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Three-dimensional nonlinear random wave groups in intermediate water depth

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ABSTRACT

High waves at ocean occur during a complex space–time evolution of wave groups. In this paper the nonlinear structure of three-dimensional sea wave groups at intermediate water depth is investigated. To this purpose, the Boccotti's Quasi-Determinism theory is firstly applied to describe the linear wave groups when a given exceptionally high crest occurs. Then, the second-order correction to the linear solution is derived for the general condition of three-dimensional wave groups, at a finite water depth. Several numerical applications, finally, have been carried out in order to show how both the spectral bandwidth and the directional spreading modify the nonlinear high waves at different water depth.

1. Introduction

Linear theory of wind-generated waves (Longuet-Higgins, 1963, Phillips, 1967) models ocean waves as a superposition of a large number of periodic wave components. Therefore the free surface displacement is a Gaussian process in time domain (Longuet-Higgins, 1952).

Boccotti (1981, 1982, 1983), in his first formulation of the Quasi-Determinism theory, showed that in a Gaussian sea, when a high crest occurs at some fixed time and location, the free surface displacement tends to be proportional to the autocovariance function. The first version of the linear theory gives what most probably would be the mechanics of a wave group, if a wave of given exceptionally large crest height should occur at some fixed point. The structure of the surface displacement near a local maximum was also analyzed by Lindgren (1970, 1972) and by Tromans et al. (1991), who introduced the NewWave model to describe the sea wave groups.

Boccotti (1989, 1997) proposed then the second formulation of the Quasi-Determinism theory, which gives what most probably would be the mechanics of the wave group to the first-order, if a wave with a given exceptionally large crest-to-trough height should occur at some fixed point. Furthermore, he showed as the two formulations are consistent to each other (for a complete review see Boccotti, 2000), giving also a fully verification of the theory (Boccotti et al., 1993) with a small scale field experiment. An additional field verification off the Atlantic coast of USA was proposed by Phillips et al. (1993a,b).

The nonlinear modeling of random ocean waves was introduced by Longuet-Higgins (1963). The second-order solution for the theory of wind-generated waves was then proposed by Sharma and Dean (1979).

The second-order analytical solution of the QD theory was derived recently for the hypotheses of both long-crested (Arena, 2005; Arena and Fedele, 2005; Fedele and Arena, 2005) and short-crested waves (Pavone and Arena, 2004), in deep water.

The second order conditional expected structure of large waves was also investigated by Jensen (2005) who adopted an approximated approach based on the edgeworth form of Gram-Charlier series.

In this paper, starting from the linear Quasi-Determinism theory and the Sharma and Dean solution, the second order wave correction is obtained, to model the most probable nonlinear three-dimensional space–time evolution of a wave group in finite water depth if a very large crest should occur at some fixed point.

Finally the effects of both finite bandwidth and directional spreading are investigated, as well as the effect of finite water depth.

2. Linear theory of wind-generated waves

Following the theory of wind-generated waves, given by Longuet-Higgins (1963) and Phillips (1967), a random sea state is defined as a sum of a very large number N of periodic components, with infinitesimal amplitudes α_i, frequencies ω_i different from each other and phase angles ε_i uniformly distributed on $(0, 2\pi)$ and stochastically independent to each other.

Consequently, the linear free surface displacement and the velocity potential are stationary Gaussian random processes in time domain and their expressions (for the general condition of three-dimensional random waves) are respectively provided by:

$$\eta(x,y,t) = \sum_{i=1}^{N} \alpha_i \cos\left(k_i x \sin\theta_i + k_i y \cos\theta_i - \omega_i t + \varepsilon_i\right) \tag{1}$$

$$\phi(x,y,z,t) = g\sum_{i=1}^{N} \alpha_i \omega_i^{-1} \frac{\cosh\left[k_i(d+z)\right]}{\cosh\left(k_i d\right)} \sin\left(k_i x \sin\theta_i + k_i y \cos\theta_i - \omega_i t + \varepsilon_i\right) \tag{2}$$

where, for the ith component, the direction of wave advance makes the angle θ_i with the y-axis, and the wave number k_i related to the angular

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frequency ω_i, for any fixed depth d, by the well-known linear dispersion relation:

$$k_i \tanh(k_i d) = \omega_i^2/g \tag{3}$$

with g the acceleration due to gravity.

Furthermore, the amplitudes α_i, the angular frequency, ω_i, and the directions of advance θ_i of every elementary wave component, are such to form a directional wave spectrum $S(\omega, \theta)$.

By defining $S(\omega)$ the frequency spectrum and $D(\theta;\omega)$ the directional spreading function, we have that

$$S(\omega,\theta) = S(\omega)D(\theta;\omega). \tag{4}$$

A typical directional spectrum is the JONSWAP-Mitsuyasu spectrum (Hasselmann et al., 1973; Mitsuyasu et al., 1975), whose mathematical form is:

$$S(\omega,\theta) = \alpha g^2\omega^{-5}\exp\left[-\frac{5}{4}\left(\frac{\omega_p}{\omega}\right)^4\right] \times \exp\left\{\ln\gamma\exp\left[-\frac{(\omega-\omega_p)^2}{2\sigma^2\omega_p^2}\right]\right\}K(n)\left|\cos\left[\frac{1}{2}\left(\theta-\overline{\theta}\right)\right]\right|^{2n} \tag{5}$$

where ω_P is the peak angular frequency, $\bar{\theta}$ is the angle between the y-axis and the dominant direction of the spectrum, α is the Phillips' parameter, γ and σ are shape parameters (for the mean JONSWAP, they are equal to 3.3 and 0.08 respectively), $K(n)$ is the normalizing factor

$$K(n) = \left[\int_0^{2\pi}\cos^{2n}\left(\frac{1}{2}\theta\right)d\theta\right]^{-1} \tag{6}$$

and n is a parameter depending on the frequency

$$n = n_p(\omega/\omega_p)^5 \text{ if } \omega\le\omega_p, \quad n = n_p(\omega_p/\omega)^{2.5} \text{ if } \omega>\omega_p, \tag{7}$$

with

$$n_p = 7.5\cdot10^{-3}\left(\frac{gF_e}{u^2}\right)^{0.825} \tag{8}$$

where F_e is the fetch and u the wind speed.

3. 'Quasi-determinism' theory for three-dimensional random linear wave groups when a very high crest occurs

The theory of 'Quasi-Determinism' was derived by Boccotti for the mechanics of linear three-dimensional wave groups when either a very high crest (first formulation [1981–1982]) or a very large crest-to-trough wave height (second formulation [1983, 1989, 1997, 2000]) occurs.

In this paper the theory is applied in its first formulation and thus the most probable evolution in the space–time domain of random three-dimensional linear wave groups is described when a very high crest occurs, in a Gaussian sea, at any fixed time and location.

In more detail, if a wave crest with a given exceptionally large height H_C occurs at some point x_0, y_0 at a time instant t_0, during a sea storm, with probability approaching to 1 (Quasi-Determinism) the random free surface displacement around point x_0, y_0, for a span of time before and after t_0, will be very close to the following deterministic wave function

$$\overline{\eta}(x_0+X, y_0+Y, t_0+T) = \frac{\Psi(X,Y,T)}{\Psi(0,0,0)}H_C \tag{9}$$

and the deterministic velocity potential to

$$\overline{\phi}(x_0+X, y_0+Y, z, t_0+T) = \frac{\Phi(X,Y,z,T)}{\Psi(0,0,0)}H_C \tag{10}$$

where $\Psi(X,Y,T)$ and $\Phi(X,Y, z,T)$ are the space–time covariance functions, which are respectively defined as

$$\Psi(X,Y,T) \equiv \langle\eta(x_0,y_0,t)\eta(x_0+X,y_0+Y,t+T)\rangle \tag{11}$$

$$\Phi(X,Y,z,T) \equiv \langle\eta(x_0,y_0,t)\phi(x_0+X,y_0+Y,z,t+T)\rangle \tag{12}$$

η and ϕ being respectively the free surface displacement and the velocity potential of the wind wave field [i.e. Eqs. (1) and (2)].

The linear deterministic functions of water surface and velocity potential for three-dimensional wave groups, given by expressions (9) and (10) respectively, may be rewritten as a function of the directional spectrum:

$$\overline{\eta}_1(x_0+X, y_0+Y, t_0+T) = \frac{H_C}{\sigma^2}\int_0^\infty\int_0^{2\pi}S(\omega,\theta)\cos(kX\sin\theta+kY\cos\theta-\omega T)d\theta d\omega \tag{13}$$

$$\overline{\phi}_1(x_0+X, y_0+Y, z, t_0+T) = \frac{H_C}{\sigma^2}g\int_0^\infty\int_0^{2\pi}S(\omega,\theta) \times \frac{\cosh[k(d+z)]}{\omega\cosh(kd)}\sin(kX\sin\theta+kY\cos\theta-\omega T)d\theta d\omega \tag{14}$$

where

$$\sigma^2 \equiv \int_0^\infty\int_0^{2\pi}S(\omega,\theta)d\theta d\omega \tag{15}$$

is the variance of the random free surface displacement. Note that the origin of the Cartesian co-ordinate system (X,Y) is at (x_0,y_0).

4. Second-order solution of the quasi-determinism theory for three-dimensional wave groups with a very high crest

The second-order correction to the linear solution is obtained by considering the free surface displacement and the velocity potential, which satisfy the set of partial differential equations for an irrotational flow with an incompressible and inviscid fluid.

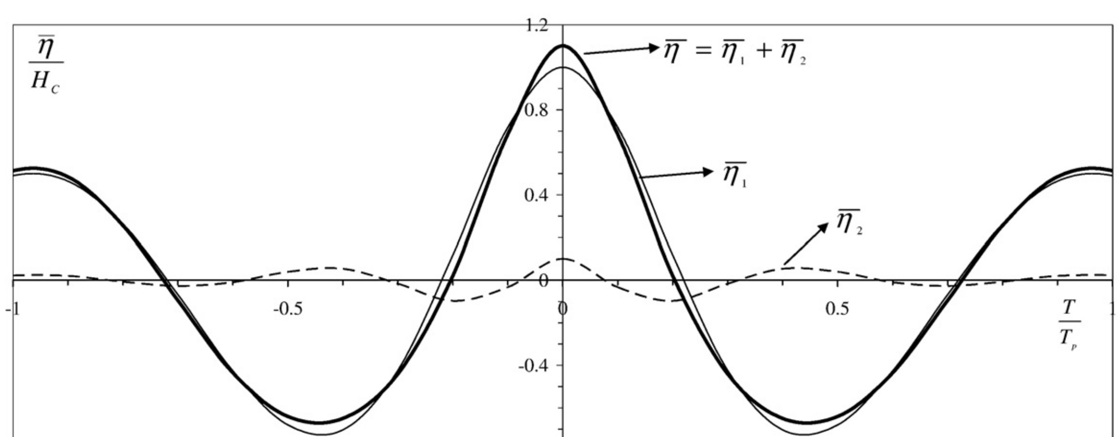

Fig. 1. The free surface displacement when a large crest height H_C=8σ occurs in deep water: linear profile $\overline{\eta}_1$ second-order term $\overline{\eta}_2$, total second-order profile $\overline{\eta}$.

In more detail, the surface displacement and the velocity potential are approximated by a truncated perturbation series

$$\overline{\eta}(x_0+X,y_0+Y,t_0+T)\equiv\sum_{i=1}^{2}\varepsilon^i\overline{\eta}_i(x_0+X,y_0+Y,t_0+T)+o(\varepsilon^2) \quad (16)$$

$$\overline{\phi}(x_0+X,y_0+Y,z,t_0+T)\equiv\sum_{i=1}^{2}\varepsilon^i\overline{\phi}_i(x_0+X,y_0+Y,z,t_0+T)+o(\varepsilon^2) \quad (17)$$

where the nonlinearity parameter is $\varepsilon=kH_C\ll 1$, k being the wave number and H_C the crest amplitude.

The linear free surface displacement $\overline{\eta}_1$ (Eq. (13)) and velocity potential $\overline{\phi}_1$ (Eq. (14)), when the high crest of amplitude H_C occurs, may be written in a discrete form as

$$\overline{\eta}_1(x_0+X,y_0+Y,t_0+T)=\sum_{i=1}^{N}\sum_{j=1}^{M}a_{ij}\cos(\varphi_{ij}) \quad (18)$$

$$\overline{\phi}_1(x_0+X,y_0+Y,z,t_0+T)=g\sum_{i=1}^{N}\sum_{j=1}^{M}a_{ij}\omega_i^{-1}\frac{\cosh[k_i(d+z)]}{\cosh(k_id)}\sin(\varphi_{ij}) \quad (19)$$

where

$$\varphi_{ij}\equiv k_i\sin\theta_jX+k_i\cos\theta_jY-\omega_iT, \quad (20)$$

$$a_{ij}\equiv\frac{H_C}{\sigma^2}S(\omega_i,\theta_j)\Delta\omega_i\Delta\theta_j. \quad (21)$$

Let us consider the second-order system of differential equations governing an irrotational flow with a free surface, in a three-dimensional field. Following the line of reasoning of Sharma and Dean (1979 — see also Longuet-Higgins, 1963), the solution for $\overline{\phi}_2$ is given starting from the combination of the dynamic and the kinematics free surface boundary conditions. It is obtained that:

$$\begin{aligned}\overline{\phi}_2&(x_0+X,y_0+Y,z,t_0+T)\\&=\frac{1}{4}\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{l=1}^{N}\sum_{m=1}^{M}a_{ij}a_{lm}B^+_{ijlm}\frac{\cosh\left[k^+_{ijlm}(d+z)\right]}{\cosh\left(k^+_{ijlm}d\right)}\sin(\varphi_{ij}+\varphi_{lm})\\&+\frac{1}{4}\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{l=1}^{N}\sum_{m=1}^{M}a_{ij}a_{lm}B^-_{ijlm}\frac{\cosh\left[k^-_{ijlm}(d+z)\right]}{\cosh\left(k^-_{ijlm}d\right)}\sin(\varphi_{ij}-\varphi_{lm})+CT\end{aligned} \quad (22)$$

where

$$B^\pm_{ijlm}(\omega_i,\theta_j,\omega_l,\theta_m)\equiv\frac{D^\pm_{ijlm}}{(\omega_i\pm\omega_l)}\frac{g^2}{\omega_i\omega_l} \quad (23)$$

$$\begin{aligned}D^\pm_{ijlm}(\omega_i,\theta_j,\omega_l,\theta_m)\equiv&\frac{[\sqrt{r_l}(k_i^2-r_i^2)\pm\sqrt{r_i}(k_l^2-r_l^2)](\sqrt{r_i}\pm\sqrt{r_l})}{(\sqrt{r_i}\pm\sqrt{r_l})^2-k^\pm_{ijlm}\tanh\left(k^\pm_{ijlm}d\right)}\\&+\frac{2(\sqrt{r_i}\pm\sqrt{r_l})^2\left[\vec{k}_{ij}\cdot\vec{k}_{lm}\mp r_ir_l\right]}{(\sqrt{r_i}\pm\sqrt{r_l})^2-k^\pm_{ijlm}\tanh\left(k^\pm_{ijlm}d\right)}\end{aligned} \quad (24)$$

with

$$r_i(\omega_i)=\frac{\omega_i^2}{g}=k_i\tanh(k_id), \quad (25)$$

$$k^\pm_{ijlm}(\omega_i,\theta_j,\omega_l,\theta_m)=\sqrt{k_i^2+k_l^2\pm 2k_ik_l\cos(\theta_j-\theta_m)}. \quad (26)$$

Finally, the second-order correction, $\overline{\eta}_2$, to the linear free surface displacement (13), is given by:

$$\overline{\eta}_2=\frac{1}{4}\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{l=1}^{N}\sum_{m=1}^{M}a_{ij}a_{lm}[A^+_{ijlm}\cos(\varphi_{ij}+\varphi_{lm})+A^-_{ijlm}\cos(\varphi_{ij}-\varphi_{lm})]-\frac{C}{g} \quad (27)$$

with

$$A^\pm_{ijlm}(\omega_i,\theta_j,\omega_l,\theta_m)=\frac{B^\pm_{ijlm}(\omega_i\pm\omega_l)+(\omega_i^2+\omega_l^2)}{g}-g\frac{\vec{k}_{ij}\cdot\vec{k}_{lm}}{\omega_i\omega_l}\pm\frac{\omega_i\omega_l}{g}. \quad (28)$$

It is noteworthy that Eqs. (27) and (22) in the Sharma and Dean formulation represent the second-order contribution to the free surface displacement and velocity potential respectively, in a random wave field (see Longuet-Higgins, 1963). In this paper, ($\overline{\eta}_2$, $\overline{\phi}_2$) are the second order correction to the linear free surface displacement and velocity potential ($\overline{\eta}_1$, $\overline{\phi}_1$), which give the quasi-deterministic mechanics of wave groups when an exceptionally high wave crest H_C occurs at some point x_0, y_0 at a time instant t_0, during a sea storm; to this purpose the amplitudes a_{ij} are achieved by a discretization of the directional wave spectrum (see Eq. (21)).

Note that, with respect to the original formulation of Sharma and Dean, in the right hand side of Eq. (22) the term CT has been added, to satisfy the above mentioned system of differential equations. The value of the unknown coefficient C is calculated by imposing the zero mean of the free surface displacement process in order to assure the principle of mass conservation associated to the absence and to the presence of wave motion. We obtain:

$$C=-\frac{g}{4}\sum_{i=1}^{N}\sum_{j=1}^{M}a_{ij}^2\frac{2k_i}{\sinh(2k_id)}. \quad (29)$$

The final expressions of the second-order deterministic components both of the free surface displacement $\overline{\eta}_2$ and of the velocity potential $\overline{\phi}_2$, for short-crested wave groups, are respectively:

$$\begin{aligned}\overline{\eta}_2=&\frac{1}{4}\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{l=1}^{N}\sum_{m=1}^{M}a_{ij}a_{lm}[A^+_{ijlm}\cos(\varphi_{ij}+\varphi_{lm})+A^-_{ijlm}\cos(\varphi_{ij}-\varphi_{lm})]\\&+\frac{1}{4}\sum_{i=1}^{N}\sum_{j=1}^{M}a_{ij}^2\frac{2k_i}{\sinh(2k_id)}\end{aligned} \quad (30)$$

$$\begin{aligned}\overline{\phi}_2=&\frac{1}{4}\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{l=1}^{N}\sum_{m=1}^{M}a_{ij}a_{lm}B^+_{ijlm}\frac{\cosh\left[k^+_{ijlm}(d+z)\right]}{\cosh\left(k^+_{ijlm}d\right)}\sin(\varphi_{ij}+\varphi_{lm})\\&+\frac{1}{4}\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{l=1}^{N}\sum_{m=1}^{M}a_{ij}a_{lm}B^-_{ijlm}\frac{\cosh\left[k^-_{ijlm}(d+z)\right]}{\cosh\left(k^-_{ijlm}d\right)}\sin(\varphi_{ij}-\varphi_{lm})\\&-\frac{g}{4}\sum_{i=1}^{N}\sum_{j=1}^{M}a_{ij}^2\frac{2k_i}{\sinh(2k_id)}T\end{aligned} \quad (31)$$

and they both may be rewritten in an integral form as

$$\begin{aligned}\overline{\eta}_2=&\frac{H_C^2}{4\sigma^4}\int_0^\infty\int_0^\infty\int_0^{2\pi}\int_0^{2\pi}S(\omega_1,\theta_1)S(\omega_2,\theta_2)\Big\{A^+_{1122}\cos(\varphi_{11}+\varphi_{22})\\&+A^-_{1122}\cos(\varphi_{11}-\varphi_{22})+\delta_{12}\frac{2k_1}{\sinh(2k_1d)}\Big\}d\theta_2d\theta_1d\omega_2d\omega_1\end{aligned} \quad (32)$$

$$\begin{aligned}\overline{\phi}_2=&\frac{H_C^2}{4\sigma^4}\int_0^\infty\int_0^\infty\int_0^{2\pi}\int_0^{2\pi}S(\omega_1,\theta_1)S(\omega_2,\theta_2)\\&\times\Big\{B^+_{1122}\frac{\cosh[k^+_{1122}(d+z)]}{\cosh(k^+_{1122}d)}\sin(\varphi_{11}+\varphi_{22})\\&+B^-_{1122}\frac{\cosh[k^-_{1122}(d+z)]}{\cosh(k^-_{1122}d)}\sin(\varphi_{11}-\varphi_{22})\\&-g\delta_{12}T\frac{2k_1}{\sinh(2k_1d)}\Big\}d\theta_2d\theta_1d\omega_2d\omega_1\end{aligned} \quad (33)$$

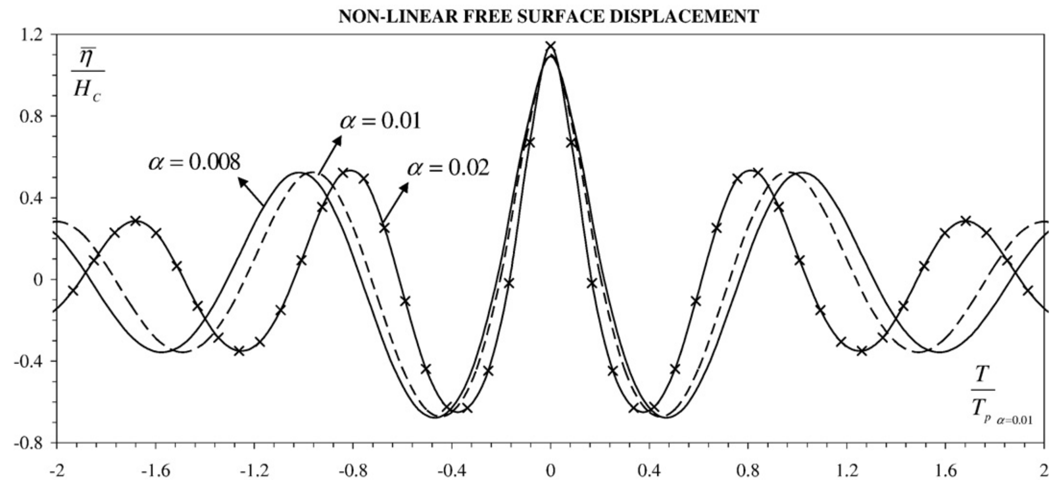

Fig. 2. The second-order total free surface displacement ($\overline{\eta}_1+\overline{\eta}_2$) when a large crest height H_C=8σ occurs in deep water, for different values of the Phillips' parameter.

where

$$\delta_{12} = \begin{cases} 1 & \text{if } \omega_1 = \omega_2 \\ 0 & \text{otherwise.} \end{cases} \tag{34}$$

The expression (32) enables us to determine the most probable nonlinear structure up to the second-order of three-dimensional wave groups, when a high crest of given elevation H_C occurs.

Note that in the nonlinear velocity potential a term proportional to T is considered. It comes from the Whitham (1974) discussion on the second-order problem of Stokes waves in finite depth, and it highlights as the free surface and the velocity potential cannot be zero-mean processes simultaneously (see also Dalzell, 1999; Boccotti, 2000).

5. Nonlinear short-crested wave groups in deep water

The structure of nonlinear three-dimensional wave groups in deep water is obtained from Eqs. (13) and (32). The nonlinear free surface displacement, for the mean JONSWAP-Mitsuyasu spectrum [Eq. (5)], is shown in Fig. 1 compared to the linear profile (H_C=8σ is assumed): the

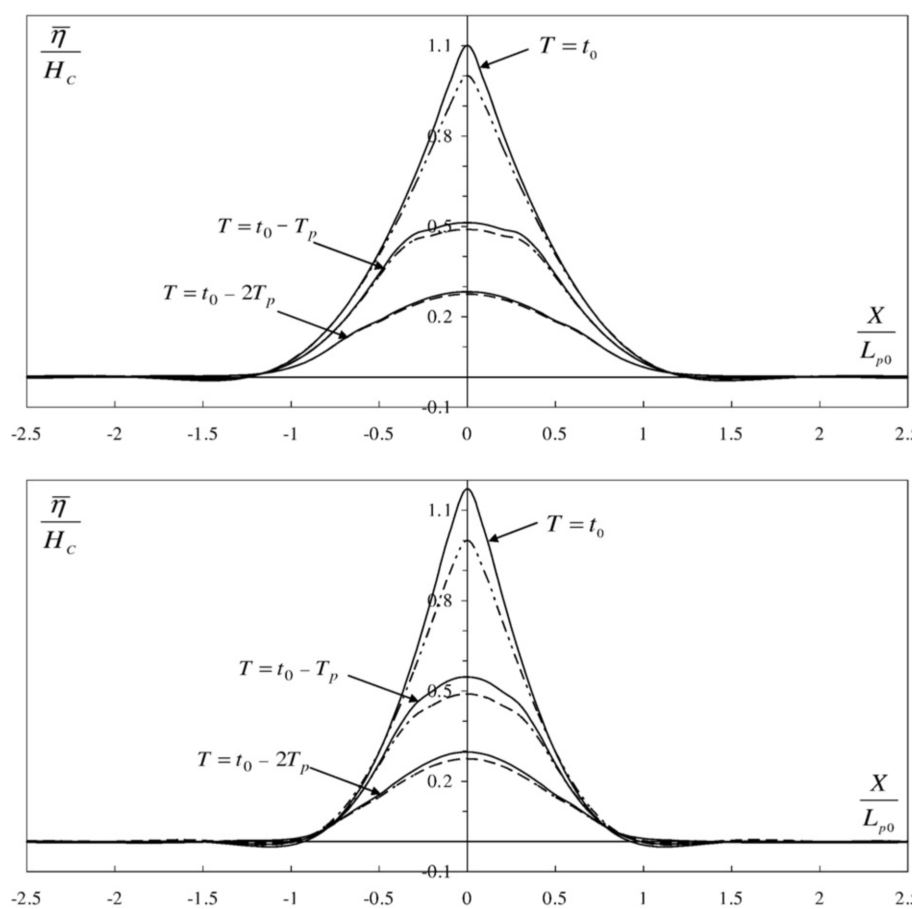

Fig. 3. Wave-front profiles along x-axis, y=y_0. Upper panel: deep water; lower panel: water depth d equal to 0.15 times L_{p0}. Continuous line represents the second-order surface displacement, the dashed one the linear surface displacement (H_C=8σ).

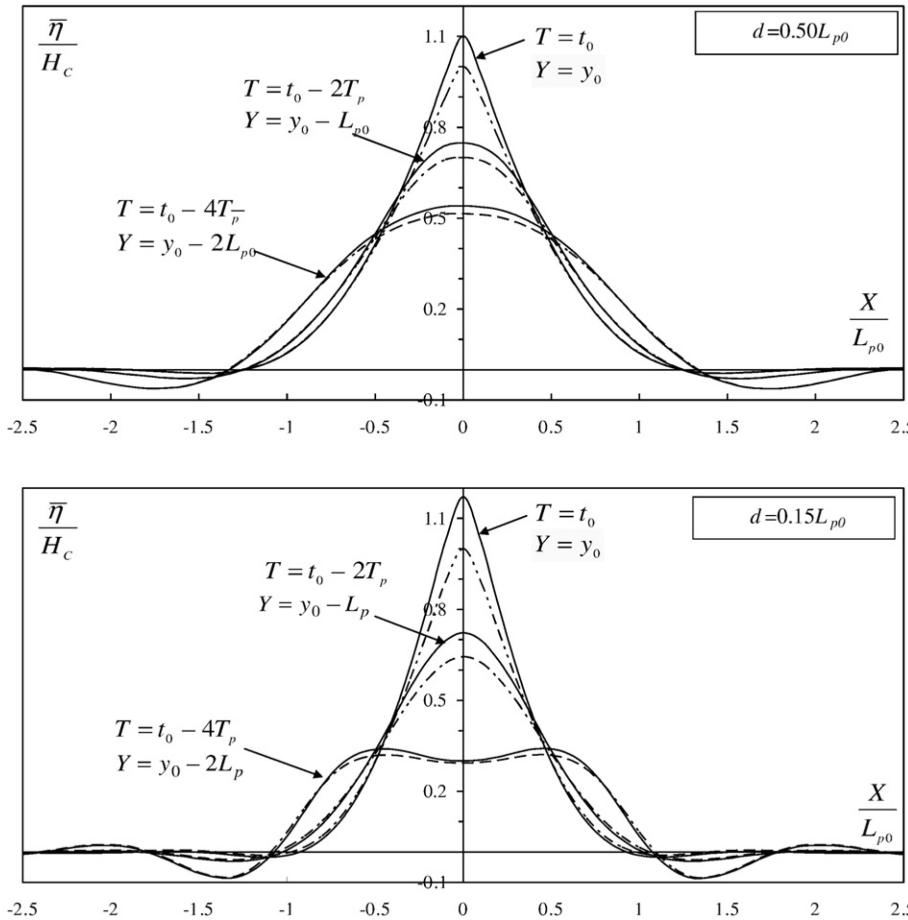

Fig. 4. Front profiles of the wave that at different times is at the centre of the group envelope. Upper panel: deep water; lower panel: water depth d equal to 0.15 times L_{p0}. Continuous line represents the second-order displacement, the dashed one the linear displacement (H_C=8σ).

nonlinear crest is higher and sharper, the nonlinear trough is lower and flatter. The nonlinear effects depend upon the wave steepness (or, equivalently, upon the Phillips' parameter α of the spectrum): if we define the steepness as $S=H_s/L_{p0}$, with $L_{p0}=gT_p^2/(2\pi)$, it is easy to verify that for the mean JONSWAP-Mitsuyasu spectrum

$$S = 0.35\sqrt{\alpha}. \tag{35}$$

For α ranging between 0.008 and 0.02, the steepness S varies between $3.1\cdot10^{-2}$ and $5.0\cdot10^{-2}$ and the maximum amplitude of the second-order term $\overline{\eta}_2$ ranges between 0.09 and 0.14 times the linear crest height respectively (see Fig. 2).

By varying α the most important change in the structure of the wave groups is in the wave period of the individual waves, as we can appreciate in Fig. 2. Anyway, this effect is well predicted by the linear theory too.

A characteristic value of the Phillips' parameter α is 0.01 [note that for this value of α the peak period is $T_p=4.25\pi\ (H_s/g)^{0.5}$]. In this condition, the second-order wave group, for short-crested waves, is shown in Fig. 1 and the total second-order amplitude of the highest crest is 1.10 times the linear one.

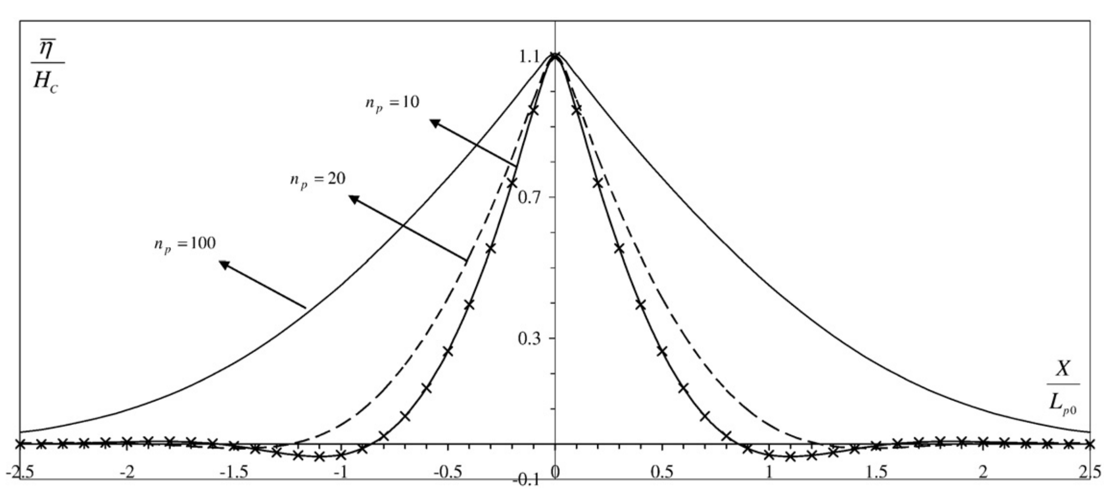

Fig. 5. Second-order wave-fronts at $t=t_0$, $y=y_0$, corresponding to different values of the directional spreading parameter n_p (H_C=8σ).

Fig. 3 (upper panel) shows the profile of the wave fronts along a direction perpendicular to the wave propagation at a fixed location in deep water, for varying times. It can be noticed that the second-order effects do not involve a modification in the transversal extent of the individual waves: indeed, both to the first and to the second-order, the length of wave fronts is about 2.5 times the wavelength L_{p0}. It could be also appreciated that, along all the wave front, the total second-order free surface displacement is always higher than the linear one and the ratio between these two quantities increases as we move towards the centre: at time t_0 it is equal to 1.02 for $X=0.5L_{P0}$ and to 1.10 for $X=0$. Nonlinearities at different time instants exhibit that the total second-order highest crest is 1.04 times the linear one at time t_0-T_p, and 1.02 at time t_0-2T_p.

Fig. 4 (upper panel) gives the front profile of the wave, which is at the centre of the group envelope at any instant. From this Figure, the group evolution may be followed: there is the first developing stage (for $T<0$) during which the central wave increases its height; afterwards, the apex stage of the group occurs at the time t_0 (that is $T=0$), then the decay stage (for $T>0$) during which the group brings down.

The ratio between the total second-order and the linear crest increases during the developing stage and decreases after the time $T=0$, when the highest crest occurs. In particular, the above ratio is equal to 1.05 at time t_0-4T_P, 1.07 at time t_0-2T_P and finally reaches a maximum equal to 1.10 at time t_0.

On the whole, we can conclude that the second-order effects affect the space time evolution of wave groups, pointing out as the maximum nonlinearity occurs when the groups reach the apex of their development.

Finally, in order to evaluate the influence of the directional spreading on the evolution of wave groups, in Fig. 5 three different

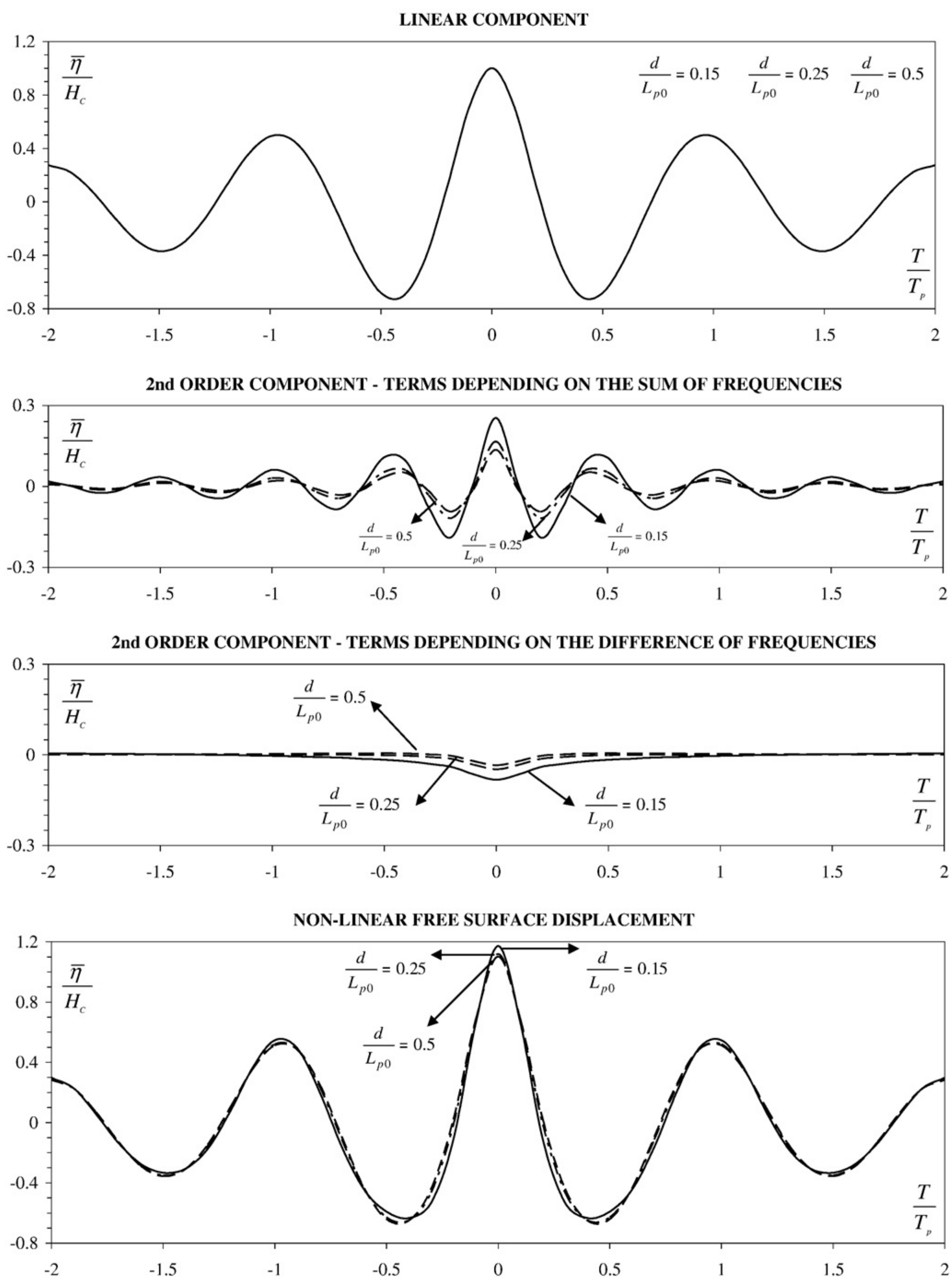

Fig. 6. Free surface displacement time-histories, at three different relative depths, when a large crest height $H_C=8\sigma$ occurs: (i) linear prediction; (ii–iii) second-order terms; (iv) total nonlinear profile.

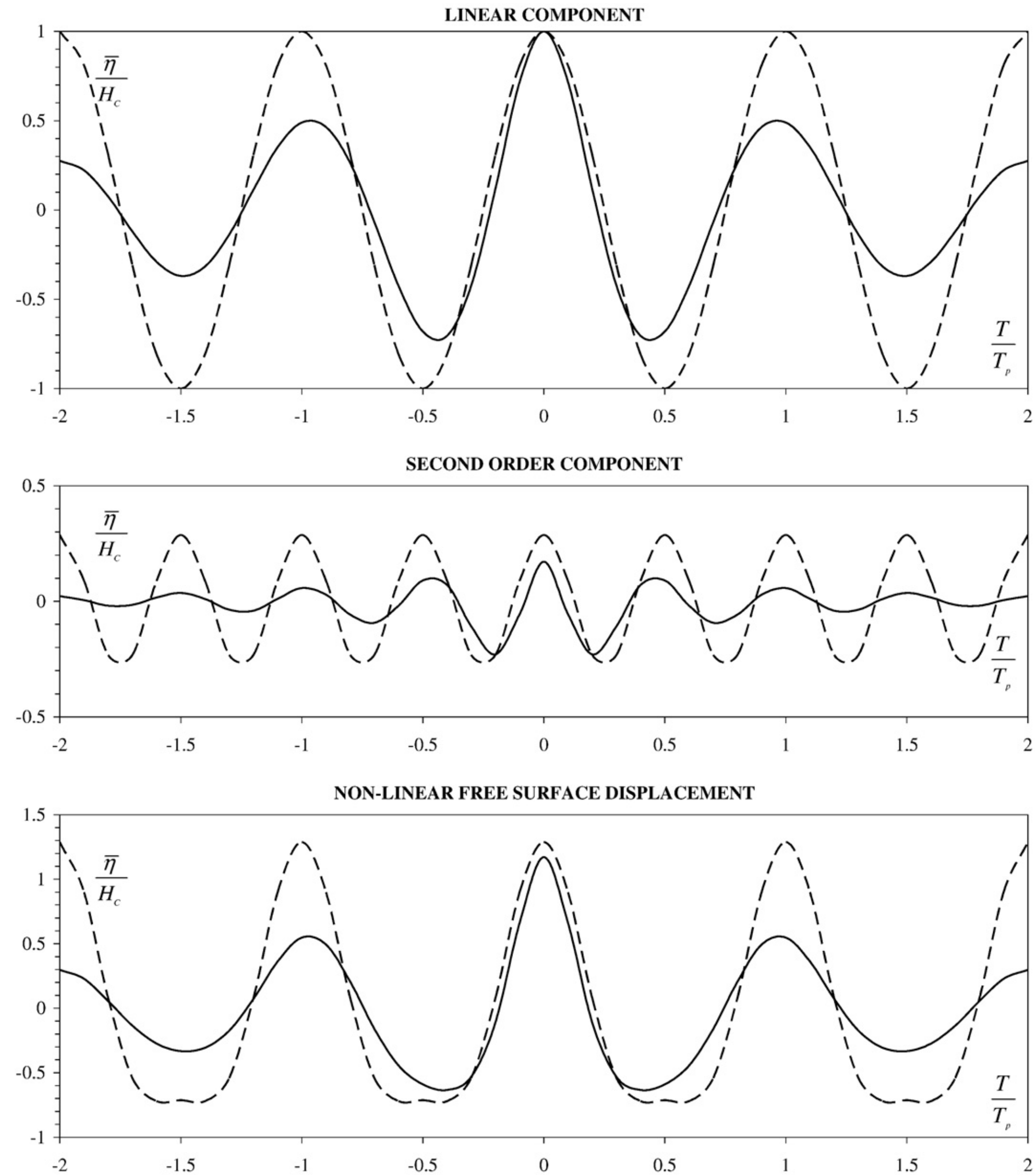

Fig. 7. Random waves when a large wave crest of given height $H_C=8\sigma$ occurs at the relative depth $d/L_{p0}=0.15$: comparison between infinitely narrow spectrum (dashed lines) and mean JONSWAP-Mitsuyasu (continuous lines). Upper panel: linear components; middle panel: second-order components; lower panel: total second-order free surface displacement.

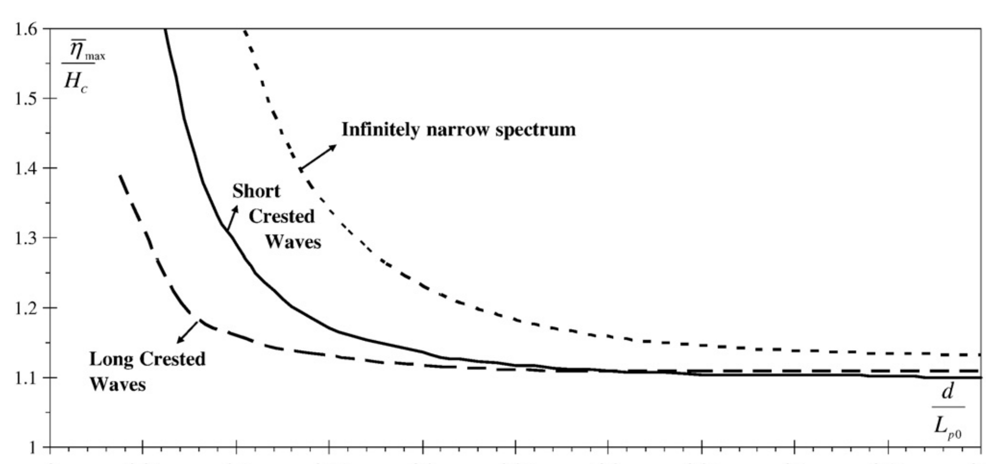

Fig. 8. Ratio between heights of nonlinear and linear largest wave crest, when a crest of given height $H_C=8\sigma$ occurs, as a function of bottom depth d: comparison among narrow-band, long-crested and short-crested waves.

wave fronts are shown, each one corresponding to a different value of the parameter n_p in Eq. (7). It can be observed that as the directional spreading decreases (larger value of n_p) the wave front becomes wider.

Note that Figs. 2, 3, 4 and 5 have been obtained by assuming the crest height H_C equal to 8σ.

6. Nonlinear short-crested wave groups in finite water depth

In this section, the influence of finite depth on the nonlinear free surface displacement for random sea wave groups is examined. In particular, the second-order correction to the wave-group profile, at different relative depths d/L_{p0}, is highlighted.

The wave front, which has been plotted in Figs. 3 and 4 in deep water (upper panels), is also shown on a finite water depth d equal to 0.15 L_{p0} (lower panels). As we can see the wave front width is nearly equal to $2L_{p0}$, and, therefore, it is narrower than that one observed in deep water.

In order to appreciate just the finite-depth effects, the free surface displacement is plotted in the time domain (see Fig. 6) considering a wave group when a high crest with height H_C=8σ occurs at time T=0.

In the upper panel of Fig. 6, the linear free surface displacement time-history is reported for three different values of d/L_{p0}; it can be understood easily that all the linear time histories are the same, at any value of the water depth d. Some modifications may be instead appreciated in the second and third panel of Fig. 6, where the time-histories of the second-order component at three different relative depths are shown.

In the lower panel of this figure we can see that the amplitude of both the crest and trough of the total second-order component

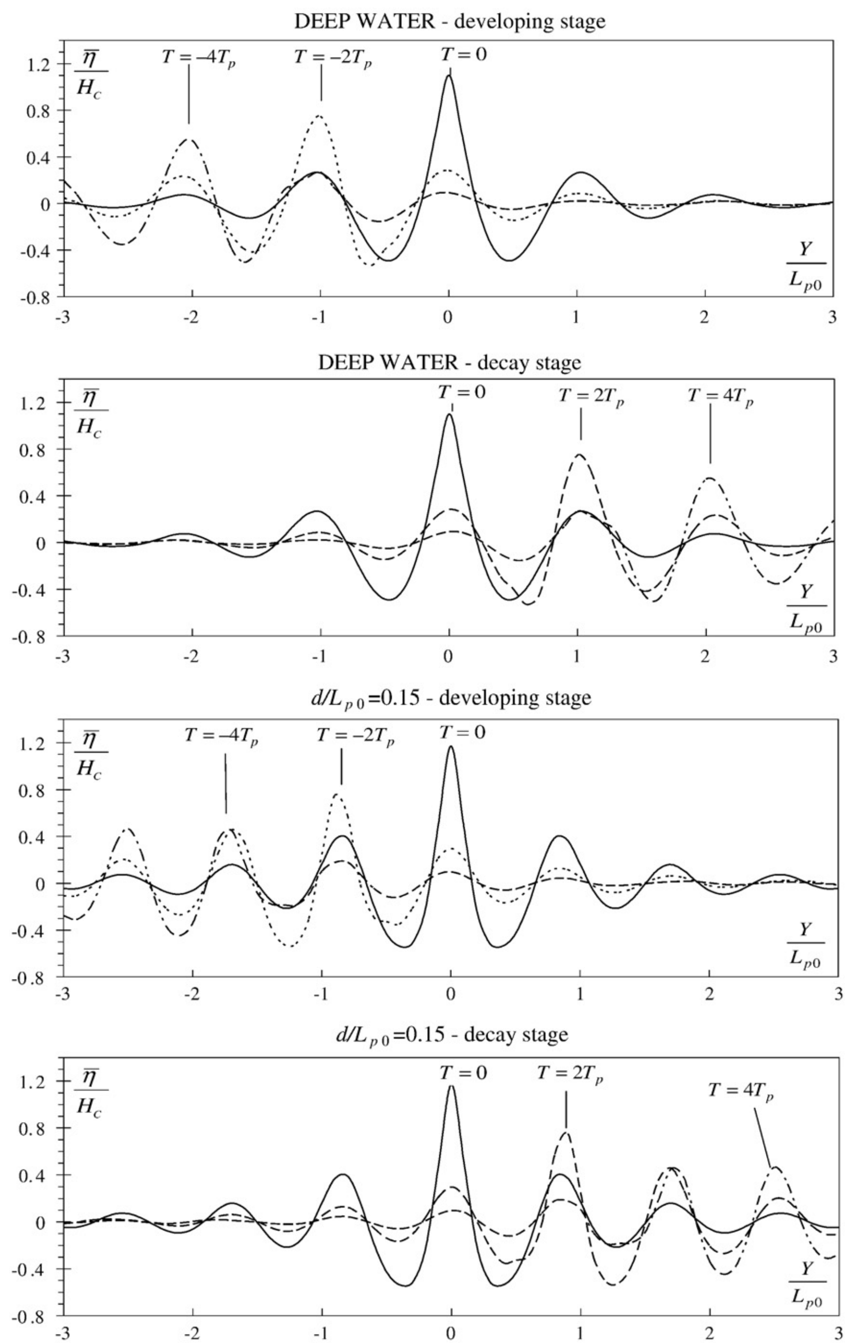

Fig. 9. Space time evolution of wave groups in the development stage (T<0) and in the decay stage (T>0), either for deep water or for d/L_{p0}=0.15.

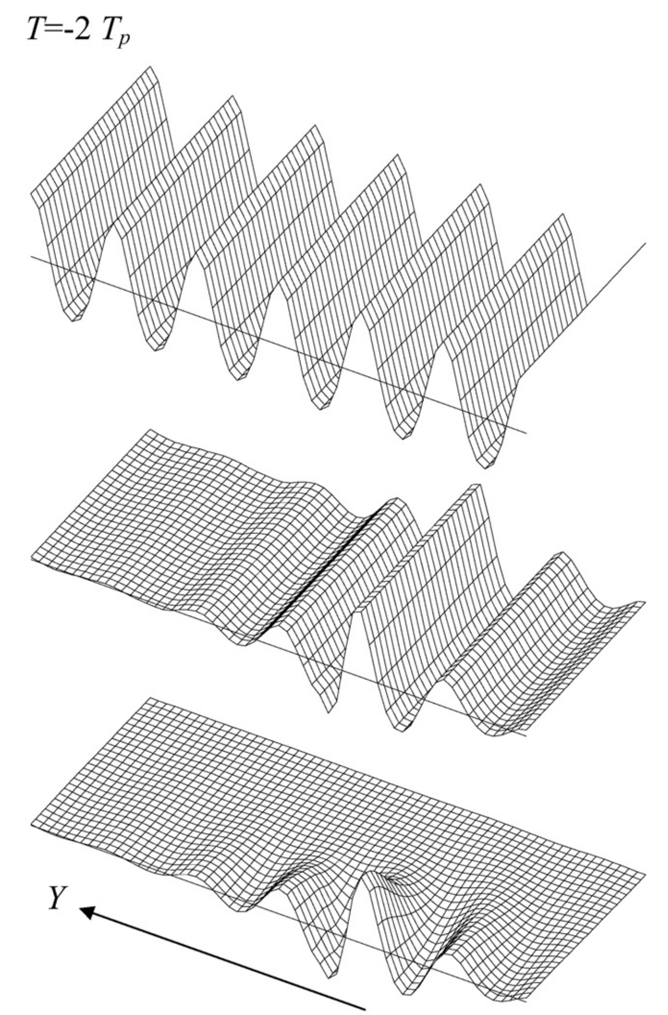

Fig. 10. Wave group in the space domain at time T equal to $-2T_p$, in deep water. Upper panel: narrow-band waves; middle panel: broad band long-crested random waves; lower panel: short-crested waves. The dominant wave direction is given by Y-axis (H_C=8σ).

increases by reducing the water depth: the highest crest amplitude is equal to 1.10, 1.12 and 1.17 times the linear crest height H_C, for water depth d/L_{p0}=0.50, 0.25 and 0.15 respectively. The deepest trough amplitude is, with respect to H_C, equal to 0.65 in deep water and to 0.63 for d equal to $0.15L_{p0}$.

Furthermore, it is noteworthy that the second-order component is strongly influenced by the set-down term, associated to the difference of frequencies and, therefore, proportional to the B^{-}_{ijlm} coefficient of Eq. (23) (its trend is represented in the third panel of Fig. 6): the differences on the water profiles of wave groups with either narrow-band or finite bandwidth of the spectrum are related just to this term.

To appreciate the effects of finite bandwidth of the spectrum, in the middle panel of Fig. 7, the second-order component of the water surface of random wave groups at d/L_{p0}=0.15 is reported, calculated for the narrow-band and for the mean JONSWAP-Mitsuyasu spectrum. As a consequence of the set-down term, the second-order correction gives a different behavior in finite water depth with respect to the trends on deep water: for finite bandwidth a lowering of about 11.5% on the maximum crest (lower panel of Fig. 7) is noticed.

This different behavior between random wave groups with narrow-band and finite bandwidth of the spectrum may be appreciated in Fig. 8, where the ratio between the nonlinear and the linear height of the largest wave crest is plotted as a function of bottom depth d. In deep water both curves, calculated for the narrow-band and mean JONSWAP-Mitsuyasu spectra, tend to be closer one another; the ratio is much greater for the infinitely narrow spectrum rather than for finite bandwidth of the spectrum, in intermediate water.

In other words, the effect of nonlinearity for the highest crest increases by decreasing the water depth, whichever the spectrum is. At any water depth, the nonlinearities for narrow band spectra are larger than those achieved for a finite bandwidth of the spectrum; this difference is greater as smaller the water depth is.

Finally, Fig. 8 compares also the effect of nonlinearity, given by the ratio $\overline{\eta}_{\max}/H_C$, for short-crested and long-crested waves. Second-order effects are slightly greater for two-dimensional waves in deep water. An opposite trend is observed by reducing the water depth, when the nonlinearities are greater for short-crested than for long-crested waves. This result is in full agreement with conclusion of Forristall (2000) who analyzed the short-term wave statistics of nonlinear crests by means of numerical simulations of second-order random waves.

7. Finite-depth influence on space-time evolution of free surface displacement for short-crested waves

In this section the space–time evolution, in a three-dimensional domain, is analyzed. Fig. 9 shows the wave group evolution both in deep water and at a water depth d/L_{p0}=0.15, when a crest with height H_C=8σ occurs. In detail, the free surface displacement is plotted for different time instants, in space domain along Y-axis (that is for X=0); the waves are short-crested with dominant wave direction $\bar{\theta}$=0. In the

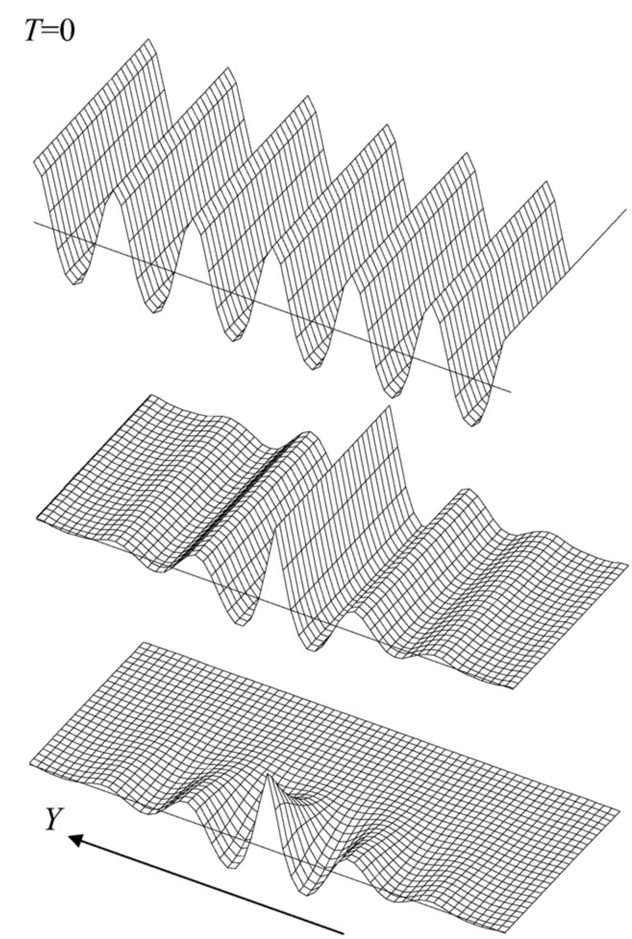

Fig. 11. Wave group in the space domain at time T equal to 0 (apex of the developing stage), in deep water. Upper panel: narrow-band waves; middle panel: broad band long-crested random waves; lower panel: short-crested waves. The dominant wave direction is given by Y-axis (H_C=8σ).

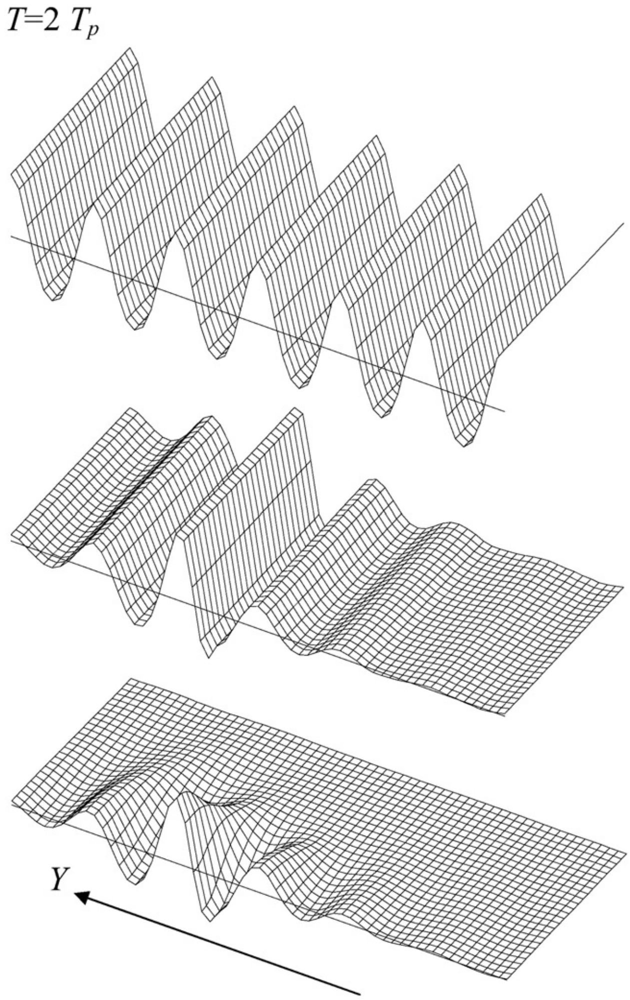

Fig. 12. Wave group in the space domain at time T equal to $2T_p$, in deep water. Upper panel: narrow-band waves; middle panel: broad band long-crested random waves; lower panel: short-crested waves. The dominant wave direction is given by Y-axis (H_C=8σ).

upper panel, the wave group is represented during its development stage, which occurs for $T<0$: for T approaching 0 the height of the largest wave of the group increases; in the lower panel, the wave group is given during its decay stage, which occurs for $T>0$, when the height of the highest wave of the group decreases as T increases.

The effects of finite water depth may be appreciated by comparing the plots: in finite water depth the length of the group envelop is slightly smaller, with respect to the deep water condition.

Finally, to consider the propagation of the wave groups in the three-dimensional domain, they have been plotted at some fixed time instants: $-2T_p$, 0 and $2T_p$. More specifically, for each time instant (see Figs. 10, 11 and 12), the free surface displacement has been calculated for an infinitely narrow spectrum [upper panels], for long-crested waves [mean panels] and for short-crested waves [lower panels]. The dominant wave direction is assumed to be coincident with the Y-axis.

8. Conclusion

The paper proposes the second-order solution for the mechanics of three-dimensional wave group at any arbitrary water depth, starting from the Boccotti's Quasi-Determinism theory that is exact to the first order in a Stokes expansion.

The space–time evolution of wave groups has been investigated and the effects associated to the finite bandwidth of the wave spectrum have been analyzed: the results obtained by assuming the narrow-band condition have been compared with those achieved for both long-crested ocean waves and short-crested waves.

To appreciate the directional spreading effects, the space time evolution of the wave front has been analyzed for different directional spreading degrees and at any fixed water depth.

A first result has been that the main properties for the mechanics of wave groups are well described by linear theory (see Boccotti, 2000, Section 10).

By analyzing second-order contributions, it has been found that, in the space time evolution of wave groups, nonlinear effect is maximum when the highest crest occurs and that the nonlinearity increases as the water depth becomes smaller. As effect of the finite bandwidth of the spectrum, nonlinearity is reduced at any fixed water depth with respect to second-order results derived for the narrow-band condition.

Finally, comparison has been proposed between nonlinearities in sea states with short-crested and long-crested waves. In the latter nonlinearities are slightly greater in deep water, whereas by reducing the water depth second-order effects become greater for short-crested waves.

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