NOTES AND CORRESPONDENCE

Some Aspects of the Forced Wave Response of Stratified Coastal Regions

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ABSTRACT

A two-layer model with an idealized continental shelf and slope bottom topography is utilized to study some properties of the response of stratified coastal regions to meteorological forcing with variations in the alongshore direction. The model is such that the coastline is straight, there are no alongshore variations in the bottom topography, and the parameter $\lambda=\delta_R'/\delta_B'$ is small, i.e., $\lambda\ll 1$, where δ_R' is the internal Rossby radius of deformation and δ_B' is a scale length of the bottom topography. In that case, the inviscid response to forcing by an alongshore wind stress is composed of uncoupled baroclinic and barotropic components. The baroclinic component consists of forced internal Kelvin waves, with an offshore scale of the order of δ_R' (~15 km) and the barotropic component consists of forced continental shelf waves, with an offshore scale the order of the width L of the continental shelf and slope ($L\approx 100$ km). The alongshore scale of the forcing is assumed to be greater than L and the method of solution of Gill and Schumann and of Gill and Clarke is used. As a result, the alongshore and time-dependent behavior of the baroclinic and barotropic components is governed by forced, first-order wave equations. The response to an impulsively applied, upwelling-favorable wind stress with a specialized alongshore structure, i.e., a constant value for a distance of limited extent, is studied to give insight into the qualitative nature of the behavior of the forced time-dependent, baroclinic and barotropic components. The solutions show clearly how the region of forced upward motion of density surfaces may propagate alongshore to locations distant from that of the wind stress which causes the upwelling. They also illustrate how the barotropic onshore flow to the coast is influenced by the propagation of forced continental shelf waves such that the region of onshore flow from the interior to the slope and shelf may also propagate alongshore.

1. Introduction

The coastal model described by Allen (1975) is utilized here to study, with the methods of Gill and Schumann (1974) and Gill and Clarke (1974), some of the properties of the response of stratified coastal regions to meteorological forcing. In particular, the behavior of the time-dependent response to an idealized wind stress forcing with variations in the alongshore direction is investigated to give insight into the nature of the resultant alongshore variation in the oceans response.

A two-layer f-plane model with an idealized continental shelf and slope bottom topography is utilized (Allen, 1975). The bottom topography is given by an exponential depth variation in the offshore direction for the shelf and slope region as in Buchwald and Adams (1968). This coastal region adjoins a constant depth interior. The boundary at the coast is vertical and the heavy fluid covers the entire shelf such that the density interface intersects the vertical section at the coast. The coastline is straight and there are no alongshore variations in the bottom topography. The motion is driven by an alongshore component of the wind stress τ through the suction of fluid into the surface layer at the coast.

It is assumed that the parameter $\lambda=\delta_R'/\delta_B'$ is small, i.e., $\lambda\ll 1$, where δ_R' is the internal Rossby radius of deformation and $\delta_B'=H'/H_{x'}'$ is a scale length of the bottom topography. (H' is the depth, x' is a coordinate in the offshore direction, and the subscript x' denotes differentiation.) In that case, the response to forcing by an alongshore component of the wind stress is composed, in the first approximation, of baroclinic and barotropic components which are uncoupled. The baroclinic response feels only a flat bottom to the lowest order and consists of forced internal Kelvin waves, with an offshore scale of the order of δ_R' (~15 km). The barotropic component consists of forced continental shelf waves, with an offshore scale of the order of the width L of the continental shelf and slope ($L\approx 100$ km).

The alongshore scale of the forcing $\delta y_F'$, and therefore that of the response $\delta y'$, is assumed to be greater than L and the time scales of interest $\delta t'$ are assumed to be larger than an inertial period, i.e.,

$$\delta y'\gg L, \quad \delta t'\gg f^{-1}, \qquad (1.1a, b)$$

where f is the Coriolis parameter.

The method of solution of Gill and Schumann (1974) and Gill and Clark (1974) is utilized. As a consequence, the alongshore and time-dependent behavior of the baroclinic and barotropic components is governed by forced first-order wave equations. The solutions presented here are simply the result of solving these forced first-order wave equations, with a particular idealized stress distribution, and making some inquiries about the consequences of the solutions on the nature of the flow.

One particular coastal phenomenon of considerable interest is that of coastal upwelling where an upwelling-favorable alongshore component of the wind stress drives water in the surface layer offshore. This creates a suction of fluid from the shelf into the surface layer near the coast. The simplest conceptual picture of this upwelling process is that of a two-dimensional response in a plane normal to the coast, i.e., with no gradients in the flow in the alongshore direction (e.g., O'Brien and Hurlburt, 1972). In that case, the offshore flow in the surface layer is balanced by an onshore flow toward the coast in the same onshore-offshore plane. With respect to the flow below the surface layer in the present two-layer model, the motion outside a distance δ_R' from the coast is barotropic, i.e., depth-independent. The onshore flow is fed to the shelf-slope region from the interior and a barotropic alongshore current develops over the shelf and slope. Within a region of scale δ_R' from the coast, the flow also has a baroclinic component. The density interface rises in this region as the flow is modified by vertical motion such that a mass flux, equal to that of the barotropic inflow, is pumped out of the top layer at the coast and into the surface layer. A baroclinic alongshore current develops simultaneously.

This two-dimensional response is the picture that is commonly visualized for coastal upwelling. Since features which can lead to alongshore variations in the flow, e.g., alongshore variations in coastline, bottom topography and wind stress, are present in upwelling regions, it is important to appreciate under what conditions the above two-dimensional picture of upwelling may be valid. Here a simple example is used to illustrate the alongshore variations in the flow that result in the present model from an alongshore variation in the wind stress.

2. Formulation

We utilize a linear two-layer model situated on an f-plane. The equations are formulated in detail in Allen (1975) and we use the same notation here with the additional definition of a dimensionless wind stress $\tau=\tau'/\tau_0$, where τ_0 is a characteristic value of the surface wind stress and where the characteristic horizontal velocity $U=\tau_0/(\rho_2 f H_0')$. The model geometry is also the same as given in Allen (1975). The coordinate system is placed with the y axis aligned in the alongshore direction and the x axis pointing offshore, with the origin $x=0$ at the coast. The equation for the total depth, in dimensionless coordinates, is

$$H_T=H_{T(0)}\exp(x/\delta_B),\quad 0\leqslant x\leqslant 1,\tag{2.1a}$$

$$H_T=H_{T(0)}\exp(\delta_B^{-1})=H_{T(1)},\quad 1\leqslant x,\tag{2.1b}$$

where H_T is independent of y.

We will utilize assumptions (1.1a, b) which in dimensionless form are

$$\delta y\gg 1,\quad \delta t\gg 1.\tag{2.2a, b}$$

As a result of (2.2a, b), the alongshore component of the velocity is assumed to be in geostropic balance.

Two equations may be derived for the mass transport streamfunction ψ and the interface height h. As is discussed in Allen (1975), if

$$\lambda=\delta_R/\delta_B\ll 1,\tag{2.3}$$

where δ_R is the dimensionless internal Rossby radius of deformation and $\delta_B=H_T/H_{Tx}$, the barotropic problem for ψ and the baroclinic problem for h are uncoupled.[1] We will assume that (2.3) holds and, more specifically, that $\delta_B=O(1)$ and $\delta_R\ll 1$. With (2.3), the resulting equations for ψ and h are

$$(\psi_{xx}-\delta_B^{-1}\psi_x)_t+\delta_B^{-1}\psi_y=0,\tag{2.4}$$

$$(h_{xx}-\delta_R^{-2}h)_t=0,\tag{2.5}$$

where the subscripts (x,y,t) denote differentiation.

The boundary conditions at the coast are

$$u_1=-\tau(y,t)/H_1,\quad u_2=0\quad \text{at}\quad x=0,\tag{2.6a, b}$$

where (2.6a) is required to balance the offshore transport in the surface Ekman layer. In terms of ψ and h, Eqs. (2.6a, b) imply

$$\psi_y=-\tau(y,t)\qquad \text{at } x=0,\tag{2.7}$$

$$h_y+h_{xt}=-\tau(y,t)/H_1\quad \text{at } x=0.\tag{2.8}$$

The other boundary conditions in x for ψ and h are

$$\psi_x=0\quad \text{at } x=1,\tag{2.9}$$

$$h\to 0\quad \text{for } x/\delta_R\gg 1.\tag{2.10}$$

The condition (2.9) has been explained by Gill and Schumann (1974). We point out that it is essentially based on the *assumption* (see Allen, 1976) that the onshore flow to the shelf-slope region at $x=1$ is geostrophically balanced, i.e., that

$$\psi_y=-(H_1p_{1y}+H_2p_{2y})\quad \text{at } x=1.\tag{2.11}$$

We will consider initial-value problems where the wind stress forcing τ is applied impulsively at $t=0$ and

[1] The free wave solutions calculated by perturbation methods for $\lambda\ll 1$ by Allen (1975) required frequency corrections dependent on λ. This implies that in the forced problem the weak coupling will cause non-uniformities in the solution for long time scales dependent on λ. We do not consider these effects here. We essentially assume that λ is small enough that, for the time scales of interest for these examples, the non-uniformities are negligible.

where the initial value of h is equal to zero, i.e.,

$$h=0 \quad \text{at } t=0. \tag{2.12}$$

The initial value for ψ is such that the offshore flow in the surface Ekman layer, which is set up instantaneously in this problem, is balanced by an onshore flow with an equal mass flux, i.e.,

$$\psi_y=-\tau \quad \text{at } t=0. \tag{2.13}$$

The alongshore distribution of the wind stress is a rectangular function

$$\tau(y, t>0)=\begin{cases}0, & 0<y\\ \tau_w, & -|y_0|<y<0\\ 0, & y<-|y_0|\end{cases} \tag{2.14}$$

where τ_w is a constant and $\tau_w>0$ so that the wind stress is upwelling-favorable. The alongshore distribution is chosen deliberately to have the rather unrealistic form (2.14) since this introduces, very strongly, the effects of alongshore variations in τ and since solutions of the first-order wave equations by the method of characteristics are especially easy to obtain and to understand in this case. The discontinuities of τ at $y=0$ and $-|y_0|$ would undoubtedly generate a response with short alongshore wavelengths which would violate assumption (2.2a). However, since the first-order wave equations that result with assumptions (2.2) admit solutions with discontinuous forcing functions and since this type of forcing results in a sharp division in y and t of the flow properties, which helps clarify the results, we will utilize it. If the variation of the forcing is sufficiently smoothed in the alongshore direction, the resulting solutions will be smoothed so that (2.2a) is satisfied. Similar qualitative results will follow, but the change in flow properties will take place more gradually in y and t.

3. The baroclinic problem

We first consider the baroclinic problem for h which is defined by Eq. (2.5), boundary conditions (2.8) and (2.10), and initial condition (2.12). With the small λ assumption [Eq. (2.3)], this problem is the two-layer analogy of the continuously stratified, flat bottom upwelling problem considered by Gill and Clarke (1974).

The appropriate solution to (2.5), which satisfies (2.10), is

$$h=G(y,t)\exp(-x/\delta_R). \tag{3.1}$$

The satisfaction of (2.8) requires that G obey the equation

$$c_1^{-1}G_t-G_y=\tau(y,t)/H_1, \tag{3.2}$$

where $c_1=\delta_R$ and where, from (2.12),

$$G=0 \quad \text{at } t=0. \tag{3.3}$$

Eq. (3.2) is a forced first-order wave equation. It is the two-layer equivalent of the equation found for the baroclinic modes in the continuously stratified case by Gill and Clarke (1974). The solution to this two-layer problem may be thought of as corresponding to the solution for the first baroclinic mode in the continuously stratified case. With $\tau\equiv 0$, Eq. (3.2) governs the propagation of free internal Kelvin waves. These nondispersive free waves propagate toward the negative y direction (toward the north on the west coast of the United States) with a wave velocity of c_1. A typical dimensional value for c_1 is approximately 50 km day^{-1} (Kundu *et al.*, 1975).

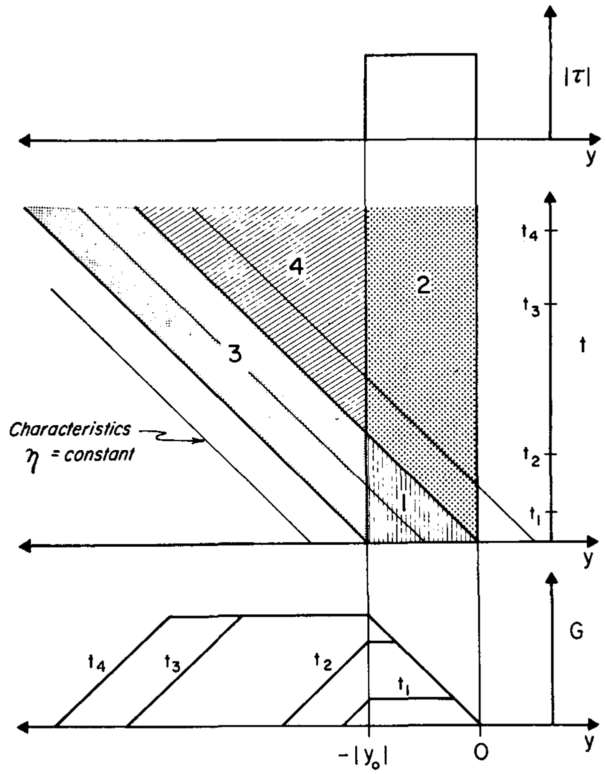

FIG. 1. The stress τ for $t>0$ as a function of y (top plot). Below that is a y-t diagram for the solution G of (3.2). The solution has different behavior, which is summarized in Table 1, in the four marked regions. Outside of these regions the solution is $G=0$. The bottom plot is G as a function of y [Eq. (3.6)] for the four values of t indicated in the y-t diagram.

The forced wave equation (3.2) may be most easily solved by the method of characteristics. If the variables

$$\eta=y+c_1t, \quad s=t, \tag{3.4a, b}$$

are defined, (3.2) transforms to

$$G_s(\eta,s)=\tau c_1/H_1, \tag{3.5}$$

which may be readily integrated along the characteristics $\eta=$ constant.

The wind stress, defined by (2.14), is plotted for $t>0$ as a function of y in Fig. 1. Below that plot in Fig. 1, a y-t diagram is given with some of the characteristics $\eta=$ constant plotted and with the region $-|y_0|<y<0$, in which the stress τ acts, indicated.

TABLE 1. Features of the balances in (3.8) in the four regions of Fig. 1.

Region	Solution	Eq. (3.8a)	Eq. (3.8c)	Character
1	(3.6d)	$\bar{u}_x=-\delta_R^{-2}h_t$ $(\bar{v}_y=0)$	$\bar{v}_t+\bar{u}=0$ $(h_y=0)$	time-dependent, two-dimensional
2	(3.6c),(3.6f)	$\bar{u}_x+\bar{v}_y=0$ $(h_t=0)$	$\bar{u}=-\bar{h}_y$ $(\bar{v}_t=0)$	steady, geostrophic
3	(3.6e),(3.6h)	$\bar{v}_y=-\delta_R^{-2}h_t$ $(\bar{u}_x=0)$	$\bar{v}_t=-\bar{h}_y$ $(\bar{u}=0)$	time-dependent, free internal Kelvin wave front
4	(3.6g)	$(\bar{u}_x=0,$ $\bar{v}_y=0,$ $h_t=0)$	$(\bar{v}_t=0,$ $\bar{u}=0,$ $h_y=0)$	$\bar{v}=h_x\neq 0$, the alongshore velocity is in steady geostrophic balance

Four regions in which the solution has a different character are also marked. These will be referred to shortly. It is easy to see, from this y-t diagram, how the solution G behaves as (3.5) is integrated along the characteristics. Initially, at $s=t=0$, we have $G=0$. When (3.5) is integrated with respect to s, for η = constant, G increases linearly with s when the characteristic is in the region $-|y_0|<y<0$ and G remains constant when the characteristic is outside this region.

The solution is

$$G=0 \quad \text{for } 0<y, \quad y<-|y_0|-c_1t, \tag{3.6a, b}$$

and, for $c_1t<|y_0|$,

$$G=|y|\tau_w/H_1 \quad \text{for } -c_1t<y<0, \tag{3.6c}$$

$$G=c_1t\tau_w/H_1 \quad \text{for } -|y_0|<y<-c_1t, \tag{3.6d}$$

$$G=(y+|y_0|+c_1t)\tau_w/H_1 \quad \text{for } -|y_0|-c_1t<y<-|y_0|, \tag{3.6e}$$

while, for $c_1t>|y_0|$,

$$G=|y|\tau_w/H_1 \quad \text{for } -|y_0|<y<0, \tag{3.6f}$$

$$G=|y_0|\tau_w/H_1 \quad \text{for } -c_1t<y<-|y_0|, \tag{3.6g}$$

$$G=(y+|y_0|+c_1t)\tau_w/H_1 \quad \text{for } -|y_0|-c_1t<y<-c_1t. \tag{3.6h}$$

The solution for G as a function of y, at the four different times marked on the y-t diagram, is plotted at the bottom of Fig. 1. Note that $h(x=0)=G$ and that G may be thought of as the height of the density interface at the coast.

It will be useful, in discussing the solution (3.6), to look at the equations for the baroclinic velocity components which may be defined by

$$\bar{v}=v_2-v_1, \quad \bar{u}=u_2-u_1. \tag{3.7a, b}$$

These components obey the equations

$$\bar{u}_x+\bar{v}_y=-\delta_R^{-2}h_t, \tag{3.8a}$$

$$\bar{v}=h_x, \tag{3.8b}$$

$$\bar{v}_t+\bar{u}=-h_y. \tag{3.8c}$$

We point out that the question of what type of balance, two-dimensional or otherwise, is present in the baroclinic component of the flow may be asked with regard to which terms are important in the continuity equation (3.8a) and in the y momentum equation (3.8c). For example, in the two-dimensional response discussed in Section 1, the terms in (3.8a, c) with y derivatives are equal to zero and do not enter in the balance.

We note that the solution (3.6) has a different character in the four regions shown in the y-t diagram in Fig. 1, and that outside these regions the solution is $G=0$. The balances in (3.8) are also different in these four regions. These features are summarized in Table 1. We can see the following points. In region 1, the response is time-dependent and is purely two-dimensional. There is no y dependence in the flow in this region, because the stress in $-|y_0|<y<0$ is y-independent and because information about the y-dependence of τ has not had time to propagate, with the velocity c_1, from the point $y=0$.

In region 2, on the other hand, the flow is steady and is geostrophically balanced. With this particular stress distribution, the propagation of information about the finite extent of τ, along the characteristic $\eta=0$, is accompanied, within the region $-|y_0|<y<0$, by the immediate set-up of a steady geostrophic flow. In the steady flow, the baroclinic onshore component of velocity $\bar{u}$ is balanced by h_y, i.e., by alongshore pressure gradients.

The solution in region 3 is essentially a free internal Kelvin wave front which is generated in response to the directly forced solution in $-|y_0|<y<0$ and which propagates toward the north (negative y) with velocity c_1. The onshore velocity components are zero in free internal Kelvin waves and in this region $\bar{u}=0$.

The vertical motion of density surfaces takes place in regions 1 and 3, as is evident from the plot of G for different times in Fig. 1. The vertical velocities in these two regions are balanced in the continuity equation (3.8a) in different ways. In region 1, with two-dimensional flow, it is onshore gradients in the onshore component $\bar{u}$ which balance the vertical motion $\delta_R^{-2}h_t$. In region 3, however, it is alongshore gradients in the alongshore component $\bar{v}$ which balance $\delta_R^{-2}h_t$. Note that the upwelling of the density interface that occurs in region 3 takes place at an alongshore location which is different from that of the wind stress which is causing the upwelling.

With the constraints of continued wind stress forcing and a barotropic inflow to the coast outside of $x=O(\delta_R)$, there must continue to be an upward motion of the density interface at some alongshore location. For $c_1t>|y_0|$, all of the time-dependent upward motion of the density interface occurs in region 3, i.e., in the internal Kelvin wave front which is propagating toward the north with velocity c_1.

Region 4 connects the motion in the internal Kelvin wave front with the steady motion in region 2, where both the onshore and alongshore velocity components are geostrophically balanced. In region 4, the density interface at the coast is elevated, compared with its initial value, but it is independent of time and of y. As a result, an alongshore velocity component $\bar{v}$, which is in steady geostrophic balance, exists in this region, and the onshore component $\bar{u}=0$.

The description of the two-dimensional problem, given in Section 1, applies to the velocity field in region 1. In contrast to this picture, however, for $c_1t>|y_0|$, e.g., at times t_3 and t_4 in Fig. 1, the baroclinic velocity field that is set up within δ_R of the coast is described in the following way. There is a barotropic onshore flow in $-|y_0|<y<0$ for $x>O(\delta_R)$. Within $0<x<\delta_R$, a steady baroclinic component exists such that in (3.8a) $\bar{u}_x$ is balanced by $\bar{v}_y$. The onshore flow in the bottom layer turns toward the north (toward $-y$). The velocity in the bottom layer is northward (in region 4) up to the location of the internal Kelvin wave front where it falls to zero as $\bar{v}_y$ balances $\delta_R^{-2}h_t$ and the density interface moves vertically upward. The corresponding alongshore flow in the top layer, from the location of the wave front to the region where the stress acts, is toward the south. Within $-|y_0|<y<0$, the alongshore flow in the top layer is turned and is fed horizontally to the coast to satisfy the boundary condition (2.6a).

We emphasize that the motion of fluid into the surface Ekman layer at the coast $x=0$ occurs within $-|y_0|<y<0$ as it must to satisfy the boundary flux condition (2.6a) set by the local wind stress. However, the upward motion of the density interface which occurs in region 3 is not determined by the local wind stress. This property of the response was pointed out by Gill and Clarke (1974) and the present example provides a simple specific example of it. The solutions also illustrate clearly some of the general features found in the numerical studies of Suginohara (1974).

As far as a conceptual picture of the motion involved in coastal upwelling is concerned, perhaps one of the more useful results here is the reminder that, as well as the familiar two-dimensional mass balance situation in which onshore gradients in the onshore velocity balance the vertical motion (region 1), it is also possible to find cases where alongshore gradients in the alongshore velocity component balance the vertical motion (region 3).

4. The barotropic problem

The barotropic problem is defined by Eq. (2.4), boundary conditions (2.7) and (2.9), and initial condition (2.13). With the small λ approximation (2.3), this problem is similar to the forced barotropic shelf wave problem considered by Gill and Schumann (1974).

To find the solution, it is convenient to define

$$\bar{\psi}=\psi+\int_{-\infty}^{y}\tau(\hat{y},t)d\hat{y}, \tag{4.1}$$

and to solve for $\bar{\psi}$.

The equation and boundary conditions, in terms of $\bar{\psi}$, are

$$(\bar{\psi}_{xx}-\delta_B^{-1}\bar{\psi}_x)_t+\delta_B^{-1}\bar{\psi}_y=\delta_B^{-1}\tau, \tag{4.2}$$

$$\bar{\psi}_y=0 \quad \text{at } x=0, \quad \bar{\psi}_x=0 \quad \text{at } x=1, \tag{4.3a, b}$$

$$\bar{\psi}=0 \quad \text{at } t=0. \tag{4.4}$$

The problem is now identical in form to that treated by Gill and Schumann (1974) and we follow their procedure. We expand $\bar{\psi}$ in terms of the free shelf wave eigenfunctions $\phi_n(x)$ (Buchwald and Adams, 1968), i.e.,

$$\bar{\psi}=\sum_{n=1}^{\infty}Y_n(y,t)\phi_n(x), \tag{4.5}$$

where ϕ_n satisfies the eigenvalue problem

$$\phi_{n_{xx}}-\delta_B^{-1}\phi_{nx}+(a_n\delta_B)^{-1}\phi_n=0, \tag{4.6a}$$

$$\phi_n=0 \quad \text{at } x=0, \quad \phi_{nx}=0 \quad \text{at } x=1. \tag{4.6b, c}$$

Substituting (4.5) into (4.2) and utilizing the expansion

$$1=\sum_{n=1}^{\infty}b_n\phi_n(x), \tag{4.7}$$

we find that Y_n satisfies the forced first-order wave equation

$$a_n^{-1}Y_{nt}-Y_{ny}=-b_n\tau, \tag{4.8}$$

where, from (4.4),

$$Y_n=0 \quad \text{at } t=0 \tag{4.9}$$

and where a_n is the phase velocity of free continental shelf wave modes in the long-wave (nondispersive) limit (2.2). Typical dimensional values of a_n (Cutchin and Smith, 1973) are $a_1' \approx 400$ km day^{-1} and $a_2' \approx 100$ km day^{-1}.

With (4.9), the solutions to (4.8) for Y_n are similar to the solution for G [Eq. (3.6)] in Section 3, except that the characteristics are modified by the change in wave speed from c to a_n and the stress τ in (4.8) has a different multiplicative factor. In particular, the solution for Y_n will have the same general behavior as G in the four different regions in the y-t diagram in Fig. 1. The interpretation of the resulting flow here will differ somewhat, however, as a consequence of the difference in the relation of the velocity components to the functions h and ψ.

Barotropic velocity components may be defined in a manner similar to the baroclinic components in (3.7) and they are

$$\hat{u} = H_T^{-1}\psi_y = H_T^{-1}(\sum_n Y_{ny}\phi_n - \tau) = \sum_n \hat{u}_n - H_T^{-1}\tau, \quad (4.10a)$$

$$\hat{v} = -H_T^{-1}\psi_x = -H_T^{-1}\sum_n Y_n\phi_{nx} = \sum_n \hat{v}_n, \quad (4.10b)$$

where

$$\hat{u}_n = H_T^{-1}Y_{ny}\phi_n, \quad \hat{v}_n = -H_T^{-1}Y_n\phi_{nx}. \quad (4.10c, d)$$

The solutions for Y_n in the regions 1–4 may be interpreted in terms of the resulting consequences for the velocity components $\hat{u}$ and $\hat{v}$.

In region 1, the solution for Y_n is y-independent and we find a two-dimensional response. The $\hat{v}$ component of velocity grows due to the onshore flow forced by the wind stress. This follows from the balance $\hat{v}_t + \hat{u} = -\hat{p}_y$ in the y-momentum equation where in region 1, $\hat{u} = -H_T^{-1}\tau$, $\hat{u}_n = 0$, and $\hat{p}_y = H_{T(1)}^{-1}\tau$. Note that, if all modes Y_n are in region 1, i.e., if $a_1t < |y_0|$ and $-|y_0| < y < -a_1t$, this response may be directly described by the solution to (4.2), (4.3) and (4.4) with $\bar{\psi}_y \equiv 0$. That solution is

$$\hat{v} = -H_T^{-1}\psi_x = tH_T^{-1}\tau_w\{1 - \exp[(x-1)/\delta_B]\}, \quad (4.11)$$

and it represents a barotropic coastal jet. The jet structure in $\hat{v}$ is formed by the increase in magnitude of the onshore velocity $\hat{u} = -H_T^{-1}\tau$ as H_T decreases and by the assumption (2.9) that $\hat{v}(x=1) = 0$.

In region 2, the flow is steady and the balance in (4.8) is

$$Y_{ny} = b_n\tau. \quad (4.12)$$

To see what this implies we include the expansion (4.7) in (4.10a), which gives

$$\hat{u} = H_T^{-1}\sum_n (Y_{ny} - b_n\tau)\phi_n. \quad (4.13)$$

Note that it is not appropriate to draw conclusions on $\hat{u}$ at $x=0$ from (4.13), since $\hat{u}(x=0) = -H_T^{-1}\tau$, but the eigenfunctions $\phi_n(x=0) = 0$. It will be helpful, therefore,

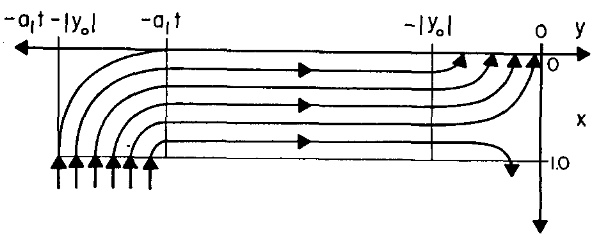

FIG. 2. A schematic of the flow pattern that develops in the barotropic problem from the first mode, i.e., from $\hat{u} \approx \hat{u}_1 - H_T^{-1}\tau$, $\hat{v} \approx \hat{v}_1$, at a time $a_1t \gg |y_0|$.

to look specifically at the behavior of the onshore flow at the slope boundary $x=1$ where at the initial time $t=0$, and in region 1,

$$\hat{u}(x=1) = -H_T^{-1}\sum_n b_n\tau\phi_n(x=1). \quad (4.14)$$

We see from (4.13) that when the balance (4.12) is achieved in region 2 for a mode n, the contribution of that mode to the onshore flow in region 2 is equal to zero. The loss of a contribution to the onshore flow in region 2, however, is accompanied by the development of an equivalent contribution in region 3. This may be seen in the bottom plot of Fig. 1 where the y gradients of Y_n are of equal magnitude but of opposite sign in regions 2 and 3. The total contribution of a given mode to $\hat{u}(x=1)$, therefore, remains constant and equal to its initial value, although the alongshore location changes. For $a_nt < |y_0|$, the contribution to $\hat{u}(x=1)$ from the nth mode occurs in regions 1 and 3, whereas for $a_nt > |y_0|$ it occurs entirely in region 3, where the solution represents a free continental shelf wave front propagating northward with velocity a_n.

The motion in region 4 again connects that in regions 2 and 3. It is characterized by a steady alongshore velocity $\hat{v}_n$ and zero onshore velocity $\hat{u}_n = 0$. The steady alongshore motion exists from the location of the propagating shelf wave front in region 3 to the steady flow in region 2. This alongshore flow provides the mass flux between region 3, where there is onshore or offshore flow[2] to the slope at $x=1$, and region 2, $|y_0| < y < 0$, where the total flow is turned onshore to satisfy the flux condition (2.7) which is set by the local wind stress.

The flow pattern that develops from the first mode is the simplest and probably the most important since that mode has the largest amplitude and the highest wave speed. It includes an onshore flow at $x=1$ in the shelf wave front in region 3. For $a_1t > |y_0|$, this onshore flow in region 3 is linked, by a steady alongshore velocity in region 4, to the steady motion in $-|y_0| < y < 0$ in region 2, where the flow is turned onshore to satisfy (2.7). The onshore-offshore motion in region

[2] The contribution $\hat{u}_n(x=1)$ of the modes to $\hat{u}(x=1)$ alternates in sign for increasing n.

2 results from a superposition of $-H_T^{-1}\tau$ and $\hat{u}_1$ and, since $|\hat{u}_1(x=1)|>|H_T^{-1}\tau|$, a small amount of flow near $x=1$ is turned offshore. A schematic of the flow pattern is given in Fig. 2.

We point out that for increasing t, as the higher modes achieve the steady balance (4.12), the total alongshore current $\hat{v}$ in $-|y_0|<y<0$ concentrates close to the coast at $x=0$. In the limit $t\to\infty$, $\hat{v}$ has a delta function-like behavior at $x=0$ for $-|y_0|<y<0$.

The important point here is that the region of largest onshore flow to the shelf-slope region, which occurs in connection with the first mode, propagates northward with velocity a_1. As a result, fluid is drawn from the ocean interior onto the shelf-slope region at alongshore locations that differ from that of the wind stress. This propagation also results in the set-up of steady alongshore barotropic currents (region 4) at locations northward of the wind stress that forces them.

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