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Similarity Theory for Turbulence, Induced by Orbital Motion of Surface Water Waves

Alexander V. Babanin

University of Melbourne, Parkville, Melbourne 3010 Australia

Abstract

Similarity theory of isotropic turbulence induced by waves on the water with free surface is proposed. Scaling is obtained from experimental and numerical observations of dissipation rates for surface waves, and then used to estimate the turbulent viscosity of the locally-isotropic turbulence.

Keywords: surface water waves; wave-induced turbulence; similarity theory

Water motion induced by waves excited at the free surface should produce turbulence at large enough Reynolds Numbers, like every other motion in viscous fluids with velocity gradient. The concept of wave-induced turbulence is not new [eg. 1-6], but since the early 60s it has been neglected and largely forgotten. This can be explained by breakthroughs in new potential wave theories which at about the same time demonstrated remarkable success in describing previously elusive or even unknown dynamic features such as instabilities and resonances in nonlinear wave fields [7-12]. The potential theories assume zero viscosity, their respective motions by definition produce no vorticity and hence turbulence, and such approaches have dominated the wave science ever since.

This includes the oceanography where the turbulence, however, is of great importance for the upper ocean dynamics. Various other phenomena were held responsible for this upper-ocean turbulence, such as breaking of surface and internal waves, wind and current shear, Langmuir circulation. In the meantime, the wave orbital-motion turbulence research continued in background, both experimental and theoretical [eg. 13-18]. It remained quite marginal until recently when the apparent need for more accurate description of wave-coupled effects in the lower atmosphere and upper ocean revived the interest to the missing source of turbulence [eg. 19-25].

Here we will propose a Kolmogorov-Obukhov theory for the locally isotropic turbulence due to such water motion, i.e. due to waves on the free surface of the water. For the general case of viscous fluids, original concepts and similarity theory were offered in [26-29], and here we will follow [30] for a modern fluid mechanics interpretation.

Like any similarity theory, the Kolmogorov theory is qualitative, subject to an unknown proportionality coefficient. For the water waves, we will try and determine this coefficient on the basis of available experimental

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and field observations, as well as numerical simulations. This way, we will also be able to offer quantitative conclusions of the Kolmogorov turbulence theory for the orbital motion of surface water waves (unrelated to wave breaking), which may have important practical value in the ocean modelling.

Dimensionally, volumetric dissipation rate ε of the Kolmogorov turbulence is defined by velocity scale Δu and length scale l, where Δu specifies change of the mean velocity over distance l:

$$\varepsilon \sim \frac{(\Delta u)^3}{l} = \frac{(b_2 \Delta u)^3}{b_2 l} = b_2^2 \frac{(\Delta u)^3}{l}. \tag{1}$$

In [30], it was argued that while Δu and l are meant to be the external (largest) scales of the turbulence induced by a mean motion, in reality these scales of the largest turbulent fluctuations are several times less. As a result, a substantial proportionality coefficient b_2 can be expected in (1), which depends on this ratio (note that we reserve parameter b_1 for the swell-decay coefficient in (3) as it had been introduced before).

For the water particles participating in wave orbital motion with radian frequency ω, velocity in any direction changes from $u=a\omega$ to 0 over the orbit radius a, i.e. the scales are $\Delta u = u = a\omega$ and $l=a$. Therefore

$$\varepsilon = b_2^2 \frac{(a\omega)^3}{a} = \frac{b_2^2}{ak} k a^3 \omega^3 \tag{2}$$

where k is wavenumber connected with frequency, in the deep water, by dispersion relationship $\omega^2 = gk$ (g is gravity constant), $s=ak$ is wave steepness. Note that wave amplitude (orbital radius a) decays exponentially away from the surface $z=0$, as $a = a_0 \exp(-kz)$ where a_0 is wave amplitude (radius of wave orbit) at the surface.

In [22], based on a broad range of laboratory experiments [31], numerical simulations [24], satellite observations of swell propagation in the ocean [32,33], it was argued that

$$\varepsilon = b_1 k a_0^3 \omega^3 = 0.0014 k a_0^3 \omega^3. \tag{3}$$

Hence, $b_1 \cong 0.0014 = \frac{b_2^2}{ak}$, i.e. the main velocity and spatial scales of wave-induced turbulence have to be $b_2 = \sqrt{b_1 ak} = 0.12 - 0.13$ assuming mean steepness of wind-generated waves of s=0.1-0.12. Thus, the main velocity and spatial scales of wave-induced turbulence scales are of the order of $b_2 = \frac{1}{10}$ th of the external scales adopted in (1).

Now, we are prepared to apply the Kolmogorov theory to the wave-induced turbulence quantitatively. Coefficient of turbulent viscosity is

$$K = b_2 \Delta u \cdot b_2 l = b_2^2 a^2 \omega = (b_1 ak) a^2 \omega = b_1 \frac{a^3 \omega^3}{g} = \frac{b_1}{g} u_{orb}^3 \tag{4}$$

where u_{orb} is wave orbital velocity.

Figure 1 demonstrates magnitude of K in the full range of wave conditions, $H=2a$=0.02-20 m and $f = \omega/(2\pi)$ =0.05-1 Hz, ie. from very small to full-scale ocean seas. Obviously, the small waves produce negligible eddy viscosity, and for large and steep waves it reaches K~$10^{-2}\ m^2/s = 10^2\ cm^2/s$. Experimental guidance for such wave-induced turbulence viscosity is not available, but numerical large-scale ocean-circulation models, e.g. [21], suggest values of ~100 cm^2/s in the ocean areas with large waves, such as the Southern Ocean, in summer when the wave-induced mixing should prevail over vertical convection.

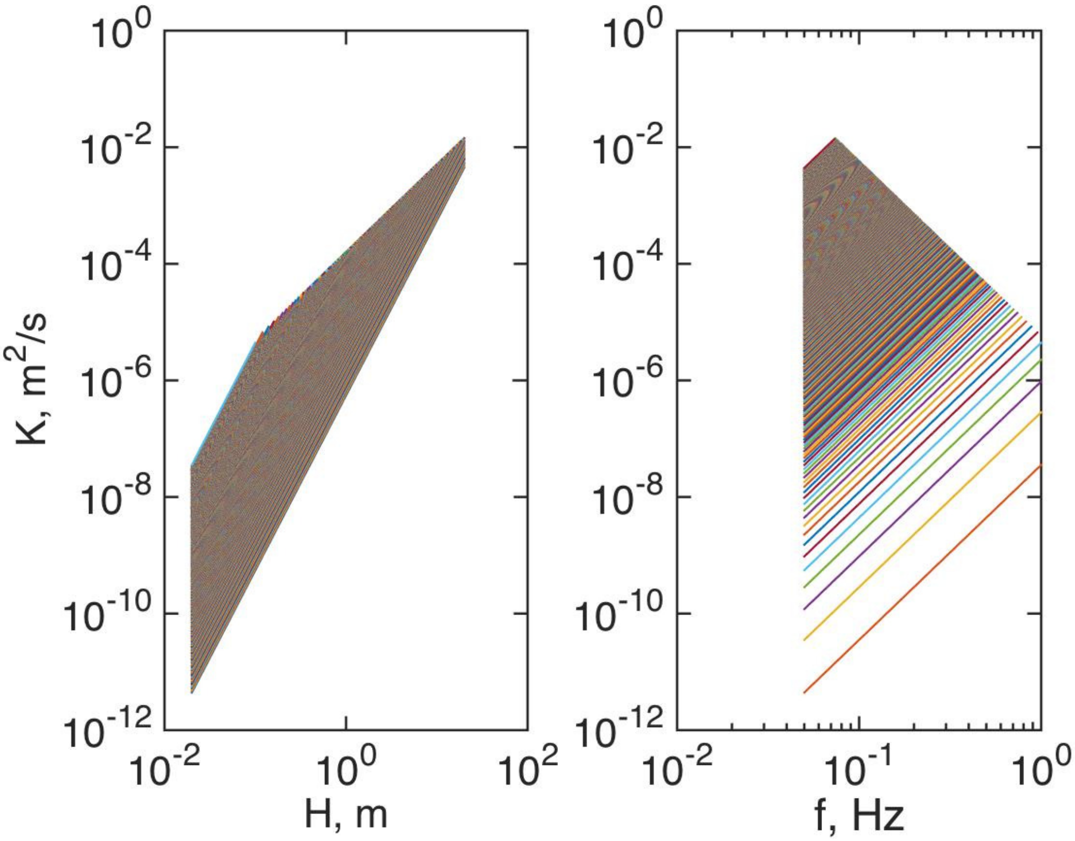

Fig. 1. Dependence of the coefficient of turbulent viscosity on (left) wave height for frequency f=0.05-1 *Hz*; (right) frequency for wave heights H=0.02-20 *m*

Summary. In the paper, a similarity theory of turbulence induced by waves on the water with free surface is proposed, based on the Kolmogorov-Obukhov theory for the locally isotropic turbulence. Scaling is obtained from a broad range of observations of wave dissipation rates unrelated to the breaking, and then used to estimate the turbulent viscosity. The estimates are consistent with values of the wave-induced eddy viscosity in ocean-circulation models.

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