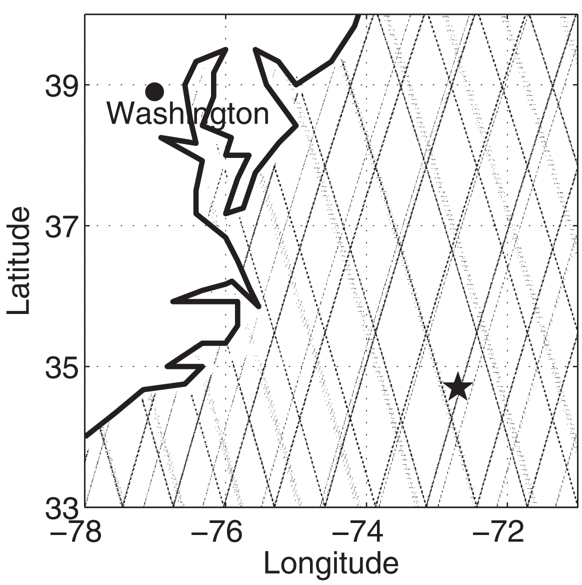

Figure 2. ENVISAT tracks (2002–2012) in the study area. NDBC buoy East Hatteras (41001) is shown by symbol ⋆.

Star symbol in Figure 2 shows location of NDBC buoy East Hatteras (41001). This buoy is one of NDBC network used in the collocated data set of the most recent model of wave periods by *Mackay et al*. [2008]. Our first step study in coordinate box $1^\circ\times1^\circ$ centered at this buoy showed no essential difference with features of sea waves in the

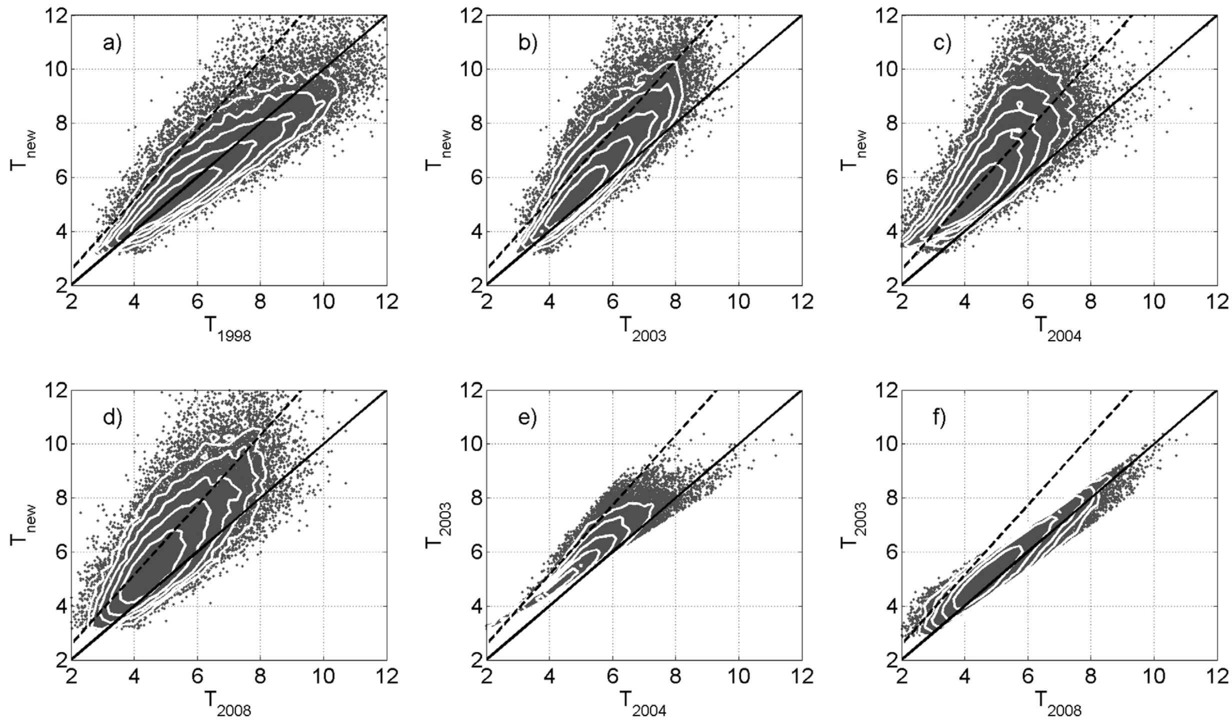

Figure 3. Comparison of wave periods calculated with different models for the study area (map in Figure 2) in the range of altimeter winds $5 < U_{10} < 20$ m/s derived using *Abdalla* [2007]. Totally 45,408 counts are used in each panel, contour lines are given with decrement 1/2 (i.e., $1/2, 1/4, 1/8\ldots$) from the maximal density of counts. Dashed line tangent 1.29 corresponds to ratio of spectral peak T_p and mean period T_m [*Mackay et al*., 2008, definitions and comments; *Hwang et al*., 1998]. (a–d) New model (ordinate) versus previously proposed algorithms (abscise) by *Hwang et al*. [1998]; *Gommenginger et al*. [2003]; *Quilfen et al*. [2004]; *Mackay et al*. [2008]; (e and f) *Gommenginger et al*. [2003] (ordinate) versus *Quilfen et al*. [2004] and *Mackay et al*. [2008].

whole study area. It can be seen as an argument for extending the parametric models mentioned above for the whole area.

All the quality flags of the L2P altimeter data have been taken into account in order to have clean data sets. Wave heights below 10 cm have been ignored. Two data subsets have been considered: the first one for altimeter wind range $5 < U_{10} < 20$ m/s (45,408 counts), the second one—for $0 < U_{10} < 50$ m/s (60,096). The wind estimates followed the algorithm by *Abdalla* [2007].

The spatial gradients for the new method (6) were estimated by simple differences between consecutive points of the clean tracks. Globwave data use default averaging of altimeter pulses over 1 s, i.e., slightly <6 km distance between consecutive counts. Skipping defective counts gives longer distances. The effect of these skips is twofold. First, it can underestimate spatial gradients and, hence, overestimate periods in (6). Second, the underestimated gradients can have a real physical meaning when they are responsible for smooth seas with small steepness μ (8). Totally, these cases give $<20\%$ of the clean data sets.

Comparison of wave periods calculated with different models is given in Figure 3. Contours in scatter plots are log-spaced with decrement 1/2. In addition to the diagonal line in Figure 3 that shows exact coincidence, we add a line with tangent 1.29 as an average ratio of the peak period T_p and the mean one [*Hwang et al*., 1998].

The new model estimates are close to results by *Hwang et al*. [1998] (Figure 3a) in spite of the fact that estimated periods are different: mean-over-spectrum period T_z in *Hwang et al*. [1998] and spectral peak one T_p of the new algorithm.

Other methods (Figures 3b–3d) show better correspondence to definitions of zero-upcrossing period T_z [*Gommenginger et al*., 2003; *Mackay et al*., 2008] and mean one T_m [*Quilfen et al*., 2004]. Comparison between previously proposed models (Figures 3e and 3f) gives lower variance of estimates but scattering of points is still high on the periphery of distributions.

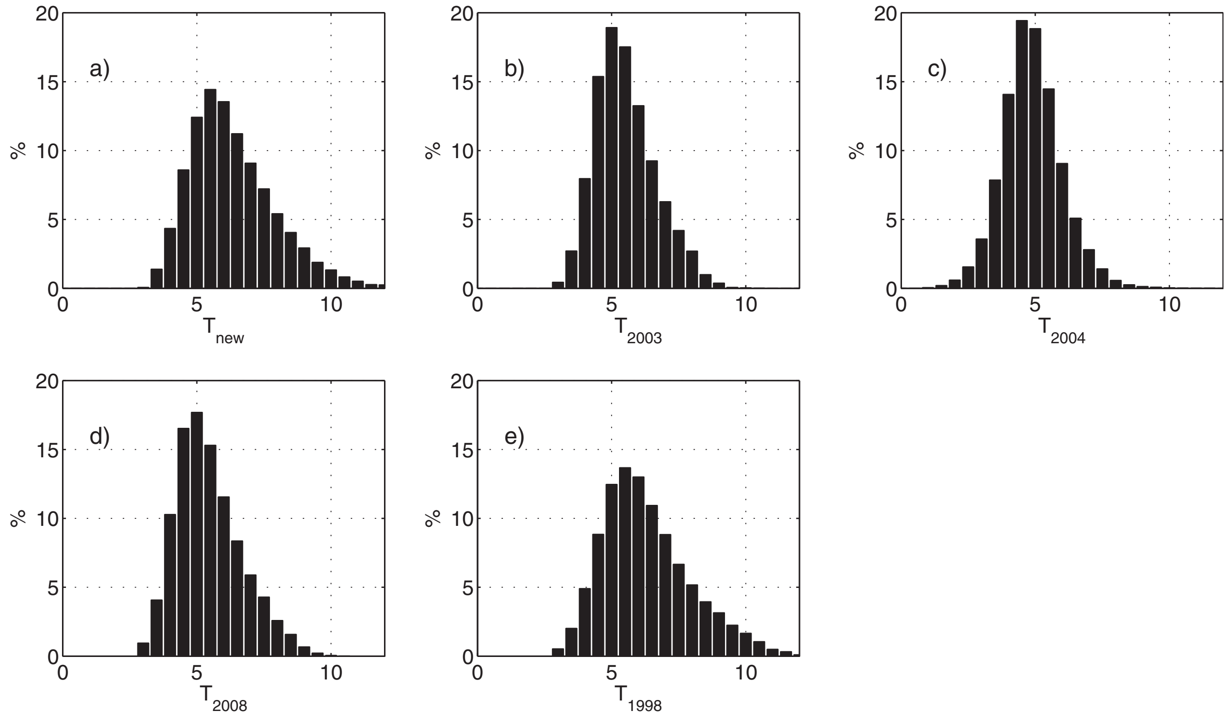

Figure 4. Distribution of wave periods for the study area. Data for altimeter winds $5 < U_{10} < 20$ m/s (45,408 counts) and bins 0.5 s are presented. (a) new method; (b) *Gommenginger et al*. [2003]; (c) *Quilfen et al*. [2004]; (d) *Mackay et al*. [2008]; and (e) *Hwang et al*. [1998].

Tendency to overestimate wave periods and high scattering in Figures 3a–3d are expectable for the new method. First, the vector derivative in (6) can be calculated along the satellite track only, i.e., it is generally underestimated and, correspondingly, wave period in (6) is overestimated, though low exponent 1/10 mitigates this effect. Adequacy of the along-track processing can be evaluated in further study by using data of match-up points where both components of spatial variation of wave height H_s can be found accurately. As a glance at the problem, we analyzed scatter diagrams for descending and ascending tracks separately. No visible difference has been found.

Inherently noisy altimeter data (even with 1 s averaging) can explain high variance in scatter plots in Figures 3a–3d. Again, effect of low exponent 1/10 attenuates this problem. The new algorithm (6) appears to be robust for the imperfect data.

One more evidence of consistency of the new method with previously proposed approaches is presented in Figure 4 in the form of probability distributions of estimated periods. The distribution for the new method (Figure 4a) is quite close to the one by *Hwang et al*. [1998] (Figure 4e), while three other methods show their resemblance with more peaked shapes and slightly lower periods of the distributions maxima (Figures 4b–4d). All the histograms are qualitatively consistent with distributions presented by [*Quilfen et al*., 2004, their Figure 2] for collocated data sets.

The case study showed rather good agreement of the new method with its counterparts. However, in no circumstances should this result be regarded as an indication that the different models are equivalent. Quantitative agreement in terms of wave period T only constitutes a superficial analysis of the problem. Conceptually, the models use different physics. These differences should be realized for better understanding of their limitations and applicability.

3.2. New Method and Physics of Sea Waves

The result of the above case study yields more than a simple quantitative agreement (even with high scattering of estimates). First, in contrast to its counterparts, the new method does not contain any empirical parameters. Second, it operates with spatial gradient of wave height in contrast to conventional models that rely upon backscatter coefficient σ_0. This reflects the fundamental feature of the new method based on physics of nonlinear interactions in a random wave field. The key physical mechanisms of inverse [*Zakharov and Zaslavsky*, 1983] and direct cascades [*Zakharov and Filonenko*, 1966] govern the balance of energy content and flux of energy coming to/from waves (4) very similar to the balance of strong hydrodynamic turbulence. Alternatively, reference to σ_0 implies complex physics that links sea state on scales of few or tens centimeters to characteristics of much longer wind waves of tens and hundreds meters.

In order to show this difference of physical backgrounds, consider the new formula (6) and algorithm by *Gommenginger et al*. [2003]. The latter one parameterizes wave period as function of a combination of wave height H_s and σ_{00} (backscatter coefficient in linear non-dB form) as follows

$$T_z \sim (H_s^2 \sigma_{00})^b \tag{9}$$

where exponent b is close to 1/4. Thus, dependencies on H_s in (6) and (9) are nearly identical and effects of wave height gradient in the new model can be related to one of backscatter coefficient σ_{00} by *Gommenginger et al*. [2003] straightforwardly. Results of the analysis are presented in Figure 5 for the range of altimeter wind speed $0 < U_{10} < 50$ m/s estimated from algorithm by *Abdalla* [2007] (totally 60,096 counts in the study area). Isolines of count density are given with decrement 1/2 from maximal value and formal limitation of each method is shown by dashed lines. Vertical lines s_5 and s_{20} correspond to wind speed estimates 5 and 20 m/s, i.e., outline conventional range where parameterizations of wind speed and wave periods are well supported by in situ measurements. Horizontal line represents value of $|\nabla H_s|^{-1/10}$ for wave steepness μ=0.04 in (8). Limitations of the backscatter coefficient σ_{00} gives about 30% of variance of wave period due to factor $\sigma_{00}^{1/4}$. Similar variations are provided by spatial gradient from the reference value M in Figure 5 down to minimal observed values. As it is seen in Figure 5, there are many points above $|\nabla H_s|^{-1/10}=M$. The corresponding low magnitudes of the spatial gradient $|\nabla H_s|$ are provided by estimates over longer distances than standard one of 1 s altimeter counts (about 6 km) when skipping low quality data. "Extra-low" magnitudes of spatial gradient are evidenced by "strips" in Figure 5. In some cases, the low spatial gradients can be related to low wave steepness, e.g., to swell conditions.

The most striking feature of Figure 5 is absence of visible correlation between two quantities of the new physical model $|\nabla H_s|^{-1/10}$ and $\sigma_{00}^{1/4}$ of empirical model by *Gommenginger et al*. [2003]: different physical backgrounds lead to independent results. Validity of the new physical model and shortcomings of the empirical approach for retrieval wave periods should be analyzed thoroughly for better understanding this result and underlying physics of sea waves.

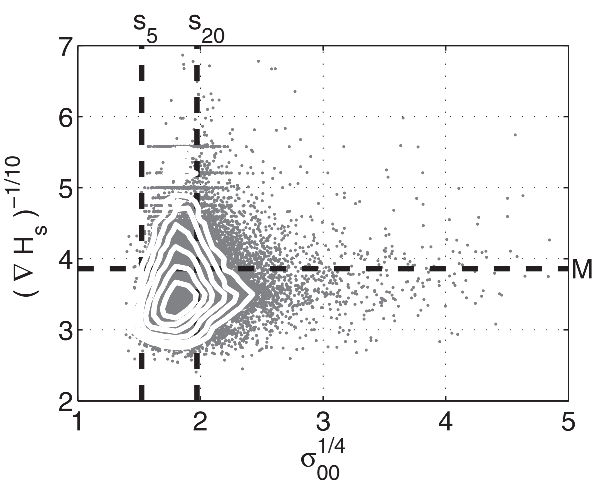

Figure 5. Dependence of measured gradients of wave height ∇H_s on backscatter coefficient σ_{00} in linear (non-dB) form. Totally, 60,096 counts are taken for the altimeter winds $0 < U_{10} < 50$ m/s [*Abdalla*, 2007]. Isolines are plotted with logarithmic decrement 1/2 from maximal density of counts. Vertical dashed lines at s_5, s_{20} outline the wind speed range $5 < U_{10} < 20$ m/s, horizontal one at $|\nabla H_s|^{-1/10}=M$ corresponds to wave steepness μ=0.04.

Analysis of wave growth in terms of conventional wind speed scaling [*Kitaigorodskii*, 1962] brings us back to motivation of the present work (see Figure 1). Figure 6 shows growth curves $\tilde{H}(\tilde{T})$ derived from different models of wave period for the data set presented in Figure 3. Hard lines fix a reference theoretical-empirical Toba's law $\tilde{H}_s \sim \tilde{T}^{3/2}$. One can see striking difference of the models in the context of this empirical law.

Method by *Hwang et al*. [1998] (Figure 6b) fixed the dependence $\tilde{H}_s \sim \tilde{T}^{5/3}$ that has been first considered by *Hasselmann et al*. [1976] and then discussed in many papers [e.g., *Resio and Perrie*, 1989; *Resio et al*., 2004; *Hwang and Wang*, 2004]. It should be stressed that it is not correct to treat the law 5/3 ($\tilde{H}_s \sim \tilde{T}^{5/3}$) as one "in excellent agreement with *Toba* [1978] power law relation" as mentioned by [*Hwang et al*., 1998]. The law 5/3 corresponds to a

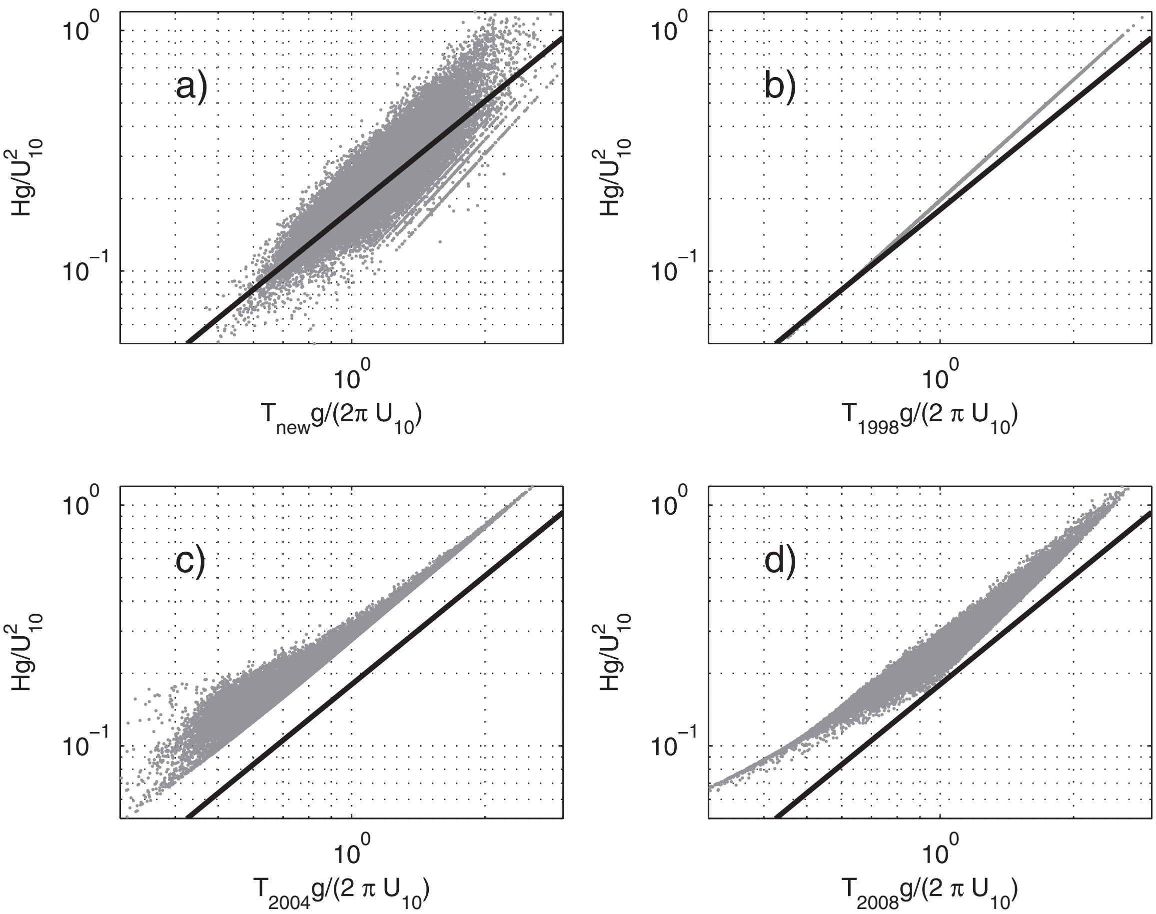

Figure 6. Dependence of nondimensional wave height on nondimensional wave period (wave age) for 45,408 measurements in the study area ($5 < U_{10} < 20$ m/s, see Figures 3 and 5): (a) new approach, algorithms by (b) *Hwang et al*. [1998], (c) *Quilfen et al*. [2004], and (d) *Mackay et al*. [2008]. Hard line corresponds to the *Toba* [1972] law $\tilde{H} \sim \tilde{T}^{3/2}$.

reference case of constant production (constant flux) of wave momentum and describes the evolution of relatively young wind waves [*Resio and Perrie*, 1989; *Resio et al*., 2004; *Gagnaire-Renou et al*., 2011; *Badulin and Grigorieva*, 2012; *Zakharov et al*., 2012]. The *Toba* [1972] law 3/2 ($\tilde{H}_s \sim \tilde{T}^{3/2}$) represents different physics of wind-sea interaction when production (flux) of wave energy remains constant.

Two other empirical dependencies in Figure 6 (bottom) are not so flat in reflection of wave growth but they definitely "empoverish" wave dynamics as compared with the new model (Figure 6a). This impoverishment is more pronounced for longer waves. Additionally, these dependencies do not follow conventional scheme of wave evolution: young waves should be growing faster than old ones. The new physical model looks "more natural" providing more freedom in terms of the conventional wind speed scaling and, at the same time, respecting weakly turbulent physics of the asymptotic relationship (4).

Previous consideration in terms of wind speed scaling (nondimensional $\tilde{H}_s$ and $\tilde{T}$) follows conventional understanding of wind-wave growth as one rigidly linked to wind forcing. Wider (currently, unconventional) view requires a correct account of the effect of nonlinear transfer in which wave properties are not related directly to external forcing. It is wave steepness μ that is well known as nondimensional parameter responsible for wave nonlinearity (7). In our discussion, μ appears to be an extremely indicative physical parameter. Figure 7 shows distributions of wave steepness calculated by different methods. Distributions of the top row look qualitatively similar. The new method and the one by *Gommenginger et al*. [2003] have maxima at close values of period. The third one for method by *Quilfen et al*. [2004] shows higher steepness. It can be made very close to one of the new method by reduction μ by factor 1.35. The latter is consistent with results of direct comparison of wave periods in Figure 3c where *Quilfen et al*. [2004] underestimate systematically the new periods T_{new} by 10–15%.

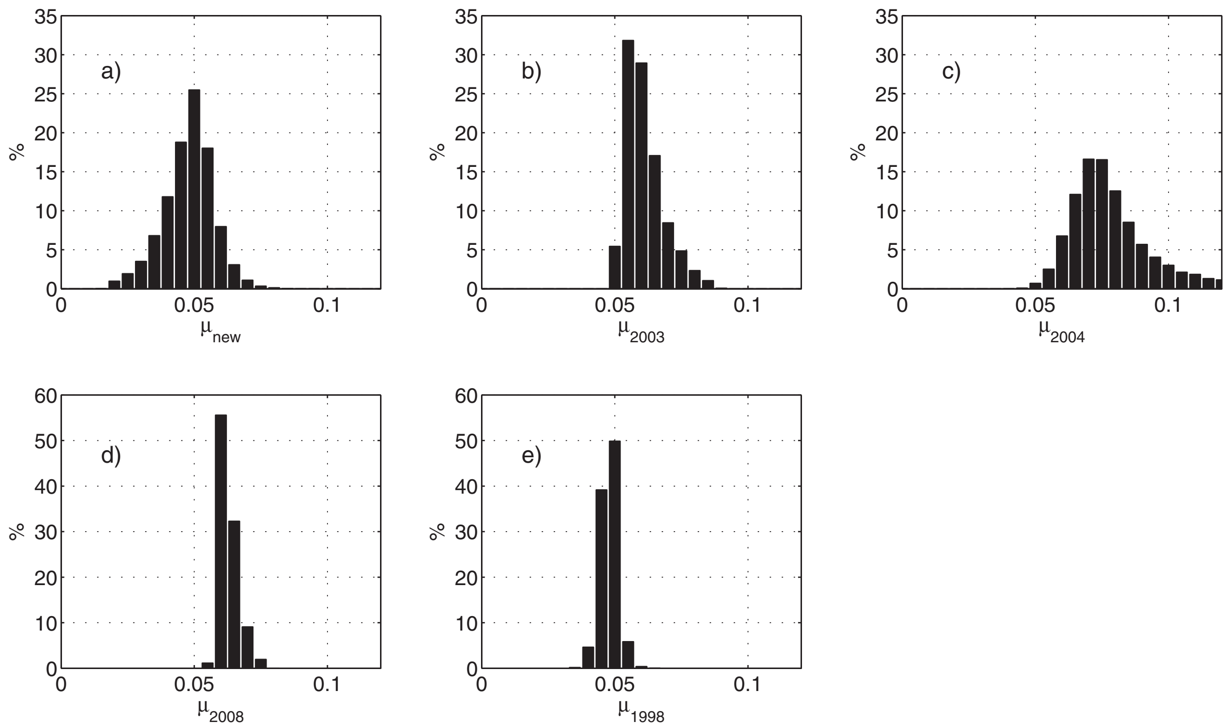

Figure 7. Distribution of wave steepness $\mu = \pi^2 H_s/(gT^2)$ for the study area. (a) new method; (b) *Gommenginger et al.* [2003]; (c) *Quilfen et al.* [2004]; (d) *Mackay et al.* [2008]; and (e) *Hwang et al.* [1998].

Figure 7 (bottom) shows two times higher peaks. Distributions collapse to a very narrow range of steepness magnitudes quite similarly to distributions of nondimensional wave heights and periods in Figure 6. Thus, impoverishment of wave dynamics by *Hwang et al.* [1998] and *Mackay et al.* [2008] manifests itself both in terms of conventional wind speed (forcing) and in terms of nonlinearity parameter—wave steepness.

Method by *Hwang et al.* [1998] requires additional comments. Its parametric formulas give immediately

$$\mu = 0.0092 \tilde{T}^{-1/3}. \tag{10}$$

Low exponent in (10) limits variations of wave steepness. For close but qualitatively different laws of wave growth by *Toba* [1972] and *Zakharov and Zaslavsky* [1983], one can get exponents $(-1/2)$ and $(-2/3)$ correspondingly and, hence, "more freedom" for wave steepness. This simple comment points out once more the deficiency of empirical parameterizations: they can corrupt essential features of wave dynamics.

4. Discussion: The Physical Model Versus Parametric Approaches

This paper introduces the new model of wave periods from altimeter data as a physical concept rather than a regular tool for monitoring sea state. This is why its validation is performed as a comparison with available empirical parameterizations but not as conventional analysis of collocated data sets of altimeter and in situ measurements [e.g., *Hwang et al.*, 1998; *Gommenginger et al.*, 2003; *Quilfen et al.*, 2004; *Mackay et al.*, 2008]. Even for a relatively small area, a single mission and maximal quality control a larger data set is available (45,408 counts) than the one commonly used for building empirical dependencies [cf. *Mackay et al.*, 2008, Table 2]. These data cover a wide range of sea state conditions as evidenced by the visual observations in the study area and by estimates of wave periods themselves.

Default parameters of the Globwave altimeter data have been used. The most critical parameter of the new method—time interval of altimeter counts—was fixed at 1 s. We used the coarsest way to quantify wave height gradient in (6) by simple differences between two counts. Even with these plain means, the new method agrees well with the results of the previously proposed empirical algorithms.

In contrast to its empirical counterparts, the new model does not require "inner" calibration: all the coefficients are given by the asymptotic theory of waves. The calibration becomes an "outer" problem: measurements of wave height H_s and its spatial gradient ∇H_s should be adequate to the theoretical background. In particular, measurements in two consecutive points should respect scaling of the asymptotic relationship (4) this model is based on. More specifically, consecutive measurements have to skip transitional processes in order to fix an instant state and to trace variations of the state as external forcing (net flux). In other words, characteristic scale of nonlinear relaxation $L_{nonlinear}$, scale of wave field variations due to external forcing $L_{forcing}$ and distance between consecutive counts ΔL have to be ranged as follows

$$L_{nonlinear} \ll \Delta L \ll L_{forcing}. \tag{11}$$

The scale of wave forcing $L_{forcing}$ relies upon a small parameter, the ratio of air and water densities $\delta = \rho_a/\rho_w \approx 1/800$, i.e., $L_{forcing} \propto \delta^{-1}\lambda$ (λ is a characteristic wavelength). On the other hand, $L_{nonlinear}$ is determined by wave steepness μ

$$L^{-1}_{nonlinear} \approx C_{nl}\mu^4\lambda. \tag{12}$$

A feature of (12) is the big coefficient C_{nl} that depends, in particular, on spectra anisotropy [*Zakharov*, 2010; *Zakharov and Badulin*, 2011]. An estimate of its minimal magnitude in isotropic case gives $C_{nl}=22.5\pi \approx 71$. These simple estimates justify the basic physical assumption of our asymptotic theory and the new method that exploits this theory: scales of nonlinear relaxation of sea waves are generally much shorter than the ones of wind forcing. *Zakharov and Badulin* [2011, Figure 3] showed a prominent gap between time scales of nonlinear relaxation and external forcing fairly well.

The success of the new method, in our opinion, results from our happy chance when default intervals of altimeter counts 1 s correspond to the condition (11) of the compromise between scales of nonlinear wave-wave interactions, wind forcing, and interval of measurements. In fact, the default 1 s implies one more physical scale of the problem: averaging of a number of pulses (1795 per 1 s for ENVISAT) provides sufficiently large footprint at sea surface for assessment statistical properties of wave field (significant wave height H_s). In our method, the corresponding scale L_f can differ from ΔL—distance between two consecutive footprints. We mentioned this option in section 2. when performance of the new method for measuring swell was discussed. Evidently, both scales of footprint L_f and interval between two consecutive measurements ΔL can be changed to fit particular physical conditions better. We consider this option to be the nearest prospect of development of the new model.

Note that scaling of different processes (e.g., equation (11)) is always implied (but, unfortunately, is not usually respected) in all physical measurements. For example, length of ocean buoy record has to be sufficiently long (at the least, much longer than the wave period) for the wave field to be considered as stationary one and, hence, an estimated wave period can be regarded as a representative mean characteristic of the wave field. On the other hand, the length of the record should be small as compared to scales of variability that we are trying to fix in our measurements. Generally, this issue is beyond discussion: parameters of measurements are usually set up at default values. Say, standard 20–40 min records of ocean buoys are considered as optimal and universal choice. For the new method, as we see, the problem of correct physical scaling in the form (11) has no such trivial solution: its efficiency is determined by physically consistent choice of parameters of data processing.

The fact itself of consistency with previously proposed algorithms is twofold. First, it can be regarded as a verification of a new tool of wind wave studies. Second, success of our model gives assurance to its physical background—theory of weak turbulence and its important particular result (4). The success is really surprising when we take "purely theoretical" value of the self-similarity parameter α_{ss} and do not use any tuning procedure in our algorithm.

Once again, we have to emphasize features of using wave height gradient ∇H_s as a physical parameter of the model. "Unrealistically" low exponent (1/10) makes the model extremely robust both to errors of measurements of H_s and to the effect of uncertainty of wave direction. Note this is true when we solve the problem of wave period retrieval. In other cases, the proposed approach may be "too coarse". For example, one can try to extend this approach to assessment of the horizontal fluxes of wave energy directly without empirical parameterizations as soon as our model operates inherently with wave field gradients (P. A. Hwang, unpublished manuscript, 2008). But these fluxes are proportional to the gradients and along-track measurements become insufficient for this important problem solution. Necessary information can be derived for the match-up points of altimeter measurements where two components of wave field variations become available. Estimated energy fluxes are heavily nonlinear functions of wave parameters and the joint analysis of different models of wave periods will be extremely informative both in quantifying these fluxes and in estimating performance of each approach. This problem is also seen as a challenging one.

Our Figures 6 and 7 evidence conceptual problem of empirical models in description wave dynamics: all the models comprise severe restrictions on wave evolution patterns both in conventional terms of wind speed scaling and in terms of wave steepness. Wave steepness being quadratic function of wave period is more sensitive to the quality of a model in use. Therefore, our first comparison of different methods requires further thorough analysis.

Acknowledgments
The author is thankful to Viktoria Grigorieva for cooperation and fruitful discussion of the results. The author expresses his gratitude to Paul Hwang for his interest in this work and sharing his expertise in wave studies from space. He also appreciates efforts of two reviewers of this work. Their comments and suggestions were extremely useful for the work revision. The research was conducted under Russian Foundation for Basic Research 11-05-01114-a and 14-05-00479, grants of the Government of the Russian Federation 11.G34.31.0035, 11.G34.31.0078, and ONR grant N000141010991. The author gratefully acknowledges continuing support of these foundations.

References

Abdalla, S. (2007), Ku-band radar altimeter surface wind speed algorithm, in paper presented at Envisat Symposium 2007, Eur. Space Agency, Montreux, Switzerland, 23–27 April.

Abdalla, S., and L. Cavaleri (2002), Effects of wind variability and variable air density on wave modeling, *J. Geophys. Res.*, *107*(C7), 3080, doi:10.1029/2000JC000639.

Badulin, S. I., and V. G. Grigorieva (2012), On discriminating swell and wind-driven seas in voluntary observing ship data, *J. Geophys. Res.*, *117*, C00J29, doi:10.1029/2012JC007937.

Badulin, S. I., A. N. Pushkarev, D. Resio, and V. E. Zakharov (2005), Self-similarity of wind-driven seas, *Nonlinear Processes Geophys.*, *12*, 891–946.

Badulin, S. I., A. V. Babanin, D. Resio, and V. Zakharov (2007), Weakly turbulent laws of wind-wave growth, *J. Fluid Mech.*, *591*, 339–378.

Badulin, S. I., A. V. Babanin, D. Resio, and V. Zakharov (2008), Numerical verification of weakly turbulent law of wind wave growth, in *IUTAM Symposium on Hamiltonian Dynamics, Vortex Structures, Turbulence. Proceedings of the IUTAM Symposium held in Moscow, 25–30 August, 2006, IUTAM Bookseries*, vol. *6*, edited by A. V. Borisov et al., pp. 175–190, Springer, Dordrecht.

Brown, G., H. Stanley, and N. Roy (1981), The wind-speed measurement capability of spaceborne radar altimeters, *IEEE J. Oceanic Eng.*, *6*(2), 59–63.

Cavaleri, L., J.-H. G. M. Alves, F. Ardhuin, A. Babanin, M. Banner, K. Belibassakis, et al. (2007), Wave modelling: The state of the art, *Prog. Oceanogr.*, *75*, 603–674.

Chelton, D. B., and P. J. McCabe (1985), A review of satellite altimeter measurement of sea surface wind speed: With a proposed new algorithm, *J. Geophys. Res.*, *90*(3), 4707–4720, doi:10.1029/JC090iC03p04707.

Donelan, M. A., A. V. Babanin, I. R. Young, M. L. Banner, and C. McCormick (2005), Wave follower field measurements of the wind input spectral function: Part I: Measurements and calibrations, *J. Atmos. Oceanic Technol.*, *22*, 799–813.

Gagnaire-Renou, E., M. Benoit, and S. I. Badulin (2011), On weakly turbulent scaling of wind sea in simulations of fetch-limited growth, *J. Fluid Mech.*, *669*, 178–213.

Goldhirsh, J., and E. Dobson (1985), A recommended algorithm for the determination of ocean surface wind speed using a satellite-borne radar altimeter, *Rep. JHU/APL SIR-85-U005*, App. Phys. Lab., Johns Hopkins Univ., Laurel, Md.

Gommenginger, C. P., M. A. Srokosz, P. G. Challenor, and P. D. Cotton (2003), Measuring ocean wave period with satellite altimeters: A simple empirical model, *Geophys. Res. Lett.*, *30*(22), 2150, doi:10.1029/2003GL017743.

Grigorieva, V., S. Badulin, and S. Gulev (2012), Voluntary Observing Ship (VOS) data as an experimental background of wind-sea studies, *Geophys. Res. Abstr.*, *14*, EGU2012–8542.

Hasselmann, K. (1962), On the nonlinear energy transfer in a gravity wave spectrum: Part 1: General theory, *J. Fluid Mech.*, *12*, 481–500.

Hasselmann, K., D. B. Ross, P. Müller, and W. Sell (1976), A parametric wave prediction model, *J. Phys. Oceanogr.*, *6*, 200–228.

Hwang, P. A., and D. W. Wang (2004), Field measurements of duration-limited growth of wind-generated ocean surface waves at young stage of development, *J. Phys. Oceanogr.*, *34*, 2316–2326.

Hwang, P. A., W. J. Teague, G. A. Jacobs, and D. W. Wang (1998), A statistical comparison of wind speed, wave height and wave period derived from satellite altimeters and ocean buoys in the Gulf of Mexico region, *J. Geophys. Res.*, *103*(10), 10,451–10,468.

Hwang, P. A., D. W. Wang, W. J. Teague, G. A. Jacobs, J. Wesson, D. Burrage, and J. Miller (2002), Anatomy of the ocean surface roughness, *NRL Formal Rep. No. NRL/FR/7330-02-10036*, 45 pp.

Hwang, P. A., B. Zhang, J. V. Toporkov, and W. Perrie (2010), Comparison of composite Bragg theory and quad-polarization radar backscatter from RADARSAT-2: With applications to wave breaking and high wind retrieval, *J. Geophys. Res.*, *115*, C08019, doi:10.1029/2009JC005995.

Kitaigorodskii, S. A. (1962), Applications of the theory of similarity to the analysis of wind-generated wave motion as a stochastic process, *Bull. Acad. Sci. USSR, Geophys. Ser., Engl. Transl.*, *N1*, 105–117.

Mackay, E. B. L., C. H. Retzler, P. G. Challenor, and C. P. Gommenginger (2008), A parametric model for ocean wave period from Ku-band altimeter data, *J. Geophys. Res.*, *113*, C03029, doi:10.1029/2007JC004438.

Pierson, W. J., and L. A. Moskowitz (1964), A proposed spectral form for fully developed wind seas based on the similarity theory of S. A. Kitaigorodsii, *J. Geophys. Res.*, *69*, 5181–5190.

Quilfen, Y., B. Chapron, and M. Serre (2004), Calibration/validation of an altimeter wave period model and application to TOPEX/Poseidon and Jason-1 altimeters, *Mar. Geod.*, *27*, 535–549.

Resio, D. T., and W. A. Perrie (1989), Implications of an f^{-4} equilibrium range for wind-generated waves, *J. Phys. Oceanogr.*, *19*, 193–204.

Resio, D. T., C. E. Long, and C. L. Vincent (2004), Equilibrium-range constant in wind-generated wave spectra, *J. Geophys. Res.*, *109*, C01018, doi:1029/2003JC001788.

Toba, Y. (1972), Local balance in the air-sea boundary processes: Part I: On the growth process of wind waves, *J. Oceanogr. Soc. Jpn.*, *28*, 109–121.

Toba, Y. (1978), Stochastic form of the growth of wind waves in a single-parameter representation with physical implementations, *J. Phys. Oceanogr.*, *8*, 494–507.

Vignudelli, S., A. G. Kostianoy, P. Cipollini, and J. Benveniste (Eds.) (2011), *Coastal Altimetry*, Springer, Berlin, doi:10.1007/978-3-642–12796-0.

Young, I. R. (1993), An estimate of the Geosat altimeter wind speed algorithm at high wind speeds, *J. Geophys. Res.*, *98*(C11), 20,275–20,285.

Zakharov, V. E. (2005), Theoretical interpretation of fetch limited wind-driven sea observations, *Nonlinear Processes Geophys.*, *12*, 1011–1020.

Zakharov, V. E. (2010), Energy balance in a wind-driven sea, *Phys. Scr.*, *T142*, 014052, doi:10.1088/0031–8949/2010/T142/014052.

Zakharov, V. E., and S. I. Badulin (2011), On energy balance in wind-driven seas, *Dokl. Earth Sci.*, *440*(2), 1440–1444.

Zakharov, V. E., and N. N. Filonenko (1966), Energy spectrum for stochastic oscillations of the surface of a fluid, *Sov. Phys. Dokl., Engl. Transl.*, *160*, 1292–1295.

Zakharov, V. E., and M. M. Zaslavsky (1983), Dependence of wave parameters on the wind velocity, duration of its action and fetch in the weak-turbulence theory of water waves, *Izv. Atmos. Ocean. Phys., Engl. Transl.*, *19*(4), 300–306.

Zakharov, V. E., D. Resio, and A. N. Pushkarev (2012), New wind input term consistent with experimental, theoretical and numerical considerations, *ArXiv e-prints*, (1212.1069).

Zhao, D., and Y. Toba (2003), A spectral approach for determining altimeter wind speed model functions, *J. Oceanogr.*, *59*, 235–244.

