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#### **Key Points:**

- The "hum" is caused by the interaction of ocean waves with the bottom slope
- Our model yields maps of seismic sources at all periods from 3 to 300 s
- Sources of the hum are strongest along shelf breaks, on the east side of oceans

Supporting Information:

Text S1 and Figures S1–S4

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# How ocean waves rock the Earth: Two mechanisms explain microseisms with periods 3 to 300 s

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**Abstract** Microseismic activity, recorded everywhere on Earth, is largely due to ocean waves. Recent progress has clearly identified sources of microseisms in the most energetic band, with periods from 3 to 10 s. In contrast, the generation of longer-period microseisms has been strongly debated. Two mechanisms have been proposed to explain seismic wave generation: a primary mechanism, by which ocean waves propagating over bottom slopes generate seismic waves, and a secondary mechanism which relies on the nonlinear interaction of ocean waves. Here we show that the primary mechanism explains the average power, frequency distribution, and most of the variability in signals recorded by vertical seismometers, for seismic periods ranging from 13 to 300 s. The secondary mechanism only explains seismic motions with periods shorter than 13 s. Our results build on a quantitative numerical model that gives access to time-varying maps of seismic noise sources.

#### 1. Introduction

Ocean waves provide most of the energy that feeds the continuous vertical oscillations of the solid Earth. In these, three period bands are usually identified. The hum contains periods longer than 30 s, and the primary and secondary peaks are centered around 15 and 5 s, respectively. Motions in all three bands are recorded everywhere on our planet [Webb, 1998; Nishida, 2013] and can provide information on both the solid Earth structure [Shapiro et al., 2005], the ocean wave climate over the past century [Bernard, 1990; Grevemeyer et al., 2000], and the properties of short-period ocean waves [Farrell and Munk, 2010]. Yet the use of seismic data is limited because where and how ocean waves rock the Earth is not known, in particular for the hum. The hum is the least understood part of Earth's oscillations. Recently discovered in seismometer records [Nawa et al., 1998; Suda et al., 1998], it has been associated with ocean waves for periods shorter than 300 s [Tanimoto, 2005; Rhie and Romanowicz, 2004; Bromirski, 2009; Nishida, 2013]. For any period, a Fourier analysis of the motions in the solid Earth and ocean layer shows that the transfer of energy from ocean waves to seismic waves is significant if ocean wave motions match both the wavelength and periods, and hence the speed, of seismic waves [Hasselmann, 1963]. This necessary matching of the speeds of different wave trains applies to all sorts of wave motions [Hasselmann, 1966], for example, the generation of atmospheric waves by tsunamis [e.g., Artru et al., 2005]. Both primary and secondary mechanisms can be the matchmakers between slow ocean waves and much faster seismic waves. Without any of these two mechanisms, ocean waves propagating over a flat bottom only produce pressure oscillations in the water, at the frequency and wave number of the ocean waves, hence distinct from seismic waves. The crust deformation under such a pressure field is proportional to the local ocean wave amplitude and is known as compliance [e.g., Crawford et al., 1991] and does propagate like seismic waves.

The secondary mechanism is best known, and corresponds to the nonlinear interaction of pairs of ocean wave trains with frequencies  $f_1$  and  $f_2$  and wave number vectors  $\mathbf{k}_1$  and  $\mathbf{k}_2$ . It gives pressure patterns with sum and difference frequencies  $f_1 \pm f_2$ , wave numbers  $\mathbf{k}_1 \pm \mathbf{k}_2$ , and thus, phase velocities  $C_s = 2\pi |f_1 \pm f_2| / |\mathbf{k}_1 \pm \mathbf{k}_2|$ . We will not consider the difference interactions, which have been proposed in other studies [Uchiyama and McWilliams, 2008; Traer and Gerstoft, 2014], because only the sum interactions can yield velocities matching those of seismic waves [Hasselmann, 1963; Webb, 2008] (see also supporting information Text S1). With the secondary mechanism, velocity matching occurs for nearly opposing directions and nearly equal periods (Figure 1a).

In that case, seismic waves have a frequency that is twice the ocean wave frequency [Longuet-Higgins, 1950; Hasselmann, 1963; Ardhuin and Herbers, 2013]. That secondary mechanism explains the most energetic

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**Figure 1.** The two mechanisms that produce seismic noise. (a) In the secondary mechanism, opposing waves produce wave groups with fast traveling envelopes, in green. The second-order surface pressure (in red) is proportional to the surface velocity squared; hence, it contains Fourier components that have the same speed as the envelope. (b) The most simple case of the primary mechanism involves a wave with wave number  $\mathbf{k}_2$  propagating over a sinusoidal bottom profile with almost the same wave number  $\mathbf{k}_1$ . The result is a large-scale oscillation of the bottom pressure, here in purple. As  $k = \mathbf{k}_1 - \mathbf{k}_2$  goes to zero, the wavelength and speed of that oscillation go to infinity. (c) A realistic depth profile D(x) contains all possible wavelengths, giving rise to pressure patterns with all possible speeds. The large-scale average of this pressure has a phase which changes sign where kD = 0.76, which is where the effect of an increase in wave amplitude toward shallow water is exactly compensated by the effect of a reduction in wavelength. Here the amplitude of the large-scale average was exaggerated for readability. Our choice of an infragravity wave propagating from the shoreline to open ocean corresponds to the most energetic hum sources off west coasts. In Figures 1b and 1c the time series of pressure at each point is a sinusoid.

seismic band with typical periods of 3 to 10 s [*Kedar et al.*, 2008; *Ardhuin et al.*, 2011; *Stutzmann et al.*, 2012; *Gualtieri et al.*, 2013]. At these periods, there is still a debate on the amplitude of the waves transmitted from the oceanic waveguide to the continental crust [e.g., *Obrebski et al.*, 2012; *Bromirski et al.*, 2013], which may vary with the shape of each continental margin.

In the primary mechanism, the bottom topography is like a frozen wave train, with  $f_1 = 0$ , but it contains wave numbers  $\mathbf{k}_1$  that can take any value. Bottom interference with ocean waves of frequency  $f_2$  and wave numbers  $\mathbf{k}_2$  gives energy in the bottom pressure spectrum at the same period as the ocean waves (Figures 1b and 1c) but at wave numbers  $\mathbf{k}_1 \pm \mathbf{k}_2$ , which can match seismic wavelengths. This mechanism was invoked by *Hasselmann* [1963] to explain the "primary microseism" peak, with periods usually between 10 and 20 s. The only test of this theory considered periods around 20 s, 6 days of data for a single location [*Hasselmann*, 1963], and a constant bottom slope that was adjusted to fit seismic observations.

The generation of hum (30 to 300 s) by the primary or the secondary mechanism has been controversial [e.g., *Rhie and Romanowicz*, 2004; *Tanimoto*, 2006; *Webb*, 2007, 2008; *Nishida*, 2013]. Here we focus on vertical motions which are dominated by Rayleigh waves, but horizontal motions corresponding to Love waves can also be very energetic [*Kurrle and Widmer-Schnidrig*, 2008]. *Saito* [2010] and *Fukao et al.* [2010] have shown how a distributed random array of bottom topographic features could generate both Rayleigh and Love waves, suggesting that bottom slopes can be important. Also, *Nishida* [2013, 2014] showed that the maximum hum acceleration around 10 mHz suggests a generation by infragravity waves of the same frequency.

Yet without global quantitative information on long-period ocean waves, and a poor resolution of source locations estimated from measurements, both primary and secondary mechanisms appeared plausible [*Bromirski*, 2009]. Now, with a numerical wave model extended to the infragravity frequencies by *Ardhuin et al.* [2014], we evaluate the seismic sources predicted by both mechanisms, for all periods, and we model the seismic energy of the fundamental mode of Rayleigh waves, recorded by vertical seismometers.

#### 2. The Primary Mechanism for Realistic Ocean Bottom Profiles

Because the interaction of waves with a sloping bottom has received little theoretical attention, we recall here how pressure fluctuations appear at large wavelengths when short ocean waves propagate over a sloping bottom, following Hasselmann [1963] with minor corrections. Further details are given in the supporting information Text S1. For straight and parallel depth contours, only waves propagating nearly perpendicular to them are able to produce seismic wavelengths. When depth variations occur at scales much smaller than the seismic wavelength, the seismic source amplitude is proportional to the horizontal integral of the ocean bottom pressure field. Decomposing this integral into sums over pairs of neighboring wave trough and crest reveals a particular pattern (Figure 1c). For a monotone bottom slope, the ocean waves shorten and, assuming that the energy is conserved, the wave amplitude increases. Over half a wavelength, the integrated bottom pressure is reduced by the shorter wavelength, corresponding to a higher wave number k, but increased by the larger amplitude. For any wave period, these two opposing effects cancel exactly at the depth  $D_0$  where  $kD_0$  equals 0.76. In deeper water, the amplification dominates over the shortening, with a nearly constant phase shift of the average pressure relative to the local pressure. For shallower water, this phase shift changes sign. As a result, the large-scale averaged pressure field is an oscillating dipole around the depth  $D_{0}$ , with two poles that do not exactly cancel. For any period, the seismic sources correspond to pressure oscillations at scales of the seismic wavelength, much larger than the ocean wavelength. These sources are thus distributed around  $D_0$ , provided that bottom slopes are significant. For periods of 16, 50, 100, and 200 s, D<sub>0</sub> is 30, 300, 1200, and 4800 m, respectively.

Hence, for periods longer than 50 s, sources are mostly located around shelf breaks. For shorter periods, depth variations on the scale of the ocean wavelength can strongly amplify the seismic source (Figures 1b and S2). Small-scale topographic features on the shallow part of continental shelves are the likely dominant sources of the primary microseismic peak, for periods between 10 and 20 s. In the following applications, the effect of the ocean bottom shape is represented by an effective slope that we take constant at s = 6%, based on calculations for a wide variety of depth profiles (see supporting information Text S1, equation (S21), and Figure S3). The seismic source power is proportional to *s*.

### 3. Numerical Modeling and Seismic Data Processing

Four consecutive steps yield modeled spectra of the vertical ground acceleration. First, from analyzed winds over the oceans, we compute ocean wave spectra, with a spatial resolution of half a degree in longitude and latitude [*Ardhuin et al.*, 2014]. The results of this first step are distributions of wave energy in frequency and direction, every 3 h, at each point of the spatial grid. From these, in a second step, we obtain power spectral densities of pressure in wave number and frequency space at the sea surface [*Hasselmann*, 1963; *Ardhuin and Herbers*, 2013] and bottom [*Ardhuin and Herbers*, 2013]. Third, these pressure spectra are converted into sources of seismic Rayleigh waves [*Longuet-Higgins*, 1950; *Hasselmann*, 1963; *Ardhuin and Herbers*, 2013]. Our fourth and last step is the propagation of the Rayleigh wave energy along great circles, accounting for geometrical spreading and seismic attenuation [*Ardhuin and Herbers*, 2013; *Stutzmann et al.*, 2012]. Details about assumptions and technical implementation can be found in the supporting information Text S1.

Our numerical wave model is a global configuration (78°S to 80°N) of the spectral model WAVEWATCH III in its version 5.01 [*Tolman et al.*, 2014]. This configuration has a spatial resolution of half a degree in longitude and latitude, and a spectral grid that contains 36 evenly spaced directions, and 58 frequencies exponentially distributed between 3.3 mHz and 0.72 Hz. The energy reflection coefficient  $R^2$  is parameterized from the wave height, frequency, and the local shoreface slope  $s_f$  [*Ardhuin and Roland*, 2012]. This parameterization has  $R^2$  increasing with wave period and decreasing with wave height; here we assumed a constant slope  $s_f = 10\%$ , and  $R^2$  is limited to a maximum value of 80%. This 80% is generally the value applied in the infragravity wave band. The reflected spectrum is defined as the maximum, and not the sum, of the



**Figure 2.** Measured spectra of the vertical ground acceleration, and modeled result for the primary and secondary mechanisms. (a) Median ground acceleration power spectra (LHZ channel) in March 2008 at the SSB seismic station (Geoscope Network) using records uncontaminated by earthquakes with magnitudes 5.6 and larger [*Ekström*, 2001]. The green dashed lines indicate different power law asymptotes. (b) Spectrogram of modeled ground acceleration and (c) measured spectrogram. Light blue to red vertical stripes correspond to earthquakes. The dotted magenta lines in (a) represent the New Low-Noise Model and New High-Noise Model of *Peterson* [1993], which is a familiar lower bound of microseismic activity.

reflection of incident waves and the expected local source of free infragravity waves. This procedure allows a smooth transition from the infragravity (IG) to the swell band, and the IG results are consistent with analyses of in situ data [*Ardhuin et al.*, 2014].

The output of this wave model includes standard parameters such as the significant wave height, used to verify the model against remote sensing data, and frequency-direction spectra of surface elevation  $E(f, \theta)$  every 3 h at a selected list of points (10,000 points located every half degree along all the shorelines and the 300 m depth contour, including subgrid islands in the full resolution shoreline database compiled by *Wessel and Smith* [1996]). On the full model grid, we compute the second-order pressure spectrum at the sea surface and at near-zero seismic wave number.

These pressure spectra are transformed into seismic sources. These seismic sources are added and attenuated along great circles, including multiple orbits around the Earth, as described in the supporting information Text S1. That propagation of seismic energy is controlled by the product of the attenuation Q and the group velocities U. For frequencies above 70 mHz we have used Q = 240 with U = 1.8 km/s, and a propagation coefficient P = 1, already adjusted to measurements at the SSB station [Stutzmann et al., 2012]. For frequencies under 30 mHz and all seismic stations, Q values are given by the QL6 model [Durek and Ekstrom, 1996], and we use seismic group velocities from the Preliminary Reference Earth Model [Dziewonski and Anderson, 1981]. Between 30 and 70 mHz the high-frequency and low-frequency values of UQ are combined as shown in Figure S1. At 3 mHz this gives an e-folding distance of 30,000 km, and 9000 km at 7 mHz. Seismic data from the LHZ channel of the seismometers were obtained from Incorporated Research Institutions for Seismology (IRIS) Data Management Center and corrected for the instrument response. Spectra of the vertical displacement, every 3 h centered on the hour (e.g., from 10:30 UTC to 13:30 UTC), were given by the median of the spectra obtained with 1024 point Fourier transforms with half-overlapping windows. These spectra were automatically edited for earthquakes and glitches, removing all spectra which had a sudden relative increase or drop of power from one 3 h time step to the next, with a fixed threshold. Further removal of earthquake-contaminated records in Figures 2a and 3b uses the empirical editing criterion of Ekström [2001]. Namely, after each earthquake of magnitude greater than  $M_w = 5.6$ , a time segment was removed of duration

$$T = 2.5 + 40(M_w - 5.6) \tag{1}$$

where T is in hours.



**Figure 3.** Noise time series and noise source distributions. (a) Measured root-mean-square vertical ground acceleration in the hum frequency band (7–20 mHz) at the SSB seismic station (Geoscope Network), obtained from the data and our hum model. Grey segments correspond to records contaminated by earthquakes with magnitudes  $M_{W} \ge 5.6$  and larger. (b) Daily median root-mean-square acceleration without earthquake at the seismic stations SSB (Geoscope, France, in red), PAB (IU, Spain, in black), and KONO (IU, Norway, in blue). 1 nGal is  $10^{-11}$  m s<sup>-2</sup>. The arrow marks the noise event of 10–11 March, for which (c) the modeled seismic source distributions are shown. The source power shown here is the value of the spectral density of bottom pressure  $F_p(\mathbf{k}_s = 0, f)/(\rho_W g)^2$ , evaluated on 11 March at 00:00 UTC with equation (S22) given in the supporting information.

#### 4. Results

We evaluated the primary and secondary mechanisms for the year 2008 at the locations of several seismic stations, with a first focus on the French Geoscope station SSB in March (Figure 2). As already shown [Stutzmann et al., 2012; Gualtieri et al., 2013], the secondary mechanism explains the main microseismic peak for periods between 3 and 10 s. For deep water waves, the predicted seismic acceleration power spectrum grows like the *n*th power of the frequency, with n close to 8 for periods longer than 25 s (Figure 2a and supporting information Text S1). This slope and the low spectral level are not consistent with measurements at periods longer than 13 s, contrary to previous estimates of deep water sources by Webb [2008]. Ardhuin and Herbers [2013] showed that for vanishing water depths compared to the ocean wave wavelength, the wave-induced pressure at the bottom exactly cancels the pressure at the surface. Because this bottom pressure was ignored by Webb [2007, 2008], it seemed possible that IG waves on the shelf could explain the measured hum levels. Taking bottom pressure into account here, we predict very low energy level at the lowest frequencies.

The primary mechanism, instead, reproduces the main features of the vertical seismic spectra, with a clear peak around 15 s, and a nearly constant acceleration spectrum from 50 s to 250 s

(Figure 2a). A further verification of the theory and model is given by the time evolution of the hum amplitude (Figures 2b and 2c). The resemblance between model (Figure 2b) and measurements (Figure 2c) is clear at frequencies above 40 mHz, but lower frequencies are obscured by the many transient signatures of earthquakes. In particular, two events with magnitude  $M_w = 6.9$  and  $M_w = 7.1$  occurred on 3 March at 14:11 UTC and on 20 March at 22:33 UTC, according to the centroid moment tensor catalog [*Dziewonski et al.*, 1981; *Ekström et al.*, 2012]. The largest predicted hum event coincides with the strongest infragravity wave event, during the Johanna storm on 10–11 March 2008. That storm generated waves with a peak period of 16 s and significant wave height up to 12.8 m recorded at the Spanish buoy of Estaca de Bares, the highest value in the previous 12 years at that location. That storm also affected the entire Bay of Biscay [e.g., *Senechal et al.*, 2011]. In our model, the strongest source of infragravity waves is predicted along the North-West Spanish coast. The measured spectrogram in Figure 2c exhibits higher energy around 10 mHz at the time of this storm, extending over 36 h. This long duration is not compatible with the expected decay from the largest recent earthquake, a  $M_w = 5.6$  event at 9:43 UTC on 10 March. We thus conclude that the measured seismic activity on 11 March around 00:00 UTC corresponds to the modeled infragravity waves and hum caused by the storm.



**Figure 4.** Same as Figure 3a, for the full year 2008: measured root-mean-square vertical ground acceleration in the hum frequency band (7–20 mHz) at the SSB seismic station (Geoscope Network), obtained from the data and our hum model, every 3 h. Grey segments correspond to records contaminated by earthquakes with magnitudes  $M_{W} = 5.6$  and larger, according to *Ekström* [2001].

We performed a more quantitative analysis by integrating the acceleration around 80 s period (7 to 20 mHz), the peak of the predicted hum. The hum model provides an accurate lower bound of the seismic record when earthquake activity is low (Figure 3a). We estimated the root-mean-square acceleration for each 3 h record, and the median of these for each day, excluding records contaminated by earthquakes with magnitude  $M_w = 5.6$  and larger, as described above [see also Ekström, 2001]. Results of this processing are shown in Figure 3b and extended to other seismic stations. The model reproduces the higher-hum energies during events that are strongest and closest to the seismic station, with a decrease in amplitude from station PAB in Spain, to KONO in Norway (Figure 3b). The infragravity bursts originate at the shoreline when long-period swells arrive there, typically in the eastern part of ocean basins. These infragravity waves propagate offshore from east to west [Rawat et al., 2014], and hum sources are generally lower to the west of ocean basins (Figure 3c).

Model and observations have a similar decrease of hum amplitude by a factor 3 between the peak on 11 March, and the low level on March 19. Thus, observations are consistent with the linear dependence of the hum with infragravity wave height predicted by the primary mechanism. This consistency persists for the entire year (Figure 4). The second largest event of

the year occurs on 5 and 6 January, with a strong infragravity wave event in the Gulf of Cadiz, described by *Rawat et al.* [2014].

Because IG waves produce seismic energy over bottom slopes, provided that the depth is shallow enough, mid-oceanic ridges or seamounts [*Fukao et al.*, 2010] may also be significant noise sources for periods larger than 100 s. When considering the average pressure across the mid-Atlantic ridge, we found effective slopes typically 10–100 times smaller than along shelf breaks (Figure S4). A quantitative statistical treatment of seamounts, such as proposed by *Saito* [2010] and *Fukao et al.* [2010] will be needed to determine the relative contributions of shelf topography, shelf break, mid-oceanic ridges, and seamounts.

#### 5. Conclusion

Combining the correction for shallow water effects given by *Ardhuin and Herbers* [2013] for seismic noise generation by ocean waves, and an extension of a numerical ocean wave model for periods up to 300 s [*Ardhuin et al.*, 2014], we have shown that the nonlinear interaction of nearly opposing wave trains is a significant source of microseisms for periods shorter than 13 s and is negligible for longer periods. Waves propagating over a sloping bottom can generate the observed microseisms at periods from 13 to 300 s. Together, the two mechanisms provide a realistic model of the full microseism spectrum of Rayleigh waves,

recorded on vertical seismometers. An important prediction is that most hum sources, with periods 50 to 300 s, are located along the shelf breaks, and are generated by ocean waves of the same period. More work will be needed to expand the present analysis to Love waves and investigate the model-data discrepancies between 20 and 50 mHz. The seismic energy released in this process varies by up to 7 dB on the scale of a few days, associated with predictable infragravity wave events caused by long-period swells. A broadband numerical modeling of both ocean and seismic waves, as described here, should be accurate enough to guide the analysis of seismometer data for Earth-monitoring and tomographic studies.

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# Geophysical Research Letters

Supporting Information for

# How ocean waves rock the Earth:

# two mechanisms explain microseisms with periods 3 to 300 s

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# Introduction

The text S1 gives more details and some corrections on the theory by Hasselmann (1963) as well as the pratical implementation. The four figures illustrate various aspects related to the expected seismic sources on realistic bathymetric profiles.

# Text S1.

# Detailed theoretical aspects of the generation of seismic waves by ocean waves

We give here a few important details and corrections to the general theory laid out by Hasselmann [1963], with a particular attention to its application for long ocean waves with periods larger than 30 s. Both primary and secondary mechanisms enable the short surface gravity waves to generate much longer wavelengths, and thus much faster waves that can couple to seismic waves. The question we address is rather quantitative: how much seismic energy is radiated by each of these mechanism?

After giving the expression for transforming the wave-induced pressure into seismic ground displacement, which is common to both mechanisms, we detail the expressions of the long-wavelength pressure spectrum as a function of the usual ocean wave spectrum, beginning with the secondary mechanism because this is the most studied mechanism. This follows the plan of Hasselmann [1963], with less mathematical detail for some parts, and a final expression in terms of ocean wave spectra, as given by usual numerical models.

For both mechanisms the conversion of pressure in the ocean to seismic amplitudes uses the coupling coefficients that were first determined by Longuet-Higgins [1950, his figure 2], and recomputed by Gualtieri et al. [2013] using normal modes. This follows exactly what has been done for the secondary mechanism by Kedar et al. [2008], Ardhuin et al. [2011], Stutzmann et al. [2012] and others. In the case of the primary mechanism we apply the same formulation to the wave-induced pressure at the bottom because of the shallow water depth. Hence the oscillating bottom elevation is transformed into a flat bottom with a pressure at the sea surface.

# 1) From the wave-induced pressure spectrum at large wavelengths to seismic waves

For both primary and secondary mechanisms, the source of seismic waves *S* is a function of the wave-induced pressure spectrum at seismic wavelengths. Since these wavelengths are much larger than those of surface gravity waves, we may practically estimate the pressure spectral density at near-zero wavenumber. For linear waves in a homogeneous ocean (i.e. constant depth and current velocity), this spectral density would be zero. Its non-zero value is due to either nonlinearity (secondary mechanism) or spatial non-

homogeneity (primary mechanism). For a solid half space of density  $\rho_s$  and shear wave speed  $\beta$ , Longuet-Higgins [1950] and Hasselmann [1963] give the local seismic source as a function of the seismic frequency  $f_s$  and of the wave-induced pressure spectral density near zero wavenumber  $F_p(k_x \approx 0, k_y \approx 0, f_s)$ ,

$$S(f_{s}) = 4\pi^{2} f_{s} c^{2} F_{p}(k_{x} \approx 0, k_{y} \approx 0, f_{s}) / (\beta^{5} \rho_{s}^{2})$$
(S1)

where *c* is a non-dimensional coefficient for microseismic generation [Longuet-Higgins 1950; Ardhuin and Herbers 2013], which is *c*=0.2 for  $\beta$ =2.2 km/s,  $\rho_s = 2.6$  kg/m<sup>3</sup>,  $\rho_w = 1.0$  kg/m<sup>3</sup>, and *T*=200 s with a water depth of 2000 m. The following sections give the details on how to compute  $F_p$ , based on the secondary mechanism considering  $F_{p,2}$  (eq. S5) or based on the primary mechanism considering  $F_{p,1}$  (eq. S22).

The pressure spectrum in eq. (S1) corresponds to the wave-induced spectrum at the sea surface. However, in the case of the primary mechanism, the water depth at the source is small compared to the acoustic wavelength and the bottom pressure is equivalent to a surface pressure.

This source *S* has S.I. units of m/Hz. Here we integrate these sources along great circle paths for all azimuths  $\varphi$ , and all epicentral distances  $\Delta$ , taking into account the seismic attenuation and geometrical spreading [e.g. eq. (4.36) in Ardhuin and Herbers 2013]. The integral over  $\Delta$  goes from zero to  $2\pi$ , covering one full orbit around the Earth in each direction. The power spectral density of the seismic displacement is

$$F_{\delta}(f_{s}) = \int_{0}^{2\pi} \int_{0}^{2\pi} S(\Delta, \phi, f_{s}) \frac{\exp(-2\pi f_{s} \Delta R_{E} / UQ)}{(1-b)R_{E} \sin \Delta} R_{E}^{2} \sin \Delta d \Delta d \phi, \qquad (S2)$$

where *b* is the attenuation over one orbit around the Earth,

 $b = \exp\left(-2\pi f_s 2\pi R_E/UQ\right)$ , hence the factor  $1/(1-b) = 1+b+b^2+b^3$ ... corresponds to the incoherent sum of the energies of all the orbits. This incoherent sum for epicentric distances larger than one orbit is consistent with the coarse spectral resolution used in figure 2, in which the normal mode structure is not resolved. For spectra with frequency resolution finer than  $U/(2\pi R_E)$  [e.g. Webb 2007], one could first perform a coherent sum over the orbits. Values of U and Q are discussed in the article and illustrated in figure S1.

A simple estimation of the shape and order of magnitude of the expected microseismic response to the ocean wave forcing in any water depth is provided by assuming a uniform source distribution S. In this case S reduces to a function of the seismic frequency  $f_s$  only. Under these assumptions the frequency spectrum of ground vertical displacement  $F_{\delta}$  is uniform, and eq. (S2) writes,

$$F_{\delta}(f_{s}) = S(f_{s}) \int_{0}^{2\pi} \frac{\exp\left(-2\pi f_{s} \varDelta R_{E} / UQ\right)}{(1-b)R_{E} \sin \varDelta} R_{E}^{2} 2\pi \sin \varDelta d\varDelta = S U Q / f_{s}(1-b),$$
(S3)

where U is the group speed of the considered seismic mode,  $\Delta$  is the epicentric distance between the source and the seismic station,  $R_E$  is the radius of the Earth and Q is the nondimensional seismic attenuation factor.

Eq. (S3) is strictly valid for underwater measurements, within a region of homogeneous sea state. This was shown by Ardhuin et al. [2013] to apply for microseisms with periods of 3 s and less. For the general case of a variable sea state, we expect this equality to hold only for time and spatial averages. Since the measurements discussed here are generally made on land, and the oceans only cover part of the Earth surface, eq. (S3) is only expected to provide a reasonable order of magnitude. This approach is consistent the analysis in Webb [2007]. In this context we may neglect the (1-*b*) factor, with b = 0.35 at 3 mHz, decreasing rapidly towards higher frequencies, with b=0.01 at 7 mHz.

Combining (S1) and (S3), for either the primary or secondary mechanism, we thus note that the seismic displacement power spectrum  $F_{\delta}$  is directly proportional to the wave-induced pressure power spectrum  $F_{p}$ , and the  $f_{s}$  term in (S1) cancels the  $1/f_{s}$  in (S3). This will be important for the asymptotic shapes of the seismic acceleration power spectrum for the primary and secondary mechanisms.

## 2) How short ocean waves make long seismic waves: Secondary mechanism

The seismic noise source given by the secondary mechanism is proportional to the square of the wave-induced orbital velocity at the sea surface [eq. 2.11 in ref Hasselmann 1963]. As a result, any pair of wave trains with velocities given by  $u_1\cos(k_1x-2\pi f_1t)$  and  $u_2\cos(k_2x-2\pi f_2t)$  will yield a surface pressure pattern that contain the term

$$u_1 u_2 \{ \cos[(k_1 + k_2)x - 2\pi(f_1 + f_2)t] + \cos[(k_1 - k_2)x - 2\pi(f_1 - f_2)t] \}.$$
(S4)

This nonlinear interaction of waves involves sum and difference interactions, which are given by the two parts of this interaction term.

# **2.a)** Why the difference interaction of ocean waves cannot produce seismic waves in a horizontally homogeneous environment.

It is well known that difference interactions, the second part of eq. (S4) with the frequency  $(f_1-f_2)$  can have very long periods. Recent publications by Uchiyama and McWilliams [2008] and Traer and Gerstoft [2014] have used that result to suggest this interaction could directly generate seismic noise in the hum frequency band. However, only sum interactions, the first part of eq. (S4) with  $(f_1+f_2)$ , were considered for seismic wave generation theory. The reason for neglecting difference interactions, is that they cannot produce fast traveling waves in a horizontally homogeneous environment. This was already mentioned by Hasselmann [1963], and discussed in Webb [2008].

Namely, the difference interactions cannot match the wavenumbers of seismic

components. This is clearly established by considering any pair of frequencies  $f_1$  and  $f_2$ , with  $f_2 > f_1$ . The speed  $C_2 = 2\pi (f_2 - f_1)/|\mathbf{k}_2 - \mathbf{k}_1|$  is fastest for  $\mathbf{k}_1$  and  $\mathbf{k}_2$  aligned and in the same direction. For a constant depth *D*, these frequencies  $f_i$  are functions of the norms  $k_i$  of  $\mathbf{k}_i$  given by the linear dispersion relation  $f_i = h(k_i) = g k_i \tanh(k_i D)/(2\pi)$ , where g is the acceleration of gravity. Applying the Taylor-Lagrange formula, there is a wavenumber  $k_3$  such that

$$[f_2 - f_1]/(k_2 - k_1) = h'(k_1) + (k_2 - k_1)h''(k_3)/2.$$

The group speed of ocean waves is  $2\pi h'(k_1)$ , known to be slower than the ocean phase speed. Because h''(k) is strictly negative, then  $C_2$  is less than this ocean phase speed, itself much slower than the seismic waves.

## 2.b) Surface and bottom pressure induced by sum interactions

Ocean wave properties are usually represented by a wave frequency-directional spectrum  $E(f,\theta)$ , where *f* is the wave frequency, and  $\theta$  is the azimuth of ocean waves. Only the direction-integrated spectrum E(f) is really well known,

$$E(f) = \int_{0}^{2\pi} E(f, \theta) d\theta.$$

Very few estimates [Ardhuin et al., 2012; Ardhuin et al. 2013] are available for the directional "overlap integral", defined here using the convention of Webb [2008], which is twice the value used in Ardhuin and Herbers [2013],

$$I(f) = \int_{0}^{2\pi} E(f, \theta) E(f, \theta + \pi) d\theta / E^{2}(f).$$

In deep water, the spectral density at  $k_x = k_y = 0$  of the equivalent second order pressure at the sea surface that generates seismo-acoustic noise is given by Hasselmann [1963] and Ardhuin and Herbers [2013],

$$F_{p,2}(k_x=0,k_y=0,f_s) = (\rho_w g)^2 f E^2(f) I(f)$$
(S5)

where  $f_s = 2f$  is the seismic frequency, equal to twice the ocean wave frequency f, g is the acceleration of gravity and  $\rho_w$  is the water density.

#### Finite depth effect in the secondary mechanism

Because we are also dealing with very long waves that also modify directly the bottom pressure, we need to consider the effect of wave-induced bottom pressure together with the wave-induced pressure at the sea surface. That additional bottom pressure term was not considered in previous investigations on the hum by Webb [2007, 2008]. On the full model grid we compute the second order pressure spectrum at the sea surface and at near-zero seismic wavenumber  $k_s$ . This is given by eq. (2.29) in Ardhuin and Herbers [2013]. Ardhuin and Herbers [2013] showed that considering the wave-induced pressure at the ocean bottom gives a reduction by a factor

$$G(kD) = [\tanh(kD)]^2 [1+2kD/\sinh(2kD)],$$
(S6)

where the ocean wavenumber k is related to the wave frequency f by the linear wave

dispersion relation

$$(2 \pi f)^2 = gk \tanh(kD),$$

with *D* the water depth. For long periods and shallow water,  $kD \ll 1$ , and G(kD) can be replaced by 2 tanh<sup>2</sup>(kD). In practice, this correction has no significant effect for seismic waves with periods around 5s, for which most of the sources are in deep enough water for the approximation  $G \approx 1$  to be accurate as used by Stutzmann et al. [2012], but it changes the results for the hum by several orders of magnitude.

Because the orbital velocities induced by ocean waves are much larger for a given height when waves are in shallow water, it was expected by Webb [2007] that microseismic sources would be amplified compared to deep water. Instead, the reduction in bottom pressure is fundamentally related to the Bernoulli effect, in which the pressure drops proportionally to the square of the velocity. Applied at the bottom and in the limit  $kD \rightarrow$ 0, this effect exactly cancels the surface velocity effect. This cancellation is a well known property in the incompressible context [Herbers et al. 1991; Ardhuin and Herbers 2013] and is supported by bottom pressure observations that resolve both the short and long ocean waves [Herbers et al. 1991]. The consistency between the compressible and incompressible theories is detailed in the appendix A of Ardhuin and Herbers [2013], with the sum of surface and bottom contributions given there by eq. (A4).

### Power law behavior of wave and microseismic spectra for the secondary mechanism

Combining eqs. (S1), (S3) and (S5) gives a dependence of the vertical displacement power spectrum on the seismic frequency that is proportional to  $f_s$  if  $E(f,\theta)$  is constant, giving a power spectrum of the acceleration proportional to  $f_s^5$ . In our model,  $E(f,\theta)$ increases like  $f_s$  around  $f_s = 10$  mHz. That increase and the factor G(kD) combine to give an acceleration power spectrum that is closer to  $f_s^8$ . At higher wave frequencies f = 0.15Hz, the wave spectrum typical decreases like  $f^{-n}$ 

with *n* between 4 and 5. Taking *n*=4 gives the observed shape of the acceleration power spectrum, decreasing like  $f_s^{4+1-2n} = f_s^{-3}$  (Figure 2a).

### 3) How short ocean waves make long seismic waves: Primary mechanism

This mechanism is a linear transformation of ocean waves into seismic waves of the same frequency f. For a practical application we use the wave model output of  $E(f,\theta)$  along shorelines, and more particularly its value in the direction perpendicular to shore. The primary mechanism was previously explored in the case of constant bottom slopes by Hasselmann [1963]. We give here a few more details and show general results for realistic bottom profiles. We estimate the spectral density at  $k_x = k_y = 0$  in order to give, for the primary mechanism, the equivalent of the secondary source of eq. (S5). We estimate a similar pressure spectrum at large wavelengths, for the first order pressure field in the presence of a sloping bottom. The final result is eq. (S22). We first characterize the bottom pressure field for a monochromatic wave train, and then apply that result to random waves and express the pressure spectrum as a function of the wave spectrum  $E(f,\theta)$ .

#### **3.a)** Bottom pressure for monochromatic waves

A monochromatic wave train propagating over depths varying only in direction x gives field of surface elevation and bottom pressures, with amplitudes that are modulated as shown in figure 1.c. In order to normalize all the wave-related fields, we use the wave amplitude  $a_A$  for a reference depth that we set at  $D_A = 4000$  m. The assumption of alongshore uniformity is appropriate if the depth varies alongshore on scales much larger than the seismic wavelength. For cases where the depth varies in the y direction at scales comparable or shorter than the seismic wavelength, we only expect a qualitative agreement, which is supported by preliminary tests with a phase-resolving refraction-diffraction model. We note that for linear ocean waves over a flat bottom, the dispersion relation is

$$(2\pi f)^2 = gk \tanh(kD), \tag{S7}$$

where k is the wavenumber, f is the frequency, D is the water depth, and g is the vertical acceleration due to gravity.

We will see that one effect of the bottom slopes is to put a very small fraction of the ocean wave energy at wavenumbers that are very far from this linear dispersion relation. With an alongshore wavenumber  $k_y$  we consider the bottom pressure  $P_0(x, k_y, f)$  induced by our monochromatic wave train, normalized by the wave amplitude  $a_A$ . Because of this normalization,  $P_0$  has units of Pa/m. Its Fourier transform in the x dimension over a distance  $L_x$  is

$$K(f, k_x, k_y) = \frac{1}{2\pi} \int_{-L_x}^0 P_0(x, k_y, f) e^{ik_x x} dx$$

(S8) which has units of Pa. This spectrum K is used with the same definition by Hasselmann [1963]. The actual bottom pressure spectrum is  $a_A K$ , with units of Pa.m. In Hasselmann, the eq. (4.23) is an integral from minus infinity to plus infinity. Here we have defined our Fourier transforms over a finite domain because this is how it is used for practical applications in the next section. Because the shoreline is at x=0,  $P_0$  is zero for x > 0. As a result our definition of K is the same as Hasselmann's, in the limit  $L_x \rightarrow \infty$ . Figure S2 shows examples of the pressure field at the bottom for waves propagating in one dimension.

If  $k_y$  is smaller than the magnitude of seismic Rayleigh wavenumbers, then the broad spectrum of K overlaps with wavenumbers of seismic Rayleigh waves. In practice the spectrum K is nearly white, i.e. weakly dependent on wavenumbers near  $k_x = k_y = 0$ . We can thus replace K by its value at  $k_x=0$ , which corresponds to the x-average of the pressure signal, and  $k_y=0$ , which corresponds to waves exactly perpendicular to the depth contours.

The alongshore wavenumber  $k_y$  is the same for ocean and seismic waves, and is unchanged during cross-shore propagation. Since we are considering only the spectral components of the ocean wave field that can couple to seismic Rayleigh waves, with a typical phase speed  $C_s = 3$  km/s, the ocean waves must have propagation angles that are nearly perpendicular to the depth contours, namely the ratio of the alongshore wavenumber  $k_y$  to the wavenumber magnitude k should be less than the ratio of the ocean wave to seismic wave phase speeds  $C / C_s$ , which is always less than 0.1.

## 3.b) Theoretical expression for the pressure spectrum

The bottom pressure power spectral density is, by definition, the ratio of the bottom pressure variance and the spectral increment,

$$F_{p,1}(f, k_x, k_y) = \frac{\left|a_A K(k_x, k_y, f) \Delta k_x\right|^2}{\Delta k_x \Delta k_y \Delta f}.$$
(S9)

Mathematically, eq. (S9) is correct in the limit of small spectral increments, which correspond to spectra estimated over long times and distances. In practice, due to the spatial inhomogeneities and temporal variability of the sea state, we estimate (S9) with finite spectral increments  $\Delta k_x = 2\pi/L_x$ ,  $\Delta k_y = 2\pi/L_y$ ,  $\Delta f = 1/\tau$ , where  $L_x$ ,  $L_y$ , and  $\tau$  are the lengths over which the Fourier transform is computed in the three dimensions. The definition of the wave directional spectral density is again the ratio of the variance (this time of the surface elevation) divided by the spectral increment,

$$E_{A}(f, k_{y}) = \frac{|a_{A}|^{2}}{\varDelta k_{y} \varDelta f}.$$
(S10)

We now change the spectral coordinates  $(f,k_y = k \sin\theta)$  to the more usual  $(f,\theta)$  where  $\theta$  is the azimuth of the ocean waves. The Jacobian of this spectral coordinate transform is  $k \cos\theta$ . For  $\theta=0$ ,  $\cos\theta=1$  and eq. (S10) gives

$$E_{A}(f, \theta=0) = k_{A}E_{A}(f, k_{y}=0),$$
(S11)

where, at location A, the depth is  $D_A$ , and the norm of the wavenumber vector is  $k_A$ , as given by eq. (S7), namely  $(2 \pi f)^2 = g k_A \tanh(k_A D_A)$ .

We can now replace the wave amplitude  $a_A$  in eq. (S9) by the wave spectrum (S11), using (S10), and we arrive at the first order bottom pressure spectrum. We particularly consider the power spectral density relevant for seismic wave generation, with  $k_x = 0$ ,  $k_y = 0$ ,

$$F_{p,1}(k_x=0,k_y=0,f) = \frac{E_A(f,\theta=0)}{k_A} |K^2(k_x=0,k_y=0,f)| \frac{2\pi}{L_x}.$$
(S12)

This expression is inversely proportional to  $L_x$ , the length over which the spectrum is evaluated. This is so because the relevant pressure perturbation is localized at a scale much smaller than  $L_x$ .

For an alongshore distance  $L_y$ , and a frequency bandwidth df the bottom pressure spectrum given by (S12) is equivalent, for small k, to the one produced by a vertical force oscillating with a frequency f and an amplitude F [Hasselmann 1963, Gualtieri et al. 2013]. This amplitude is simply the area  $L_x L_y$  multiplied by the square root of twice the corresponding pressure variance,

$$F = 2\pi \sqrt{2L_x L_y F_{p,1}(k_x=0, k_y=0, f) df}.$$
(S13)

Given (S12), this force F does not depend on  $L_x$ , and varies like the square root of  $L_y$ 

which corresponds to an incoherent sum of sources in the *y* direction. For  $k_x = 0$ ,  $k_y = 0$ , eq. (S8) becomes a spatial average

$$K[k_{x}=0,k_{y}=0,f]=\frac{1}{2\pi}\int_{-L_{x}}^{0}P_{0}[x,k_{y}=0,f]dx.$$
(S14)

The usual spectral density is obtained when the length  $L_x$  goes to infinity, but for practical estimates, this length is always finite. Without any wave dissipation, the pressure field at the bottom is given by the following equations. First the local group speed is defined from the local water depth D and the frequency f

$$C_{g} = \frac{2\pi f}{k} \left( 0.5 + \frac{kD}{\sinh(2kD)} \right)$$
(S15)

in which k is estimated from D and f using eq. (S6). The conservation of the energy flux gives the local surface elevation amplitude a,

$$C_{\rm g} a^2 = C_{\rm gA} a_{\rm A}^2$$
 (S16)

The phase is obtained by integrating the local ocean wavenumber k, [e.g. Mei 1989],

$$S(x) = \int_{0}^{x} k(x') dx'.$$
(S17)

These amplitudes and phase can be used to determine the surface elevation,

$$\zeta(x,k_y=0,f)=a_A\sqrt{\frac{C_g}{C_{gA}}}e^{iS}.$$

Using linear wave theory gives the conversion factor from surface elevation to the bottom pressure, which, divided by the amplitude  $a_A$  gives

$$P_0(x, k_y = 0, f) = \frac{\rho_w g}{\cosh(kD)} \sqrt{\frac{C_g}{C_{gA}}} e^{iS}.$$
(S18)

All this is enough to estimate the pressure spectrum defined by (S12). We have used this eq. (S18) to compute the pressure field over the bottom, as shown by the red curve in figure 1.c.

#### **3.c)** *Practical calculation*

In general there is no simple expression for the pressure integral (S14). For a constant bottom slope D', Hasselmann [1963] has evaluated (S14), and proposed that

$$|K|^{2} = \frac{\rho_{w}^{2} g^{4}}{(2\pi f)^{4} 64\pi} D'.$$
(S19)

 $|K_1|^2 = \frac{0.01 \rho_w^2 g^4}{(2\pi f)^4 64 \pi}.$ 

we will in particular use his value for D' = 1%,

We now revise this analytical expression, which is not exact but is still a good

(S20)

approximation of (S14), for slopes *D*' between 1.5 and 4%. Near the shoreline, where *x*=0 and *S*=0, the integrand in (S14) is proportional to  $\sqrt{S} e^{iS}$ . Under the incorrect assumption that the integral of  $\sqrt{S} e^{iS}$  from *S* = 0 to infinity is defined, eq. (S19) was obtained by Hasselmann [1963] using contour integration on the complex plane [Hasselmann, personal communication 2013]. In practice, the integral (S14) defined. For D' between 1.5 and 4% we found that (S19) is within 10% of our numerical estimates using the full expression for (S14). We also verified that (S8), with the non-zero wavenumber components  $k_x$  and  $k_y$  expected for Rayleigh waves, gives similar results.

We now consider any bottom depth profile, with the only assumption that the depth is constant in the y direction. To simplify the notations, we introduce a normalized seismic source s, using eqs. (S14) and (S20),

 $s = |K|^2 / |K_1|^2.$ (S21)

This parameter *s* contains all the effects of the bottom topography and is an "effective slope". *s* is a non-dimensional parameter that depends on the depth profile and the wave frequency f. We finally use a wave-induced pressure power spectrum expressed as follows,

$$F_{p,1}(k_x=0,k_y=0,f)=s\frac{E_A(f,\theta=0)}{k_A}\frac{\rho_w^2 g^4}{(2\pi f)^4 3200 L_x}$$

This equation is directly used to compute seismic sources in figure 3, with the direction  $\theta=0$  replaced by the local shore-normal direction  $\theta_n$ , and, to take into account waves propagating towards the shore and away from the shore we replace  $E(f,\theta=0)$  by  $[E(f,\theta_n) + E(f,\theta_n+\pi)]$ ,

$$F_{p,1}(k_{x}=0,k_{y}=0,f)=s\frac{\rho_{w}^{2}g^{4}[E_{A}(f,\theta_{n})+E_{A}(f,\theta_{n}+\pi)]}{k_{A}(2\pi f)^{4}3200L_{x}}.$$
(S22)

Estimates of *s* for different bottom profiles are shown in Figure S3.

Eq. (S22) can be combined to eq. (S3) to estimate the shape of the seismic response. Taking A in 4000 m depth, then  $k_A$  is proportional to f for f smaller than 10 mHz. A constant spectrum  $E_A$  gives a pressure spectrum  $F_{p,1}$  predicted by the primary mechanism that decreases like  $f^{-5}$ . Combined with eq. (S3) this gives a vertical displacement power spectrum that decreases like  $f^{-5}$  and an acceleration power spectrum that follows  $f^{-1}$ . Since our numerical model in the infragravity wave band actually predicts a spectrum  $E_A$  that grows like f from 5 to 15 mHz, this explains the nearly constant acceleration spectrum in that frequency range. At higher frequencies, a decrease of the wind wave spectrum like  $f^{-4}$  around f=0.1 Hz gives a  $f^{-5}$  seismic acceleration power spectrum.

#### **3.d)** Interpretation of the source dependence on the bottom profile

Eq. (S22) is interesting because is gives a practical expression for the pressure spectral density. However, the use of Fourier transforms removes the spatial resolution, and

prevents us from understanding which is the part of the depth profile that is responsible for the seismic source. In order to discuss spatial distribution of sources, we thus define a partial spatial integration s(x),

$$s(x) = \frac{1}{(2\pi)^2 |K_1|^2} \left| \int_x^0 P_0(x', k_y, f) dx' \right|^2.$$

(S23)

Combining eqs. (S13) and (S20), this definition gives  $s(-L_x) = s$ , namely, the limit of s(x) when reaching the end of the depth profile is the noise source previously defined. The quantity s(x) is plotted infigures S1g and S1h. The locations where s(x) varies most rapidly can be interpreted as the locations where the seismic sources are strongest.

## 3.e) Surf zone parameterization

An important detail for wave periods between 10 and 50 s is the treatment of the surf zone.

The ocean wave height decreases rapidly to zero for wave periods shorter than 30 s, and becomes roughly constant in the infragravity wave band. Hasselmann [1963] had proposed to represent the effect of wave dissipation in shallow water by an abrupt reduction of the bottom pressure to zero at a position  $x_b$ . Such a sharp jump in bottom pressure would give an order of magnitude increase of *s*. Instead, we prefer to reduce gradually the wave amplitude, and thus the bottom pressure, by a factor proportional to the water depth, giving the more realistic amplitude and pressure distribution shown in Fig. (S1.f). In the case of a smooth bottom topography, this treatment of the surf zone reduces *s* by one order of magnitude. Using a constant amplitude, more appropriate for the infragravity wave band (i.e. T > 30 s), did not significantly change the results.

### 3.f) Effects of unresolved bathymetric features

By taking into account realistic depth variations on the scale of the ocean wavelength for sandy bottoms, the value of s is typically increased by an order of magnitude, for periods in the range 10 to 30 s. Here we have used an empirical spectrum of the bottom elevation,

$$F_{B}(k_{b}) = 1.5 \times 10^{-4} k_{b}^{-3}, \qquad (S24)$$

which is consistent with depth soundings over sandy continental shelves [Hino 1968; Ardhuin and Magne 2002]. This impact of the depth oscillations, on the scale of the ocean wavelength, can be interpreted as a classical wave-wave interaction process [Hasselmann 1966] with the ocean waves of wavenumber vector  $\mathbf{k}_1$  coupling to bottom depth perturbations of wavenumber vector  $\mathbf{k}_b$  to generate seismic waves of the same frequency but wavenumber  $\mathbf{k}_1 + \mathbf{k}_b$ .

In practice we have estimated the ratio  $s = |K|^2/|K_1|^2$  for many different bottom profiles and wave periods. Our simple choice s = 6 is consistent with these calculations, within one order of magnitude (figures S2 and 2), and gives a reasonable fit to the measured hum, as shown in figures 2, 3 anf 4. We have used east-west depth profiles are a 1 degree interval around 41° N [mid-Atlantic ridge, Lourenco et al. 1998], and 0.2° intervals around 46.47°N [Bay of Biscay, Anonymous 2008], 41.473°N [Oregon shelf, U.S. DoC NOAA/NGDC 2006], and north-south profiles around 157.8°W across the Oahu north shore, Hawaii [Anonymous 2011].

# **3.g)** Choice of s for our calculations

When evaluating the integral (S14) with the linear wave-induced bottom pressure (S18) we generally found a slightly larger value of *s* for frequencies below 10 mHz, with a maximum value obtained for the Hawaii bathymetry profile s = 40 at 5 mHz. The fundamental reason why *s* is larger at these low frequencies, is that there are no strong enough slopes for depth such that kD > 0.76. Indeed, for f=5 mHz, kD = 0.76 corresponds to D=4800 m.

For short periods (typically less than 30 s), the value of *s* critically depends on the magnitude of the bottom depth perturbations at wavelengths similar to the ocean surface wavelength (around 100 m), and would otherwise be as low as 0.1 for a smooth bottom on these scales. The constant value s = 6, gives good results for the hum amplitude (Fig. 2). The good amplitude for the primary peak also should not be overstated. At these shorter periods we would expect *s* to be lower (figure S2) and we know that the seismic attenuation model is not well constrained. Finally we also expect that the alongshore variation in water depth lead to quantitative deviations from the previous result. This aspect has not been explored in the present paper.

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# Figure S1.



Supporting information, figure S1. Attenuation of seismic waves in our model. a, Values of UQ obtained by combining QL6 for Q and PREM for U (computed with a water depth of 2 km), for periods larger than 30 s with a constant U=1.8 km/s and Q=240 for shorter periods. b, Corresponding distance over which the seismic energy is reduced by a factor 2.7. For a period of 250 s ( $f_s$ =4mHz) this distance is half-way round the globe, which allows the constructive interference that yields the modal structure seen in high-resolution spectra (e.g. Webb 2007).



Supporting information, figure S2. How ocean waves propagating over a sloping bottom produce very long wavelength components capable of coupling to seismic **Rayleigh waves.** Example with waves of period 100 s and 16 s, over a continental shelf. a,b Example bottom topography in the Bay of Biscay at 47° N. c Snapshot of the bottom pressure computed using linear wave theory and energy conservation - except in the surf zone, for a period of 100 s and **d**, 16 s. **e**, Schematic showing how the pressure in a trough, with negative values, does not exactly cancel the pressure in the adjacent crest, due to a combination of wavelength shortening and increase in wave amplitude, giving a systematic phase shift between the wavelength-averaged pressure and the local pressure. For a non-dimensional depth kD > 0.76, corresponding to x > 250 km in **c.e.g**, the amplitude changes dominate. In shallower water the shortening dominates, giving opposing contributions to the mean pressure. f, close-up on the surf zone and reduction in wave amplitude due to breaking near the shore. g,h, In red, expected normalized seismic source power s(x) due to waves between the shoreline and the local position x as defined in eq. (S22). In g the areas of black bars are proportional to the increment in source power over one ocean wavelength.



**Supporting information, figure S3.** Amplification factors for primary seismic sources. The effective bottom slope *s* defined by eq. (S22, Supporting information text) is shown for different bottom depth profiles and for frequencies ranging from 5 to 100 mHz. These estimates are median values over 11 contiguous depth profiles taken from (28-31). For the calculations shown in figures 1 and 2, we have used a constant *s* = 6. The exact location of east-west depth profiles are a 1 degree interval around 41° N (mid-Atlantic ridge), and 0.2° intervals around 46.47°N (Bay of Biscay), 41.473°N (Oregon shelf), and north-south profiles around 157.8°W across the Oahu north shore, Hawaii. The seismic source power is proportional to *s*.

# Figure S4.



Supporting information, figure S4. Primary seismic sources over the mid-Atlantic ridge. Results for 100 s infragravity waves over a depth profile taken from the mid-Atlantic ridge, north of the Azores, at 41°N. **a**, Bottom topography, **b**, bottom pressure computed using linear wave theory and energy conservation. **c**, seismic source power due to waves between the local position x and x=1500 km, estimated from the variance of the spatially-averaged bottom pressure.