Wave Breaking in Directional Fields

A. V. BABANIN

Faculty of Engineering and Industrial Sciences, Swinburne University of Technology, Melbourne, Australia

T. WASEDA AND T. KINOSHITA

Department of Ocean Technology, Policy and Environment, University of Tokyo, Tokyo, Japan

A. TOFFOLI

Faculty of Engineering and Industrial Sciences, Swinburne University of Technology, Melbourne, Australia

(Manuscript received 23 February 2010, in final form 26 August 2010)

ABSTRACT

Wave breaking is observed in a laboratory experiment with waves of realistic average steepness and directional spread. It is shown that a modulational-instability mechanism is active in such circumstances and can lead to the breaking.

Experiments were conducted in the directional wave tank of the University of Tokyo, and the mechanically generated wave fields consisted of a primary wave with sidebands in the frequency domain, with continuous directional distribution in the angular domain. Initial steepness of the primary wave and sidebands, as well as the width of directional distributions varied in a broad range to determine the combination of steepness/directional-spread properties that separates modulational-instability breaking from the linear-focusing breaking.

1. Introduction

Wave breaking has routinely been perceived as a phenomenon hard to understand, predict, and describe. Although the limiting steepness for two-dimensional surface waves has been analytically obtained long ago (i.e., Stokes 1880; Michell 1893), this steepness of

$$ak \approx 0.44 \tag{1}$$

was not treated by oceanographers in any consistent way (here a is wave amplitude and k is wavenumber). Some believed it indeed to be the breaking criterion; for instance, an entire set of wave-dissipation theories was built on this criterion (see review of probabilistic theories in Donelan and Yuan 1994). Others argued that such steepness of individual waves is unrealistic for oceanic wave fields whose typical average steepness is

$$ak \sim 0.1, \tag{2}$$

particularly as the waves were, indeed, reported as breaking while having steepness much lower than (1) [e.g., Holthuijsen and Herbers (1986) in the field and Rapp and Melville (1990) in the laboratory]. Search for more plausible geometric, kinematic, and dynamic criteria has continued, partly because a criterion based on steepness alone also seemed to be not sophisticated enough (i.e., observations would reveal that breaking waves often had steepness smaller than those nonbreaking within the same wave train, etc.). A review of this topic is given, for example, by Babanin (2009).

Recently, however, both experimental and theoretical evidence started reinstating the steepness (1) as both realistic and, in fact, a robust criterion for wave breaking. Brown and Jensen (2001), for the breaking achieved by linear focusing, and Babanin et al. (2007), for breaking due to modulational instability, both showed that in their laboratory experiments the waves would break when $Hk/2 \approx 0.44$, where $H = 2a$ is the wave height. In Babanin et al. (2007), where the front and rear troughs were not symmetric, the criterion is based on the rear

Corresponding author address: Alexander Babanin, Swinburne University of Technology, P.O. Box 218, Hawthorn, Melbourne VIC 3122, Australia.
E-mail: ababanin@swin.edu.au

DOI: 10.1175/2010JPO4455.1

steepness: that is, height H is estimated as the vertical distance between the crest and rear trough. The same limiting steepness was revealed in numerical simulations by means of fully nonlinear wave models (Dyachenko and Zakharov 2005; Babanin et al. 2007).

The contradictions between these new results and the earlier measurements mentioned above, however, seem to be rather straightforward to reconcile. Babanin et al. (2010) argued that the breaking onset is an instantaneous, that is, very short-lived, stage of wave evolution to the breaking; thus, both before and after the moment when the water surface starts to collapse, the steepness of the wave to break and the breaker in progress is below that specified by (1). Therefore,so as to detect steepness (1), the very instant of the breaking onset has to be measured rather than any other phase of wave breaking, which is apparently not the case in experiments other than those explicitly dedicated to studying the incipient-breaking point. For example, Brown and Jensen (2001), who used ''essentially the same experimental setup to generate and study breaking waves'' as Rapp and Melville (1990), indeed found that ''wave breaking occurs when the instantaneous local wave steepness $(ka)_{\max}$ exceeds ~0.44,'' whereas Rapp and Melville did not observe such large steepness as mentioned above.

The field experiments, as far as observations of the breaking onset are concerned, suffer from even more serious shortcomings. As discussed in detail in Babanin et al. (2010), the absolute majority of such measurements rely on whitecapping and associated features to detect the breaking, and the waves only produce whitecapping when the breaking is already well in progress. Therefore, most of the field observations deal with the developing and subsiding phases of the breaking process (Liu and Babanin 2004), and steepness of breakers at these phases can be well below the limiting steepness (1) and even below the mean steepness of the wave field (Liu and Babanin 2004; Babanin et al. 2010).

Therefore, while measuring the breaking onset in controlled laboratory conditions is difficult, measuring it in the field is a challenge, given the sporadic and random nature of breaking occurrence. That is, if a wave is already detected as breaking, then it is unknown what its steepness was before. When the steepnesses of nonbreaking waves are measured, they cannot be readily associated with the breaking inception. Limiting steepness of the value of (1), however, is observed in the field. V. A. Dulov (2008, personal communication) monitored a wave starting to break within an array of wave probes, and at the incipient point its rear steepness was $Hk/2 \approx 0.45$. Toffoli et al. (2010a) investigated the probability distribution of the steepness of individual waves, based on two different field datasets and two sets obtained in different directional wave tanks, and found a threshold for the ultimate steepness as

$$\frac{H_{\text{front}}k}{2} = 0.55 \tag{3}$$

for the front steepness and

$$\frac{H_{\text{rear}}k}{2} = 0.45 \tag{4}$$

for the rear steepness. Since the probability function is terminated at the threshold value, that is, does not extend into the higher values of steepness, this result should be interpreted as the ultimate steepness beyond which the directional waves will certainly break.

Both (3) and (4) steepness obtained in the directional wave fields are somewhat higher than the limiting steepness (1) of two-dimensional waves. We see two possible reasons behind this. First, there can be some mechanism of stabilization of wave crests in three-dimensional wave fields leading to a shift of the breaking onset to higher slope values. Second, the ultimate values (3) and (4) may be indicative of transient maximal steepness rather than the breaking onset as such: that is, even if the water-surface collapse starts once steepness (1) is reached, some waves can keep growing for some time and reach higher steepness, up to those of (3) and (4), while already breaking. Once the breaking starts, the wave height will inevitably drop eventually, but both the superposition and modulational-instability scenarios may allow for the transient interim wave growth.

Which of the two reasons mentioned above—or maybe another one or perhaps all of them together—are responsible for the steepness (3) and (4) reached cannot be answered now and is not essential here. As far as the water-surface collapse is concerned, it is not an instability of wave trains or wave superpositions that cause the wave to break. Any physical mechanism that makes two-dimensional waves higher than (1) or directional waves higher than (3) and (4) will result in wave breaking. The same mechanism, if it only leads to a steepness below the critical, will not bring about a breaking.

Here we have very broadly placed such potential mechanisms into two groups: instability mechanisms and superposition mechanisms. However, within these two phenomenological groups further subdivisions are obvious, which will also correspond to different physics. For example, the classical Benjamin–Feir/McLean instability (Zakharov 1966, 1967; Benjamin and Feir 1967; McLean 1982) does result in very tall waves rising within wave groups, provided the average wave steepness is large enough, but once an individual-wave steepness is higher than 0.42, the wave crest instability (also called second-type instability) becomes important (Longuet-Higgins and Dommermuth 1997; Longuet-Higgins and Tanaka 1997). It is not, however, the first or second type of instability that causes the wave to break; it is the ultimate steepness that they help to achieve.

In terms of wave superpositions, these can be subdivided into frequency focusing (e.g., Longuet-Higgins 1974; Rapp and Melville 1990; among many others), amplitude focusing (Donelan 1978, Pierson et al. 1992), and directional focusing (e.g., Fochesato et al. 2007). Investigation of the breaking by means of such superpositions have been a useful tool for decades (for a review see Babanin 2009), but how likely are they in the field? The answer is perhaps not that encouraging.

The linear focusing is a superposition taking place due to different phase speeds of waves with different frequencies. In a wave field with continuous spectrum, these can happen often, but the problem is that waves with different speeds have very different amplitudes. As an approximate rule if the Phillips spectrum of f^{-5} is used as a scale, then, if frequency f is doubled (i.e., phase speed is decreased 2 times), the amplitude a on average will drop by some $\sqrt{32} \approx 5.7$ times. With the typical mean steepness of ocean waves of (2), this would require superposition of a great number of individual waves to make a dominant wave reach steepness (1), which then appears a low-probability event.

The amplitude focusing and directional focusing bring together waves of close frequencies and, thus, amplitudes. Such focusing, therefore, would require fewer waves superposed to achieve the ultimate steepness (1), (3), or (4). The current paper looks into the issue of the directional focusing and demonstrates that it does happen randomly in directional wave fields, but superposition of more than two waves is not observed. It is not impossible of course but it is unlikely, which means that, in order to break, such waves have to be at least $Hk/2 = 0.22$ steep initially. In the field, individual waves with similar steepness are a rare occurrence; hence, their random superposition is again a low-probability event.

If the superposition breaking is possible but unlikely, then the instability-caused breaking appears the feasible mechanism responsible for majority of the field waves breaking. Indeed, in two-dimensional wave trains steepness (2) would be sufficient for waves inevitably starting to break within such trains. In fact, the breaking caused by a linear superposition and that due to the modulational instability bear a variety of signatures and impacts, which allows one to distinguish between them. These start from shrinkage of the wavelength prior to the breaking onset in the case of instability growth and end with different spectral distribution of the breaking-caused dissipation, with many options in between. A great part of breaking features observed in the field point to the modulational-instability cause (see Babanin et al. 2010). There is, however, a serious scientific argument against the nonlinear wave group instability in directional fields, which needs to be investigated and answered before any conclusions about the plausibility of such mechanism are made.

The convincing modulational-instability breaking onset simulations (Dyachenko and Zakharov 2005; Babanin et al. 2007) are strictly two dimensional, and the corresponding measurements (Babanin et al. 2007, 2010) are quasi two dimensional. In the meantime, there are theoretical and experimental evidences (Onorato et al. 2002, 2009a,b; Waseda et al. 2009a,b) that demonstrate the non-Gaussian properties; therefore, the modulational instability in directional fields is impaired or perhaps even suppressed. If so—that is, if the focusing is unlikely and the modulational instability is suppressed in pure conditions (i.e., in absence of shoaling, adverse currents and other physical interactions)—then how do waves in a wave field with average steepness (2) reach the steepness (3) and (4) identified by Toffoli et al. (2010a)?

The current paper presents results of a laboratory experiment in a directional wave tank, which was intended to answer the question whether the modulational instability is still active in directional wave fields with typical angular spreads and typical mean steepness. The steepness is a recognized key property for this instability that is routinely identified by the modulational index (i.e., Benjamin and Feir 1967; Onorato et al. 2002; Janssen 2003; among others),

$$M_I = \frac{ak_0}{\Delta f/f_0}, \tag{5}$$

where k_0 and f_0 are characteristic wavenumber and frequency in a wave train or wave field and Δf is the bandwidth that defined the wave groups in this field. As an approximate rule, for the instability to develop, there should be

$$M_I \gtrsim 1. \tag{6}$$

Babanin et al. (2007, 2010) argued, based on two-dimensional experiments with no initial sidebands imposed, that the bandwidth in (5) is not an independent property and that it is determined by the steepness. Thus, steepness appeared as the singlemost important parameter that defined whether and what kind of instability develops. In directional fields, however, Waseda et al. (2009a,b) demonstrated that for typical Joint North Sea Wave Project (JONSWAP) frequency spectra the modulational instability also depended on the directional spread and was only detected if the inverse integral of this spread was

$$A > 4. \tag{7}$$

Here A is the inverse normalized directional spectral width, defined according to Babanin and Soloviev (1987, 1998a) as

$$A(f)^{-1} = \int_{-\pi}^{\pi} K(f, \varphi)\, d\varphi \tag{8}$$

in which φ is the wave direction and $K(f, \varphi)$ the normalized directional spectrum,

$$K(f, \varphi_{\max}) = 1; \tag{9}$$

that is, higher values of A correspond to narrower directional distributions. Experimental conclusion (7) was also consistent with numerical simulations of Onorato et al. (2002) [for correction of a typo made in the original paper by Onorato et al., see Babanin et al. (2010)].

Thus, Babanin et al. (2010) suggested a directional analog of M_I as the directional modulational index

$$M_{Id} = Aak_0, \tag{10}$$

which is effectively a ratio of steepness and normalized directional bandwidth and would signify whether the modulational instability takes place in the directional wave fields. That is, if the directional spreading broadens, this can be compensated by increasing characteristic steepness, and vice versa.

In a way, this logic is indirectly supported by observations of behavior of the modulational instability at surface currents. Tamura et al. (2008), Janssen and Herbers (2009), and Toffoli et al. (2010b) all showed that this kind of instability is enhanced in the presence of wave currents. Janssen and Herbers (2009) explicitly demonstrated that such activation of the nonlinear group behavior can be caused by the directional wave spectra becoming narrower due to focusing effects over the currents.

In this paper, we are attempting to answer this issue directly. Section 2 describes the experimental facility and setup. Section 3 is dedicated to the investigation of limiting characteristics of short-crested waves at breaking and the connection between directionality and breaking probability and, most importantly, to identifying parametric limits of existence of the linear focusing and modulational-instability breaking events. Section 4 formulates conclusions.

2. Experimental facility, setup, and observations

The experiments were conducted in the Ocean Engineering Tank of the Institute of Industrial Science of the University of Tokyo (Kinoshita Laboratory and Rheem Laboratory). It is described in detail by Waseda et al. (2009a), and here we will only mention features and specifications of the facility that are most important for the present study.

The tank is 50 m long, 10 m wide, and 5 m deep. It is equipped with a multidirectional wave maker, which consists of 32 triangular plungers that are 31 cm wide. The plungers are computer controlled to generate waves of periods of 0.5–5 s initially propagated with predetermined spectrum and directional spread. In our experiments, no periods longer than 1.1 s were used; therefore, deep water conditions were always satisfied.

The directional wave array employed is shown in Fig. 1. This is a centered pentagon, with radius $r = 10$ cm, which allows very good resolution of wavelengths $\lambda \sim 1$ m used in this study. Numbering the array's wires have to be noted as they will be used below when discussing particular examples of oblique wave propagation through this array. Additionally, a one-dimensional array consisting of 7 wire probes was deployed along the tank. The sensors were positioned 2.3 m from one of the sidewalls at 5-m intervals to monitor the wave train evolution along the tank. The directional array was situated on the mobile bridge and could be moved both along and across the tank. In most observations employed here, it was positioned in the center. Capacitance wire sensors measured surface elevations with an accuracy of ~1 mm, and a sampling rate of the wave profile at 100 Hz resolution was nearly continuous for the purposes of this study. Durations of the records were typically 10 minutes. A set of video cameras looking down, up, and across the tank were also used, and their records were synchronized with the wave records.

In the reported experiment, a very simple setup was adopted to avoid complications caused by a variety of physical interactions present at the same time. That is, for the majority of the records, in the frequency/wavenumber domain wave trains were generated monochromatically, with small sidebands. Different values of the frequency (in the range of wavelengths from 0.8 to 2 m) were used, as well as a set of magnitudes for sidebands and modulational indexes M_I, but what was varied broadly were the initial monochromatic steepness ak and directional spreading A: it is the *combined effect* of the last two properties on the cause of the breaking that was investigated.

Apart from recording the waves, the main observational task was to distinguish the breaking between those caused by linear directional focusing and by modulational instability. This was done visually, and visual observations were verified and confirmed by other means directly and indirectly as described below.

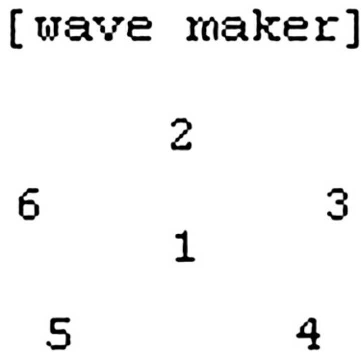

FIG. 1. Directional array configuration.

As discussed in the introduction, the wave breaking expected was due to either focusing or evolution of nonlinear wave groups. These two types were easy to distinguish visually. In the case of linear superposition, the two directionally converging wave trains provide a critical steepness at some location, and then every subsequent wave coming to the area of this location breaks. In the case of modulational instability, on the contrary, only one or two waves that break at the top of subsequent groups; for example, if the group consists of seven waves, the observer watching a breaking location will see one or two breakers followed by six or five nonbreaking waves, and then the pattern is repeated.

As one could expect, there were records when both linear focusing and modulational instability were present. These would be either coexisting or replacing each other patterns. Such records were labeled transitional. Thus, in the logging journal, the wave trains that did bring about breaking events were classified into modulational-instability, linear-focusing, or transitional records. The transitional records, therefore, now represent the third group of data. It should be mentioned, however, that for an individual breaking there was no ambiguity, and it was apparent whether it is due to modulational instability or linear focusing.

As a side remark, it is interesting to mention that in such a simple setup some other geometric/dynamic behaviors of the directional focusing and modulational breaking were easily observed. In Fig. 2 (left) numerical simulation of directional superposition by Fochesato et al. (2007) is reproduced. Such concave/convex patterns were abundant visually in cases that were classified as the linear-focusing breaking: one of the photos is shown in Fig. 2 (right). If a strong enough modulational breaking happened underneath the observation bridge, it was very interesting to visually see the downshifting (i.e., Tulin and Waseda 1999; Babanin et al. 2010). Prior to breaking, the wave length in such case is expected to shorten (i.e., Babanin et al. 2007, 2010), which it apparently did. Then, visually, it rapidly accelerated in the course of breaking as its wavelength rebounded—increasing to the point of catching up and merging with the preceding individual wave. The number of waves in the group, thus, was reduced; that is, the downshifting occurred.

Attempts were also made to directly identify the class of a record based on statistical or dynamic properties that can be attributed to the modulational instability. Most prominent of the statistical properties and most frequently employed is kurtosis K, the fourth-order moment of the probability density function of surface elevations (e.g., Yasuda et al. 1992; Onorato et al. 2009a,b; Waseda et al. 2009a). If the wave field is random phased and

$$K > 3, \tag{11}$$

then nonlinear wave groups are present, whose dynamics is thus driven by the modulational instability.

Such a criterion, however, did not appear robust in the wave tank records where initial phases are not random and marginal on/off cases are exercised. In such conditions, strongly nonlinear groups are not uniformly distributed along the surface of the tank, and their locations are few. As will be shown below, the transition from quasi linear to a very skewed group is rapid, and therefore detecting such groups with a limited number of wave probes in the tank is not easy. In other words, if (11) is true, the modulational instability is indeed present but, if (11) is not measured, this fact cannot prove its absence.

As an example, kurtosis estimated by the probes of the wave array shown in Fig. 1 can be given. This is done for record 63, which was visually determined as such where the breaking happened due to modulational instability. Kurtosis measured at probes 1–6 was $K = 3.17$, 3.51, 2.95, 2.69, 3.13 and 3.73, respectively.

This means that abnormally high waves were coming through the array obliquely, first through probes 6 and 2, then 5 and 1, then 3, and finally probe 4. At probes 6 and 2 their non-Gaussianity was massive ($K = 3.7$ and 3.5), at probes 5 and 1 marginal $K \sim 3.1$, and then it disappeared. Visually, differences in the surface elevation records were not striking by any means, even when the kurtosis was high.

The entire observed transformation of the wave train kurtosis happened within the 20-cm expanse of the array, which is some $1/8$ of the wavelength. Obviously, there is a good amount of luck necessary to detect the high kurtosis. Out of the other seven wave probes deployed along the 50-m tank, only two exhibited kurtosis greater than 3 in this record; that is, $K = 3.2$ and $K = 3.46$ at probes 5 and 6 of the long array, respectively.

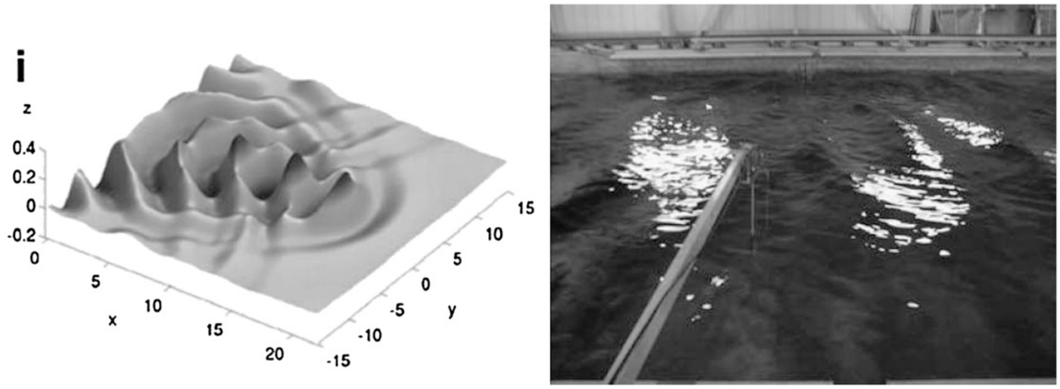

FIG. 2. (left) Directional focusing/defocusing (reproduced from Fochesato et al. 2007) and (right) photo of the directional focusing/defocusing (from the Ocean Engineering Tank).

When maximal kurtosis appeared at any of the 13 probes in the respective wave records was used, it did not help to separate the modulational-instability from the linear-focusing breaking. No statistically significant correlation with wave steepness and directionality was established; in fact, the trend was actually negative toward high initial values of ak and akA. This means, most likely, that the frequent breaking, which decreases the background wave steepness, affects the modulational instability and reduces the kurtosis, as higher values of ak and akA should signify higher breaking rates (verified in section 3).

A dynamic property that can identify active modulational instability in monochromatic wave trains is the growth of sidebands. In most of the tests run in our experiments, sidebands were seeded such that their amplitude b was $b = 0.01a$. A limited number of records were conducted unseeded, to allow the sidebands to grow from the background noise, and in the other records it was set $b = 0.1a$ so as to accelerate the nonlinear evolution of wave groups. The latter experiment was intended to accelerate the modulational growth and, in fact, led to very rapid breaking within the first 10 m: therefore, it was excluded from subsequent analysis because initial steepness of such waves was not measured by the probes located farther along the tank. Frequency separation of the sidebands and primary wave was dictated by $M_I = 1$ in (5), unless other values of the modulational index are explicitly stated.

In Fig. 3, the spectrum measured at the seventhmost distance (40 m) from the wavemaker probe is shown. In the left panel, the modulational instability is very strong and the sideband has grown to nearly the same energy as the carrier wave. Almost no growth of the initial sidebands is visible in the right panel, and the middle panel is apparently transitional. The upper and lower horizontal lines signify 20% and 5% of the energy of the primary wave peak and were used as an approximate indicator for separating the modulational instability, transitional, and linear cases.

In the majority of situations this separation agreed with visual observation classification of the breaking mentioned above. The wave breaking due to modulational instability and the instability itself, however, is not the same feature. The instability may be present and active but does not necessarily lead to breaking, that is, if the mean wave steepness is below the threshold (e.g., Babanin et al. 2007). Also, obviously, in the presence of initially steep waves with broad angular distribution, linear superposition of the directionally propagating crests can bring about the linear-focusing breaking, while the modulational instability is also active. Therefore, in our paper the dynamic feature of sideband growth will be used to identify the instability, transitional, and linear evolutions in the general investigation below (the dataset including 9, 7, and 8 records, respectively); however, when it comes to analysis of the breaking, the separation of breaking mechanisms will be based on the visual observations (dataset of 13, 14, and 18 records, respectively; in Fig. 8).

3. Experiment

One of the key arguments used above was based on the existence of limiting steepness: that is, since the waves break when they reach such steepness, then, as far as a breaking occurrence is concerned, it does not matter which physics caused such steepness. The waves break not because of these physical mechanisms but, rather, because the water surface cannot sustain slopes higher than critical. Therefore, the first subsection of this experimental section will be dedicated to analysis of geometrical characteristics of the short-crested waves observed in our directional trains.

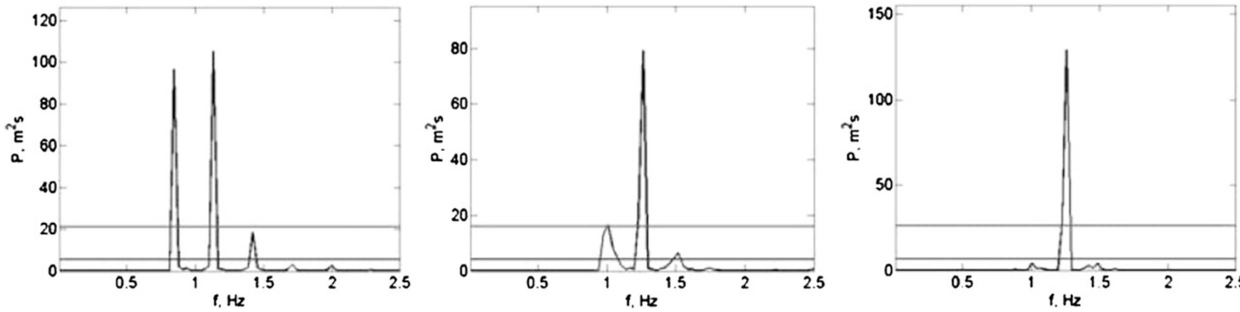

FIG. 3. Spectral energy density P vs frequency f measured at distance 40 m from the wavemaker. The upper (lower) horizontal line signifies 20% (5%) of the carrier-wave peak. (left) Strong growth, (middle) transitional growth, and (right) marginal growth of the modulational sidebands.

The second subsection is the main part of the paper. It is intended to answer the question of whether the modulational instability can occur in typical, that is, not-so-narrow directional wave fields. The last subsection will describe connection between directionality, steepness, and breaking probability.

a. *Limiting characteristics of short-crested directional waves at breaking*

As a short notice in the beginning of this subsection, we would like to mention that short crestedness and directionality are often used interchangeably in literature but are not the same property in a general case. Indeed, linear superposition of long-crested directional waves leads to a short-crested appearance of the surface. However, short-crested waves are not necessarily directional: for example, steep unidirectional waves in wave tanks and their counterparts in nature (nonlinear quasi-unidirectional wave groups that consist of short-crested individual waves). Since in this paper we are particularly interested in the latter, we should make this distinction.

Therefore, the question whether the limiting steepness (1) found in two-dimensional wave trains is applicable to three-dimensional waves was the first question to answer: it was done by Toffoli et al. (2010a) based on a comprehensive field/laboratory dataset, which also included the data of this experiment. The answer was limiting criteria (3) and (4) for the front and rear steepnesses, both somewhat higher than two-dimensional steepness (1) [note that this limiting steepness (1) is applicable to unidirectional waves regardless whether they are short (Babanin et al. 2007) or long crested (Brown and Jensen 2001)].

Here, we would like to see whether the geometrical properties of directional waves, including their steepness ak, skewness S_k, and asymmetry A_s, are different in case of prebreaking waves and breaking in progress. For precise definitions of the geometrical properties, we refer the reader to Babanin (2009), and here mention that (i) skewness characterizes asymmetry of a wave with respect to the horizontal axis (i.e., positive skewness means crests sharper than troughs) and (ii) A_s is asymmetry with respect to the vertical axis (i.e., negative asymmetry means a wave is tilting forward). In this paper, these geometrical properties were obtained by means of zero-crossing analysis, that is, for each individual wave separately. Since the sampling rate of the wave profile was very high, estimates of the steepness, skewness, and asymmetry of individual waves are precise.

In Fig. 4 (left), such individual wave data points are shown for three records that included breaking events. The scatter is large, but for the skewness the trend is positive, for asymmetry it is negative, and for asymmetry versus skewness the trend is negative (i.e., as the crests grow sharper, the waves are leaning forward). The latter, in particular, indicates that breaking-in-progress waves are embedded in the data points used, at different phases of their breaking progress (Babanin et al. 2010). Steepness ak of some waves reaches beyond the value of 0.5.

For record 101 it was noticed that waves were breaking immediately after the directional array. Therefore, this record does not contain breaking events, but rather prebreaking waves very close to the breaking onset. Individual characteristics of those are shown in view graphic Fig. 4 (right set of panels). The most obvious feature is the reduced scatter of skewness. Thus, skewness is some asymptotic indicator of the breaking onset. Once the waves start breaking, their skewness can be anything, as in the left set of panels in Fig. 4, but, when a wave is approaching breaking onset, its skewness narrowly asymptotes to

$$S_k \sim 0.7. \tag{12}$$

This is somewhat lower than the limiting skewness $S_k \sim 1$ observed in two-dimensional breaking onset (Babanin et al. 2007).

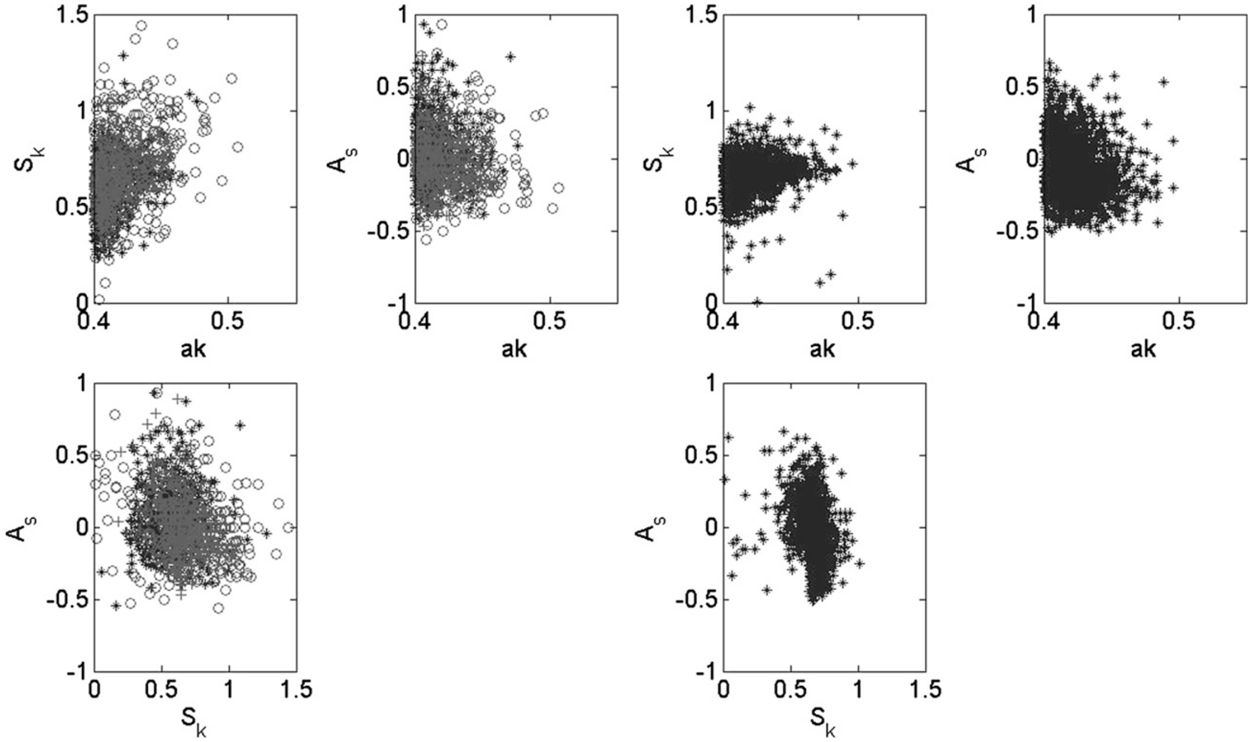

FIG. 4. (left) Characteristics of individual waves from records that include breaking events. (right) Wave array data of record 101, prebreaking waves. In each tripanel subset is shown (left) skewness S_k vs steepness ak, (right) asymmetry A_s vs steepness ak, and (bottom) A_s vs S_k.

Also, when the waves break (left set of panels in Fig. 4), their steepness can be above 0.5, but for the breaking onset a more robust limit seems applicable,

$$\frac{Hk}{2} = 0.46 - 0.48. \tag{13}$$

In this regard, the higher ultimate steepness values observed by Toffoli et al. (2010a) should relate to transient waves already breaking, as discussed above. That is, short-crested directional waves start breaking if the steepness (13) is reached, but in the course of breaking they can achieve even higher steepness (3). The limiting steepness (13) for directional waves, however, is only slightly higher than that of (1) for two-dimensional wave trains.

b. *Directionality and steepness thresholds for the modulational instability and directional superposition*

As discussed in the introduction, Waseda et al. (2009a,b) concluded that in a full-spectrum wave field modulational instability ceases to be active if directional spread is so broad that $A < 4$, Eq. (7). When measuring directional spectra in various field conditions, Babanin and Soloviev (1987, 1998a) demonstrated that typically in field conditions $A \sim 1$ and for dominant waves it can reach up to $A \sim 1.8$. In both cases, directional spectra were measured by means of wave arrays, but different methods of data processing were used: the wavelet directional method (WDM) (Donelan et al. 1996) in Waseda et al. (2009a,b) and the maximum likelihood method (MLM) (Capon 1969) in Babanin and Soloviev (1987, 1998a). Therefore, before conclusions are made, comparisons of the two methods need to be done.

This is conducted in Fig. 5. For the Ocean Engineering Tank directional records, values of A were obtained both by means of WDM and MLM, and for WDM A on average is 3 times larger. This means that, based on the same directional-array input data, WDM indicates much narrower spectra. Although the WDM estimates should be more accurate and have other advantages (e.g., Waseda et al. 2009a; Young 2010), here we will use the MLM estimates so as to compare with field-observed directional spectra of Babanin and Soloviev (1987, 1998a), which were done by means of MLM.

An important implication of the difference observed in Fig. 5 is immediately obvious if we boldly divide criterion (7) of Waseda et al. (2009a,b), obtained with WDM, by 3: then the transition from no visible modulational

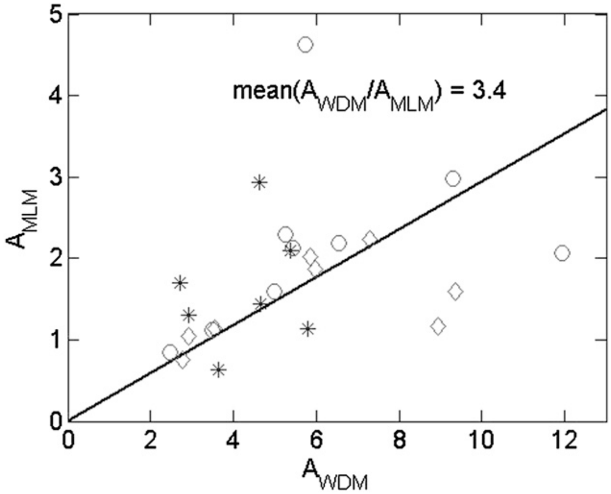

FIG. 5. Comparison of A_{MLM} and A_{WDM} for the Ocean Engineering Tank records. Circles correspond to modulational-instability, diamonds to linear-focused, and stars to transitional cases.

instability to detected modulational instability happens at $A \sim 1.3$, which, according to Babanin and Soloviev (1987, 1998a), falls into the range of directional spreads typical for dominant waves [and the modulational instability should only be considered for the dominant waves because waves in other parts of the spectrum may not exhibit narrow-banded behavior; for discussion, see Babanin (2009)]. In Fig. 6, this limit is investigated on the basis of directional data of the present experiment.

In the top subplot, directional-spread parameter A is plotted versus ak. Clearly, the instability exists at values of A as low as

$$A \approx 0.8, \tag{14}$$

which are quite realistic directional widths in field conditions—in fact quite broad by any standards, certainly at the spectral peak where the modulational instability is expected to work. In terms of $M_{Id} = Aak$ in the bottom panel, the modulational instability limit is

$$Aak \approx 0.18, \tag{15}$$

which is again absolutely feasible. With $A \sim 1$, there should be $ak \sim 0.2$, which is possible, and, for $A \sim 1.8$, there should be $ak \sim 0.11$, which is a typical steepness of ocean waves.

It is instructive to notice that the linear focusing only exists at $ak > 0.24$, which signifies superposition of two waves leading to the breaking at limiting steepness (13). Thus, superposition of three waves is unlikely—at least it did not happen in the course of our records, which encompassed some 500–700 waves—whereas breaking did happen. This is an important observation since, in typical conditions with much less steep waves and dispersive rather than directional focusing, superpositions of even much greater number of waves would be required, as discussed above. Such observation indicates that the probability of linear superposition, which would lead to the limiting steepness (13), is low: therefore, the linear focusing is hardly to be expected as the main cause of wave breaking in directional fields. On the other extreme, the directional focusing does not happen for very narrow (i.e., near unidirectional) spectra of $A > 2.25$ ($A > 9.4$ for WDM-estimated spectra).

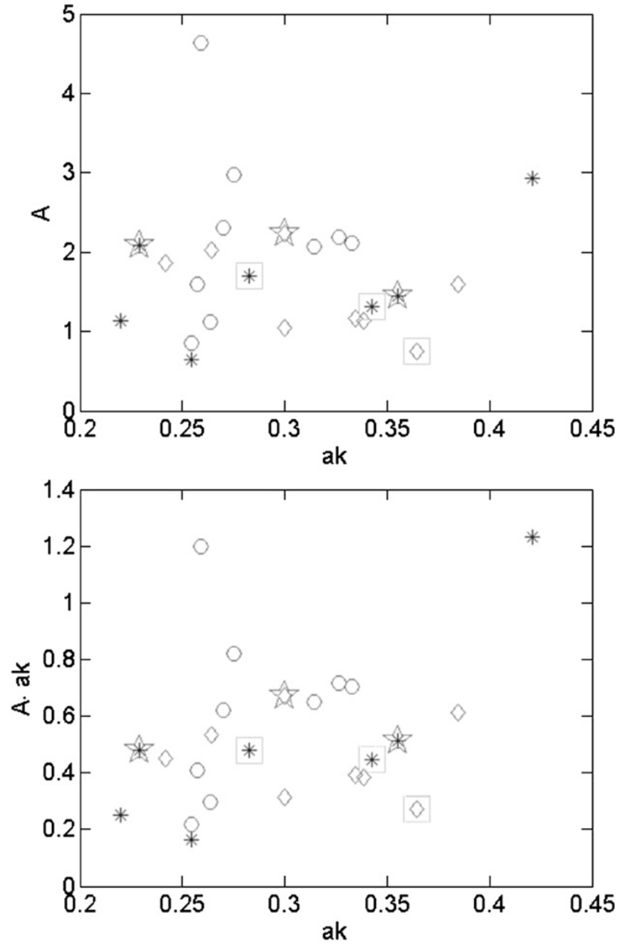

FIG. 6. Plot of (top) A vs ak and (right) Aak vs ak: symbols as in Fig. 5. Squares signify $M_I = 2$ and pentagrams $M_I = 0.5$.

Since $M_I = 1$ was imposed in most of the records, influence of the imposed modulation was further verified. In Fig. 6, squares signify records with $M_I = 2$ and pentagrams those with $M_I = 0.5$. These enforced modulations of different scales and different instability rates did not lead to any outliers. The squares and pentagrams are in the middle of the overall scatter, which means that

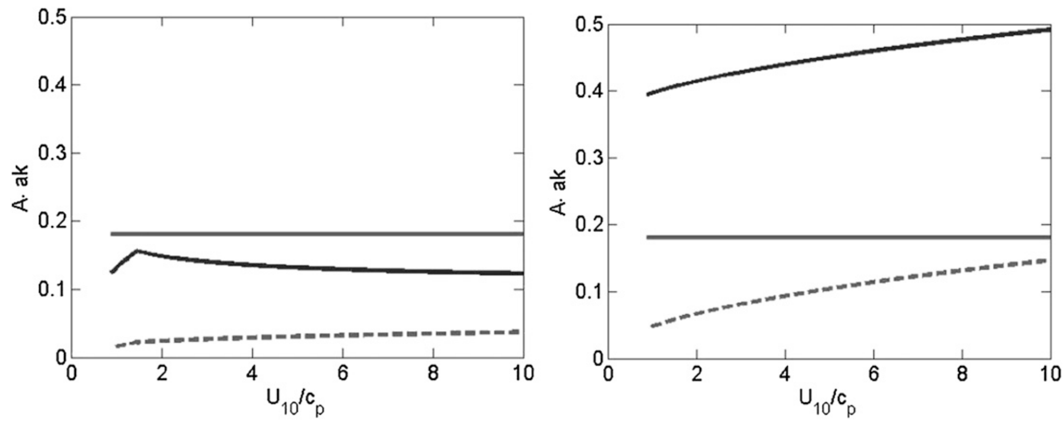

FIG. 7. The straight solid line is the experimental threshold (15), the curved solid line is the Babanin and Soloviev (1998a,b) parameterization for the JONSWAP and directional spectra combined and the dashed line corresponds to the isotropic directional spread: (left) default average dependences and (right) maximal dependences (within 95% confidence limit).

the choice of a type of the nonlinear group is not a principal matter as long as instability is active, but on average the $M_I = 2$ data points are below those with $M_I = 0.5$.

Feasibility of the new experimental threshold (15) is now verified in Fig. 7 against the average wave-age field dependences of Babanin and Soloviev (1998a,b). As already mentioned, Babanin and Soloviev (1998a) provided such dependence for the directional wave spectra, and Babanin and Soloviev (1998b) for frequency spectra based on JONSWAP shape. Owing to steepness and directional spread, the key properties of threshold (15), they are therefore expected to vary as a function of wave age.

Although default average dependences (left) give Aak marginally under the threshold, the maximal possible (within 95% confidence limit) dependences give Aak much greater than the threshold across the entire range of wave ages. This makes the modulational instability, as a course of the wave breaking in realistic field conditions, again quite feasible.

c. *Breaking probability*

Distance to breaking D was estimated by measuring the closest distance from the wavemaker to the first visually observed breaker in a record. If presented in terms of carrier wavelength D/λ, this property is inversely related to the traditional breaking probability, that is, the percentage of breaking crests in a wave train (Babanin et al. 2007; Babanin 2009).

Figure 8 verifies dependence of distance to breaking D/λ on ak and Aak. The plot for ak exhibits such a dependence, whereas the Aak graph does not show any clear connection on average. This means that, although the combination Aak is important to determine whether the modulational instability happens, once it does, the breaking probability only depends on steepness regardless of the directional spreading. As before, the breaking conditions are separated into those due to modulational-instability, directional-superposition, and transitional breaking (or both types present in the same record) but, as discussed above, the separation of the breaker types is now based on visual observations.

In Fig. 8 (top), for the modulational-instability circles, the distance to breaking is clearly correlated with steepness. This dependence is approximated by the straight dashed line, and the best fit is

$$D/\lambda = -144ak + 62. \qquad (16)$$

As a consistency check, in this panel dependence (16), obtained for the three-dimensional waves, is compared with the empirical parameterization of the distance to breaking versus steepness obtained by Babanin et al. (2007) for quasi-two-dimensional waves (solid line). The latter imposed low- and high-steepness limits, which correspond to no-breaking/immediate-breaking physical circumstances rather than being the best fit like dependence (16).

The two parameterizations are consistent in general, but the 3D dependence is steeper in the relevant range of parameters. When extrapolated to $D = 0$, that is, instantaneous breaking, this dependence crosses the line at $ak = 0.43$, which is somewhat lower than (13). When extrapolated to $ak = 0$, this dependence gives modulational-instability breaking at $D = 62\lambda$, which is of course unphysical. Therefore, extrapolations of this dependence into extremes have obvious limitations and should be done with caution.

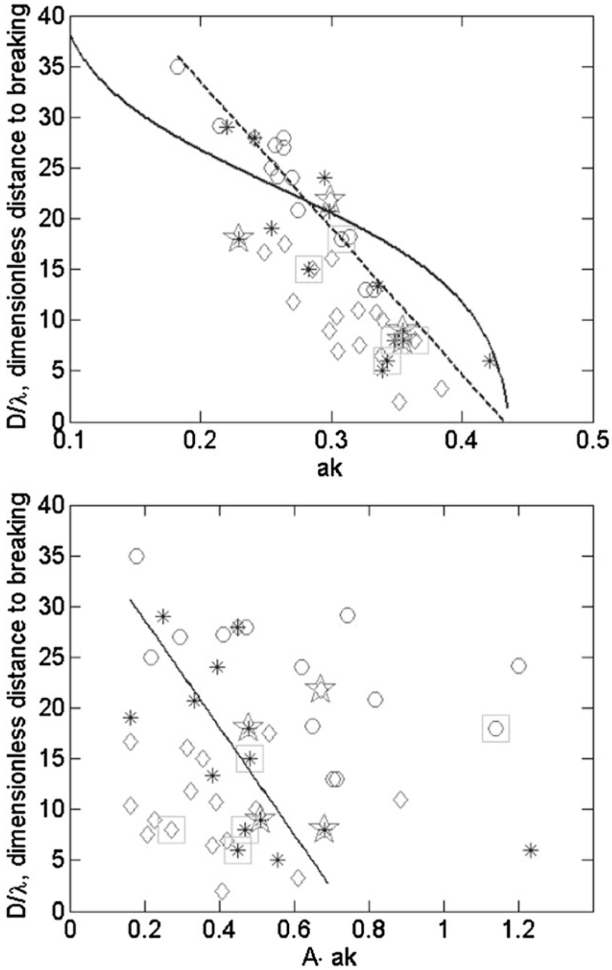

FIG. 8. Distance to breaking vs (top) steepness ak and (bottom) Aak for the 2D (solid line), Babanin et al. (2007), and 3D (dashed line) dependence (16) in the top panel and for dependence (17) (solid line in the bottom panel). Symbols as in Fig. 6.

Most interesting, however, Fig. 8 (bottom) finally provides means to separate directional-focused breaking from the modulational-instability breaking. The line approximately goes through the fit to asterisk points only, that is, through the data points even visually detected as transitional,

$$D = -53akA + 39. \qquad (17)$$

As before, since $M_I = 1$ was imposed in most of the records, cases with $M_I = 2$ (squares) and $M_I = 0.5$ (pentagrams) are highlighted for comparison. These cases do not exhibit any essential trend, being basically in the middle of the data cloud. Marginally, perhaps, higher M_I cases (longer groups) lead to transitional points being under the curve (i.e., in the focused-breaking area) and $M_I = 0.5$ to the transitional points being in the modulational-breaking area, but this needs further statistically significant verification.

4. Discussion and conclusions

In the paper, the main question was whether the modulational instability can still be active in directional wave fields with broad enough angular distribution. Wave breaking was observed in a laboratory experiment with waves of realistic average steepness and directional spread. Intentionally, so as to isolate the impact of parameters of interest, a very simple experimental setup was chosen: that is, wave trains monochromatic in frequency domain, with small initial sidebands. Parameters of interest were steepness and width of the directional distribution of such wave trains, and these parameters were varied broadly. It was expected that the combination of these parameters, that is, the directional modulation index (10), will allow us to define a threshold for the instability breaking to occur in directional fields.

This threshold was identified as (15), which permits both steepness and directional spreads to be quite realistic for typical oceanic conditions. We realize, of course, the potential difference between quasi-monochromatic waves investigated here and continuous-spectrum oceanic fields and the need for the conclusions made herein to be verified in full-spectrum circumstances. With some caution, we mention that the modulational index (5), analytically proposed for monochromatic modulated wave trains, was also applied to wave fields by analogy and appeared valid (e.g., Onorato et al. 2002; Janssen 2003).

The issue of modulational instability present/absent in directional wave fields is of major importance for the topic of wave breaking. Lately, a number of experimental studies pointed to a limiting steepness around the value of (13) as a condition for wave breaking onset (i.e., Brown and Jensen 2001; Babanin et al. 2007; Toffoli et al. 2010a). In the presented experiments, even for waves with the close frequency and amplitudes, superposition of more than two waves—to reach this steepness—appeared unlikely. Therefore, in the absence of other apparent physical mechanisms capable of providing steepness of (13) magnitude, we would expect the modulational instability to be the main cause of breaking for the dominant waves.

The breaking also provides a possibility to distinguish modulational-instability and directional-focused cases. The steepness ak, directional spreading A, and their combination $M_{Id} = Aak_0$ identify limits beyond which breaking due to one cause or another does not happen. That is, if $A < 0.8$ or $Aak < 0.18$, modulational instability did not lead to breaking, and linear focusing did not happen if $ak < 0.24$ or $A > 2.25$. Between these limits, the breaking due to either cause is possible. The breaking probability dependence (17), however, appears to separate the two major reasons for the dominant breaking in its parametric space.

Acknowledgments. Alex Babanin and Alessandro Toffoli gratefully acknowledge financial support of the Australian Research Council and Woodside Energy Ltd. through the Linkage Grant LP0883888 and of the Australian Research Council through the Discovery Grant DP1093517. Alex Babanin was Visiting Professor to the University of Tokyo at the time of research.

REFERENCES

Babanin, A. V., 2009: Breaking of ocean surface waves. *Acta Phys. Slovaca,* **59,** 305–535.

——, and Y. P. Soloviev, 1987: Parameterization of the width of angular distribution of the wind wave energy at limited fetches. *Izv. Akad. Nauk SSSR, Fiz. Atmos. Okeana,* **23,** 868–876.

——, and ——, 1998a: Variability of directional spectra of wind-generated waves, studied by means of wave staff arrays. *Mar. Freshwater Res.,* **49,** 89–101.

——, and ——, 1998b: Field investigation of transformation of the wind wave frequency spectrum with fetch and the stage of development. *J. Phys. Oceanogr.,* **28,** 563–576.

——, D. Chalikov, I. Young, and I. Savelyev, 2007: Predicting the breaking onset of surface water waves. *Geophys. Res. Lett.,* **34,** L07605, doi:10.1029/2006GL029135.

——, ——, ——, and ——, 2010: Numerical and laboratory investigation of breaking of steep two-dimensional waves in deep water. *J. Fluid Mech.,* **644,** 433–463.

Benjamin, T. B., and J. E. Feir, 1967: The disintegration of wave trains in deep water. Part 1. Theory. *J. Fluid Mech.,* **27,** 417–430.

Brown, M. G., and A. Jensen, 2001: Experiments in focusing unidirectional water waves. *J. Geophys. Res.,* **106,** 16 917–16 928.

Capon, J., 1969: High-resolution frequency-wavenumber spectrum analysis. *Proc. IEEE,* **57,** 1408–1418.

Donelan, M. A., 1978: Whitecaps and momentum transfer. *Turbulent Fluxes through the Sea Surface, Wave Dynamics and Prediction,* A. Favre and K. Hasselmann, Eds., NATO Conference Series, Vol. 1, Plenum Press, 74–94.

——, and Y. Yuan, 1994: Wave dissipation by surface processes. *Dynamics and Modelling of Ocean Waves,* G. J. Komen et al., Eds., Cambridge University Press, 143–155.

——, W. M. Drennan, and A. K. Magnusson, 1996: Nonstationary analysis of the directional properties of propagating waves. *J. Phys. Oceanogr.,* **26,** 1901–1914.

Dyachenko, A. I., and V. E. Zakharov, 2005: Modulation instability of Stokes wave–freak wave. *JETP Lett.,* **81,** 318–322.

Fochesato, C., S. Grilli, and F. Dias, 2007: Numerical modeling of extreme rogue waves generated by directional energy focusing. *Wave Motion,* **26,** 395–416.

Holthuijsen, L. H., and T. H. C. Herbers, 1986: Statistics of breaking waves observed as whitecaps in the open sea. *J. Phys. Oceanogr.,* **16,** 290–297.

Janssen, P. A. E. M., 2003: Nonlinear four-wave interaction and freak waves. *J. Phys. Oceanogr.,* **33,** 863–884.

Janssen, T. T., and T. H. C. Herbers, 2009: Nonlinear wave statistics in a focal zone. *J. Phys. Oceanogr.,* **39,** 1948–1964.

Liu, P. C., and A. V. Babanin, 2004: Using wavelet spectrum analysis to resolve breaking events in the wind wave time series. *Ann. Geophys.,* **22,** 3335–3345.

Longuet-Higgins, M. S., 1974: Breaking waves in deep or shallow water. *Proc. 10th Conf. Naval Hydrodynamics,* Cambridge, MA, Massachusetts Institute of Technology, 597–605.

——, and D. G. Dommermuth, 1997: Crest instabilities of gravity waves. Part 3. Nonlinear development and breaking. *J. Fluid Mech.,* **336,** 51–68.

——, and M. Tanaka, 1997: On the crest instabilities of steep surface waves. *J. Fluid Mech.,* **336,** 33–50.

McLean, J. W., 1982: Instabilities of finite-amplitude water waves. *J. Fluid Mech.,* **114,** 315–330.

Michell, J. H., 1893: On the highest waves in water. *Philos. Mag.,* **5,** 430–437.

Onorato, M., A. R. Osborne, and M. Serio, 2002: Extreme wave events in directional, random oceanic sea states. *Phys. Fluids,* **14,** 25–28.

——, and Coauthors, 2009a: Statistical properties of mechanically generated surface gravity waves: A laboratory experiment in a three-dimensional wave basin. *J. Fluid Mech.,* **627,** 235–257.

——, and Coauthors, 2009b: On the statistical properties of directional ocean waves: The role of the modulational instability in the formation of extreme events. *Phys. Rev. Lett.,* **102,** 114502, doi:10.1103/PhysRevLett.102.114502.

Pierson, W. J., M. A. Donelan, and W. H. Hui, 1992: Linear and nonlinear propagation of water wave groups. *J. Geophys. Res.,* **97,** 5607–5621.

Rapp, R. J., and W. K. Melville, 1990: Laboratory measurements of deep-water breaking waves. *Philos. Trans. Roy. Soc. London,* **311A,** 735–800.

Stokes, G. G., 1880: Considerations relative to the greatest height of oscillatory irrotational waves which can be propagated without change of form. *On the Theory of Oscillatory Waves,* Cambridge University Press, 225–229.

Tamura, H., T. Waseda, and Y. Miyazawa, 2008: Freakish sea state and swell-windsea coupling: Numerical study of the *Suwa-Maru* incident. *Geophys. Res. Lett.,* **36,** L01607, doi:10.1029/2008GL036280.

Toffoli, A., A. V. Babanin, M. Onorato, and T. Waseda, 2010a: The maximum steepness of oceanic waves. *Geophys. Res. Lett.,* **37,** L05603, doi:10.1029/2009GL041771.

——, and Coauthors, 2010b: Extreme waves in sea states crossing an oblique current. *Proc. 29th Int. Conf. on Ocean, Offshore and Arctic Engineering (OMAE 2010),* Shanghai, China, American Society of Mechanical Engineers, 8 pp.

Tulin, M. P., and T. Waseda, 1999: Laboratory observations of wave group evolution, including breaking effects. *J. Fluid Mech.,* **378,** 197–232.

Waseda, T., T. Kinoshita, and H. Tamura, 2009a: Evolution of a random directional wave and freak wave occurrence. *J. Phys. Oceanogr.,* **39,** 621–639.

——, ——, and ——, 2009b: Interplay of resonant and quasi-resonant interaction of the directional ocean waves. *J. Phys. Oceanogr.,* **39,** 2351–2362.

Yasuda, T., N. Mori, and K. Ito, 1992: Freak waves in a unidirectional wave train and their kinematics. *Proc. 23rd Int. Conf. on Coastal Engineering,* Venice, Italy, American Society of Mechanical Engineers, 751–764.

Young, I. R., 2010: The form of the asymptotic depth-limited wind-wave frequency spectrum. Part III—Directional spreading. *Coastal Eng.,* **57,** 30–40.

Zakharov, V. E., 1966: The instability of waves in nonlinear dispersive media (in Russian). *Zh. Eksp. Teor. Fiz. Pis'ma Red.,* **51,** 1107–1114.

——, 1967: The instability of waves in nonlinear dispersive media. *Sov. Phys. JETP,* **24,** 744–744.