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Variability of directional spectra of wind-generated waves, studied by means of wave staff arrays

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Abstract. Parameterization of the directional distribution of the wave energy obtained by means of arrays of wave probes is suggested. The parameterization encompasses a wide frequency band both above and below the peak frequency as well as a broad range of wave ages. The parameterization is developed for the integral width of the directional spectrum, which does not depend on a specific form of the approximating function. The width of the directional distribution depends on the stage of wave development and relative frequency. The narrowest directional spectra are observable, on average, 5% below the peak frequency. Above that frequency, the width of the directional spectrum increases as the frequency rises, and the rate of widening does not depend on the wave age. On the forward face of the spectrum, the rate of widening of the directional distributions towards low frequencies depends strongly on the stage of wave development and, at early stages, the directional spectra in this range are unchangeable. In general, the directional distributions of wave energy become narrower as the waves develop.

Introduction

The frequency-directional spectrum usually employed to study spatial characteristics of wind waves can be presented in the form

$$\chi(f,\theta) = S(f)\phi(f,\theta) \qquad (1)$$

where f is frequency, θ is direction, $S(f)$ is frequency spectrum, and $\phi(f,\theta)$ is a function characterizing the directional distribution of spectral components. At present, the directional distribution function is less studied than $S(f)$ because it generally requires far more complex methods of measurement and data processing.

According to experimental results, in the case of steady winds, the principal travel direction of waves coincides with the wind direction. The narrowest directional distributions are usually accepted to correspond to the peak frequency f_m, and they broaden for frequencies both greater than and less than f_m (e.g. Mitsuyasu *et al.* 1975; Davidan *et al.* 1978; Hasselmann *et al.* 1980; Holthuijsen 1983; Donelan *et al.* 1985). Since, in practice, it is sufficient to know the average directional distribution of energetic waves in a broad range of conditions, $\varphi \sim \cos^n\theta$ is usually used where n is 2 to 4. Similar estimates are often employed in wave models (Hasselmann *et al.* 1976).

It is difficult to draw definite conclusions on the character of the function $\phi(f,\theta)$ based on theoretical representations. According to Hasselmann *et al.* (1976), evolution of the wind wave field is described by the transfer equation in which special attention is given to non-linear interactions of spectral components. Joint analysis of experimental data with functions of energy sources led to the conclusion that the stabilizing influence of non-linear interaction helps retain the self-similar form of the frequency spectra of developing waves (JONSWAP spectrum). Based on the same consideration, a similar assumption was made for the directional distribution which is in general supposed to be determined by the ratio f/f_m and stage of wave development U/c_m (U is wind speed and c_m is phase speed of waves with frequency f_m). Experimental evidence for this contention is available in Hasselmann *et al.* (1980) where it is concluded that the directional distribution at frequency f_m is not dependent on wave development stage. However, this result obtained for values $U/c_m \sim 1$ does not necessarily apply in a wide range of possible values of U/c_m, since directional distribution is determined by the joint action of various physical processes, e.g. wind-induced energy flux, dissipation and non-linear interactions, the relative contribution of which varies at different stages of wave development.

As a result of the theoretical works of Zakharov and Smilga (1981), Komen *et al.* (1984) and Tsimring (1989), properties of directional distributions have become more important for understanding of the basic physical mechanisms of generation and evolution of wind waves. Analysis of the kinetic equation by Zakharov and Smilga (1981) showed that non-linear interactions result in an energy 'spread' in various directions. Later, Tsimring

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(1989) used the kinetic equation, taking into account the processes of non-linear resonant wind–wave interactions and obtained self-similar solution for the directional spectrum with width decreasing in time as $t^{-\frac{1}{3}}$. A computer simulation of the stationary solution of the transfer equation for fully developed waves with the Pierson–Moscowitz spectrum was developed by Komen *et al.* (1984); this demonstrated that a solution exists only under certain parameters of the directional distribution form, which significantly differ from those proposed by Hasselmann *et al.* (1980). In that 1984 paper it was for the first time proposed that the narrowest directional spectra may not be located at the peak frequency f_m.

Accuracy of experimental estimates of the function $\phi(f, \theta)$ depends on the choice of methods for data measurement and processing (Young 1994). The two most widely adopted *in situ* methods are based on measurements of either wave slopes in a point or simultaneous surface elevations at several points. The wave slopes are measured by means of buoys of special construction, pitch-and-roll and cloverleaf, within the frequency range 0·05 to 0·5 Hz (Longuett-Higgins *et al.* 1963; Ewing 1969). The method of data processing is based on the calculation of parameters of an assigned function $\phi(f, \theta)$ and its accuracy depends on the degree of compliance of a model and properties of the process itself. These conditions explain the limited range of frequencies for which the most well-known approximations of the directional distribution (Mitsuyasu *et al.* 1975; Hasselmann *et al.* 1980) were obtained. Later, more effective methods of estimation of $\phi(f, \theta)$ based on measurements of wave slope were proposed by Oltman-Shay and Guza (1984) and Lygre and Krogstad (1986).

In another, usually more precise, method, sensors fixed to a stationary base are employed for measurements of directional spectra by means of arrays of wave staffs. Elaborate mathematical methods of estimation of the directional characteristics, which are reviewed in Regier and Davis (1977), allow us to determine in advance, using numerical modelling, the optimum geometry of an array. At present, only a few estimates obtained by this method are available, and they reveal strong variability of $\phi(f, \theta)$ (Leikin and Rozenberg 1971; Fujinawa 1975; Davis and Regier 1977; Pawka 1983; Donelan *et al.* 1985; Babanin and Soloviev 1987).

An independent branch of directional spectra research is remote sensing (e.g. Jackson *et al.* 1985; Beal 1991) but this is outside the scope of the present paper.

The objective of this work was to study properties of the directional distribution of wind wave energy in a wide range of conditions, under steady wind conditions. Wave staff arrays used varied for different wave-length bands. The spectra were obtained in the following range of wave development stages U/c_m, relative frequency f/f_m and wave steepness $\varepsilon = \eta/\lambda_m$ (here U is wind speed at the standard height of 10 m, wave length λ_m corresponds to the peak frequency f_m, and η^2 is wave variance):

$$1 \cdot 1 \le U/c_m \le 5 \cdot 3 ; \tag{2}$$

$$0 \cdot 7 \le f/f_m \le 3 \cdot 5 ; \tag{3}$$

$$0 \cdot 5 \le \varepsilon \cdot 10^2 \le 1 \cdot 5 . \tag{4}$$

In situ data

Measurements were performed in different regions of the Black Sea (Fig. 1), mainly in the north-western part of the sea. A detailed description of the region of measurements is provided by Efimov *et al.* (1986). The bottom topography in this region is approximately level with 30 m water depth and 30 to 150 km distance to the shore. Anemorumbographs M62-MP, mounted at heights from 20 to 70 m, were employed for wind measurements. Wind speed and direction were recorded continuously with 1 min averaging.

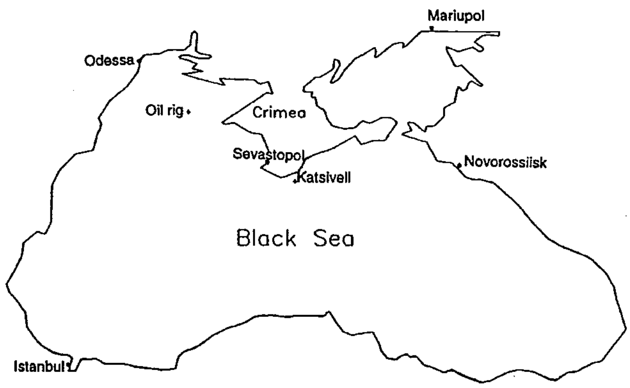

Fig. 1. Locations of the research platforms in the Black Sea.

Steady speed and direction of winds served as the main criteria for selection of measurement data for wind waves. Cases of preceding changes of wind direction or attenuation of wind were rejected. Analysis of meteorological charts showed that, under steady winds with a speed in excess of 6 m s^{-1} and a direction in the sector from north to south-east, the wind field can be treated as stable in a perpendicular direction from the shore.

Wire resistive wave gauges, with nichrome wire of ~0·3 mm diameter and 10 m length (Verkeev *et al.* 1983), were employed for reading surface elevation. Arrays consisted of 4–6 sensors with a distance of 0·6 to 20 m between them. The radiation pattern and resolution of the arrays was adjusted for a chosen sector of wave directions and range of wave lengths. Signals were synchronized and recorded on an analogue recorder or in digital form on magnetic tape. In the first case, the accuracy was ~2% of wave amplitude, and in the second it was ~1 mm.

Since very well developed waves (U/c_m at 1–2) occur in the open sea with stable wind, the measurements in the coastal part of the Black Sea (Katsiveli village) were performed from a platform mounted at the depth of 30 m to obtain data at initial stages of development. With offshore winds up to 15 m s^{-1}, characteristic values of U/c_m were 3–5 at fetches of 0·6 to 1 km depending on wind speed and direction. In general, measurement methods were the same as those in the open sea, with the exception of size of the arrays which consisted of four sensors with bases of 1–7 m.

About 30 records were selected for processing after analysis of meteorological conditions and visual observations of several hundred wave records. In further analysis of frequency-directional spectra, records that revealed the presence of swell were excluded.

Table 1. Summary of data used

U, wind speed at 10 m height; η^2, wave variance; f_m, peak frequency; U/c_m, parameter of wave development stage; ϵ, wave slope

No. of record	U (m s^{-1})	η^2 (cm^2)	f_m (Hz)	U/c_m	$\epsilon \times 10^2$
1	6·0	122	0·38	1·54	1·02
2*	9·2	515	0·23	1·36	0·77
3	8·0	475	0·28	1·45	1·10
4	10·6	785	0·23	1·58	0·95
5	11·4	1270	0·20	1·46	0·91
6*	15·1	3790	0·18	1·69	1·28
7	7·0	6	0·83	3·72	1·08
8*	9·2	9	0·78	4·62	1·17
9	10·8	22	0·68	4·73	1·39
10	11·8	23	0·70	5·29	1·51
11*	9·0	1000	0·20	1·16	0·81
12	10·3	1035	0·18	1·18	0·67
13	8·3	487	0·20	1·06	0·57
14	11·4	1288	0·20	1·46	0·92
15	8·6	539	0·24	1·32	0·86
16	9·2	331	0·22	1·30	0·56
17	10·3	1121	0·20	1·32	0·86
18	10·0	1028	0·20	1·28	0·82
19	8·5	351	0·21	1·15	0·53
20	10·2	899	0·22	1·44	0·93
21	11·2	1820	0·20	1·44	1·09
22	13·0	1323	0·18	1·49	1·76
23	9·2	432	0·27	1·78	0·97

* Records employed only for parameterization of the dependence on U/c_m.

Spatial characteristics of short wind waves can be heavily distorted by surface currents which were measured regularly with submerged floats. In general, prolonged three-hourly measurements of surface currents in the Black Sea (Babanin 1988) demonstrated weak (on average 1% of the wind speed U) wind-induced current. Influence of similar currents on directional spectra is insignificant even at relatively high frequencies. For the short fetches, records obtained while the currents were absent or weak and perpendicular to the wind were selected from the data.

In all, 23 records were selected for final processing (Table 1); several were records for which reliable estimates of $\phi(f,\theta)$ in a wide range of f/f_m were not obtained owing to a variety of reasons, although they were employed in the analysis of the dependence on U/c_m.

Processing methods

At stage one of processing, estimates for elements of the cross-spectral matrix were obtained by traditional methods of spectral analysis (Bendat and Piersol 1974). The parameters were as follows: durations of records were from 6 min up to several hours, sampling intervals were 0·1–0·2 s, frequency resolution was 0·017 Hz and angular resolution was 10°.

At the next stage, estimates of $F(\mathbf{k}, f)$ according to the maximum likelihood method (MLM) (Capon 1969; Isobe *et al.* 1984) were calculated at a frequency for wave numbers k at short bands around wave number k_0 following from the linear dispersion relation $(2\pi f)^2 = gk_0$:

$$F(\mathbf{k}, f) = \frac{Q(f)}{\Sigma_m \Sigma_n C_{mn}^{-1}(f) \exp(i\mathbf{k}(\mathbf{x}_n - \mathbf{x}_m))} \quad (5)$$

where $Q(f)$ is a normalization factor such that the total energies of the wave-number and frequency spectra are equivalent, $C_{mn}(f)$ values are cross-spectra between any two of the wave gauges, and $\mathbf{x}_i$ is the vector of a gauge position. The connection between spectra $F(\mathbf{k}, f)$ and $\chi(f,\theta)$ is determined by the dispersion relationship, which was not studied specially in this investigation (see Efimov and Babanin 1990). Note, however, that the estimates of $F(\mathbf{k}, f)$ are in good agreement with the dispersion relation for the purpose of the present work.

The directional distribution $\phi(f,\theta)$ was determined as the cross-section of the spectrum $F(k_0, \theta, f)$ at a particular frequency. Here, k_0 corresponds to the spectrum maximum at defined f. For convenience of comparisons, all directional distributions were normalized with the maximum value in the dominant direction in which the waves were travelling, i.e. $K(f,\theta_0) = 1$. Connection of $K(f,\theta_0)$ with the function $\phi(f,\theta)$ in Eqn (1) is determined from the conditions of the normalization

$$\int_{-\pi}^{\pi} \phi(f,\theta) d\theta = 1, \phi(f,\theta) = A(f) K(f,\theta),$$

$$A^{-1} = \int_{-\pi}^{\pi} K(f,\theta) d\theta . \quad (6)$$

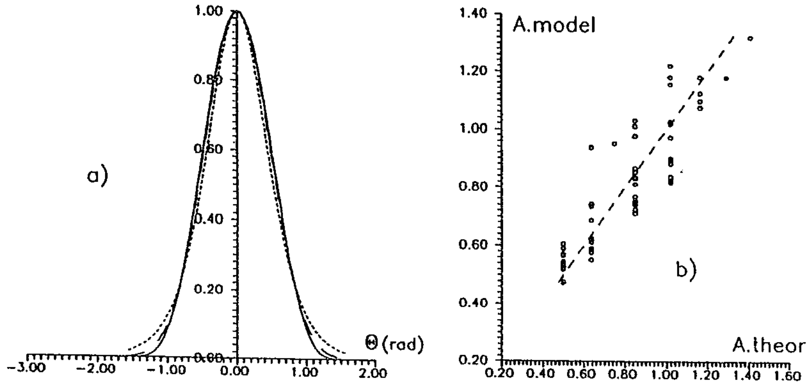

Fig. 2. *(a)* Comparison of directional spectra models (7)–(9). *(b)* Modelling scattering of the parameter A (6).

The estimates were averaged in the band of the spectral window $\Delta f = 0{\cdot}05$ Hz. More than 200 independent estimates of directional distributions were obtained, and 125 stable estimates were employed for further analysis.

Obviously, with a limited number of sensors available, it is impossible to provide reliable resolution over a wide range of wave lengths of arbitrary direction. The optimum positions of sensors were determined by modelling. As it is accepted, $\phi(f,\theta)$ has a unimodal shape for steady wind; the model spectrum was taken in the form

$$\frac{kF(\mathbf{k},f)}{S(f)}\Big|_{\mathbf{k}(f)=\mathbf{k}_0} = \begin{cases} A(n)\cos^n(\theta-\theta_0)\delta(k-k_0), & |\theta-\theta_0| \le \frac{\pi}{2}, \\ 0, & |\theta-\theta_0| > \frac{\pi}{2}, \end{cases} \tag{7}$$

where δ is the delta-function. Accurate values of cross spectra for a given array were calculated by means of inverse Fourier transformation. They were then employed to estimate the model spectrum according to the MLM method. Geometry of the array was adjusted to provide steady resolution for a predetermined frequency band taking f_m and θ_0 as typical of a region of measurements. It is worth mentioning that a set shape of model directional spectrum does not affect the final result. Any function with spatial scales typical of wind waves could be taken as a model. For example, functions of the form

$$K(f,\theta) = \cos^{2s}\frac{\theta-\theta_0}{2} \tag{8}$$

(Longuett-Higgins *et al.* 1963) or

$$K(f,\theta) = \mathrm{sech}^2[\beta(\theta-\theta_0)] \tag{9}$$

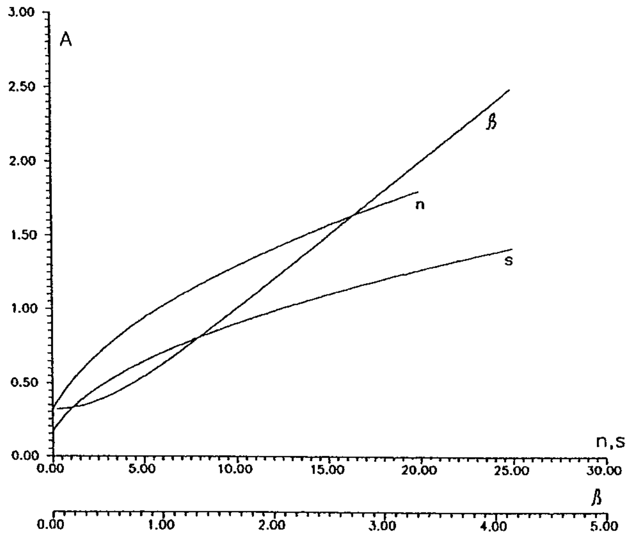

Fig. 3. Interdependences of directional width parameter A on the directional spread parameters n, s, and β.

(Donelan *et al.* 1985) as well as Eqn (7) are often employed. Eqns (7)–(9) do not differ significantly (Fig. 2*a*).

Let us consider one more important step of processing the data. Usually, parameterization of values n, s or β in (7), (8) and (9) is an objective in investigating $K(f,\theta)$. However, this can present problems in selection of intervals θ–θ_0 to estimate the coefficients in (7), (8) and (9) and possible distortions of the form of $K(f,\theta)$. On the basis of modelling, it was found that the area under the graph of $K(f,\theta)$, i.e. the integrand of $K(f,\theta)$, is less sensitive to small distortions of the form of directional distributions, such as small asymmetry of the form, small false peaks, etc. Thus, the normalization factor A, with its value determined by Eqn (6) in inverse proportion to the area of the $K(f,\theta)$ function, was taken as a directional distribution parameter. Such an approach allows a researcher to avoid selecting a specific form of $K(f,\theta)$ and still determines parameters of functions of type (7), (8) and (9) (Fig. 3):

$$A=\frac{\Gamma(n/2+1)}{\pi^{\frac{1}{2}}\Gamma(n/2+1/2)},\ A=\frac{\Gamma(s+1)}{2\pi^{\frac{1}{2}}\Gamma(s+1/2)},$$

$$A=(\beta/2)\coth(\pi\beta/2) \qquad (10)$$

where Γ is the gamma-function. Results of the modelling scattering of parameter A can be seen in Fig. 2*b*.

For practical purposes, it is possible to approximate the dependences (10) with less than 5% error for the intervals of the most probable values $2\le n\le 10$, $5\le s\le 20$ and $1\cdot 2\le\beta\le 3\cdot 4$:

$$n\approx 12\cdot 4A-6\cdot 3,\ s\approx 24\cdot 1A-11\cdot 4,$$

$$n\approx 0\cdot 5s-0\cdot 5,\ \beta\approx 2A\,. \qquad (11)$$

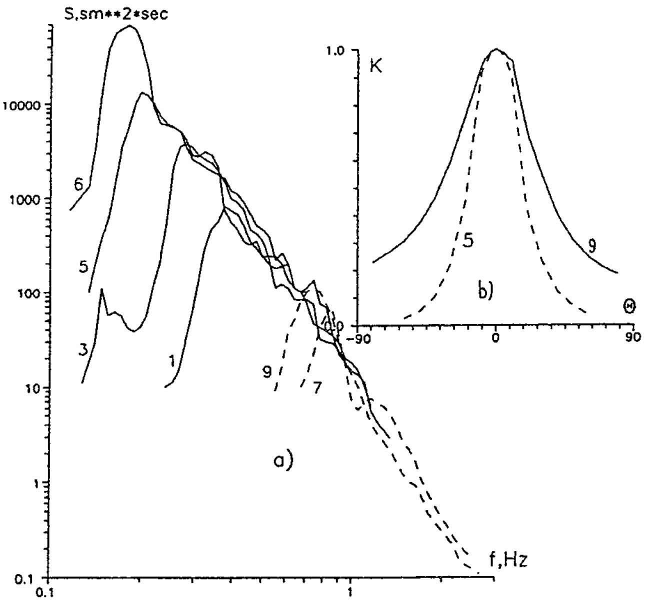

Fig. 4. Samples of *(a)* frequency and *(b)* directional spectra used in the paper. Numbers of records correspond to those in Table 1. Directional spectra are taken at a relevant peak frequency.

Experimental results

Data listed in Table 1 were obtained at wind speeds of $\approx$ 6–15 m s^{-1} at various stages of wave development. Fig. 4*a* shows several frequency spectra averaged over 4–5 synchronous realizations. Jointly with the table data, frequency spectra demonstrate a variety of wind wave parameters. Examples of directional distribution estimates for various stages of wave development are given in Fig. 4*b*. Directional distributions are of unimodal almost symmetric form relative to the wind direction, typical of generation conditions by a steady wind. Note the narrowing $K(f,\theta)$ for more developed seas.

Preliminary analysis of all the estimates for directional distribution showed that:

(1) Directional maxima $K(f,\theta)$ for the selected situations of stable wind coincide with wind direction with an accuracy of directional quantification of $\Delta\theta=\pm 10°$ in all the bands of frequencies.

(2) The narrowest directional distributions are, on average, located at frequency $f_0=0\cdot 95f_m$, below the peak frequency f_m.

In general, these conclusions agree with others available on the properties of directional distribution, though the average narrowest directional spectrum is located slightly below the peak frequency.

Behaviour of directional spectra below and above f_0 is essentially different. Firstly, consideration is given to the region of $f\ge f_0$.

For further analysis, we provide the known parameterizations of $K(f,\theta)$ proposed in Mitsuyasu *et al.* (1975), Hasselmann *et al.* (1980) and Donelan *et al.* (1985) subsequently for the rear face of a frequency spectrum:

$$s=11\cdot 5\left(\frac{U}{c}\right)^{-2\cdot 5},\quad f/f_m\ge 1, \qquad (12)$$

$$s=9\cdot 77\left(\frac{f}{f_m}\right)^{-2\cdot 33-1\cdot 45(U/c_m-1\cdot 17)},\quad f/f_m\ge 1\cdot 05, \qquad (13)$$

$$\beta=\begin{cases}2\cdot 44\left(\dfrac{f}{0\cdot 95f_m}\right)^{-1\cdot 3}, & 0\cdot 95\le f/f_m\le 1\cdot 6,\\ 1\cdot 24, & f/f_m\ge 1\cdot 6\end{cases}. \qquad (14)$$

Donelan *et al.* (1985), whose data extended only up to $f/f_m=1\cdot 6$, assumed β to be constant beyond this frequency. Banner (1990), on the base of stereo photography data, concluded that this assumption was unreasonable and proposed his high-frequency extension of dependence (14):

$$\beta=10^{-0\cdot 4+0\cdot 8393\exp(-0\cdot 567\ln((f/f_m)^2))},\ f/f_m>1\cdot 6. \qquad (15)$$

In spite of using parameters of the specific forms (8)–(9), approximations (12)–(15) describe variability of directional spectrum width rather than form, because the exact functional shape of the spectrum is unknown, and it is expedient to use the integral width parameter A (6) for comparisons and further analysis. This does not place any restrictions on a specific form of $K(f,\theta)$.

It is noteworthy that in Mitsuyasu *et al.* (1975) and Hasselmann *et al.* (1980) location of the narrowest directional spectra is near the peak frequency f_m, but

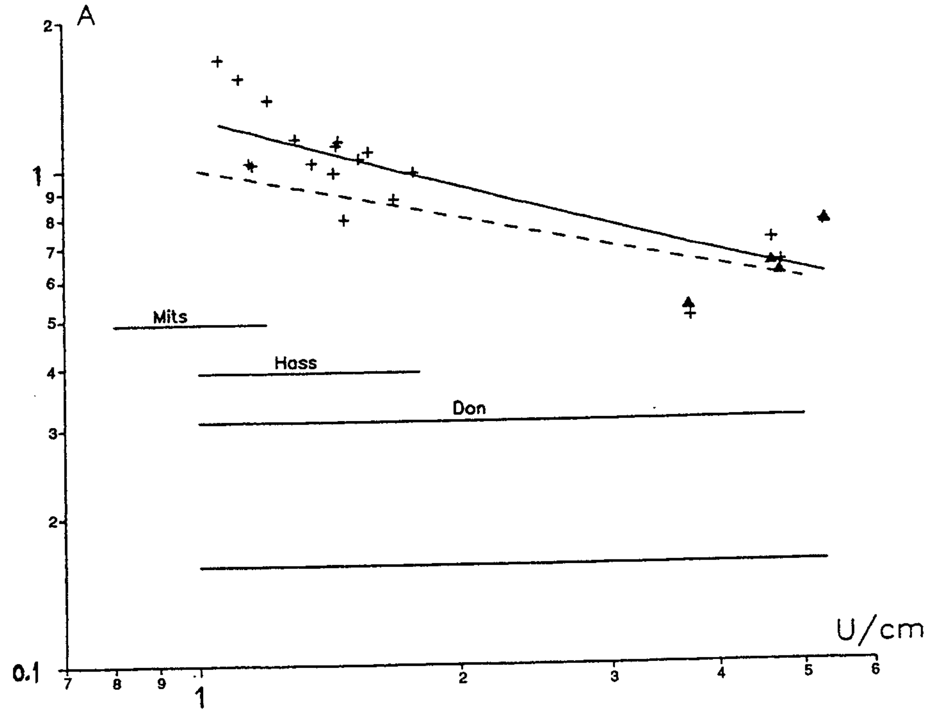

Fig. 5. Dependence of parameter A on wave development stage U/c_m at peak frequency f_m. Dashed line is the dependence of the parameter A on parameter U/c if directional spectra at all frequencies f are taken into account. Level of isotropic directional spectra is shown at the bottom as well as data ranges of the parameterizations of Mitsuyasu *et al.* (1975), Hasselmann *et al.* (1980) and Donelan *et al.* (1985). Triangles show positions of the theoretical spectra (31) with $a = 2.6 \times 10^{-3}$.

Donelan *et al.* (1985) conclude that this point is somewhat below f_m at frequency $0 \cdot 95\ f_m$ which corresponds to the frequency f_0 above. Komen *et al.* (1984) demonstrated that, for the existence of fully developed frequency spectra, the narrowest directional spectrum should be located at frequency $0 \cdot 95\ f_m$. In this sense, the $0 \cdot 95\ f_m$ location of the spectra used in this paper is an average, not an absolute empirical frequency. It is possible for the narrowest directional spectra to migrate along the frequency axis as waves develop. From the experimental data one can conclude that the narrowest directional spectra are not necessarily located at the peak frequency f_m as was previously supposed.

Parameterizations (12)–(15) differ qualitatively. According to (12) (Mitsuyasu *et al.* 1975), directional spectrum width at a fixed frequency does not depend on other frequencies and is simply determined by the wind. Hasselmann *et al.* (1980) assumed that if directional spectrum is mainly formed by non-linear interactions, then its width at a fixed frequency f should depend strongly on dimensionless frequency f/f_m and less on development stage parameter U/c_m; however, if the main forming factor is wind influence, then dependence of the width should reduce to similar dependence to (12) with the single parameter U/c. It seemed that parameterization (13) of Hasselmann *et al.* (1980) corroborated their conclusion about the predominant role of non-linear interactions under a weak wind influence. However, parameterization (14) of Donelan *et al.* (1985) altogether denies participation of wind in the formation of directional distribution, and postulates that width of a directional spectrum for any frequency depends only on its position relative to a peak frequency. Donelan *et al.* (1985) emphasize that strong conformity between parameters f/f_m and U/c_m and the source functions cannot be implied because they both pertain to wave steepness ϵ which, in turn, influences wind input, non-linear interactions and dissipation.

We could not find any real impact of wave steepness ϵ in the parametric dependences, in spite of the wide variations of the latter in the data used (4). That does not mean that directional spectra width has no dependence on the steepness; however, if such dependence exists, it must be very weak. To obtain quantitative data would require extensive experimental material.

Dependence of directional spectra width on wave development stage

To demonstrate dependences of width of directional spectra on wave development stage, Fig. 5 shows values of parameter A at $f/f_m = 1$ as a function of U/c_m. Ranges where Mitsuyasu *et al.* (1975), Hasselmann *et al.* (1980) and Donelan *et al.* (1985) obtained their parameterizations are also indicated.

The data of Mitsuyasu *et al.* (1975) were obtained for developed waves in the narrow band where U/c_m is $0\cdot7$–$1\cdot3$ and, possibly because of this, expression (12) contains only one parameter U/c, i.e. dependences on U/c_m and f/f_m coincide in the whole frequency band. Rapid broadening of the directional distribution as U/c_m increases results from formula (12) and would lead to directional spectra that are close to isotropic at $U/c_m \sim 5$.

However, Hasselmann *et al.* (1980) postulate independence of width of directional distribution on U/c_m for $f = f_m$ although almost all their data belong to the narrow interval $U/c_m = 1\cdot0$ to $U/c_m = 1\cdot4$.

Our data were obtained in a broader range of wave development stages (2). To approximate the dependence at $f = f_m$, the exponential function

$$A = a\left(\frac{U}{c_m}\right)^{-b} + (2\pi)^{-1} \tag{16}$$

was used, where the last term was introduced formally and corresponds to an isotropic distribution as $U/c_m \to \infty$. Coefficients in (16) and their RMS deviations were determined by the least-squares method:

$$a = 1\cdot12,\quad \sigma_{lna} = 0\cdot10,\quad b = 0\cdot50,\quad \sigma_b = 0\cdot11. \tag{17}$$

Ratios (12) and (16) are of the same functional form. To compare our data with dependence (12), dependence of A only on U/c, regardless of different wave development stages, was obtained from the full data set and is plotted in Fig. 5. The result was that the predicted values of A were lowered in the region of $U/c_m \sim 1$ in comparison with real data, thus indicating the necessity of involving the second parameter. Dependence of the directional spectra width on wave development stage (16)–(17) as obtained has intermediate behaviour compared with (12) and (13). Directional spectra even at initial development stages are not isotropic as would follow from (12) but, in contrast to (13), they can significantly alter at peak frequency.

Some explanations are necessary to meet the fact that approximation (14) (Donelan *et al.* 1985), which was obtained in quite a broad range of wave development stages, $U/c_m = 1$ to $U/c_m = 6$, does not include a parameter to account for wind and indicates that directional spectra are dependent only on dimensionless frequency f/f_m. In Donelan *et al.* (1985), when the width parameter s was calculated in the same way as in Mitsuyasu *et al.* (1975) and Hasselmann *et al.* (1980), using Fourier coefficients of the polar distribution, the dependence s on U/c_m took place. It vanished when s and then β were determined by the half-height of directional spectra. It may be that this transition not only reduces scattering but makes the dependence of spectra width less sensitive. Nevertheless, according to fig. 32 of Donelan *et al.* (1985), one can still conclude that β is dependent on U/c_m. When recalculated in terms of the parameter of integral width A (6), the latter changes from $1\cdot22$ for U/c_m values of 1–2 to $1\cdot07$ for U/c_m values of 4–6. Therefore, dependence of β on U/c_m in (14) must have been averaged.

Dependence of directional spectra width on dimensionless frequency

To approximate dependence of A on f/f_m, an exponential function, similar to (16), was used:

$$A = c\left(\frac{f}{f_m}\right)^{-d} + (2\pi)^{-1}. \tag{18}$$

The choice of function (18) is determined not only by analogy with (16), although that considerably simplifies construction of a final approximation; it is determined first of all by the criterion of variance minimum. For different realizations, exponents d are approximately equal, and no dependence of d on U/c_m or ϵ was found. Using (16)–(17) to exclude dependence on U/c_m, one can approximate connection of parameters A and f/f_m.

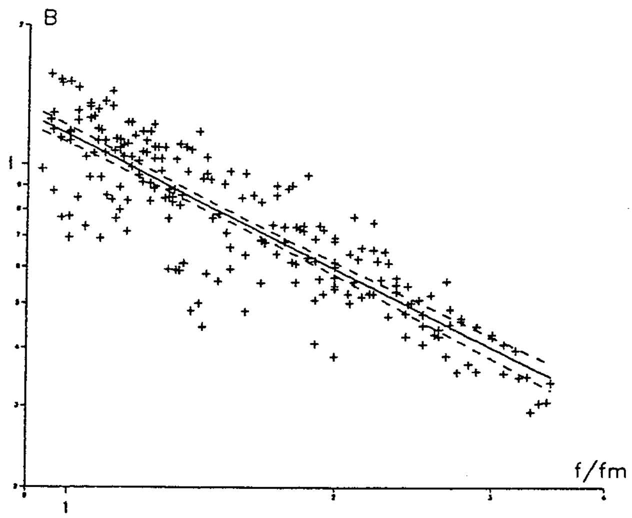

Fig. 6. Dependence of parameter B on relative frequency f/f_m. Dashed lines show 95% confidence intervals.

Fig. 6 shows approximation of dependence of $B = (A-(2\pi)^{-1})(U/c_m)^{0\cdot50}$ on dimensionless frequency f/f_m. Good agreement of experimental data with the resulting curve confirmed the appropriateness of this approach. The final formula is:

$$A = 1\cdot12\left(\frac{U}{c_m}\right)^{-0\cdot50}\left(\frac{f}{f_m}\right)^{-0\cdot95} + (2\pi)^{-1} \tag{19}$$

$(\sigma_{0\cdot95} = 0\cdot05,\ \sigma_{ln1\cdot12} = 0\cdot03)$.

Fig. 6 and Eqn (19) show permanent broadening of directional spectra with frequency; however, they do not lead one to expect that an isotropic level is feasible, at least in the interval of gravitational waves.

As was shown earlier for typical values of A, employment of (11) made it easy to proceed to more common presentations of directional distributions (7)–(9). For example,

$$s = 27 \cdot 0 \left(\frac{U}{c_m}\right)^{-0 \cdot 50} \left(\frac{f}{f_m}\right)^{-0 \cdot 95} - 7 \cdot 6. \quad (20)$$

From comparison of dependencies (12), (13) and (19) in Figs 7 and 8, the most obvious descrepancies arise in the region of large values of U/c_m, i.e. areas where the data of Mitsuyasu *et al.* (1975) and Hasselmann *et al.* (1980) are absent. The descrepancies decrease at $(U/c_m, f/f_m) \sim 1$; however, directional distributions (19) are narrower than with (12) or (13). This may be due to poorer resolution of the methods of obtaining directional spectra by means of a buoy and calculation of the parameter s using first Fourier coefficients of the polar distribution. Different roles of parameter U/c_m in (13) and (19) should be noted. If the value of U/c_m in (13) determines the rate of broadening of directional distributions in the region $f > f_m$, variations of $K(f, \theta)$ with the frequency in (19) does not depend on U/c_m.

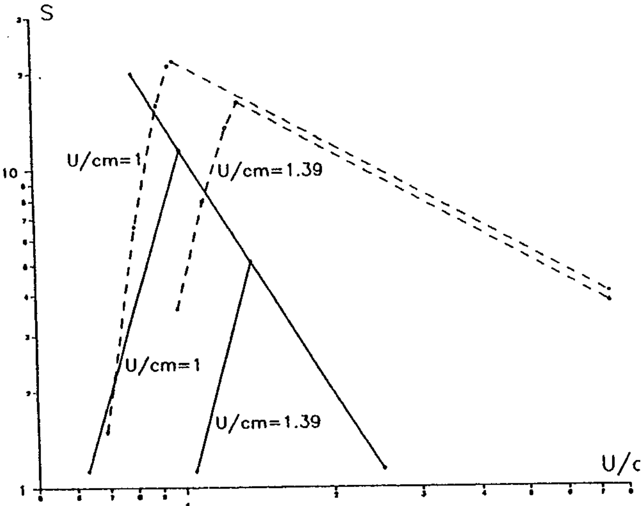

Fig. 7. Comparison of (dashed lines) results of the present paper with (unbroken lines) the results of Mitsuyasu *et al.* (1975).

Fig. 9 shows comparison of the present authors' parameterization with the parameterization of Donelan *et al.* (1985) and Banner (1990). As discussed above, dependence (14) must have averaged U/c_m and therefore dependence (19) for $U/c_m = 1$ is higher than (14), and the one for $U/c_m = 4$ is lower than (14).

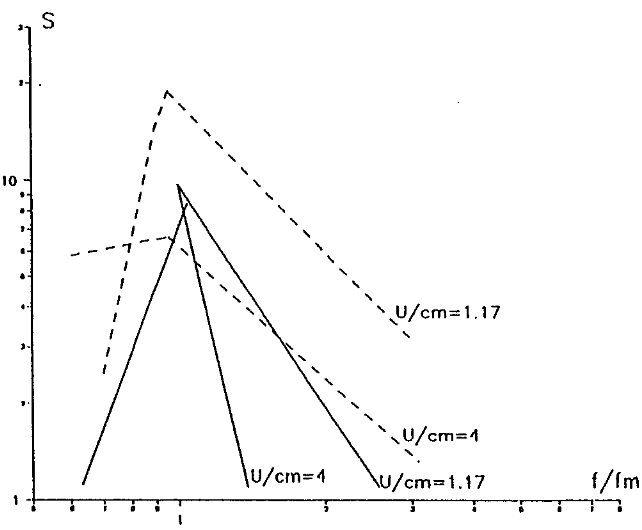

Fig. 8. Comparison of (dashed lines) results of the present paper with (unbroken lines) the results of Hasselmann *et al.* (1980).

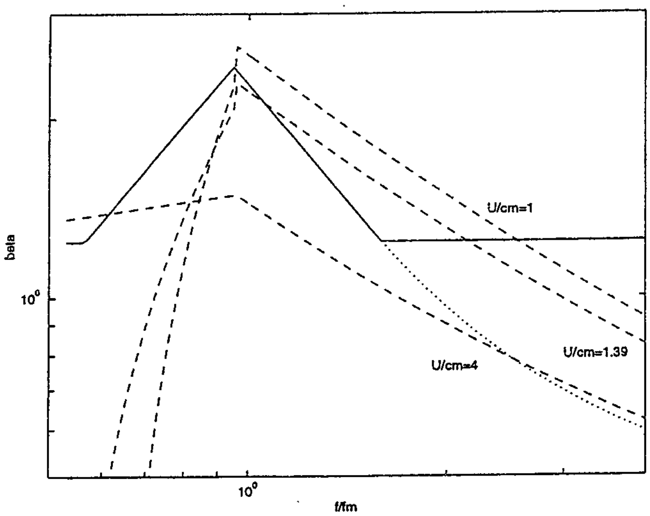

Fig. 9. Comparison of (dashed lines) results of the present paper with (unbroken line) the Donelan *et al.* (1985) dependence and (dotted line) the Banner (1990) high-frequency extension.

For our data, the average wave development stage for developed waves corresponds to $U/c_m = 1 \cdot 39$; this curve is also shown on Fig. 9 and conforms rather well to the dependence of Donelan *et al.* (1985). However, the present results showed that directional spectra continue to broaden up to $3 \cdot 5 f/f_m$, which was the highest frequency measured. These spectra agree with the Banner (1990) extension for the case of younger waves.

Holthuijsen (1983) and Ewing and Laing (1987) described another attempt to evaluate the width of directional spectra. Holthuijsen (1983) used stereophotography to obtain directional spectra at four stations for fetch-limited conditions. Obtained in developed seas, his observations agree fairly well with the conclusions in Mitsuyasu *et al.* (1975) and Hasselmann *et al.* (1980) for similar conditions, although providing rather narrow directional spectra for ideal wind situations.

Ewing and Laing (1987) obtained directional spectra near full wave development, by means of a buoy. They did not provide a final expression, but comparisons of their results with the conclusions of Hasselmann *et al.* (1980) and Donelan *et al.* (1985) and the present authors for fully developed seas shows that their directional spectra have intermediate width and that their average $-1 \cdot 33$ power in f/f_m-dependence is quite close to (19).

Directional spectra in the low-frequency region

On the forward face of the frequency spectrum, parameterizations of Mitsuyasu *et al.* (1975), Hasselmann *et al.* (1980) and Donelan *et al.* (1985) are as follows:

$$s = 11 \cdot 5\left(\frac{U}{c_m}\right)^{-7 \cdot 5}\left(\frac{U}{c}\right)^5, \quad f/f_m \leq 1, \tag{21}$$

$$s = 6 \cdot 97\left(\frac{f}{f_m}\right)^{4 \cdot 06}, \quad f/f_m \leq 1 \cdot 05, \tag{22}$$

$$\beta = \begin{cases} 2 \cdot 44\left(\dfrac{f}{0 \cdot 95 f_m}\right)^{1 \cdot 3}, & 0 \cdot 56 \leq f/f_m \leq 0 \cdot 95, \\ 1 \cdot 24, & f/f_m < 0 \cdot 56. \end{cases} \tag{23}$$

All the parameterizations here retain only dependence on dimensionless frequency f/f_m since (21) can be rewritten in the form

$$s = s_m \frac{f}{f_m}^5, \tag{24}$$

where $s_m = 11 \cdot 5(U/c_m)^{-2 \cdot 5}$ determines the width of directional spectrum at peak frequency f_m. However, the fact that the spectrum-broadening rate below the peak frequency does not depend on wave development stage U/c_m is not obvious. For developed waves, the directional structure at low frequencies is probably formed only by non-linear interactions, but for younger waves, wave components below peak frequency move much slower than wind, and are actively wind forced, so wind speed must be taken into account in a directional spectra-width parameterization.

In the present work, experimental data for frequencies $f < f_0$ split naturally in two groups based on U/c_m (Fig. 10):

$$1 < \frac{U}{c_m} < 1 \cdot 5,$$

$$\frac{U}{c_m} > 3. \tag{25}$$

In the intermediate region, data are practically absent. For $f \geq f_0$, that did not prevent the construction of the joint weak dependence (19), but for the low-frequency band, directional spectra behave in a qualitatively different manner in subranges (25*a*) and (25*b*).

Fig. 10*a* shows data from subrange (25*a*). There, directional spectra of developed waves broaden very quickly toward lower frequencies. To split data based on wave age within that group is not possible, and connection of the spectra width A and dimensionless frequency f/f_m is approximated by the linear function:

$$A/A_0 = 2 \cdot 11\frac{f}{f_m} - 0 \cdot 99 \tag{26}$$

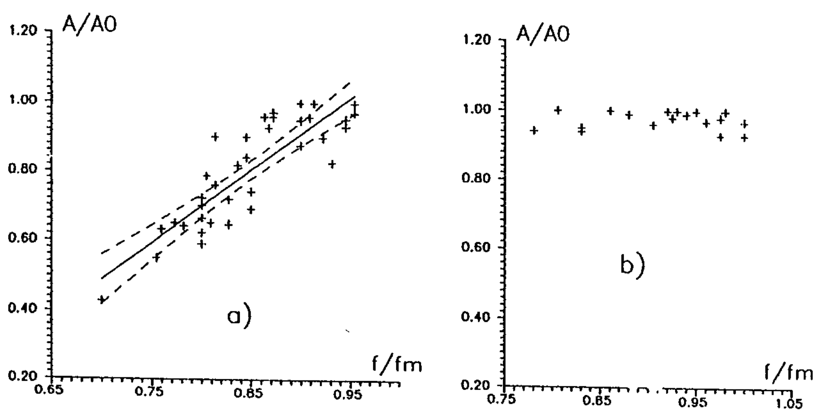

Fig. 10. Dependence of relative width of directional distributions A/A_0 on relative frequency f/f_m for $f < 0.95f_m$: *(a)* for developed waves ($1 \leq U/c_m \leq 1 \cdot 5$); *(b)* for young waves ($U/c_m > 3 \cdot 5$). Dashed lines show 95% confidence intervals.

($\sigma_{2\cdot 11} = 0\cdot 21$, $\sigma_{0\cdot 99} = 0\cdot 18$). Here, A_0 is the width of the narrowest directional spectrum in a realization, i.e. on average it is a width at frequency f_0. Empirical dependences (19) and (26) practically coincide at the frequency f_0.

Dependence (26) would intercept the isotropic level at $f/f_m = 0\cdot 54$, but at similar frequencies spectral density of wind-generated waves is hundreds of times less than at peak frequency f_m, i.e. the waves are practically absent. Below $0\cdot 7 f_m$ directional spectra were not possible to obtain. Dependences (21)–(23) differ little from (26); this corroborates the conclusion about rapid broadening of directional spectra in the low-frequency band for $U/c_m \sim 1$.

Fig. 10*b* shows data of the (25*b*) subrange for frequencies $f < f_0$. In this case, directional spectra for all practical purposes neither broaden nor become more narrow than at frequency f_0. Thus, wave development stage is a significant factor for the directional structure for low frequencies. A final parameterization within the interval $0\cdot 7 \le f/f_m \le 0\cdot 95$ is

$$A/A_0 = 2\cdot 05\left(\frac{f}{f_m}\right)^{\exp(1\cdot 39 - U/c_m)} - 1\cdot 05. \qquad (27)$$

The dependence on U/c_m in (27) is such that for $U/c_m = 1\cdot 39$, which is the average value of U/c_m for our developed wave data, Eqn (27) becomes (26) within error limits. For large values U/c_m, $A \approx A_0$ in accordance with the data of subrange (25*b*).

Thus, the parameterization of the width of the directional distribution of wind-generated wave energy in the range (2)–(4) of wave conditions is

$$A = 1\cdot 12\left(\frac{U}{c_m}\right)^{-0\cdot 50}\left(\frac{f}{f_m}\right)^{-0\cdot 95} + (2\pi)^{-1}, \quad f \ge 0\cdot 95 f_m,$$

$$A/A_0 = 2\cdot 05\left(\frac{f}{f_m}\right)^{\exp(1\cdot 39 - U/c_m)} - 1\cdot 05, \quad f < 0\cdot 95 f_m,$$

$$A_0 = 1\cdot 18\left(\frac{U}{c_m}\right)^{-0\cdot 50} + (2\pi)^{-1}, \quad f = 0\cdot 95 f_m. \qquad (28)$$

Discussion

Parameterization (28) allows one to conclude:

(1) The width of $K(f_0, \theta)$ in general decreases as waves develop.

(2) For $f > f_0$, the width of directional spectra increases with frequency, and the rate of broadening depends only on the location of a particular frequency relative to the peak frequency f_m and does not depend on the wave development stage parameter U/c_m.

(3) For $f < f_0$, the rate of $K(f_0, \theta)$ broadening to lower frequencies depends essentially on U/c_m and for $U/c_m > 3$ directional spectra stay practically constant relative to a spectrum at frequency f_0.

(4) The narrowest directional distribution at a fixed frequency f is observed in the case of $f = f_0$ independently of the stage of wave development.

The empirical parameterization allows calculation of $K(f, \theta)$ in a wide range of conditions but leaves unsolved the question of mechanisms of formation and maintenance of directional spectra.

It can be assumed that dissipation does not influence directional spectra significantly. Indeed, dependence on the wave steepness ϵ is not found and, hence, wave breaking does not play an essential role in $K(f, \theta)$ variability and viscous attenuation of the observed waves (as a rule not shorter than 1 m) is small.

Let us consider formation of the directional spectrum by constant wind, not taking into account its pulsations in speed and direction. Suppose, then, energy of wave components grows according to the Miles law

$$E = E_0 \exp(\gamma t) \qquad (29)$$

where E_0 is the energy at the initial moment of the wind action $t = 0$ with isotropic directional distribution, γ is the growth increment, and the directional distribution $K(f, \theta)$ at a point is a superposition of waves coming from various directions θ and having consequently different fetches $x/\cos\theta$, x is a distance offshore (Fig. 11). If the wind is normal to the shoreline, wave components travelling from θ direction are forced by the wind component $U\cos\theta$.

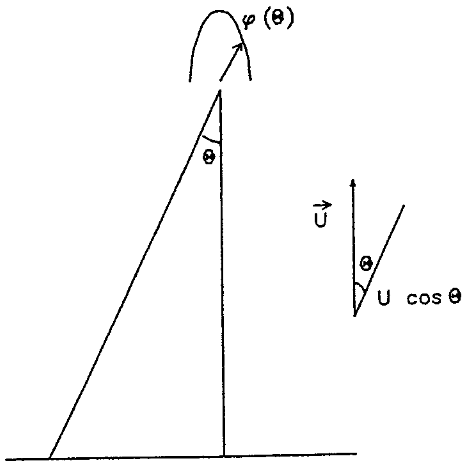

Fig. 11. Directional distribution around a point as a superposition of waves coming from various directions.

According to Mitsuyasu and Honda (1982)

$$\gamma/f = a\left(\frac{U}{c}\right)^2 \tag{30}$$

where $c = g/(2\pi f)$ is the phase speed of component f, a is a coefficient of proportionality and g is the gravitational constant. Direct estimates of a in Mitsuyasu and Honda (1982) by means of tank measurements yield $a = 5{\cdot}4 \times 10^{-3}$ for actively growing high-frequency components; their computations, based on the Miles theory, gave $a = 2{\cdot}6 \times 10^{-3}$ for this correspondent frequency band. To transit from friction velocity u_* to U, they used the approximate relation $U = 25u_*$.

From Eqns (29)–(30), we can obtain a directional spectrum formed by the action of an average wind only:

$$K(f,\theta) = \exp\left(\frac{af}{c_g}\left(\frac{U}{c}\right)^2 x(\cos\theta - 1)\right)$$

$$= \exp\left(\frac{akx}{\pi}\left(\frac{U}{c}\right)^2 (\cos\theta - 1)\right). \tag{31}$$

Here, development in time is replaced with development in space $t = x/c_g$, where c_g is group velocity, $k = (2\pi f)^2/g$ in accordance with the linear dispersion relation.

Thus, for experimental estimates of directional structure of the coefficient γ on measurements at a point under the accepted conditions, one may anticipate, according to (31),

$$\gamma = af\left(\frac{U}{c}\right)^2 \cos\theta\,. \tag{32}$$

Identical dependence was obtained by Plant (1980) with $a = (4{\cdot}0 \pm 2{\cdot}0) \times 10^{-3}$.

Note that the shape of the directional spectrum (31)

$$K(f,\theta) = \exp\Big(b\big(1 - \cos(\theta - \theta_0)\big)\Big), \tag{33}$$

where b is a parameter of the width, describes well the natural distributions, as the shapes (7)–(9) do, and is convenient in different applications even though it was obtained for a particular case.

Directional spectra (31) for a definite component must broaden as kx increases and become more narrow while U/c grows. This contradicts known empirical data. Hence, it is impossible to describe directional spectra even at initial wave development stages by means of wind input only.

However, it is interesting to compare directional spectra (31) with experimental ones obtained at the initial development stages: at different tendencies of variability, width of experimental spectra should not differ much from predictions (31). Width of directional spectra (31) depends strongly on the unknown constant a, so that another aim of such comparison is an indirect estimate of the growth increment a.

In Fig. 5, values $A(f_m)$, calculated for records 7–10 (see Table 1), are shown for $a = 2{\cdot}6 \times 10^{-3}$ which follows from the Miles theory (Mitsuyasu and Honda 1982). They agree very well with the experimental points and even describe their scattering around the average curve; however, this coincidence needs, of course, careful verification. The ideal peak frequency situation does not fit other frequencies: directional spectra for $f < f_m$ remain constant according to (28) but from (31) they must broaden quickly; for $f > f_m$, directional spectra broaden according to (28) but they must become narrower to accord with (31).

Similar variability can be explained, for example, by transmission of energy by non-linear interactions from main direction components with frequencies $f > f_m$ to main direction components of $f < f_m$; however, there are no available results of non-linear computations for initial wave development stages. So we may suppose the role of non-linear interactions in the formation of directional spectra to be important for all wave ages. No systematic computations of this influence are available. We may just as surely suggest that in the range $f < f_m$, $U/c_m \sim 1$, where the role of all mechanisms apart from non-linear interactions is insignificant, initially narrow directional spectra broaden very quickly. This qualitatively confirms the theoretical conclusions of Zakharov and Smilga (1981).

A general tendency for narrowing of $K(f,\theta)$ while U/c_m decreases (waves grow older), testifies to fluctuations of wind direction not being cumulative even at long fetches (Mitsuyasu *et al.* 1975). An explanation for this could be the short duration of fluctuations, compared with time necessary to generate waves in a new direction.

An interesting consequence of the parameterization pertains to a comparison of results of research of wave number spectra of wind-generated waves. Banner (1991), investigating the equilibrium level of spatial frequency, obtained the result

$$\Phi(k,\theta_{\max}) = \alpha\left(\frac{U}{c_m}\right)^{\frac{1}{2}} k^{-4}\,. \tag{34}$$

Here, $\theta_{\max}$ is the main direction of wave propagation, so the normalized directional distribution is expressed as

$$D(k,\theta) = \frac{\Phi(k,\theta)}{\Phi(k,\theta_{\max})} \tag{35}$$

and the frequency spectrum can be calculated as follows:

$$S(f) = 2\alpha g^2\left(\frac{U}{c_m}\right)^{\frac{1}{2}} (2\pi f)^{-5} \int_{-\pi}^{\pi} (D(k,\theta)\mathrm{d}\theta)_{k=(2\pi f)^2/g}\,. \tag{36}$$

Substituting his formula for wavenumber-directional energy distribution, Banner (1991) concluded that the equilibrium interval of frequency wave spectrum should have subintervals with both f^{-4} and f^{-5} power laws (the latter one belongs to the higher frequency band).

This result follows from the present parameterization (19) combined with (36). The integral in Eqn (36) is equal to the inverse value of the width directional parameter A. Thus, for $f \sim f_m$ from (19), $A \approx (U/c_m)^{-1/2}(f/f_m)^{-1}$ pertains and, using Banner's (1991) result (36), yields:

$$S(f) = 2\alpha g^2\left(\frac{U}{c_m}\right)^{\frac{1}{2}}(2\pi f)^{-5}\left(\frac{U}{c_m}\right)^{\frac{1}{2}}\left(\frac{f}{f_m}\right)$$
$$= 2\alpha g U(2\pi f)^{-4}\,. \quad (37)$$

This equilibrium spectrum corresponds to the well-known spectrum proposed by Toba (1972).

For high frequencies, $f \gg f_m$, $A \approx (2\pi)^{-1}$ pertains, and combining this with (36) derives

$$S(f) = 2\alpha g^2\left(\frac{U}{c_m}\right)^{\frac{1}{2}}(2\pi f)^{-5}2\pi\,, \quad (38)$$

the Phillips (1958) spectrum.

This result is interesting, not only because of the impressive degree of agreement with conclusions made as a result of different research approaches, but also because it predicts the equilibrium interval phenomenon observed in the field independently (e.g. Evans and Kibblewhite 1990).

A comprehensive approach to the study of wave directional spectra was undertaken by Wen *et al.* (1993*a*, 1993*b*) by means of a spectrum derived analytically under some restricted conditions, and it is interesting to compare some conclusions in the present paper and those papers. The derived frequency-directional spectrum was obtained by Wen *et al.* (1993*a*, 1993*b*) in a manner similar to that used in the present paper to obtain (31). After frequency spectrum variability along fetch is determined, the frequency-directional spectrum at a point is considered as a superposition of the corresponding frequency spectra of waves coming from different directions. In this approach, non-linear interactions are not taken into account; however, let us look at the predictions of the theory outlined below.

The Wen *et al.* (1993*b*) directional spectrum demonstrates dependence on wave development stage and frequency: the more developed the waves, the more narrow the directional spectra; the narrowest directional spectra are located in the vicinity of the peak frequency at approximately $0{\cdot}95 f_m$; the further the frequency is from that value, on both sides, the broader are the directional distributions of energy. All features mentioned also follow from our field parameterization, as shown in the conclusions earlier in this section. Comparisons which Wen *et al.* (1993*b*) performed with parameterizations of Mitsuyasu *et al.* (1975), Hasselmann *et al.* (1980) and Donelan *et al.* (1985) are also in agreement with this paper. Spectra of Wen *et al.* (1993*b*) repeat behaviour of the parameterized spectra, but are in general narrower than those of Mitsuyasu *et al.* (1975) and Hasselmann *et al.* (1980), and are close to those of Donelan *et al.* (1985).

Discrepancies exist at low-frequency and equilibrium bands. The spectra of Wen *et al.* (1993*b*) do not demonstrate constancy in width below the peak frequency at initial wave development stages. This is impossible from a linear superposition approach, as we discussed above. For the equilibrium range of the wind-wave spectrum, Wen *et al.* (1993*b*) provide the directional spreads, which are independent of frequency but change with the wave stage. This conclusion is in conflict with both the present paper and Donelan *et al.* (1985).

Thus, although non-linear interactions are definitely important because they are involved in directional spectra formation (Hasselmann *et al.* 1980; Zakharov and Smilga 1981; Komen *et al.* 1984; Tsimring 1989), it is possible to describe directional spectra in a simplified analytical manner with many qualitative features adjusted, without taking non-linear interactions into consideration.

Conclusions

Parameterization (28) for the width of the directional distribution, dependent on the stage of wave development U/c_m and relative frequency f/f_m, is proposed on the basis of analysis of experimental data in a wide range of wave types (2)–(4). It is shown that broadening of $K(f, \theta)$ at higher frequencies depends only on f/f_m but the width of directional spectrum at $f \approx f_m$ is determined by U/c_m. The narrowest directional spectra are on average observed at $0{\cdot}95 f_m$, and below this frequency, behaviour of the spectra is strongly dependent on both U/c_m and f/f_m.

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