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TWO-PHASE BEHAVIOUR OF THE SPECTRAL DISSIPATION OF WIND WAVES

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Abstract: Two different experimental methodologies were used to investigate the spectral dissipation function on the basis of field data. If the wave energy dissipation at each frequency were due to whitecapping only, it should be a function of the excess of the spectral density above a threshold spectral level, below which no breaking occurs at this frequency, as was shown in a number of recent experimental studies. This was found to be the case around the wave spectral peak. A more complex mechanism appears to be driving the dissipation at scales different to those of the dominant waves. Dissipation at a particular frequency above the peak demonstrates a cumulative effect, depending on the rates of spectral dissipation at lower frequencies. The nature of the induced dissipation above the peak can be due to either enhanced induced wave breaking or additional turbulent eddy viscosity or both. This issue is addressed.

INTRODUCTION

The radiative transfer equation

$$\frac{dF}{dt} = S_{in} + S_{nl} + S_{ds}, \quad (1)$$

where F is the wave spectrum, S_{in} is the wind input rate spectral function, S_{nl} is the non-linear interaction spectral term, and S_{ds} is the spectral dissipation rate, is widely used in both research and operational wave models. The source functions S are known with different degrees of certainty, with S_{ds} being least understood.

Theoretical studies of the spectral dissipation offer a few analytical models (see Young and Babanin 2005 for a review). None of these models, however, deals with the dynamics of wave breaking, which is responsible for dissipation. Rather, they suggest hypotheses to interpret either pre-breaking or post-breaking wave field properties. All of the hypotheses lack experimental support or validation. Results vary from the dissipation being a linear function of the wave spectrum to the dissipation being quadratic, cubic or even a function of the spectrum to the fifth power. In

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addition, some formulations represent the dissipation as being local in wavenumber space whilst others assume it is local in physical space.

Experimental studies of this key term are very few (Donelan 2001, Phillips et al., 2001, Melville and Matusov 2002, Hwang and Wang 2004, Young and Babanin 2005). Phillips et al. (2001), Melville and Matusov (2002) results produce spectral dissipation local in wavenumber space – i.e. dissipation rates at a particular spectral scale depend on properties of waves of that scale only. The experimental spectral dissipation functions of Donelan (2001) and Young and Babanin (2005) both exhibit a cumulative effect: dissipation rates at scales of shorter waves depend on what happens at scales of longer waves. Donelan (2001) suggests that it is the modulation of short waves by longer waves that affects wave-breaking rates at higher frequencies, and Young and Babanin (2005) demonstrate evidences of large breakers causing a broadband dissipation. Hwang and Wang (2004) present results which can be interpreted in favour of both local-in-wavenumber-space dissipation and the cumulative effect.

The objective of the present study was to investigate the wave spectral dissipation by experimental means, using field measurements. The measurements were conducted at Lake George, Australia, and the experimental set-up is described in detail in Young et al. (2005). To determine total dissipation in the water column, whitecap dissipation from the wave field and related properties of breaking waves, a set of multiple, even redundant, instruments were used. These included wave spatial arrays, vertical arrays of acoustic Doppler velocimeters, hydrophones, video filming of the measured wave breaking surface and electronic marking of breaking events.

Two different methodologies were used to investigate the dissipation function. The first employed the acoustic noise spectrograms to identify segments of breaking and non-breaking dominant wave trains (Babanin et al. 2001). The average power and directional spectra for breaking and non-breaking waves were thus obtained, and the difference was attributed to the dissipation due to wave breaking (Young and Babanin 2005). This method provides an estimate of the spectral effects, both in frequency and directional domains, due to the dominant wave breaking.

As an independent second approach, a passive acoustic method of detecting individual bubble-formation events was developed. This method was found promising for obtaining both the rate of occurrence of breaking events at different wave scales and the severity of wave breaking (Manasseh et al. 2005). A combination of the two methods should lead to direct estimates of the spectral distribution of wave dissipation.

In this paper, the two independent methods are applied to the same field data to show that a two-phase behaviour of the spectral dissipation is a consistent feature. At the spectral peak, the dissipation is a function of the excess of the spectral density above a threshold spectral level, below which no breaking occurs at this frequency. Dissipation at a particular frequency above the peak demonstrates a cumulative effect, depending on the rates of spectral dissipation at lower frequencies. A procedure for obtaining the spectral threshold is suggested. Also, the nature of the dissipation at different scales is discussed: whether it is due to wave breaking, or due to turbulent eddy viscosity, or both. It is generally accepted that the wave breaking is the major source of the wave energy dissipation, but it appears that the turbulent viscosity may also play a significant role at smaller wave scales.

DOMINANT-BREAKING DISSIPATION BY SPECTROGRAM METHOD

The spectrogram method was developed in Babanin et al. (2001) for detection of breaking of dominant waves and was applied by Young and Babanin (2005) to estimate the broadband spectral distribution of the dissipation due to such breaking. A field wave record with approximately 50% dominant breaking rate was analysed (record No.1 of Table 1 below). Segments of the record, comprising sequences of breaking waves, were used to obtain the "breaking spectrum", and segments of non-breaking waves - to obtain the "non-breaking spectrum". This approach was applied to both surface elevation spectra (Fig. 1, top panel) and subsurface velocity spectra (Fig. 1, bottom panel). Fig. 1, middle panel shows the ratio of the post-breaking wave spectrum and the incipient-breaking spectrum. The obvious broadband difference between the two spectra was attributed to the dissipation due to dominant wave breaking. This assumption was supported by independent measurements of total dissipation of kinetic energy in the water column at the measurement location, based on the turbulence spectra shown in Fig. 1, bottom panel.

Thus, it was shown that the dominant breaking causes energy dissipation throughout the entire spectrum at scales smaller than the spectral peak waves. The dissipation rate at each frequency, caused by the dominant breaking, is linear in terms of the wave spectral density at that frequency, with a correction for the directional spectral width. Directional spectra of the breaking and non-breaking waves were also considered, but this issue is beyond the scope of the present paper.

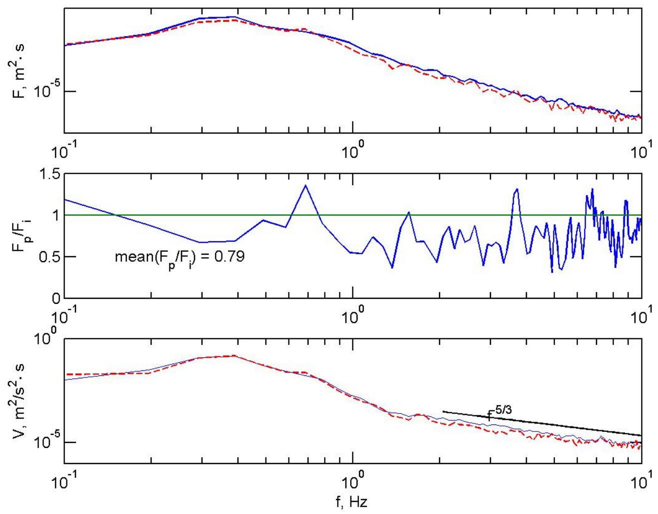

Fig. 1. (top panel) Spectra *F(f)* of incipient breaking (solid line) and post-breaking (dashed line) waves; (middle panel) Ratio of the two spectra; (bottom panel) Spectra *V(f)* of incipient breaking (solid line) and post-breaking (dashed line) subsurface velocities.

The broadband dissipation brought about by the dominant breaking suggests a two-phase behaviour of the spectral dissipation function. At the spectral peak, the dissipation should be linear in terms of the peak spectral density: when the dominant waves break due to their inherent reasons, this causes some 20% loss of this density (Fig. 1, middle panel). Simultaneously, it induces 20% loss of spectral density at higher frequencies across the spectrum. If the waves at those higher frequencies also break due to their inherent reasons (other than being induced by the larger breakers), the dissipation at the smaller scales will be larger than 20% and therefore not linear in terms of the spectral density at that frequency. In the absence of the larger breakers, obviously, the dissipation at a particular scale is caused by inherent reasons only and will stay linear in terms of the spectrum. Thus, the dissipation at each frequency *f* other than the peak frequency f_p should consist of two terms: the linear term which describes the dissipation due to inherent breaking at frequency *f* and a cumulative term which is responsible for an accumulated induced dissipation due to the average number of breaking occurring at frequencies less than *f*.

A formulation for the spectral dissipation function able to accommodate these features is suggested in Young and Babanin (2005). This formulation includes an integral cumulative term responsible for the additional induced dissipation:

$$S_{ds}(f) = a_1 \cdot \rho \cdot g \cdot f \cdot (F(f) - F_{thr}(f)) \cdot A(f) + a_2 \cdot \rho \cdot g \cdot \int_{f_p}^{f} (F(q) - F_{trh}(q)) A(q) dq \quad (2)$$

In (2), • is the water density, *g* is the gravitational constant, *A(f)* is the integral characteristic of the inverse directional spectral width (Babanin and Soloviev 1998):

$$A(f)^{-1} = \int_{-\pi}^{\pi} K(f, \varphi) d\varphi \ , \quad (3)$$

where • is the wave direction, $K(f, \varphi)$ is the normalised directional spectrum:

$$K(f, \varphi_{max}) = 1 \, . \quad (4)$$

The dissipation at each frequency depends on the difference $F(f) - F_{thr}(f)$ between the wave spectrum and a threshold spectral value at that frequency. The concept of the threshold spectral value comes from recent experimental observations by Banner et. al. (2000), Babanin et al. (2001) and Banner et al. (2002) who showed that the wave breaking is a function of the excess of the wave steepness above a threshold steepness at a particular wave scale (or a threshold spectral density at that scale), below which no breaking occurs at this scale. The value of the spectral threshold has to be determined from studies of the frequency distributions of the breaking rates and will be estimated below. In Young and Babanin (2005), it was found that the coefficients

$$a_1 = a_2 = 0.0065 \, . \quad (5)$$

Since Babanin and Young (2005) considered only a single record, and for that record they obtained only a lower bound estimate of the dissipation rate, these coefficients may, in a general case, not be equal. In addition, they may be dependent on wave age, and are expected to be greater than the value in (5).

BUBBLE-FORMATION DETECTION METHOD

The Lake George wave-acoustic data set was also utilised to develop an independent passive-acoustic method of registering breaking waves of different scales based on detecting individual bubble-formation events (Manasseh et al. 2005). When such events are detected, they are assumed to be related to synchronously recorded surface waves, and thus the rate of occurrence of wave breaking events at different wave scales can be obtained. The method also showed promise for measuring the breaking severity – absolute amount of energy lost during a breaking event. With support from a separate, laboratory experiment, the estimated bubble size *R* was argued to be dependent on the severity of wave breaking (the bigger the bubble, the greater the energy loss) and therefore to provide information on the energy loss due to the breaking at the measured spectral frequencies. A combination of the breaking-probability distribution and the bubble size across the spectrum could lead to direct estimates of the spectral distribution of wave dissipation.

Table 1. Summary of wave records used. Here, f_p is peak frequency, H_s is significant wave height, U_{10} is wind speed at 10 m height

No.	Record No.	f_p, Hz	H_s, m	U_{10}, m/s	Figure 2
1	311823.oc7	0.36	0.45	19.8	circle
2	311845.oc7	0.33	0.40	15.0	cross
3	312021.oc7	0.40	0.39	13.7	diamond
4	312048.oc7	0.37	0.37	13.2	triangle
5	311908.oc7	0.35	0.37	12.9	asterisk
6	311930.oc7	0.38	0.34	12.8	square

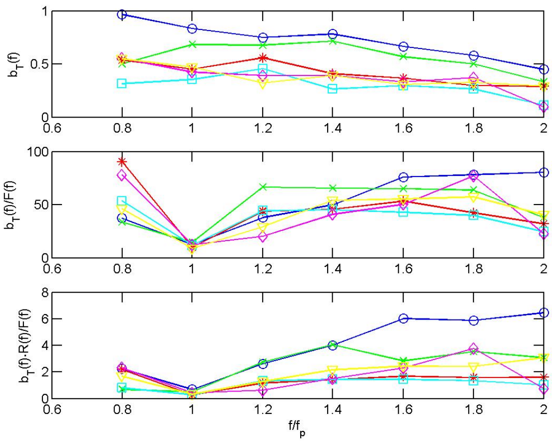

Fig. 2. Six wave records, see Table 1. (top panel) Breaking probability $b_T(f)$ versus relative frequency. (middle panle) $b_T(f)$ normalized by *F(f)*. (bottom panel) Product of bubble size *R(f)* and $b_T(f)$, normalized by *F(f)*.

Such frequency distributions are shown in Fig. 2 for 6 records summarized in Table 1. Distributions of the breaking probabilities $b_T(f)$ (ratio of the breaking crests to the total number of crest at frequency *f)* at different wind speeds (top panel) demonstrate that, in bottom-limited Lake George conditions, the highest breaking rates occur around the spectral peak and they gradually decrease towards higher frequencies. An interesting feature of these distributions is that, for wind speeds U_{10} *<14 m/s*, the breaking distributions merge, but for the wind speeds U_{10} *>14 m/s*, the level of the breaking rates appears to depend on the wind speed. Depending on the breaking severity, this may indicate that the dissipation rates at higher wind speeds are a function of the wind speed, in addition to being the function of the spectral density.

The middle plot is the most relevant for the present paper. It clearly shows the two-phase behaviour of the breaking rates. Once $b_T(f)$ is normalized by the local spectral density *F(f)*, the distributions collapse at the peak frequency and diverge elsewhere.

This is clear support for the two-phase behaviour suggested on the basis of the spectrogram-method observations above.

The bottom plot shows the frequency distribution of $b_T(f)R(f)/F(f)$. This product of the breaking probability and the bubble size gives a surrogate energy dissipation at frequency *f*, which is then normalised by the spectrum *F(f)* at this frequency. Again, such normalised dissipation collapses at the spectral peak, thus indicating a linear dependence of the dissipation on *F(f)* at f_p. At higher frequencies, dissipation rates are greater than those which could be expected if the dissipation was still linear in terms of the wave spectrum. This behaviour is consistent with the influence of a cumulative term inferred above in (2) and is further evidence of the two phase spectral dissipation function. We should point out, however, that the fact that the dimensional spectral values work well for our normalisation at the peak is apparently because the dimensional spectral peak frequencies for our six records are similar. In a general case, some dimensionless properties would be expected to be employed to bring different records together to a common dependence.

SATURATION THRESHOLD

One of the most significant uncertainties in the dissipation function (2) is the unknown threshold spectrum $F_{thr}(f)$. Existence of such a threshold, in dimensionless form, was first shown by Banner et al. (2002). They obtained dependences of the breaking probability at different frequencies as a function of the saturation spectrum $\sigma(f)$ (Phillips 1984):

$$\sigma_{Phillips}(f) = \frac{(2\pi)^4 f^5 F(f)}{2g^2}. \qquad (6)$$

Banner et al. (2002) concluded that, if the saturation is normalised by a directional spectrum width, there is a universal threshold, applicable at all frequencies for their data. If the saturation spectrum at any frequency drops below this dimensionless threshold, no breaking occurs at this frequency.

Banner et al. (2005) demonstrated the effect qualitatively and did not suggest a quantitative estimate of their threshold or a quantitative dependence for the breaking rates in terms of the saturation spectrum. We quantified data of their Figure 5 and parameterised them:

$$b_T(f) = 89.5(\sqrt{\sigma(f)} - 0.0223)^2. \qquad (7)$$

Here, the saturation spectrum is normalised by the directional spectrum parameter (3):

$$\sigma(f) = \sigma_{Phillips}(f)A(f). \qquad (8)$$

The parametric form (7) was chosen analogous to that obtained in the experiments of Banner at al. (2000), Babanin et al. (2001) for the dominant wave breaking, with the saturation spectrum now playing the role of the spectral steepness. The dimensionless spectral threshold, therefore, is:

$$\sqrt{\sigma_{thr}(f)} = const = 0.0223. \qquad (9)$$

This threshold was verified for the Lake George spectral breaking data obtained by the bubble-formation detection method (Fig. 3). In our data, the lowest value of the saturation, for which the breaking was still registered, was 0.0255. This value is not inconsistent with the threshold (9), but it leaves a window of $\sqrt{\sigma_{thr}(f)} = 0.0223 - 0.0254$ for modellers to choose from.

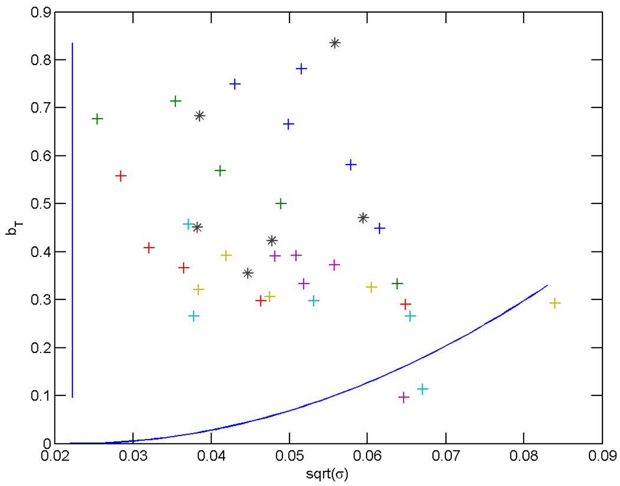

Fig. 3. Breaking probability $b_T(f)$ versus saturation parameter $\sqrt{\sigma(f)}$. Asterisks denote spectral peak points. Threshold *0.0223* and dependence (7) are shown with solid lines.

What is inconsistent, however, is the scatter of our experimental data with respect to the dependence (7). Most of the data points are positioned above the curve, and therefore the curve cannot possibly be treated as an average fit for them. It is more likely that the dependence (7) is a lower bound envelope for our data.

Another striking feature of the comparison is the large scatter of our breaking probability data, once those are plotted in terms of the saturation spectrum. The spectral peak points (asterisks) are in the middle of the cloud – with higher-frequency points being both above and below the peak points.

Analysis of the source of the scatter brought us to a conclusion that the saturation is not the most suitable parameter for such dependences in a general case. Saturation (6) is the firth moment of the spectrum, and any variations of the spectral shape, particularly at higher frequencies, causes very significant variations of the saturation. For intermediate-depth Lake George waves, the second-harmonic peak was usually noticeable and that brought about the scatter which made the saturation impossible to be accommodated in a breaking-rate dependence.

TWO-PHASE BEHAVIOR OF THE DISSIPATION

Due to the form of the dimensionless saturation spectrum (6), (8) cannot be used to parameterise our breaking probabilities, but is robust enough to be used to obtain the dimensionless spectral threshold (9). It was further used to derive the dimensional threshold $F_{thr}(f)$ from (9), (8) and (6) and to consider the breaking probability $b_T(f)$, the bubble size *R(f)* and the surrogate dissipation $R(f)b_T(f)$ as a function of $F(f) - F(f)_{thr}$ (Fig. 4). The three plots support the conclusion of two-phase behaviour for the three properties. Each of the quantities demonstrate a distinctive well-correlated linear trend for the spectral peak data points (asterisks) and a cluster of data points which belong to higher frequencies (different frequencies are denoted by different colours).

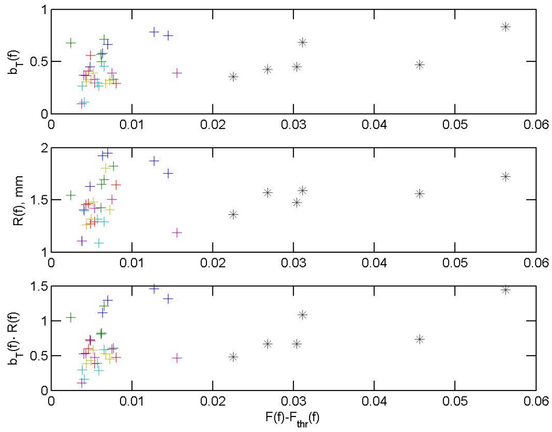

Fig. 4. (top panel) Breaking probability $b_T(f)$, (middle panel) Bubble size *R(f)*, (bottom panel) Product of bubble size *R(f)* and $b_T(f)$ - versus $F(f)-F(f)_{thr}$. Asterisks denote the spectral peak data points.

As mentioned above, the fact that the dimensional peak spectral values work well in our case is apparently because the peak frequencies for our six records are similar. In a general case, the procedure to represent the breaking rates, breaking severity or the dissipation rates across the spectrum should be as follows. The dimensionless saturation threshold value of $\sqrt{\sigma_{thr}(f)} = 0.0223-0.0254$ should be used to obtain the dimensional spectral threshold $F_{thr}(f)$ at a frequency *f*. If *f*= f_p, a linear dependence in terms of $F(f)-F(f)_{thr}$ can then be used to obtain a property. If *f*> f_p, a cumulative term should be added to the linear term (see (2)). In deep water, if *f*< f_p, the spectral density will drop rapidly below the threshold and no breaking and no dissipation due to breaking will occur at those scales. In finite depth water, (7)-like dependence should be adjusted to account for the bottom influence (Babanin et al. 2001). Such an adjustment term may become essential for longer waves below the spectral peak and result in a significant amount of breaking at those scales (see Fig. 2).

DISCUSSION

The spectral dissipation rates are usually verified by incorporating them into the right-hand side of (1) and checking against experimentally known dependences for the waves described by the spectrum *F(f)* in the left-hand side. Given the uncertainties of both the experimental dependences and, in particular, the other source functions in (1), this method is very indirect and quite unreliable.

Banner (presentation at WISE-2004, Reading, England) suggested that the dissipation functions have to be verified against their ability to conform with known spectral properties of breaking waves. It is believed that most of the wave energy dissipation, at all scales, is due to wave breaking, and therefore this should be a much more direct manner of verification.

One of such whitecapping properties, which can be easily converted into the spectral dissipation is • *(c),* the average length per unit area per unit interval of phase speed *c* (Phillips, 2001). Melville and Matusov (2002) experimentally obtained the spectral distribution of • *(c)* as a function of *c:*

$$\Lambda(c)\left(\frac{10}{U_{10}}\right)^3 = 3.3\times10^{-4} e^{-0.64c} \qquad (8)$$

which can then be converted into a dissipation function

$$S_{ds}(c) = b\rho_w g^{-1} c^5 \Lambda(c)\left(\frac{10}{U_{10}}\right)^3 \qquad (9)$$

and

$$S_{ds}(f) = \frac{g}{2\pi}\frac{1}{f^2} S_{ds}(c). \qquad (10)$$

The Melville & Matusov function (8)-(9) is based on dissipation due to breaking only. In nature, turbulent viscosity can play a role. This role would be minimal at the spectral peak where most of the dissipation, if any, would be due to the dominant breaking. At smaller scales, however, this role could be significant where the induced dissipation was observed by Young and Babanin (2005). This induced dissipation can be caused by forced breaking of shorter waves due to the dominant breaking, or by enhanced turbulent viscosity due to the dominant breaking, or both.

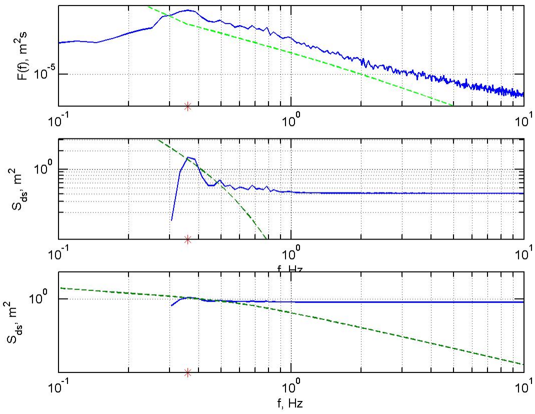

Fig. 5. (top panel) Wave spectrum (No.1, Table 1) and dimensional spectral threshold (dashed line). (middle panel) Close up of the bottom panel. (bottom panel) Young&Babanin (2005), solid line, and Melville&Matusov (2002), dashed line, dissipation functions

The enhanced level of the induced breaking is shown by Manasseh et al. (2005) and in Figs. 2 and 4. In Fig. 5, dissipation (2) is compared with dissipation (8)-(10). The coefficient *b=0.1* was chosen in (9) to make the two dissipations match at the spectral peak. The two dissipations agree quite well while the cumulative term is small, but diverge very significantly at the scales where the induced dissipation dominates (plateau in Young & Babanin S_{ds}). As is mentioned above, the experimental coefficients in both (2) and (9) need further investigation, but in any case the importance of the turbulent viscosity contribution to the cumulative dissipation is evident.

CONCLUSIONS

Two different experimental methodologies were used to investigate the spectral dissipation function on the basis of field data. The first employed the acoustic noise spectrograms to identify segments of breaking and non-breaking dominant wave trains (Babanin et al. 2001, Young and Babanin 2005). An independent second approach was a passive acoustic method based on detecting occurrences of individual bubble-formation events and the bubble sizes. This method was found promising for obtaining both the rate of occurrence of breaking events at different wave scales and the severity of wave breaking (Manasseh et al. 2005). A combination of the two methods can lead to direct estimates of the spectral distribution of wave dissipation.

If the wave energy dissipation at each frequency were due to whitecapping only, it should be a function of the excess of the spectral density above a dimensionless threshold spectral level, below which no breaking occurs at this frequency (Banner et. al. 2000, Babanin et al. 2001, Banner et al. 2002). This was found to be the case around the wave spectral peak.

A more complex mechanism appears to be driving the dissipation at scales different to those of the dominant waves. Dissipation at a particular frequency above the peak demonstrates a cumulative effect, depending on the rates of spectral dissipation at lower frequencies. This experimentally observed effect is qualitatively similar to the conclusion inferred by Donelan (2001). In terms of the dissipation function S_{ds} such an effect will mean a two-phase behaviour: S_{ds} being a simple function of the wave spectrum at the spectral peak and having an additional cumulative term at all frequencies above the peak (2).

The nature of the induced dissipation above the peak can be due to either enhanced induced wave breaking or additional turbulent eddy viscosity or both. Comparison of Young and Babanin (2005) and Melville and Matusov (2002) dissipations indicates that the turbulent viscosity becomes significant when the cumulative term dominates.

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REFERENCES

Babanin, A.V. and Soloviev, Yu.P. 1998. Variability of directional spectra of wind-generated waves, studied by means of wave staff arrays. *Marine & Freshwater Res*., **49**, 89-101.

Babanin, A.V., Young, I.R., and Banner, M.L. 2001. Breaking probabilities for dominant surface waves on water of finite constant depth. *J. Geophys. Res*., **C106**, 11659-11676.

Banner, M.L., Babanin, A.V., and Young, I.R. 2000. Breaking probability for dominant waves on the sea surface. *J. Phys. Oceanogr* ., **30**, 3145-3160.

Banner, M.L., Gemmrich, J.R., and Farmer, D.M. 2002. Multi-scale measurements of ocean wave breaking probability. *J. Phys. Oceanogr.*, **32**, 3364-3375.

Donelan, M.A. 2001. A nonlinear dissipation function due to wave breaking. In *ECMWF Workshop on Ocean Wave Forecasting, 2-4 July, 2001*, Series ECMWF Proceedings, 87-94.

Hwang, P.A. and Wang, D.W. 2004. An empirical investigation of source term balance of small scale surface waves. *Geophys. Res. Lett.*, **31**, L15301, doi:10.1029/2004GL020080.

Manasseh, R., Babanin, A.V., Forbes, C., Rickards, K., Bobevski, I., and Ooi, A. 2005. Passive acoustic determination of wave-breaking events and their severity across the spectrum, *J. Atmos. Ocean. Tech.*, submitted.

Melville, W.K. and Matusov, P. 2002. Distribution of breaking waves at the ocean surface. *Nature*, **417**, 58-63.

Phillips, O.M. 1984. On the response of short ocean wave components at a fixed number to ocean current variations. *J. Phys. Oceanogr.*, **14**, 1425-1433.

Phillips, O.M., Posner, F.L., and Hansen, J.P. 2001. High range resolution radar measurements of the speed distribution of breaking events in wind-generated ocean waves: Surface impulse and wave energy dissipation rates. *J. Phys. Oceanogr.*, **31**, 450-460.

Young, I.R., Banner, M.L., Donelan, M.A., Babanin, A.V., Melville, W.K., Veron, F., and McCormic, C. 2005. An integrated system for the study of wind wave source terms in finite depth water, *J. Atmos. Ocean. Tech.*, in press.

Young, I.R. and Babanin, A.V. 2005. Spectral distribution of energy dissipation of wind-generated waves due to dominant wave breaking, *J. Phys. Oceanogr.*, submitted.