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Comment

Momentum Balance in Shoaling Gravity Waves: Comment on 'Shoaling Surface Gravity Waves Cause a Force and a Torque on the Bottom' by K. E. Kenyon

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In a recent paper, Kenyon (2004) proposed that the wave-induced energy flux is generally not conserved, and that shoaling waves cause a mean force and torque on the bottom. That force was equated to the divergence of the wave momentum flux estimated from the assumption that the wave-induced mass flux is conserved. This assumption and conclusions are contrary to a wide body of observations and theory. Most importantly, waves propagate in water, so that the momentum balance generally involves the mean water flow. Although the expression for the non-hydrostatic bottom force given by Kenyon is not supported by observations, a consistent review of existing theory shows that a smaller mean wave-induced force must be present in cases with bottom friction or wave reflection. That force exactly balances the change in wave momentum flux due to bottom friction and the exchange of wave momentum between incident and reflected wave components. The remainder of the wave momentum flux divergence, due to shoaling or wave breaking, is compensated by the mean flow, with a balance involving hydrostatic pressure forces that arise from a change in mean surface elevation that is very well verified by observations.

Keywords:
· Shoaling waves,
· radiation stresses,
· wave force,
· wave scattering,
· set-up.

1. Wave Action, Momentum and Energy Balances

Kenyon (2004) recently hypothesized that the wave-induced mass transport should be conserved in shoaling waves and that the sea level is flat so that the wave momentum flux divergence should be balanced by a mean non-hydrostatic force on the bottom. These statements run counter to observations (e.g. Raubenheimer *et al*., 2001) of the balance between sea-level change (set-up) and wave momentum flux divergence. The fluctuating forces on a flat bottom are also well known ever since Longuet-Higgins's (1950) explanation of microseism generation. However there has as yet been no complete discussion of the mean (i.e. phase-averaged) wave momentum balance in shallow water and recent advances on this topic are a good occasion for a serious review of the question: is there a mean force acting on the bottom besides the hydrostatic pressure?

Equations are given in Cartesian coordinates with Greek indices α and β referring to horizontal coordinates. Summation is implicit over repeated indices. For simplicity the water density ρ_w and gravity acceleration g are assumed constant. The general action balance equation for a train of monochromatic surface gravity waves is given by Willebrand (1975) and Komen *et al*. (1994), and in its most general form by Andrews and McIntyre (1978b). Taking a varying wave amplitude a and wavenumber magnitude k, and assuming $\varepsilon = \max\{ka\} << 1$, the action balance equation for a train of monochromatic surface gravity waves may be written as

$$\frac{\partial A}{\partial t} + \frac{\partial}{\partial x_\beta}\left[\left(U_{A\beta} + C_{g\beta}\right)A\right] = \frac{S_{\mathrm{tot}}}{\sigma} + O\left[\left(\varepsilon + \gamma\right)\left(\frac{\gamma A}{\sigma}\right)\right], \quad (1)$$

where $A = E/\sigma = a^2/\sigma$ is the wave action, E is the wave-induced surface elevation variance, $\rho_w gE$ is the wave energy per unit horizontal surface, $\sigma^2 = gk\,\tanh(kh)$ is the linear dispersion relation, $U_{A\beta}$ is the β component of the

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wave advection velocity $\mathbf{U}_A$ due to mean currents, and $\rho_w g S_{\mathrm{tot}}$ is an energy source term made explicit below. If negative, this source is actually a sink.

Several theories have been developed for $\mathbf{U}_A$. For small wave amplitudes and weak mean current shears, an explicit expression is given by Kirby and Chen (1989). Their solution is compatible with the general expression of wave quantity fluxes given by Andrews and McIntyre (1978b) who also include the advection of wave action by the Stokes drift, a term that is formally of higher order in the wave slope, but often comparable in magnitude to advection by the mean current (see also Smith, 2006). Finally, the small parameter γ represents non-homogeneities in the wave field, and may be defined mathematically as

$$\gamma = \max\left\{\frac{\nabla\left(\tanh(kD)\right)}{k};\frac{\nabla U_i}{\left(kU_\alpha\right)};\frac{\dfrac{\partial U_\alpha}{\partial z}}{\left(kU_\alpha\right)};\frac{\nabla a}{ka};\frac{\dfrac{\partial a}{\partial t}}{(\sigma a)}\right\}, \qquad (2)$$

with U_α the mean current velocity component in direction α averaged over a wave period, and D the water depth averaged over a wave period. In other words, in Eq. (1) all the terms written explicitly are of order $\gamma A/\sigma$ and the other terms that will be neglected below are of higher order in either ε (non-linearity) or γ (non-homogeneity).

When more than one wave component or a full spectrum is considered, the conditions on the horizontal gradients of water depths can be relaxed to conditions of small changes of D on the scale of the wavelength, with a scattering term S_{bscat} included in S_{tot} that couples incident and reflected wave components (Ardhuin and Herbers, 2002, with an extension given by Ardhuin and Magne, 2006). Neglecting the higher order terms from now on, the spectral form of (1) is

$$\frac{\partial A(\mathbf{k})}{\partial t} + \frac{\partial}{\partial x_\beta}\left[\left(U_{A\beta}(\mathbf{k}) + C_{g\beta}(\mathbf{k})\right)A(\mathbf{k})\right] = \frac{S_{\mathrm{tot}}(\mathbf{k})}{\sigma}, \qquad (3)$$

with

$$S_{\mathrm{tot}}(\mathbf{k}) = S_{\mathrm{in}}(\mathbf{k}) + S_{\mathrm{ds}}(\mathbf{k}) + S_{\mathrm{nl}}(\mathbf{k}) + S_{\mathrm{bfric}}(\mathbf{k}) + S_{\mathrm{bscat}}(\mathbf{k}) \qquad (4)$$

with terms representing respectively the rate of energy input from the wind to the waves, the dissipation of waves due to breaking and interaction with ocean turbulence, non-linear scattering (also referred to as 'wave-wave interactions'), bottom friction, and bottom scattering (see Komen *et al.*, 1994; Ardhuin *et al.*, 2003).

Since the wave pseudo-momentum of component $\mathbf{k}$ is $\mathbf{M}^w(\mathbf{k}) = \rho_w \mathbf{k} A(\mathbf{k})$, with $\mathbf{k}$ the wavenumber vector, an evolution for $\mathbf{M}^w$ can be obtained by combining (1) with the equation for $\mathbf{k}$ obtained from the dispersion relation (e.g. Phillips, 1977). For one wave component, it is

$$\frac{\partial k_\alpha}{\partial t} + \left(U_{A\beta} + C_{g\beta}\right)\frac{\partial k_\alpha}{\partial x_\beta} = -k_\beta \frac{\partial U_{A\beta}}{\partial x_\alpha} - \frac{k\sigma}{\sinh 2kD}\frac{\partial D}{\partial x_\alpha}. \qquad (5)$$

The combination of (3) and (5), for a single wave component $\mathbf{k}$, gives

$$\frac{\partial M^w_\alpha}{\partial t} + \frac{\partial}{\partial x_\beta}\left[\left(U_{A\beta} + C_{g\beta}\right)M^w_\alpha\right] = \rho_w g k_\alpha \frac{S_{\mathrm{tot}}}{\sigma} - M^w_\beta \frac{\partial U_{A\beta}}{\partial x_\alpha} - \frac{S^p}{D}\frac{\partial D}{\partial x_\alpha} \qquad (6)$$

with

$$S^p = \rho_w g\left(C_g - \frac{C}{2}\right)\frac{E}{C} = \rho_w g \frac{kDE}{\sinh 2kD}. \qquad (7)$$

A direct, and more general derivation is given by Andrews and McIntyre (1978b). This wave pseudo-momentum $\mathbf{M}^w$ is often called momentum, and only represents the part of the momentum that is related to the rapidly oscillating motions due to the waves. The total wave-induced momentum also includes the secondary circulation or wave group response (McIntyre, 1981). However, neglecting higher order terms, the momentum flux tensor is given by $(U_{A\beta} + C_{g\beta})M_\alpha{}^w$.

As noted by Kenyon (2004), the wave momentum flux in the case of shoaling waves is generally divergent. For steady waves propagating at an angle θ relative to the x axis, and a topography and current field uniform along the y direction, one has

$$\frac{\partial}{\partial x}\left[\left(U_{Ax} + C_g\cos\theta\right)\cos\theta M^w\right] = \rho_w g k \frac{S_{\mathrm{tot}}}{\sigma} - \cos\theta M^w \frac{\partial U_{Ax}}{\partial x} - \sin\theta M^w \frac{\partial U_{Ay}}{\partial x} - \frac{S^p}{D}\frac{\partial D}{\partial x}. \qquad (8)$$

In cases where non-linear interactions, scattering and dissipative processes are negligible $S_{\mathrm{tot}} = 0$. The divergence of the current with a mass transport vector $\mathbf{M}^m$ is thus exactly balanced by the divergence of the Stokes transport $\mathbf{M}^w$. For waves propagating toward an impermeable beach we have $M_x{}^w = -M_x{}^m$, and U_{Ax} is of the order of these two terms, typically much smaller than C_g, so that $U_{Ax}M^w$ may be neglected, and one has

$$\frac{\partial}{\partial x}\left[C_g M^w \cos^2\theta\right] = -\frac{S^p}{D}\frac{\partial D}{\partial x}. \qquad (9)$$

As the waves enter shallower water, i.e. $\partial D/\partial x < 0$, the wave action flux $(U_{Ax} + C_g\cos\theta)A$ is conserved based on (1). This result is only true for negligible reflection, otherwise the action flux of the reflected waves must be included. Because the wave number k decreases as the waves shoal, the wave momentum flux, that is equal to $(U_{Ax} + C_g\cos\theta)\cos\theta M_w = \rho_w(U_{Ax} + C_g\cos\theta)\cos\theta kA$, increases with a rate of change given by $-(\partial D/\partial x)S^p/D$. Relative variations in the wave energy flux are thus of the order of U_{Ax}/C_g, which corresponds to an exchange of energy with the current (e.g. Phillips, 1977). Contrary to statements by Kenyon (2004), this approximate conservation of the wave energy flux is very well verified by Ardhuin *et al.* (2003) for swells of small amplitudes on the North Carolina continental shelf ($H_s < 1.5$ m). As discussed by these authors, the non-conservation of the energy flux in larger amplitude swells is likely related to bottom friction. Many other studies have given similar results (e.g. Munk and Traylor, 1947; O'Reilly and Guza, 1993).

2. Mean Flow Momentum Balance

Because waves propagate in a material medium, the mean flow associated with the wave motion cannot be ignored, and the divergence in the wave momentum flux is often associated with an opposite divergence in the mean flow momentum flux. It is thus impossible to conclude that a mean force acts on the bottom before the mean flow momentum balance is examined. Instead, Kenyon (2004) plainly discarded any possible exchange of momentum between the mean flow and the wave field, and, in particular, insisted that the mean sea level remained constant, in spite of all scientific observations to the contrary (from Saville, 1961 to Raubenheimer *et al.*, 2001).

Phillips (1977, see also Smith, 2006) gives the evolution equation for the total momentum $\mathbf{M} = \mathbf{M}^m + \mathbf{M}^w$

$$\frac{\partial}{\partial t}M_\alpha + \epsilon_{\alpha i\beta} f_i M_\beta + \frac{\partial}{\partial x_\beta}\left\{\int_{-h}^{\bar{\zeta}} \rho_w U_\beta U_\alpha \mathrm{d}z + \int\left[U_{A\alpha}(\mathbf{k})M_\beta^w(\mathbf{k}) + U_{A\beta}(\mathbf{k})M_\alpha^w(\mathbf{k})\right]\mathrm{d}\mathbf{k} + S_{\alpha\beta}^{\mathrm{rad}}\right\} = -\left(\rho_w gD + p_0^w\right)\frac{\partial\bar{\zeta}}{\partial x_\alpha} + \tau_\alpha^a - \tau_\alpha^b \qquad (10)$$

with $\mathbf{U}$ the current velocity averaged over the wave phase.* τ^a and τ^b are the surface (wind) and bottom stresses, defined as the total momentum fluxes to the ocean and to the bottom, counted positive downward. The symbol $\epsilon_{\alpha\beta i}$ represents the signature of the permutation (α, β, i), equal to 0 if one index is repeated, 1 if (α, β, i) can be obtained by shifting (1, 2, 3), and –1 otherwise. This notation allows us to write the components of the vector product of the Coriolis parameter vector (f_1, f_2, f_3) and the mass flux (M_1, M_2). In general, and hereafter, only the vertical component of the Earth rotation is retained, and the corresponding parameter f_3 is usually known as the Coriolis parameter (see e.g. Hasselmann, 1970 for a discussion of the other components). $S_{\alpha\beta}^{\mathrm{rad}}$ is the usual radiation stress tensor (noted S^{rad} by Phillips), in the absence of mean currents, which involves the wave-induced pressure and the wave momentum flux

$$S_{\alpha\beta}^{\mathrm{rad}} = -\delta_{\alpha\beta}S^p + C_{g\beta}M_\alpha^w, \qquad (11)$$

with $\delta = 1$ if $\alpha = \beta$ and $\delta = 0$ otherwise. Finally, to order ε^2, the Eulerian mean wave-induced pressure is minus the variance of the vertical velocity. Near the surface this is

$$p_0^w = -\int_{\mathbf{k}} \sigma^2 E(\mathbf{k})d\mathbf{k}. \qquad (12)$$

Note that a surface mean velocity U_α ($z = \bar{\zeta}$) appears in the equation established by Smith (2006), instead of $U_{A\alpha}$ here in (10). The difference between U_α ($z = \bar{\zeta}$) and $U_{A\alpha}$ arises from vertical shears in the mean current, which are assumed small here, and thus for our purpose the two equations are equivalent. We have used $U_{A\alpha}$ in (10) for consistency with (6), but also because (10) closely resembles the vertical integration of generalized Lagrangian mean flow equations, which allow a proper representation of the effect of the vertical current shear (Andrews and McIntyre, 1978a). This matter is further

*The use of Eulerian averages by Smith (2006) can only be justified below the wave troughs (i.e. for $z < \min\{\zeta(t)\}$) and requires an extension of the velocity field above the surface in order to have a well defined integral up to $\bar{\zeta}$. Such a problem can be avoided by using the Generalized Lagrangian Mean of Andrews and McIntyre (1978), as discussed by Ardhuin (2005), with an equivalent result to that order of approximation.

discussed by Ardhuin (2005) and Ardhuin and Rascle (manuscript in preparation).

Following Smith (2006), the subtraction of (6) from (10) gives

$$
\begin{aligned}
&\frac{\partial M_\alpha^m}{\partial t} + \rho_w \frac{\partial}{\partial x_\beta}\left(\int_{-h}^{\bar{\zeta}} U_\beta U_\alpha \mathrm{dz}\right) \\
&+\int U_{A\alpha}(\mathbf{k}) \frac{\partial M_\beta^w(\mathbf{k})}{\partial x_\beta} \mathrm{d}\mathbf{k} + \epsilon_{\alpha 3\beta} f_3 M_\beta^m + \left[\rho_w g D - p_0^w\right] \frac{\partial \bar{\zeta}}{\partial x_\alpha} \\
&= -\epsilon_{\alpha 3\beta} \int \left[f_3 + \Omega_3(\mathbf{k})\right] M_\beta^w(\mathbf{k}) \mathrm{d}\mathbf{k} - \frac{\partial S^p}{\partial x_\alpha} + \frac{S^p}{D}\frac{\partial D}{\partial x_\alpha} \\
&+\left(\tau_\alpha^a - \tau_\alpha^{\mathrm{in}}\right) - \left(\tau_\alpha^b - \tau_\alpha^{\mathrm{bfric}} - \tau_\alpha^{\mathrm{bscat}}\right) - \tau_\alpha^{\mathrm{ds}},
\end{aligned}
\quad (13)
$$

where $\Omega_3(\mathbf{k})$ is the curl of $\mathbf{U}_A(\mathbf{k})$, S^p comes from the difference between the radiation stresses and the wave pseudo-momentum flux, and the fluxes corresponding to the source terms are

$$\tau^{\mathrm{in}} = \rho_w g \int_{\mathbf{k}} \frac{\mathbf{k} S_{\mathrm{in}}(\mathbf{k})}{\sigma} d\mathbf{k}, \quad (14)$$

$$\tau^{\mathrm{ds}} = \rho_w g \int_{\mathbf{k}} \frac{\mathbf{k} S_{\mathrm{ds}}(\mathbf{k})}{\sigma} d\mathbf{k}, \quad (15)$$

$$\tau^{\mathrm{bfric}} = -\rho_w g \int_{\mathbf{k}} \frac{\mathbf{k} S_{\mathrm{bfric}}(\mathbf{k})}{\sigma} d\mathbf{k}, \quad (16)$$

$$\tau^{\mathrm{bscat}} = -\rho_w g \int_{\mathbf{k}} \frac{\mathbf{k} S_{\mathrm{bscat}}(\mathbf{k})}{\sigma} d\mathbf{k}. \quad (17)$$

The non-linear scattering due to by 4-wave interactions (Hasselmann, 1962) conserve wave energy, action, and momentum within the wave field, and thus disappears from the balance equation when these wave properties are integrated over the spectrum. τ^{in} equals the sum of surface pressure—slope correlations and correlations of shear stress fluctuations with the along-surface velocity (Longuet-Higgins, 1969). Thus ($\tau^a - \tau^{\mathrm{in}}$) is the direct turbulent flux of momentum from the atmosphere to the mean flow, equal to the mean shear stress at the surface.

We may now investigate the mean stresses acting on the bottom and draw a conclusion on the existence of a mean force acting on the bottom. Longuet-Higgins (2005, his equation (3.7)) recently showed for laminar flow that τ^{bfric} is equal to a mean additional bottom stress when waves dissipate due to bottom friction with a constant eddy viscosity in the wave bottom boundary layer. In a sense, the wave momentum lost due to bottom friction accelerates the wave bottom boundary layer, with a mean motion known as streaming. This momentum added to the bottom boundary layer entirely leaks to the bottom via the mean bottom shear acting on this streaming flow. None of that wave momentum leaks to the upper water column due to the difference in shears between the top and the bottom of the wave boundary layer.

Older results by the same author (Longuet-Higgins, 1967) and Mei (1973) can be used to reveal the nature of the bottom scattering stress τ^{bscat}. Under the single assumption of irrotational flow, and approximating the wave-induced mean pressure to second order in the wave slope, Longuet-Higgins (1967) found that Bernoulli's theorem gave the difference in sea level between the two sides of a submerged breakwater or any bottom topography (e.g. Fig. 1). His result can be re-written as

$$\bar{\zeta}_2 - \bar{\zeta}_1 = \frac{1}{\rho_w g}\left[\frac{S_1^p + S_{r1}^p}{D_1} - \frac{S_2^p}{D_2}\right]\left(1 + O(\varepsilon)\right), \quad (18)$$

where $S_1{}^p$, $S_{r1}{}^p$ and $S_2{}^p$ are the S^p terms for the incident, reflected, and transmitted waves, respectively. Thus, as given by (13), the combination of the two terms with S^p is $D\boldsymbol{\nabla}(S^p/D)$ and this balances the hydrostatic pressure gradient, $\rho_w g D \boldsymbol{\nabla}\, \bar{\zeta}$. Therefore, the reflection stress τ^{bscat} does not enter the mean flow momentum balance, and must act as a mean stress on the bottom. Although it has not been explicitly determined as such, this stress is the likely result of a correlation of a mean pressure with the bottom slope, giving a horizontal recoil force on the bottom. This is analogous to the recoil of a partially reflecting 'solar sail' hit by a beam of light, a method that may be used for the propulsion of light objects in space. We thus conclude that ($\tau^b - \tau^{\mathrm{bfric}} - \tau^{\mathrm{bscat}}$) is the direct flux of momentum from the mean flow to the bottom, in the form of a mean shear. Therefore the net water to bottom momentum flux τ^b requires momentum flux from the wave field to the bottom given by $\tau^{\mathrm{bfric}} + \tau^{\mathrm{bscat}}$.

We may finish with the S^p terms in (13). The first term with S^p, $\boldsymbol{\nabla} S^p$ in vector form, is the gradient in wave-induced mean pressure. The second term with S^p is exactly opposite to the wave momentum flux divergence term $S^p \boldsymbol{\nabla} D/D$ in (8), showing that this latter term represents an exchange of momentum between the waves and the mean flow. We now consider waves shoaling toward a straight coast defined by $x = 0$. For depth- and alongshore-uniform mean currents $\mathbf{U}(x)$, we have $\mathbf{U}_A(\mathbf{k}) = \mathbf{U}$. Using the fact that the total mass flux M_x is zero due to the impermeable shoreline, the first advection term may be expressed in terms of wave momentum advection. We thus have, for the x momentum balance,

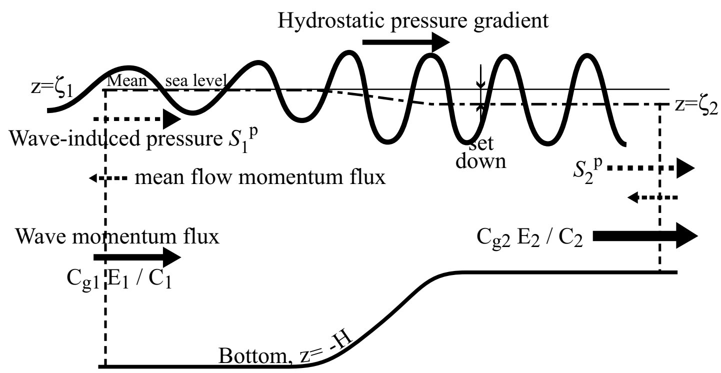

Fig. 1. Balance of forces for waves over a smooth step, in the case without dissipation or wave reflection. Fluxes of momentum across two vertical sections (dashed lines) are indicated by arrows. The force that corresponds the divergence of the wave pseudo-momentum flux C_gE/C combines with the gradient of the wave-induced pressure S^p. This combination is generally balanced by the hydrostatic pressure gradient related to the mean sea level gradient. The acceleration of the mean flow (small dashed arrow), due to a divergence in the Stokes transport, is generally much weaker.

$$-\frac{\partial}{\partial x_\beta}\left(U_\beta M_x^w\right)-M_\beta^w\frac{\partial U_x}{\partial x}+\frac{\partial S^p}{\partial x}-\frac{S^p}{D}\frac{\partial D}{\partial x}-\left(\tau_x^a-\tau_x^{\mathrm{in}}\right)$$
$$+\left(\tau_x^b-\tau_x^{\mathrm{bfric}}-\tau_x^{\mathrm{bscat}}\right)+\tau_x^{\mathrm{ds}}=-\rho_w gD\frac{\partial\bar{\zeta}}{\partial x}, \tag{19}$$

with $\mathbf{M}^w = \int\mathbf{M}^w(\mathbf{k})\mathrm{d}\mathbf{k}$. Using the wave momentum balance (6), this is simply the balance given by Phillips (1977)

$$\frac{\partial S_{xx}^{\mathrm{rad}}}{\partial x}-\tau_x^a-\tau_x^b=-\rho_w gD\frac{\partial\bar{\zeta}}{\partial x_\alpha}. \tag{20}$$

This balance is very well supported by the measurements of Saville (1961), Bowen *et al.* (1968), and Raubenheimer *et al.* (2001). Without dissipation (but with the possibility of reflection), the balance of forces is the one given by Longuet-Higgins (1967) and shown on Fig. 1,

$$-D\frac{\partial}{\partial x_\alpha}\left(\frac{S^p}{D}\right)=\rho_w gD\frac{\partial\bar{\zeta}}{\partial x_\alpha}. \tag{21}$$

In general, care should be taken to ascertain that τ^a and τ^b are the total momentum fluxes through the air-sea interface and the bottom, including the divergences in wave momentum flux due to wind-wave generation, wave reflection and bottom friction. These last two divergences are compensated by a mean momentum flux through the bottom, which is a mean force caused by the wave field on the bottom, in addition to the usual hydrostatic pressure.

In cases with either bottom friction or reflection, although the divergence of the wave momentum flux goes into the bottom and not in the mean flow, there is still a mean flow response, such as a wave set up, due to the wave-induced mean pressure (Longuet-Higgins, 2005). This effect suggests that the usual combination of both pressure and momentum flux terms in the radiation stresses (11) is physically misleading, and that it would be better to keep these two terms separated. Because bottom friction requires parameterization of the turbulent boundary layer over variable sediments, only empirical expressions for this flux can be given (e.g. Ardhuin *et al.*, 2003; Feddersen *et al.*, 2003). However, the flux induced by wave reflection can be accurately estimated from theory, at least for small bottom amplitudes (Magne *et al.*, 2005; Ardhuin and Magne, 2006).

3. Conclusions

Using a consistent momentum balance for the waves and the mean flow it was found that the wave action flux is generally conserved, leading to an approximate conservation of wave energy flux in the case of waves shoaling when approaching a beach. This conservation is well verified for small amplitude waves, when the relative wave dissipation is predicted to be weak. The observed strong convergence in wave energy fluxes for large amplitude waves is well predicted to be related to strong dissipation due to bottom friction (Ardhuin *et al.*, 2003). As a consequence, the wave-induced mass flux (also called 'Stokes transport'), as well as the related momentum flux, are generally divergent. In the absence of dissipation and reflection a mean force must explain the acceleration of the momentum flux toward the beach. It is found that this force is generally exerted by the mean flow and not the bottom. Mean forces exerted on the bottom only amount to the hydrostatic pressure and the fluxes of wave momentum related to bottom friction and wave reflection off bottom slopes. These results were established to the first order of approximation in wave steepness and bottom slope, and thus conflict with the results reported by Kenyon (2004), who found different forces, although of the same order in wave and bottom slopes. The present results are firmly established for depth-uniform currents (Phillips, 1977). Further extensions to arbitrary current profiles is being investigated using the Generalized Lagrangian Mean equations of Andrews and McIntyre (1978b).

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