Fine scale inhomogeneity of wind-wave energy input, skewness, and asymmetry

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[1] Analysis of measured sea and lake wind wave data reveals large variability of the wind energy input, as well as the waves skewness and asymmetry. The spatial and temporal third moments alternate in sign over a few wave periods and over a few wavelengths, respectively. Simulation through a 2D Wave Boundary Layer model in which the air flow is modeled by 2nd order Reynolds equations (Chalikov, 1998) conforms to these findings and exposes a rich structure. We found clear correlation of the variations of the skewness and the asymmetry with the wind input. **Citation:** Agnon, Y., A. V. Babanin, I. R. Young, and D. Chalikov (2005), Fine scale inhomogeneity of wind-wave energy input, skewness, and asymmetry, *Geophys. Res. Lett.*, *32*, L12603, doi:10.1029/2005GL022701.

1. Introduction

[2] Until recently, the bulk of wave measurements were typically carried out at a single point, producing a time series, and providing a frequency spectrum. Other, more recent, remote sensing methods provide an instantaneous image of the surface. Measurements of the wind energy input are still limited to a single point. In order to give a more complete picture of the wave field, it is often assumed that the waves are ergodic, hence the frequency spectra are assumed to be simply related to the wavenumber spectra, and likewise for the energy input and for spatial and temporal third order moments. In calculating second order quantities (the wave spectrum, the wind energy input) and third order quantities (skewness and asymmetry), it is implicitly assumed that they represent nearly stationary and homogeneous characteristics of the wave field. The linear dispersion relation is often assumed.

[3] Bispectra have been used extensively since the work of *Hasselmann et al.* [1963], to study nonlinear waves. The full bispectrum is very detailed, and provides a complex number value for each pair of frequencies (or wave-numbers). It requires long records for obtaining reliable estimates. Steep waves are known to be non-Gaussian. This is reflected by relatively large values of their skewness and asymmetry (which are related to the integrals of the real and the imaginary parts of the bispectrum, respectively). They can also be reliably computed directly in the space (or time) domain, as follows:

$$S_x(t) = E_x\left[\eta(x,t)^3\right]/\eta_{rms}^3 \quad S_t(x) = E_t\left[\eta(x,t)^3\right]/\eta_{rms}^3 \tag{1}$$

S_x and S_t are the spatial skewness and temporal skewness, respectively, of η, the free surface elevation. E_x and E_t are the spatial and temporal expected (- mean) values, respectively. η_{rms} is the rms of η. The spatial averaging is taken to be one-dimensional, since we are using a one-horizontal-dimensional numerical model. Similarly, the spatial and temporal asymmetries are given by:

$$A_x(t) = E_x\left[H_x(\eta(x,t))^3\right]/\eta_{rms}^3 \quad A_t(x) = E_t\left[\left(H_t(\eta(x,t))^3\right]/\eta_{rms}^3 \tag{2}$$

$H_x(\eta(x, t))$ and $H_t(\eta(x, t))$ are the Hilbert transforms of the spatial and temporal wave records, respectively (alternatively, the derivative of the signal can be used, instead of its Hilbert transform, with qualitatively the same results). The skewness reflects the lack of symmetry of the waves with respect to the horizontal axis, and is a measure of the waves nonlinearity. The asymmetry indicates the leaning forward or backwards of the waves. The non-Gaussian statistics of the waves is a subject of increasing interest, in particular regarding the distribution of wave-height, wavelength and crest-length, in order to predict extreme waves. *Shrira et al.* [2003] address the effect of the waves non-Gaussianity on electromagnetic wave scattering.

[4] To find out how stationary and homogenous is the wave statistics, we have analyzed two separate and distinctly different data sets of wind wave measurements. One is from the Black Sea; the other, from Lake George (NSW, Australia) - provides a faithful measurement of the wind input, by recording the pressure variations close to the instantaneous free surface. The measurements, including geometry of the wave-gauge array, ranges of wave fetches, peak

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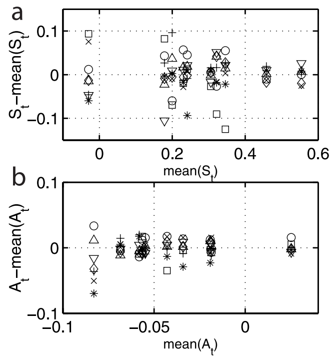

Figure 1. (a) The scatter of the skewness in simultaneous measurements (in Lake George), versus their mean value. There are 8 symbols, each representing the deviation of an individual probe, from the mean. (b) The scatter of the asymmetry.

frequencies, wave ages, as well as data processing procedures and results, including data averaging, wave steepness, spectral and other characteristics are described in detail by *Babanin and Soloviev* [1998a, 1998b] for the Black Sea and by *Young et al.* [2005] for Lake George. Here, we shall only mention the most relevant information for the present study. The data's quality in both data sets was very high (wire wave probes were used) and the records used were stationary and sufficiently long to produce very reliable estimates of statistical properties, both mean and spectral. The Black Sea data were characteristic of well-developed deep-water waves, with peak frequencies $f_p \sim 0.2$ Hz and inverse wave ages U_{10}/C_p close to 1 (U_{10} is the wind speed at 10 m height, and C_p, k_p are the phase speed and wavenumber of the f_p waves). The Lake George data represent intermediate-depth waves ($k_ph = 1.3$, h being the water depth); the waves in the current study were strongly forced, with frequencies $f_p > 0.5$ Hz and $U_{10}/C_p > 5$. Breaking rates were not high, with estimated upper bounds for the Black Sea less than 5% and for Lake George – less than 10% of the dominant waves, those which contribute most to the skewness and asymmetry.

[5] We found strong variability of the (temporal) skewness and the asymmetry. This prompted us to carry out simulations with an advanced numerical model (for the sake of simplicity we employ a 2D model presuming that the phenomena under consideration can be explored in 2D). The simulations reveal an intricate spatial and temporal structure of the skewness and the asymmetry: They exhibit a clear undulation over temporal and spatial scales slightly greater than twice the dominant waves period and wavelength, respectively. Large overall scatter of skewness has been displayed also by previous measurements [*Babanin and Polnikov*, 1994; *Prevosto et al.*, 2000; *Borgman*, 1998].

[6] Some recent studies focused on the effect that long waves have on the energy transfer from the wind to shorter waves [*Troitskaya*, 1994; *Hara and Belcher*, 2002]. *Kudryavtsev and Makin* [2002] demonstrate that accounting for the roughness variation induced by dominant long surface waves about doubles the growth rate. *Young and Babanin* [2001] have found that the intermittency of wave breaking greatly modulates the wind energy input. It has become clear that the traditional spectral description has at least to be augmented with non-Gaussian characteristics of the sea state in order to account more accurately for the waves evolution.

[7] Triad interaction leads to a significant bispectrum [cf. *Agnon and Sheremet*, 1997]. In the laboratory, *Leykin et al.* [1995] and *Feddersen and Veron* [2005] have found that the asymmetry increases consistently with the inverse wave age and concluded that it is determined by the degree of wind forcing and is an important indicator of wind wave interaction processes. *Young and Eldeberky* [1998] have observed large bispectra in Lake George, where the finite water depth makes the dispersion weaker. *Suh et al.* [2000] have studied wind-generated deep-water laboratory waves and found large positive skewness and smaller, negative asymmetry, with a strong bispectral peak representing triad interaction of the peak frequency with its double harmonic. The skewness increased at the initial stages of wave development and decreased later. They concluded that significant wave triad interaction takes place for young wind waves.

[8] Analyzing wave records, we found both positive and negative asymmetries. These are correlated with increased and decreased wind energy input, respectively, in the measurements as well as in the numerical model. The dependence of the energy input on the waves asymmetry, may help in the interpretation of the measurements which provide a partial picture of the non-homogenous sea. Accounting for these mechanisms could improve our ability to quantify the wind input.

[9] The rapid spatio-temporal oscillations of the skewness and asymmetry that we observed, suggest the need for a new perspective of wind-wave interaction. Since the waves are strongly forced by the wind, we went on to study cross-moments between the waves and the air pressure. We found visible correlation between these cross-moments and the waves third moments.

2. Third Moments and Wind Energy Input

[10] In order to get a clear picture, we start our analysis by looking at the skewness and the asymmetry. If the sea surface were stationary and homogenous, S_x (in the simplified, 2D model), and S_t, would be virtually constant in time and in space, respectively. These two constants would then be equal, implying ergodicity. The same would apply to A_x and A_t. To assess whether the temporal skewness and asymmetry are homogeneous in space, we have compared the values of S_t and A_t for an array of adjacent wave gauges. The scatter was found to be very large as seen in Figure 1 (Lake George data). The results were obtained simultaneously at 8 nearby probes (the diameter of the array is 30 cm). The "temporal variation" of S_t and A_t can be found by considering "running average" values carried out over a time interval of 400 times the peak period. Usually, spectral estimates are obtained based on such time interval. We use this interval to show that skewness and asymmetry do vary at this time scale. The durations of the records analyzed in this study were up to 19440 times the peak periods. Thus we find strong temporal variability, as seen in Figure 2. Similar variability was found for the Black Sea data.

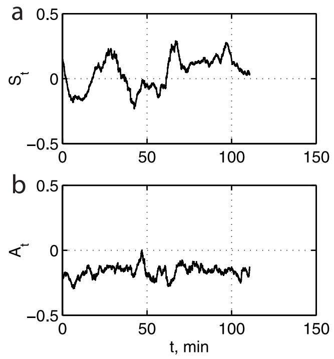

Figure 2. "Running average" values of, (a) S_t (temporal skewness) and, (b) A_t (temporal asymmetry), versus time. Lake George data. Averaging interval: 400 times T_p, the peak period (T_p = 1.4 sec).

[11] A more systematic picture of the variation of the skewness and the asymmetry can be obtained from a numerical model that simulates the coupled dynamics of water wave field (confined to 2D) and air flow. The Wave Boundary Layer model selected [*Chalikov*, 1998, 2005; *Chalikov and Sheinin*, 2005] uses a second−order Reynolds equation written in a nonstationary surface-following coordinate system, and accounts for the wind-wave interaction and for the air flow separation in great detail. The waves are described by a fully nonlinear potential flow. Exchange of matching information is done at each time step. We have modeled strongly forced waves (inverse wave age U_{10}/C_p = 6, U_{10} = 12 m/s, peak frequency = 0.53 Hz, water depth 52 cm).

[12] We show the analysis carried out for the asymmetry. The ranges of variation are about (−0.6/0.75) for the spatial asymmetry $A_x(t)$ (Figure 3a), and (−0.18/0.12) for the temporal asymmetry $A_t(x)$ (Figure 3d). The peaks of the asymmetry variation are shifted with regard to the peaks of the skewness variation (not shown). Since the waves are strongly forced by the wind, we study the link between the variation of the skewness and the asymmetry, and that of the wind-wave interaction. We denote the instantaneous energy input (per unit surface area) into the waves by $I_x(t)$. In line with our analysis, we define also the corresponding local energy input (per unit time), $I_t(x)$:

$$I_x(t) = -E_x[p\partial\eta/\partial t] \quad I_t(x) = -E_t[p\partial\eta/\partial t] \tag{3}$$

(with p the air pressure at the water surface). Employing the model we have computed various moments of p and η. The mean values over space as a function of time ($I_x(t)$), and vice versa ($I_t(x)$), are presented in Figures 3c and 3f, respectively. The results were slightly smoothed by a running average. Comparing the plots of $I_x(t)$ and $A_x(t)$, and the plots of $I_t(x)$ and $A_t(x)$, we see an overall agreement between the variations of the asymmetry and the energy input, as functions of space, and also as functions of time. Smoothing the data we find very strong coherence (reaching 1 for the peak frequency) between $I_x(t)$ and $A_x(t)$, and between $I_t(x)$ and $A_t(x)$. When the asymmetry is negative ("forward leaning waves") the energy input is maximal, and when the asymmetry is positive it is minimal. This behavior can be intuitively explained by the separation of the airflow at the lee of the wave crests, which enhances the energy transfer for forward leaning waves. The "temporal variability" of I_t is manifested also in the experimental measurements, similar to the analysis of the third moments in Figure 2.

3. Wind-Wave Cross-Moments

[13] In the present section we explore the way in which the wind forcing relates to the rapid variations of the

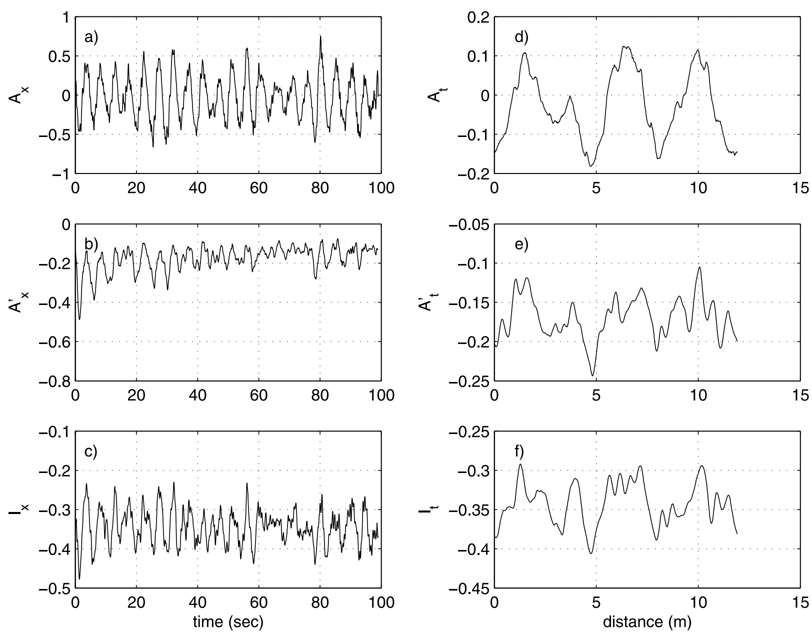

Figure 3. Temporal variation of simulated data: (a) A_x (spatial asymmetry) (b) A'_x (spatial cross-asymmetry) (c) I_x (spatial energy input). Spatial variation of simulated data: (d) A_t (temporal asymmetry) (e) A'_t (temporal cross-asymmetry) (f) I_t (temporal energy input).

skewness and the asymmetry. The rate of change of the spatial asymmetry is

$$\eta_{rms}^3 dA_x/dt = dE_x\left[H_x\left(\eta(x,t)^3\right]/dt = 3E_x\left[H_x(\eta)^2\partial H_x(\eta)/\partial t\right] \tag{4}$$

The component that is due to the wind pressure p can be (roughly) estimated using the linear approximation:

$$\partial\eta/\partial t \sim -(\rho g)^{-1}\partial p/\partial t \tag{5}$$

which leads us to define a cross moment related to the rate of change of the asymmetry due to the wind forcing:

$$A'x(t) = -3(\rho g)^{-1}E_x\left[H_x\left(\eta((x,t))^2\partial H_x(p)/\partial t\right]/\left(\eta_{rms}^2 p_{t,rms}\right) \tag{6}$$

Similarly we define the corresponding temporal cross moment:

$$A't(x) = -3(\rho g)^{-1}E_t\left[H_t(\eta)^2\partial H_t(p)/\partial t\right]\Big]/\left(\eta_{rms}^2 p_{t,rms}\right) \tag{7}$$

From Figures 3b, 3e, 3a, and 3d we may compare the variation of A'_x and A'_t, to the variation of A_x and A_t, respectively. A similar comparison was done for the corresponding skewness and cross skewness. Overall agreement is found. Note that this agreement is not due to a simple correlation between η and $\partial p/\partial t$, since these are almost out of phase with each other. Running the model without wind did not produce similar variability of the asymmetry.

4. Conclusion

[14] Using field wave measurement data and a 2D Wave Boundary Layer model, the skewness and the asymmetry of strongly forced wind waves were found to greatly vary on fine temporal and spatial scales. This variation can not be described by standard spectral theory, and may be attributed to the strong forcing and highly nonlinear waves. The wind-wave energy transfer and the wind-wave cross-asymmetry, are found to vary along the same patterns, as does the waves asymmetry, and are correlated. It seems that the wave field is less homogeneous and less stationary than assumed by many models, and new approaches are required for the interpretation of measured data, and in particular values of skewness, asymmetry and wind stress.

References

Agnon, Y., and A. Sheremet (1997), Stochastic nonlinear shoaling of directional spectra, *J. Fluid Mech.*, *345*, 79–99.

Babanin, A. V., and V. G. Polnikov (1994), About non-Gaussian properties of wind-generated waves (in Russian with English abstract), *Mar. Hydrophys. J.*, *3*, 79–82.

Babanin, A. V., and Y. P. Soloviev (1998a), Field investigation of transformation of the wind wave frequency spectrum with fetch and the stage of development, *J. Phys. Oceanogr.*, *28*(4), 563–576.

Babanin, A. V., and Y. P. Soloviev (1998b), Variability of directional spectra of wind-generated waves, studied by means of wave staff arrays, *Mar. Freshwater Res.*, *49*(2), 89–101.

Borgman, L. E. (1998), Directional wave polyspectra beyond stationarity, paper presented at 30th Offshore Technology Conference, Soc. of Petrol. Eng., Houston, Tex.

Chalikov, D. (1998), Interactive modeling of surface waves and boundary layer: Ocean wave measurements and analysis, paper presented at Third International Symposium on Ocean Wave Measurement and Analysis, Am. Soc. Civ. Eng., Virginia Beach, Va.

Chalikov, D. (2005), Statistical properties of nonlinear one-dimensional wave field, *Nonlinear Processes Geophys.*, in press.

Chalikov, D., and D. Sheinin (2005), Modeling of extreme waves based on equations of potential flow with a free surface, *J. Comput. Phys.*, in press.

Feddersen, F., and F. Veron (2005), Wind effects on shoaling wave shape, *J. Phys. Oceanogr.*, in press.

Hara, T., and S. E. Belcher (2002), Wind forcing in the equilibrium range of wind-wave spectra, *J. Fluid Mech.*, *470*, 223–245.

Hasselmann, K., W. Munk, and G. MacDonald (1963), Bispectra of ocean waves, in *Proceedings of the Symposium of Time Series Analysis*, edited by M. Rosenblatt, chap 8, pp. 125–139, John Wiley, Hoboken, N. J.

Kudryavtsev, V. N., and V. K. Makin (2002), Coupled dynamics of short wind waves and the air flow over long surface waves, *J. Geophys. Res.*, *107*(C12), 3209, doi:10.1029/2001JC001251.

Leykin, L. A., M. A. Donelan, R. H. Mellen, and D. J. McLaughlin (1995), Asymmetry of wind waves studied in a laboratory tank, *Nonlinear Processes Geophys.*, *2*, 280–289.

Prevosto, M., H. E. Krogstad, and A. Robin (2000), Probability distributions for maximum wave and crest heights, *Coastal Eng.*, *40*, 329–360.

Shrira, V. I., S. I. Badulin, and A. G. Voronovich (2003), Electromagnetic wave scattering from the sea surface in the presence of wind wave patterns, *Int. J. Remote Sens.*, *24*, 5049–5073.

Suh, K. D., S. H. Oh, N. Hashimoto, and K. Ahn (2000), Laboratory observations of triad interaction of deep water wind waves, *Coastal Eng.*, *42*(3), 321–337.

Troitskaya, Y. I. (1994), Modulation of the growth-rate of short surface capillary gravity wind–waves by a long-wave, *J. Fluid Mech.*, *273*, 169–187.

Young, I. R., and A. V. Babanin (2001), Wind wave evolution in finite depth water, paper presented at 14th Australian Fluid Mechanics Conference, Adelaide Univ., Adelaide, Australia, 10–14 Dec.

Young, I. R., and Y. Eldeberky (1998), Observations of triad coupling of finite depth wind waves, *Coastal Eng.*, *33*(2–3), 137–154.

Young, I. R., M. L. Banner, M. A. Donelan, A. V. Babanin, W. K. Melville, F. Veron, and C. McCormic (2005), An integrated system for the study of wind wave source terms in finite depth water, *J. Atmos. Oceanic Technol.*, in press.

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