Partitioning sequences for the dissection of directional ocean wave spectra: A review *

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Abstract

Coexistence of wind sea generated locally and swell radiated from distant storms often yield double-peaked or multiple-peaked wave spectra. Inter-comparison of such data and the assimilation of spectral wave data from multi-peak directional wave spectra is difficult and may give ambiguous results if the entire spectrum is treated as one wave system. However, since wave systems originating from different uncorrelated meteorological events can be assumed to be independent, a partitioning of the ocean wave spectra into components which represent physically uncorrelated wave systems can be performed. This facilitates the assimilation of spectral wave data from the entire spectrum, since each component can be interpreted as one individual wave system. In this report we review and examine the partitioning algorithms that have been developed for directional wave spectra, and show how they can be used to obtain improved algorithms for the assimilation of spectral wave data. The report also includes a computer code (Matlab) for a general partitioning algorithm.

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Contents

1 Introduction

Partitioning algorithms for the separation of wind sea and swell in a frequency wave spectrum were, among others, proposed in the mid-eighties by Earle [2], and Vartdal and Barstow [16], and later also by Wang and Hwang [18]. These methods calculate a separation frequency which distinguishes the wind sea contribution of the spectrum from the low frequency swell modes. This information can therefore be used to filter out the wind sea part of the spectrum, but can not in general discriminate between different swell wave systems created by separate meteorological events. For this to be possible, we also need to take information about the directional properties of the waves into account.

While the concept of a directional spectrum of ocean waves has existed for nearly half a century, it is only during the last two decades that good measurements of the directional spectrum of ocean waves have been available. Nowadays, many different measuring devices working on different principles provide directional wave information on an operational basis. Global directional spectra is *e.g.* provided by satellites carrying so-called Synthetic Aperture Radars (SAR)[1]. The SAR allows us to obtain global directional wave measurements for operational use, such as global wave forecasting.

Unfortunately, the inherent difficulties associated with measuring and analyzing directional spectra have not disappeared. In particular, a large amount of information is required to make a robust estimate of the full directional spectra. Moreover, currently operating satellite-borne SARs may provide more than 1000 wave spectra daily, each of which often consists of a complex superposition of waves from several generation areas. Without some sort of data reduction, this represents an effectively unmanageable data set for global operational data assimilation purposes.

In order to reduce the large number of degrees of freedom of two-dimensional directional wave spectra to a manageable number of parameters while retaining the complex structures of real ocean waves, Gerling [3] devised a spectral partitioning scheme for decomposing a given spectra into components originating from uncorrelated sources. Hasselmann *et al.*, [6, 8] modified Gerling's scheme in order to make it more amenable to compare SAR wave spectra with spectra obtained from WAM wave model. In [7] they used this modified partitioning algorithm to develop an assimilation scheme which allows multiple distinct wave systems to be characterized across space and time with a greatly reduced set of parameters. This assimilation technique was extended by Voorips et al. [17] for the assimilation of pitch-and-roll buoy wave observations into the WAM model. Recently Hanson and Phillips [5] have adapted the spectral partitioning scheme of [8] to generate a fully automated technique for wave climatology analysis.

In the present report we describe a generalized form of Hasselmann's partitioning scheme as it was presented in [5] and examine its relation to the original method proposed by Gerling [3]. We then discuss the application of partitioning sequences to comparison and assimilation of directional wave spectra data and describe the swell source identification schemes proposed by Hanson and Phillips in [5].

2 The directional wave spectrum

The *directional spectrum,* $E(\omega, \theta)$, of ocean waves may be written as a product of the *frequency spectrum*, $S(\omega)$, and the *directional distribution*, $D(\omega, \theta)$, *i.e.*

$$E(\omega, \theta) = S(\omega)D(\theta, \omega)\ ,\ 0 < \omega < \infty\ ,\ 0 \leq \theta \leq 2\pi, \tag{1}$$

where D may further be expressed as the Fourier series,

$$D(\theta, \omega) = \frac{1}{2\pi}[1 + 2\sum_{n=1}^{\infty}(a_n(\omega)\cos(n\theta) + b_n(\omega)\sin(n\theta))] = \frac{1}{2\pi}\sum_{n=-\infty}^{\infty} c_n(\omega)e^{in\theta}\ . \tag{2}$$

The directional spectrum is thus a scalar function defined in the polar (ω, θ)-plane which provides information about the energy, or intensity, of the waves traveling with an angular frequency ω at an incident angle θ relative to a predefined x-axis. We note the common convention of writing the polar integral of E (equal to the variance of the surface) as

$$\mathrm{Var}\,(\eta) = \int_{\omega=0}^{\infty}\int_{\theta=0}^{2\pi} E\,(\omega, \theta)\, d\omega d\theta.$$

Thus, the Jacobian $|\omega|$ has already been absorbed into E.

In deep water, i.e. when the water depth is larger than the typical wavelength, there are three fundamental scales for the ocean surface. The obvious *amplitude* scale is the standard deviation of the surface, or more common, the significant wave height, H_s. The *frequency* scale are given by the frequency (ω_p) of the dominating waves, and hence the *time* scale may be chosen as $T_p = 2\pi/\omega_p$. For the spatial scale, it is natural to choose the typical wavelength according to the dispersion relation, $X = 2\pi g/\omega_p^2$, where g is the acceleration of gravity. All scales can be combined into the overall *wave steepness* s:

$$s = \frac{H_s}{\lambda_p} = \frac{H_s\omega_p^2}{2\pi g} = \frac{H_s}{(g/2\pi)T_p^2}.$$

The state of the sea is furthermore depending on the wind speed U at some reference height and the fetch F over which the wind has blown. Obviously, this will frequently be a gross oversimplification, since the wind field may have large spatial variations. The parameters U and F can be combined into the dimensionless parameters *inverse wave age,*

$$\tilde{U} = \frac{U}{c_p} = \frac{U}{g/\omega_p},$$

and *dimensionless fetch,*

$$\tilde{F} = \frac{gF}{U^2}.$$

Within this simplified theory, there are thus three dimensionless parameters characterizing the surface, and it is possible to write the frequency spectrum as

$$S(\omega) = \frac{H_s^2}{16\omega_p} S_0(\omega/\omega_p, s, \tilde{U}, \tilde{F}),$$

where

$$\int_0^\infty S_0(x, s, \tilde{U}, \tilde{F}) dx = 1, \tag{3}$$

and the maximum of S_0 occurs for $x = 1$.

For purely grown wind waves at large fetches, investigations indicate a good functional relationship between s and $\tilde{U}$ [12]. Moreover, the spectral shape is not seen to vary much with $\tilde{F}$. In that case, it is therefore reasonable to assume that $S(f)$ simplifies to

$$S(f) = \frac{H_s^2}{16\omega_p} S_0(\omega/\omega_p, s).$$

Since swell is by definition independent of the local wind and the fetch, it should also be reasonable to assume a similar parametrization for swell spectra.

In a similar way, we obtain for the directional distribution,

$$D(\theta, \omega) = D_0\left(\theta - \theta_0, \sigma_\theta\left(\omega/\omega_p\right), s, \tilde{U}, \tilde{F}\right),$$

where θ_0 is the dominant (mean) wave direction and σ_θ is the directional spread With these assumptions, it is possible to write the directional spectrum for a simple wave field in the open sea as

$$E(\omega, \theta) = \frac{H_s^2}{16\omega_p} S_0(\omega/\omega_p, \cdots) D_0\left(\theta - \theta_0, \sigma_\theta\left(\omega/\omega_p\right), \cdots\right),$$

where "$\cdots$" signifies the additional parameters mentioned above. More generally, the shape of the directional distribution D_0 may vary both with ω/ω_p and the other parameters.

In practice, the directional spectrum will consist of several wave fields having different origin. There will typically be a wind sea part and one or several swell parts,

$$E(\omega, \theta) = \sum_{j=1}^{J} E_j\left(\omega, \theta\right),$$

$$E_j\left(\omega, \theta\right) = \frac{(H_s)_j^2}{16\omega_{pj}} S_0(\omega/\omega_{pj}, \cdots) D_0\left(\theta - \theta_{0j}, \sigma_\theta\left(\omega/\omega_{pj}\right), \cdots\right).$$

This will make the spectrum have a multi-modal appearance.

One should also note that there are several equivalent representations of the directional spectrum, where remote sensing groups appear to prefer the directional *wavenumber spectrum*, $\Psi(\mathbf{k})$. The connection between the wavenumber spectrum and the directional spectrum follows by observing

$$\begin{aligned} \mathrm{Var}\left(\eta\right) &= \int_{\mathbf{k}} \Psi\left(\mathbf{k}\right) d^2k = \int_{k=0}^{\infty}\int_0^{2\pi} \Psi\left(k, \theta\right) k dk d\theta \\ &= \int_{\omega=0}^{\infty}\int_0^{2\pi} \Psi\left(k\left(\omega\right), \theta\right) k\left(\omega\right) \frac{dk\left(\omega\right)}{d\omega} d\omega d\theta = \int_{\omega=0}^{\infty}\int_0^{2\pi} E\left(\omega, \theta\right) d\omega d\theta, \end{aligned}$$

and applying the dispersion relation for connecting k and ω.

3 Partitioning algorithms for directional wave spectra

3.1 The general partitioning algorithm

We shall outline the basic steps in the partitioning algorithms following Hanson and Phillips [5]. The algorithm is conveniently split into five separate steps:

1. Isolate the spectral energy peaks,
2. Identify and combine wind sea peaks,
3. Identify and combine mutual swell peaks,
4. Remove low energy partitions,
5. Calculate partition statistics.

The output of the algorithm is thus a set of wave spectrum statistics associated with a decomposition of the initial directional spectrum into distinct subsets that represent individual and uncorrelated wave fields. Ideally, the physical interpretation of each partition should be that it represents a wave system originating from a certain meteorological event which is uncorrelated with the meteorological events that created the other wave systems in the partitioning.

3.1.1 Isolation of the spectral energy peaks

The idea behind the peak isolation procedure is analogous to the concept of a *catchment area* in hydrology, if one turns the graph of the spectrum upside down. To be more precise, a partition is defined as the set of all points in the polar (ω, θ) plane whose steepest ascent paths lead to the same local maximum. Thus, there is an associated catchment area for each local maximum.

It should be stressed that in the present study the catchment area is computed based on the graph of $E(\omega, \theta)$ and not a graph of $\Psi(\mathbf{k})$, which could be another choice.

3.1.2 Locally generated wind sea

After identifying peaks (local maxima) and catchment areas, the next step will be to isolate the locally generated wind sea peaks from the swell peaks, which typically originate from distant storms. The wind sea identification is based on a local wave age criterion. The phase speed of a spectral component in deep water is given by $c = g/\omega$. Assume that the component is traveling in the direction θ. A wave component is classified as a member of the wind sea if its propagation direction is within $\pi/2$ of the wind direction θ_U, and the phase speed is less that a certain fraction of the wind speed U, i.e. if

$$|\theta_U - \theta| < \frac{\pi}{2} \quad \text{and} \quad c < \gamma U \cos(\theta_U - \theta).$$

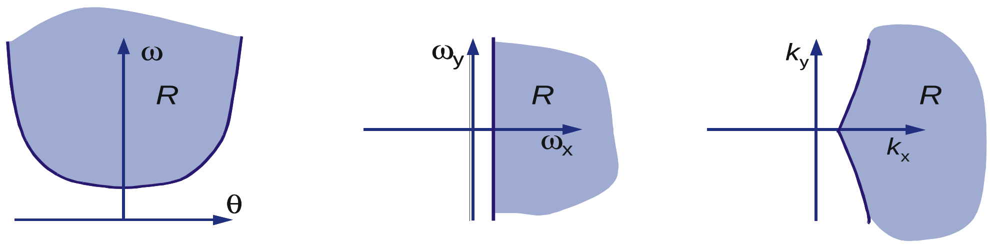

Figure 1: Typical shapes of the wind sea region, indicated by the bounds and the letter R, in the (ω, θ)-plane (left), the (ω_x, ω_y)-plane (middle), and the $\mathbf{k}$-plane (right).

In terms of peak angular frequency, the "catchment area" for wind waves is simply defined as

$$\{(\omega, \theta) : |\theta_U - \theta| < \frac{\pi}{2}, \qquad \gamma\omega \geq \frac{g}{U\cos(\theta_U - \theta)}\}.$$

Figure 1 illustrates the shape of this region in the (ω, θ) plane, and the corresponding regions in the polar, or (ω_x, ω_y)-plane, and the $\mathbf{k}$-plane. Upon classification, all wind sea peaks within this region are combined into one partition, say partition 0.

3.1.3 Combination of swell peaks

After the identification of wind sea, the next step will be to decide whether some of the swell peaks originate from the same source. Such peaks must, in order to achieve a partitioning sequence of uncorrelated wave systems, be combined into one partition. The separation criteria to be used here is (a) that the distance between two peaks is too small compared with the spectral spread of the respective peaks, and/or (b) the minimum spectral energy density on the "saddle point" between the two peaks is too high relative to the peak energy of smaller of the two.

Following Hasselmann *et al.*, [6], let p be the (vector) location of a peak

$$(\omega_p \cos\theta_p, \omega_p \sin\theta_p)\,,$$

and denote by $\Delta(p_i, p_j) = |p_i - p_j|$ the Euclidean distance between two adjacent peaks p_i and p_j. The spread of a peak at p is denoted $\delta(p)$, and amounts to usual root mean spread,

$$\delta^2(p) = \overline{(p_x - \bar{p}_x)^2} + \overline{(p_y - \bar{p}_y)^2} = \overline{p_x^2 + p_y^2} - (\bar{p}_x^2 + \bar{p}_y^2).$$

Here the overbar denotes the spectrally weighted average over the the (preliminary) partition $P = P(p)$, i.e.

$$\bar{\rho} = \frac{1}{e(P)} \int_P \rho E(\omega, \theta)\, d\omega d\theta\,,$$

where $e(P) = \int_P E(\omega, \theta)\, d\omega d\theta$ is the total energy of partition P.

Two adjacent swell partitions $P(p_i)$ and $P(p_j)$ are now combined if

$$\Delta(p_i, p_j) \leq \kappa \max\{\delta(p_i), \delta(p_j)\}\,,$$

for some suitable spread factor κ and the ratio between the saddle point value and the lowest associated maximum is larger than a certain constant.

3.1.4 Low energy partitions

Partitions whose total energy is very small compared to the total energy of the spectrum are considered to be insignificant in the sense that they do not have an important impact on any of the dominating wave systems in the spectrum. Typically, one does not wish to assimilate the data from such partitions into models. Hence, partitions with total energy E_t below $\frac{a}{f_p^4+b}$ where f_p is the peak frequency and where a and b are chosen to eliminate noise in the low-energy regions of the spectrum, are simply removed from the partitioning sequence. Alternatively, the energy may be redistributed among the other partitions so that the total energy is preserved.

3.1.5 The partition statistics

The final step is now to calculate, for each of the remaining partitions, a number of statistical parameters which capture the main characteristics of the different wave systems. As the partitions are assumed to be uncorrelated, these calculations can be carried out independently for each partition. A typical statistics file may contain

- the partition identification number
- the observation time and location
- the significant wave height
- the mean and peak frequencies
- the mean wave direction and the directional spread
- the wave age of the swell wave systems
- the angle between a swell wave system and the closest wind sea wave system

In general, the selection of parameters depends on the application of the results.

3.1.6 Some remarks on the selection of control parameters

The result of the partitioning algorithm is quite sensitive to some of the parameters involved. We therefore make some comments regarding the role of the respective parameters, and what issues one should consider when specifying appropriate parameter values.

The selection of appropriate energy threshold parameters a and b is perhaps especially important. This is because choosing a too restrictive threshold will only capture the energy peaks of the strongest wave systems so that we will only recognize a very limited number of wave systems and these will only appear in the partitioning sequence for a short period of time. On the other hand, a weak energy threshold will include many more wave systems than what one can expect is created from distinct and uncorrelated meteorological events.

Both a and b are required to be positive. The parameter a is the most significant while letting the parameter b be small preserves "medium" energy wave systems with high peak frequency. We found that $a = 0.1$ and $b = 0.002$ gave reasonable results.

The frequency threshold parameter γ for the wind sea criterion is chosen so that the phase speed of the wind sea wave systems in the wind direction is roughly less or equal to the wind speed. Hence, the parameter γ is typically chosen slightly larger than 1 to include all possible wind sea wave systems. We used a threshold parameter defined by $\frac{g}{2\pi\gamma} = 1.2$ which gives $\gamma \sim \frac{4}{3}$.

Finally we recall that two adjacent peaks are said to belong to the same wave system if (a) the distance between the two peaks is smaller than a factor κ times the directional spread of the "larger" of the two peaks, and/or (b) the minimum spectral energy density on the "saddle point" between the two peaks is higher than a factor ν times the smaller of the two peak energies. Both of these parameters may have a strong impact on the partitioning sequence, but we found it difficult to know how to determine the physically correct set of parameters. In our experimental test case we chose $\kappa = \nu = 2^{-1/2}$ which is neither a very restrictive, nor a very generous choice.

3.2 Gerling's algorithm

Though the partitioning algorithm presented above, which is essentially the one introduced by Hasselmann et al., [6, 8], has sprung out from Gerling's algorithm, [3], the original algorithm of Gerling was based on a completely different strategy. Instead of categorizing individual partitions as catchment regions, Gerling proposed that one could classify wave systems by considering the inherent tree structure seen in the graph of the spectrum. The spectral energy serves as a measure of height, and at any energy level l, the branches of the tree are associated with the *components* (i.e. the maximal connected subsets) of

$$R_l = \{(\omega, \theta);\ E(\omega, \theta) \geq l\}\,.$$

It is assumed that the spectrum is a continuous function and hence that the components vary in a smooth manner when l varies. Obviously, $R_{l_1} \subset R_{l_2}$ when $l_2 < l_1$. A branch point occurs for a component $R_{l,k}$ at the level l if the number of components within $R_{l,k}$ jump from one to a number larger than one when l increases.

The above procedure results in a tree structure that can be used as a constructive tool for further processing, analogous to steps 2-5 of Hasselmann et al.'s partitioning algorithm. The main branch structures in Gerlings algorithm play the role of partitions in Hasselmann et al.'s algorithm. We note, however, that Gerling's peak identification procedure is qualitatively different from the modified scheme of Hasselmann et al. While Hasselmann et al.'s catchment region criterion dissects the spectral plane into disjoint subregions which together make up the whole plane, Gerling's spectrum partitioning tree identifies isolated subsets of the plane corresponding to spectral peaks and sub-peaks. This means that some of the global spectral data is not distributed among the partitions, but is only contained in the root of the tree from which it is difficult to assess anything conclusive about the dynamics of the traveling wave systems.

We are not aware of comparisons between Gerling's scheme and the scheme of Hasselmann et al. However, as each of the branches of Gerling's partitioning tree are subsets of corresponding partitions obtained with Hasselmann et al.'s partitioning scheme, it seems that Hasselmann et al.'s scheme may be more robust because each partition includes more of the spectrum in the neighborhood of the peak. This is an advantage when we want to compare partitions from different spectra and makes it easier to track wave systems in space and time. This desirable feature also allows for developing supplementary swell tracking and swell source identification schemes. The key ingredient here is to introduce a *cross assignment criterion* for when two partitions of two different spectra are sufficiently similar to be classified as the same wave system.

3.3 Cross assignment of wave systems

We have assumed that different partitions within a spectrum are uncorrelated in the sense that they are created by uncorrelated meteorological events. On the other hand, two partitions from two different spectra (e.g. model and observed spectra or two measured spectra at different locations) are correlated if they correspond to the same wave system.

Thus, to decide if a partition from one spectrum represents the same wave system as a partition from another spectrum, and thereby allow us to e.g. monitor the evolution of wave systems through time, we need to define a *cross assignment* criterion. The cross assignment criterion of Voorips et al. [17] says that a partition "i" of a spectrum A should be be cross-assigned with a partition "j" of a spectrum B if i and j

- are partitions of the same type (sea or swell),
- have comparable intensity: $\nu^{-1}e(i) \le e(j) \le \nu e(i)$,
- are close in the spectral plane: $|\bar{\omega}_i - \bar{\omega}_j| \le \eta_\omega \bar{\omega}_i$, $|\bar{\theta}_i - \bar{\theta}_j| \le \eta_\theta$,

for some appropriate constants $\nu, \eta_\omega, \eta_\theta$. If several of the partitions of spectrum A fulfill the above requirements, then the one closest in wavenumber is chosen.

Clearly, it is possible that not every partition of A can be cross assigned to a partition of B. If the two spectra represent the ocean state at the same location in space, but at two different times, then non-assigned partitions can be interpreted as newly generated wave systems, or old wave systems which have faded out. However, if we are comparing wave model spectra with observed or measured spectra, then the non-existence of companion wave systems may be more difficult to explain, and may suggest that we need to adjust our model in order to achieve better coherence with the observations. A deeper discussion of how to treat non-assigned partitions in comparisons of directional wave spectra may be found in [17], Sect. 5.6.

3.3.1 Remarks on the selection of cross-assignment parameters

In the next section we will use the cross assignment criterion to track swell wave systems in time. For this purpose it is important to observe that the mutual swell system criterion which determines whether two wave systems in the same spectrum are correlated is different from the swell tracking criterion which determines whether two partitions from two different spectra represent the same wave system. This means that we can end up in a situation where two "uncorrelated" wave systems are assigned to the same partition of the previous spectrum, and thus belong to the same wave system.

In our test results for a sequence of spectra from Vøringplatået this situation did not occur frequently and was not any cause of concern because it only occurred for wave systems that we interpreted as fading wind sea wave systems that have begun to disperse in the wave spectrum. This is because these wave systems had low energy and a high, but decaying peak frequency.

When it comes to specifying proper swell-tracking parameters $\nu, \eta_\omega, \eta_\theta$ we need to keep in mind that the total energy, or equivalently the significant wave height, can vary rapidly over a rather short period of time.

Our data record from Vøringplatået contained spectra measured at three hour intervals and we found it necessary to select a very loose energy threshold, that is, we had to allow the significant wave height of one single wave system to reduce to one fourth of its original height, or to increase up to four times that height. This choice corresponds to $10 < \nu < 20$.

The frequency of the swell wave systems was much more stable than the energy was and we found that is was sufficient to require that the mean frequency from one observation time to the next should be within 50% of its original value. This criterion corresponds to $\eta_\omega = 0.5$.

Finally, the mean direction of what appeared to be the same wave system could be quite oscillatory. We therefore allowed the mean direction of a single wave system to vary as much as $\pi/3$ radians from one observation time to the next. This corresponds to $\eta_\theta = \pi/3$.

4 Applications

The primary motivation for using partitioning algorithms in processing of directional wave spectra is to identify uncorrelated wave systems and to characterize them using a reduced set of parameters. To what extent these modeled wave spectra actually reflects the observed or real ocean wave spectra depends, obviously, on the correctness of the assumption about uncorrelated wave sources, but it also depends on our ability to tune the various adjustable parameters in the algorithm.

As no rigorous mathematical theory or simple physical principles indicate how to choose the parameters, they have been selected rather *ad-hoc* in the literature. On the other hand, it has been recognized that using wave-spectrum statistics to describe complex ocean spectra can not be expected to give a complete picture of the real ocean state, and should only be interpreted as an intelligent first guess. This realization has led to the development of iterative assimilation schemes for retrieving improved directional ocean wave spectra.

Here, we shall give a brief description of iterative interpolation schemes for retrieving improved ocean wave spectra using partitioning algorithms, and show how a correctly partitioned wave spectrum can be applied to identify the source of swell wave systems.

4.1 Assimilation of wave spectra in wave models using partitioning algorithms

Assume that we want to update a family of model spectra based on a corresponding family of observed spectra. Furthermore, assume that all spectra involved, both model spectra and observed spectra, can be represented as a superposition of a uncorrelated wave systems, and that each wave system is characterized by a few spectral parameters. This assumption implies that we may benefit from the use of a spectral partitioning scheme in the sense that we only need to work with components defined by a reduced set of parameters rather than the entire spectrum.

Thus, let $p_i(x)$ denote the state vector associated with partition i of a model wave spectrum $p(x)$ at $x \in \mathcal{X}$ and let $q_j(y)$ be the state vector associated with partition j of an observed wave spectrum $q(y)$ at $y \in \mathcal{Y}$. The index sets $\mathcal{X}$ and $\mathcal{Y}$ contain the time and geographic location of the observations or the spectra that we consider. In general these $\mathcal{X}$ and $\mathcal{Y}$ may differ, but we shall for simplicity assume that they do not, and will henceforth let $\mathcal{X} = \mathcal{Y}$.

We now introduce the cross assignment identifier function,

$$\delta_{\phi_i(x),\varphi_j(y)} = \begin{cases} 1 & \text{if } \phi_i(x) \text{ is cross assigned with } \varphi_j(y) \ , \\ 0 & \text{otherwise} \ . \end{cases}$$

The general *optimal interpolation scheme* of Voorips et al. [17] is now given by

$$p_k(x) := p_k(x) + \sum_{y\in\mathcal{Y}} W_x(y) \sum_{i(y),j(y)} \delta_{p_k(x),p_i(y)} \delta_{p_i(y),q_j(y)} (q_j(y) - p_i(y)) \ ,$$

where $W_x(y)$ are statistical weight factors. Note that we also extract information from locations $y \neq x$, as these may supplement the information we are able to extract from $q(x)$.

The first cross assignment, which compares partitions of two model spectra, ensures that only innovations corresponding to the same wave system are used to increment the state vector of a model partition. Instead of $\delta_{p_k(x),p_i(y)}$ one could also choose to use $\delta_{p_k(x),q_j(y)}$, which was done by Hasselmann et al., [6], but Voorips et al. found the present choice to be more robust. The second cross assignment identifies the partition of the observed spectrum located at position y which corresponds to the same wave system as partition i of the corresponding model spectrum.

The weight factors are determined by the condition that the statistical mean square error between the analyzed model state $p_k(x)$ and the true state vector $p_k^t(x)$ is minimized. This gives,

$$W_x(y) = P(P+Q)^{-1} \ ,$$

where P and Q are the model and observation error covariance matrices

$$\begin{aligned} P(x,y) = \mathrm{Cov}(p(x),p(y)) &= E[(p(x) - E[p(x)])(p(y) - E[p(y)])] \\ Q(x,y) = \mathrm{Cov}(p(x),p(y)) &= E[(q(x) - E[q(x)])(q(y) - E[q(y)])] \ . \end{aligned}$$

Here we have used that the model and observation errors are uncorrelated, cf. Komen et al., [9], Chap. 6. Actually, a correlation between model errors and observation errors can arise through data assimilation of past errors in the observations into the model states. In fact, observation errors, particularly of satellite data, can be correlated when, for example, they are due to environmental influences that are not included in the retrieval algorithm. However, as the extent of such error correlations is difficult to access, they are normally ignored.

Note that in the definition of the covariance matrices we do not make any reference to the partitions of the respective spectra. The dependence on the partitions is dropped since we have assumed that the partitions within a spectrum are uncorrelated. Thus, two partitions $p_k(x)$ and $p_i(y)$ from two different spectra $p(x)$ and $p(y)$ are correlated only if they are cross assigned, in which case $\mathrm{Cov}(p_k(x), p_i(y)) = \mathrm{Cov}(p(x), p(y))$. Similarly, if $q_k(x)$ is cross assigned to $q_i(y)$, then $\mathrm{Cov}(q_k(x), q_i(y)) = \mathrm{Cov}(q(x), q(y))$, and zero otherwise.

4.2 Swell source identification by spectral partitioning

We first recall how one can, on the basis of simple linear wave theory, approximate the location, in space and time, of the meteorological events which created the various swell wave systems [14, 4].

Wind on the ocean surface produce a spectrum of waves that, according to linear wave theory, disperse from the generation area according to the deep water dispersion relationship $\omega^2 = gk$, implying a group (or energy propagation) velocity, $c_g(\omega) = g/2\omega$. Thus, if we release a pulse of wave energy at $x = x_s$ and at time $t = t_s$, an observer at at a distance d will, after some time, observe passing waves with a dominating frequency given by

$$\frac{g}{2\omega_p(t)} = \frac{d}{t - t_s},$$

or

$$\omega_p(t) = \frac{g}{2d}(t - t_s).$$

By fitting a linear regression line through observed pairs $\{\omega_p(t), t\}$, it is thus possible to obtain an estimate of the distance d to the generation area, as well as the time of origin, t_s. In addition, a directional measurement will give a mean direction of arrival, $\bar{\theta}$.

By applying a spectral partition algorithm and a cross assignment in time, it is now possible to carry out this analysis for each of the swell partitions. In deep water, the waves travel along great circle routes, and we deduce that the source latitude, longitude coordinates (ϕ_s, φ_s) are given by

$$\phi_s = \sin^{-1}(\sin\phi_0 \cos\theta_d + \cos\phi_0 \sin\theta_d \cos\bar{\theta}_i),$$
$$\varphi_s = \varphi_0 - \sin^{-1}\left(\frac{\sin\theta_d \sin\bar{\theta}_i}{\cos\phi}\right),$$

where ϕ_0 and φ_0 are the observation coordinates and $\theta_d = d/(\text{radius of the earth})$ is the angular distance to the source. Refinement of the above analysis may be found in [4].

5 Examples

We shall illustrate how the partitioning algorithm works, and have used data obtained from measurements on Vøringplatået in the Norwegian Sea [13] and selected a time frame which illustrates some of the important aspects. The measurements are sampled 3 hours apart, so that we have 8 spectra for each day.

We concentrates on the swell wave systems and demonstrate how the partitioning algorithm splits the spectrum into distinct components, and how the swell tracking algorithm recognizes that two partitions from two consecutive measurements correspond to a wave system which is created by the same meteorological event.

Our first objective is to show how the partitioning of directional ocean wave spectra can be used to follow "individual" wave systems in time. For this we have chosen a series of wave spectra measured at Vøringplatået in the Norwegian Sea September 27-29, 1989. Figure 2 plots the directional ocean wave spectra measured by an ocean wave buoy at three hour intervals.

We see that the first few wave spectra are nice and single peaked, but that the spectrum becomes more smeared around observation 1398. This is due to a wind sea wave system coming from south east. Around 1405 we observe that a dual peaked wave spectrum has been established representing an eastward bound wave system and a south-westward bound wave system. At the same time we have a strong north-western wind which creates a more complex wave spectrum. Thus, here there lies a challenge in dissecting the joined wave spectrum into truly uncorrelated wave systems. The most significant wave systems recognized by the partitioning algorithm are shown in Figure 3.

Figure 3 shows that the partitioning algorithm correctly identifies the main components of the directional ocean wave spectra. A total of 15 swell wave systems were identified. Seven of the wave systems appeared only for 1 time step, most of which were "low-energy" wave systems that would not appear in the partitioning sequence if more restrictive energy criteria were chosen. Selecting a stronger energy threshold would, however, also cause some of the partitions which represent one wave system to be split into two. This is because the total energy of a wave system might diminish dramatically before it again gains "strength". This is illustrated in Figure 4 where we show how the total energy of the different wave systems evolve in time. We therefore found it better to to relax the criteria slightly and rely on our ability to pick out the most significant wave systems.

Another problem was that the mutual swell partition criteria is not the same as the criteria used to determine when two wave systems from two different spectra are to be co-assigned. This led to a situation where two non-mutual wave systems from one spectrum could be co-assigned to the same wave system, thus representing the same wave system. For instance, three non-mutual wave systems of spectrum 1398 were co-assigned to the same partition of spectrum 1397.

Finally we take a closer look at the mean frequency and mean direction associated with wave spectra 1405-1410. Here we have one wave system which appears in all spectra and several other

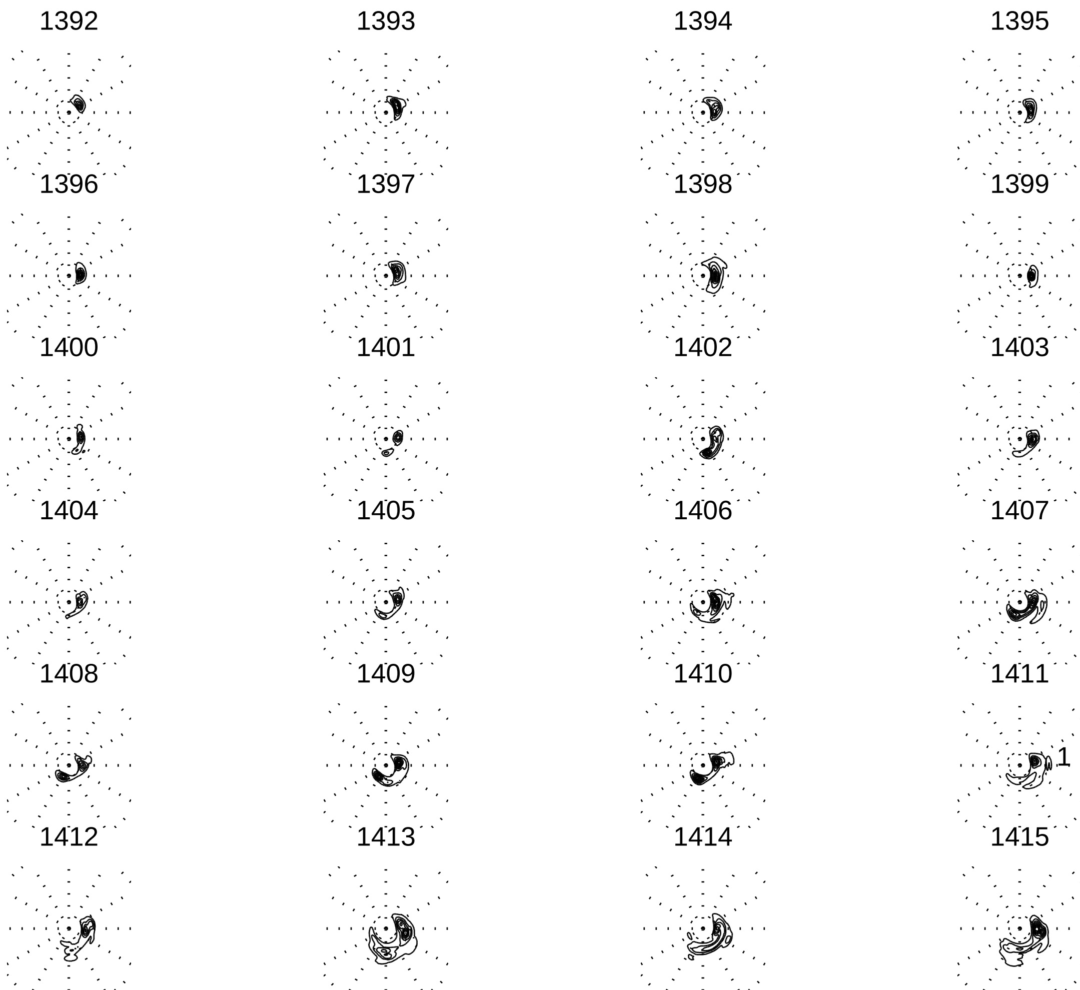

Figure 2: A subsequence (1392-1415 out of 2640) of directional ocean wave spectra taken from measurments at Vøringplatået in the Norwegian Sea in the period September 27-29, 1989.

spectra that appear over short periods of time. These spectra illustrate the complex nature of real ocean waves.

Table 1 illustrates that a directional ocean wave spectrum may consist of a multitude of different components that might or might not have been created by the same meteorological source. It is not a trivial task to dissect such a spectrum into truly uncorrelated partitions. Nevertheless, we believe, and have made an effort to show, that this kind of partitioning algorithm can be a helpful tool when attempting to interpret ocean wave spectra. However, even though this kind of partitioning algorithms are designed for automated wave spectra analysis, they can not be thought of as entirely automatic, and rely on manual interaction.

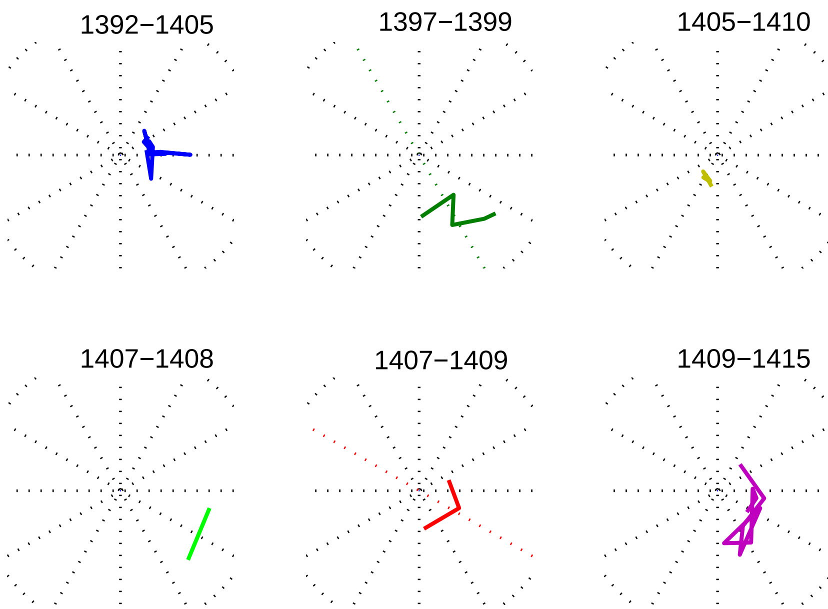

Figure 3: The peak coordinates for the dominating wave systems in the spectra depicted in Fig. 2.

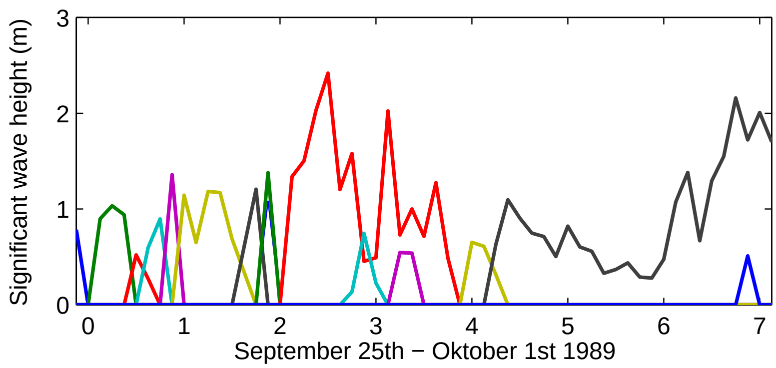

Figure 4: The significant wave height of a sequence of swell wave systems from Vøringplatået in the Norwegian Sea. The series in Figure 3 ????? can be seen between day 2 and day 5.

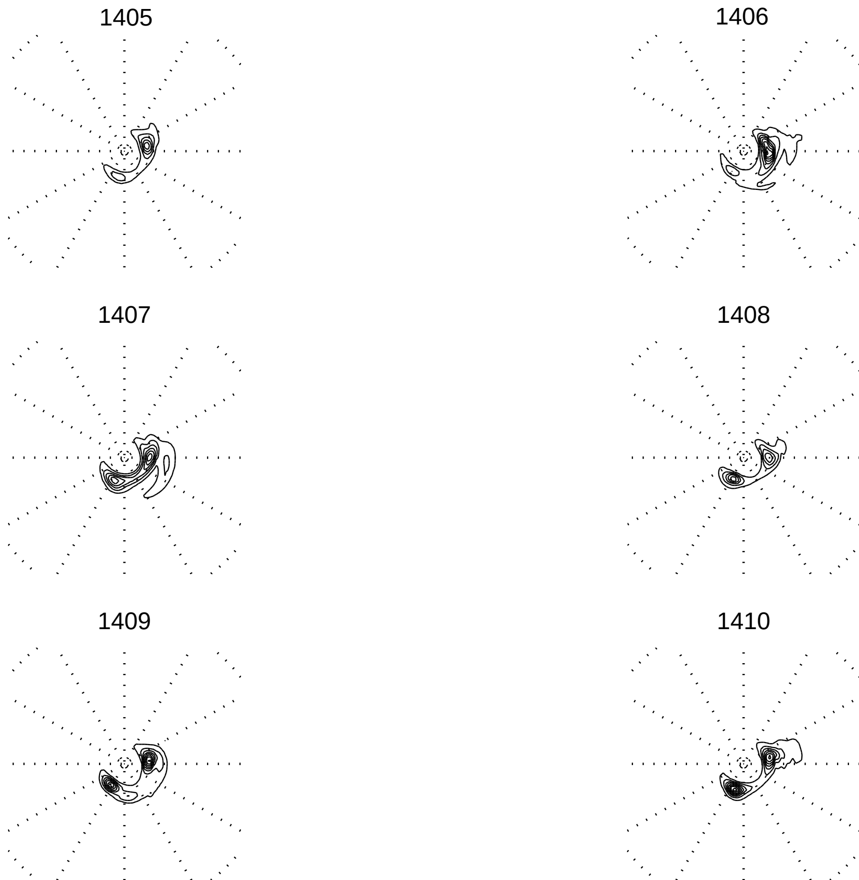

Figure 5: The figure plots the wave spectra 1405-1410 from the series depicted in Fig. 2.

	Mean frequency						Mean direction					
Spec.	*1405*	*1406*	*1407*	*1408*	*1409*	*1410*	*1405*	*1406*	*1407*	*1408*	*1409*	*1410*
Ws 1	.113	.077	.095	.088	.094	.097	260	228	254	244	237	255
Ws 2	.209						357					
Ws 3	.094						11					
Ws 4			.342	.321					314	349		
Ws 5			.267						287			
Ws 6			.112	.154	.135				20	336	277	
Ws 7			.354						29			
Ws 8					.123	.167					50	351
Ws 9						.271						10

Table 1: The mean frequency and mean direction of the identified wave systems depicted in Fig. 5

6 A Matlab code for the partitioning of ocean wave spectra

PartAlg.m - The partitioning algorithm.

```
IdentPart %Identifies partitions.

%Computing peak frequencies and peak directions.
fp=(fx(xp).^2+fy(yp).^2).^1/2;
dp=90-(180/pi)*atan2(fy(yp),fx(xp));

%Checking if partition energy is below a minimum threshold.
eth=a./(fp.^4+b);
for i=1:NP
  if EP(i)<eth(i)
    EP(i)=0;

%Identify swell and wind sea peaks.
WI=[]; SI=[];
dd=(pi/180)*(180+dp-Bdir(n-from+1,32));
for i=1:NP
  if EP(i)>0
    if gamma<cos(dd)*fp(i)*Bdir(n-from+1,28)
      WI=[WI,i]; %Partition "i" is classified as wind sea.
    else
      SI=[SI,i]; %Partition "i" is classified as swell.

%Compute swell partition statistics.
if length(SI)>0
  SPM=CombSP(...); %Combines mutual swell partitions.
  SS=CompSS(...); %Computes swell partition statistics.

%Compute wind sea statistics.
if length(WI)>0
  WS=CompWS(...); %Computes wind sea statistics.
```

IdentPart.m - Partitions the directional ocean wave spectrum into disjoint components.

```
%Step I: Locate peaks and generate steepest ascent indicator matrix.
NP=0; SAI=zeros(9,L,H);
xp=[]; yp=[]; ep=[];
for i=2:L-1
  for j=2:H-1
    me=max(max(E(i-1:i+1,j-1:j+1)));
    if E(i,j)==me %Checking if (i,j) is a local maximum.
      NP=NP+1; %Number of peaks.
      xp=[xp,i]; yp=[yp,j]; %Peak coordinates.
      ep=[ep,E(i,j)]; %Peak energy.
    for k=-1:1
      for l=-1:1
        if E(i+k,j+l)==me
          SAI(5-k*3-l,i+k,j+l)=1; %Steepest ascent path goes from (i,j) to (i+k,j+l)
SAI(5,:,:)=0;
```

```
%Step II: Follow steepest ascent paths backwards from peak.
xP=xp; yP=yp; eP=ep;
for p=1:NP
  old=1; new=0; Ne=1;
  while old>0 & Ne<1000
    for i=1:old
      e=Ne-old+i;
      for k=-1:1
        for l=-1:1
          if SAI(5+k*3+l,xP(e,p),yP(e,p))==1
            new=new+1; Ne=Ne+1;
            xP(Ne,p)=xP(e,p)+k; yP(Ne,p)=yP(e,p)+l; %Partition coordinates
            eP(Ne,p)=E(xP(e,p)+k,yP(e,p)+l); %Partition energies
    old=new; new=0;
EP=sum(eP);
```

CombSP.m - Combines mutual swell partitions and returns the swell system indicator *SPM*.

```
function SPM=CombSP(SI,fx,fy,xp,yp,ep,xP,yP,eP,EP,kappa,nu,E)

Ns=length(SI);
spx=fx(xp(SI)); spy=fy(yp(SI)); %Swell peak coordinates
sfx=zeros(1,Ns); sfy=zeros(1,Ns); %Swell peak mean energies.
sfx2=zeros(1,Ns); sfy2=zeros(1,Ns); %Swell peak mean-square energies.

for s=1:Ns
  p=SI(s); lp=nnz(eP(:,p));
  ev=eP(1:lp,p);
  Fx=fx(xP(1:lp,p));
  Fy=fy(yP(1:lp,p));
  sfx(s)=(Fx*ev)/EP(p);
  sfy(s)=(Fy*ev)/EP(p);
  sfx2(s)=((Fx.^2)*ev)/EP(p);
  sfy2(s)=((Fy.^2)*ev)/EP(p);
psp=sfx2+sfy2-sfx.^2-sfy.^2;

CM=zeros(Ns,Ns);
for s=1:Ns
  for t=1:Ns
    p1=SI(s); p2=SI(t);
    di=(spx(s)-spx(t))^2+(spy(s)-spy(t))^2;
    ms=max(psp(s),psp(t)); em=min(ep(p1),ep(p2));
    xm=min(xp(p1),xp(p2)); xM=max(xp(p1),xp(p2));
    ym=min(yp(p1),yp(p2)); yM=max(yp(p1),yp(p2));
    mu=min(min(E(xm:xM,ym:yM))); %Minimum energy between peaks.
    if di<kappa*ms | mu>nu*em %Mutual peak criterion.
      CM(s,t)=1; CM(t,s)=1; %Peak p1 is mutual to peak p2.

A1=CM; A2=spones(CM^2);
while A2-A1>0
  A1=A2; A2=spones(A2^2);
A2=rref(A2);
SPM=A2(1:rank(A2),:);
%If SPM(s,i)=SPM(s,j)=1, then partitions SI(i) and SI(j) belong to the same wave system.
```

CompSS.m - Computes swell partition statistics and stores the statistics in the matrix *SS*.

CompWS.m - Computes wind sea statistics and stores the statistics in the matrix *WS*.

Parameter.m - Control parameters for the partitioning and swell tracking algorithms.

```
%Energy threshold parameters for classification of significant partitions.
a = 0.1; %High values gives a strong energy threshold criterion.
b = 2e-3; %Lower values makes the frequency contribution more significant.

%Frequency threshold parameter for wind sea criterion.
gamma = 1.2; %1≤gamma≤1.5, low values (∼1) gives a weak wind sea criterion.

%Mutual swell peak criterion parameters.
kappa = 0.5; %Partition spread separation criterion.
nu = 0.5; %Peak separation criterion.

%Swell tracking parameters.
edt = 20; %The total energy must be within 5% of the other.
fdt = 0.5; %The mean frequency must be within 50% of the other
adt = 60; %The mean direction must be within 60 deg. of the other
```

SwellTrack.m - Tracks the evolution of the swell wave system in (space or) time.

```
%SPT(1,s) gives the first time step for which swell system "s" was observed.
%SPT(2,s) gives the final time step for which swell system "s" was observed.
%SPS(:,t,s) gives the statistics for swell system "s" at time "t".

if lso==0 %If there were no swell systems in the previous spectrum.
  for s=1:ls
    SPS(:,1,ns+s)=SS(:,s);
  SPT(1,ns+1:ns+ls)=n;
  SPT(2,ns+1:ns+ls)=n;
  ns=ns+ls;

else
  No = [];
  for t=1:ns
    if SPT(2,t)==n-1
      No=[No,t]; %Identifies the wave systems from the previous spectrum.
  nso=length(No);
  dM=zeros(3,nso);
  for s=1:ls
    for t=1:nso
      no=No(t); nt=nnz(SPS(1,:,no));
      fd=abs((SPS(7,nt,no)-SS(7,s)))/max(SPS(7,nt,no),SS(7,s));
      dd=abs((SPS(6,nt,no)-SS(6,s)));
      ed=SPS(5,nt,no)/SS(5,s);
      dM(1,t)=fd/fdt;
      dM(2,t)=dd/adt;
      dM(3,t)=max(ed,1/ed)/edt;

    sdM=sum(dM);
```

```
  for t=1:nso
    if sdM(t)==min(sdM)
      tm=No(t); %The system from the previous spectrum which is closest to "s".

  nt=nnz(SPS(1,:,tm));
  if max(dM(:,tm)) < 1 %If "s" should be cross-assigned to "tm".
    SPT(2,tm)=n; %Update final observation time.
    SPS(:,nt+1,tm)=SS(:,s); %Update statistics.
  else
    ns=ns+1; %Increment number of observed swell systems.
    SPS(:,1,ns)=SS(:,s);
    SPT(1,ns)=n;
    SPT(2,ns)=n;
```

dirspec.m

```
function E=dirspec(fx,fy,B)

% B = data record DSPEC-format.
% E = directional spectrum on a cartesian grid.

f = .01:.01:.50;
d = -pi:pi/72:pi;

(fg,tg) = meshgrid(f,d);
fg = fg'; tg = tg';

(X,Y) = meshgrid(fx,fy);
TIN = atan2(Y,X);
FIN = sqrt(X.^2+Y.^2);

ab = reshape(B(101:300),50,4);
ab(1:2,:) = zeros(2,4);
ab(46:49,:) = zeros(4,4);

(n,m) = size(ab);
D = zeros(n,max(size(d))); %MEM directional distribution.
for nn=1:n
  if ab(nn,1)>-1
    c1 = ab(nn,1)+i*ab(nn,2);
    c2 = ab(nn,3)+i*ab(nn,4);
    f1 = (c1-c2*conj(c1))/(1-abs(c1).^2);
    f2 = c2-c1*f1;
    s1 = 1-f1*conj(c1)-f2*conj(c2);
    dn = 1-f1*exp(-i*d)-f2*exp(-i*2*d);
    D(nn,:) = real(ones(size(dn))*s1./(abs(dn).^2*2*pi));

dspec = max(0.0,diag(B(51:100))*D);
E = griddata(fg,tg,dspec,FIN,TIN);
```

main.m - Main file which tracks the partitioned ocean wave spectra in time.

```
fileid=fopen(filename,'r','l'); %Read data from file.
status=fseek(fileid,(from-1)*2560,'bof');
```

```
Bdir=[fread(fileid,[640,nsamples],'float32')]';

fm=0.5; fx=-fm:fm/50:fm; fy=fx; %Generate grid coordinate vectors.
L=length(fx); H=length(fy);

ns=0; lso=0; ls=0;
for n=from:to
  E=dirspec(fx,fy,Bdir(n-from+1,:)); %Generates the directional wave spectrum.
  PartAlg %Partitioning algorithm.
  if length(SI)>0
    ls=rank(SPM);
    SwellTrack %Swell tracking algorithm.
  else ls=0;
  lso=ls;
```

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