Accuracy Improvement of Directional Spectrum Estimation with Submerged Ultrasonic Doppler-type Directional Wave Meter

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ABSTRACT

An upgraded submerged Doppler-type directional wave meter can measure 10 wave quantities related to directional wave motions, i.e. water surface elevation, 3 components of water particle velocities at each layer of 3 different water depth ranging from shallow to deep. In this study, accuracy of directional spectrum estimation is comprehensively investigated for various cases where directional spectra are estimated with various sets of different number of wave quantities. As a result, accuracy improvement is generally confirmed in various wave conditions when all the quantities are applied to the estimations, compared with the cases where conventional 4 wave quantities are applied to the estimations. Moreover, the advantage of BDM with the improved computation method was confirmed.

KEY WORDS: Directional spectrum; BDM; Wave observation; Directional wave gauge, NOWPHAS

INTRODUCTION

From the viewpoints of various activities in the coastal region such as development, utilization and disaster prevention, the ocean wave is the most variable or important external natural force. Therefore, at many coastal sites throughout Japan, observation and examination of ocean waves are continuously conducted using established information networks including the Nationwide Ocean Wave information network for Ports and HArbourS (NOWPHAS).

Desirable wave observation should include comprehensive measurements performed simultaneously and at the same point, covering not only the wave height and wave period but also mutually closely-related wave quantities such as the wave direction, current and tidal level. However, when actually observing multiple oceanographic phenomena, operating conditions of measuring instruments and environments often make it difficult to perform simultaneous and long-term observation of multiple wave quantities at the same point. With these backgrounds, the directional wave meter has been developed to “standardize the wave direction measurement”, and about 80 sites along Japanese coastal areas are now equipped with this system. As more observation sites are equipped with the directorial wave meters, a wider range of users are now provided with high-accuracy directional spectra, which used to be available only by using an array of wave height meters or large-scale wave observation system. As a result, directional spectrum with dual peaks and other interesting directional spectra of multi-directional waves are being observed and confirmed at many sites. In addition, in contrast to the previous intermittent observations, drastically increased capabilities and storage capacities of computers have enabled continuous observations of the waves as well as acquisition and processing of many raw data observed at multiple layers using directional wave meters. To cope with such changing times, further upgraded analytical techniques are now needed for processing observed wave data.

In the view of the development of improved system to proved higher-accuracy directional spectra of ocean waves for the age of high speed and large capacity, the goal of this study is to investigate the accuracy of directional spectra estimated by using the multi-layer Doppler-type directional wave meter. Thus, to contribute to the clarification of the characteristics of ocean waves and hence to provide highly reliable given conditions for the planning, designing and construction of port and shore structures, as well as to offer useful basic information for development of high-accuracy wave forecasting model or for the determination of the damaging mechanism by typhoon etc.

DOPPLER-TYPE DIRECTIONAL WAVE METER

The Doppler-type directional wave meter is equipped with a seabed mounted single sensor, which provides the capability of simultaneously measuring the various parameters of the waves,

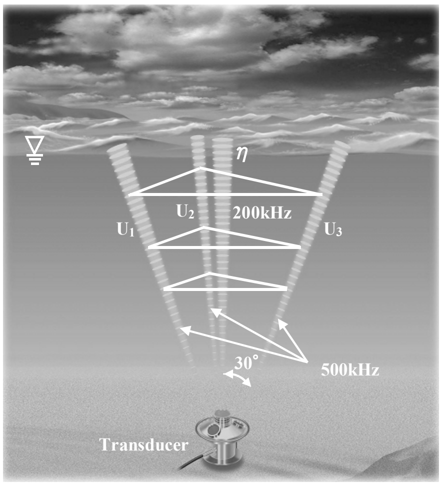

Fig. 1 Doppler-type directional wave meter

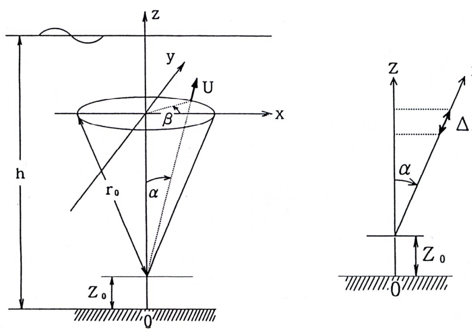

Fig. 2 Polar representation of the measurement position

current profile and offshore tidal fluctuation in the coastal region. Functionally, it is compound type wave observation equipment with the integrated capabilities of both the submerged ultrasonic wave height meter, which has often been used in Japanese traditional wave measurements and the multi-layer Doppler current meter, which has been used mainly on the ship or as mooring type equipment.

The observation system consists of a seabed mounted transducer and a control/ measuring / processing unit installed on the ground, and submerged cables. Fig. 1 shows the measuring concept of the directional wave meter.

The seabed mounted transducer is equipped with total four ultrasonic oscillators: one for detecting the water surface elevation (200 kHz), which is placed at the top center of the transducer and pointing vertically upward direction; and three for detecting water particle velocities (500 kHz), which are placed around the first one and inclined from the vertical axis by 30 degrees. As shown in Fig. 1, the first part measures the sea surface elevation by transmitting an ultrasonic to surface, and then receiving an ultrasonic reflected at the surface, and the second part measures the water particle velocities in the target layer by transmitting ultrasonic in the inclined three directions, and then analyzing Doppler frequency shifts of the received waves.

DIRECTIONAL SPECTRUM ANALYSIS

According to the theory of small amplitude waves applied to ocean waves, any wave quantities such as water surface elevation, water pressure fluctuation, water surface gradient and water particle velocities can be mutually converted to each other by using a transfer function between two relevant wave quantities. By using the polar coordinates system as shown in Fig. 2, the *r*-direction component of the water particle velocity *U* and the water surface elevation η at the position (α, β, γ) can be related by the following transfer function.

$$
\begin{aligned}
&H(\alpha,\beta,r_0,\Delta r,d,z_0;\omega,\theta) \\
&= \frac{-i\omega\exp(-i\omega\Delta t)}{\Delta rk\sinh kd}\left[\cosh\{k(r\cos\alpha+z_0)\}\right. \\
&\left.\times\exp\{ikr\sin\alpha\cos(\theta-\beta)\}\right]_{r_0-\Delta r/2}^{r_0+\Delta r/2}.
\end{aligned}
\qquad (1)
$$

where d is the water depth, k is the wave number, ω is the angular frequency, θ is the direction of the wave propagation, z_0 is the distance from the seabed to the equipment, Δt is the time difference between the measurements of water particle velocity U in each direction and the water surface elevation η, Δr is the length of the segment in r-direction along the reflected ultrasonic signal as shown in the drawing on the right of Fig. 2, and r_0 is the center position of the segment.

By using this transfer function, the cross-spectrum $\Phi_{mn}(f)$ of the wave quantities $\xi_m(t)$ and $\xi_n(t)$ is related to the directional spectrum $S(f, \theta)$ by the following integral equation.

$$\Phi_{mn}(f) = \int_0^{2\pi} H_m(f,\theta) H_n{}^*(f,\theta) S(f,\theta) d\theta \quad (2)$$

where $H_m(f,\theta)$ is an abbreviated expression of Eq. (1) representing the transfer function from the wave quantity $\xi_m(t)$ to the water surface fluctuation $\eta(t)$ at the center position, and * indicates the conjugate complex number. It is worth noting that $H_m(f,\theta)=1$ when $\xi_m(t)=\eta(t)$.

The directional spectrum can be estimated by finding a non-negative solution $S(f,\theta)$ of the simultaneous integral equations with regard to the cross-spectra given by Eq. (2).

Among several methods to estimate the directional spectrum based on Eq. (2), the Extended Maximum Likelihood Method (EMLM), the Bayesian Directional Method (BDM) and the Extended Maximum Entropy Principle Method (EMEP) are widely used in many researches and studies, because of their high accuracy and wide range applicability to any combination of wave quantities such as water surface elevation, water particle velocities and water surface gradient.

Extended Maximum Likelihood Method (EMLM)

The EMLM gives a relatively high accuracy with simple calculations. The equation for EMLM estimation is explicitly expressed as follows.

$$S(f,\theta) = \frac{\kappa}{\boldsymbol{H}^{*t}\,\boldsymbol{\Phi}^{-1}\boldsymbol{H}} \quad (3)$$

where $\boldsymbol{H}$ is the matrix consisting of transfer functions between each combination of wave quantities given by Eq. (1), $\boldsymbol{H}^{*t}$ is the complex conjugate transposed matrix of $\boldsymbol{H}$, $\boldsymbol{\Phi}^{-1}$ is the inverse matrix of $\boldsymbol{\Phi}$ that consists of cross-spectra between each combination of wave quantities $\Phi_{mn}(f)$, and κ is the constant to normalize the directional spectrum energy.

Bayesian Directional Method (BDM) and Simplified Calculation Method

The BDM assumes the directional function to be the following discretely constant value function.

$$G(\theta \mid f) \approx \sum_{k=1}^{K} \exp\{z_k(f)\} I_k(\theta) \quad (4)$$

where ,

$$I_k(\theta) = \begin{cases} 1 : (k-1)\Delta\theta \le \theta < k\Delta\theta \\ 0 : \quad \text{otherwise} \end{cases} \quad (5)$$

If unknown parameters $\{z_k\}$ are determined, then $G(\theta \mid f)$ is also determined. However, it becomes an improper inverse problem, because the number of unknown parameters K is generally greater than the number of equations expressed as Eq. (2) obtained from the observation. Therefore, it is assumed that $G(\theta \mid f)$ is a smooth continuous function with respect to the directional angle and the series $\{z_k\}$ can be locally approximated as a linear function with respect to k. In order to meet the demand, we considered the following function.

$$\sum_{k=1}^{K} (z_k - 2z_{k-1} + z_{k-2})^2 \;\; ; \;\; (z_0 = z_K,\; z_{-1} = z_{K-1}) \rightarrow \text{small} \quad (6)$$

The smaller the value of Eq. (6), the smoother the function $G(\theta \mid f)$ becomes. Therefore, desirable estimate of $G(\theta \mid f)$ should maximize the likelihood with respect to $\{z_k\}$ within the range where Eq. (6) does not become too large. These criteria can be formulated as, using an appropriate parameter u^2, to find $\{z_k\}$ that minimizes the following function.

$$\sum_{j=1}^{J} \left\{ \Phi_j - \sum_{k=1}^{K} \alpha_{j,k} \exp(z_k) \right\}^2 + u^2 \left\{ \sum_{k=1}^{K} (z_k - 2z_{k-1} + z_{k-2})^2 \right\} \rightarrow \text{minimize} \quad (7)$$

Where, the first term in Eq. (7) is obtained by substituting Eq. (4) into Eq. (2) and then discretizing it. The value of u^2 and the estimate of σ^2 can be obtained by minimizing the Akaike's Bayesian Information Criterion (ABIC) expressed as follows:

$$\text{ABIC} = -2\ln \int L(z_1,\cdots,z_K;\sigma^2)\, p(z_1,\cdots,z_K \mid u^2,\sigma^2)\, d\mathbf{z} \quad (8)$$

However, it is difficult to analytically minimize Eqs. (7) and (8). Therefore, iterations with various u^2 are carried out to obtain the parameter u^2 that minimized the ABIC. Also, an optimum solution of $\{z_k\}$ is obtained and then substituted into Eq. (4) to yield an optimum estimation of the directional function.

While searching the minimum value of ABIC, u^2 is generally changed from a large to small value, because the iterative calculation of Eq. (7) is likely to diverge if u^2 is set smaller than the optimum value, and thus the value of u^2 is changed from large to small in order to avoid the divergence of iteration before the optimum value of u^2 is found. The calculation method is described in detail by Hashimoto (1988).

As described above, Eq. (7) is minimized at each stage where u^2 is changed from a big value to a small value. Since $\{z_k\}$ is included in the exponential part of the exp function, Eq. (7) is a non-linear equation with respect to $\{z_k\}$. Therefore, Eq. (7) is approximated by linearization and then the best estimate of $\{z_k\}$ is obtained by iterative calculations. To start with the iteration, $\{z_k\}$ is set to an arbitrary initial value, and thus resulting in a large amount of calculation time required for the BDM.

In response, we have considered simplified calculation methods for BDM and confirmed that the following method gives an effective solution. To start with, at each stage of u^2: large → small, $\{z_k\}$ is set to the initial value of $\ln(1/2\pi)$ and a single iteration of linearized Eq. (7) is performed. After ABIC values are calculated at all stages, choose such u^2 that gives the minimum value of ABIC. Finally, for the chosen u^2, Eq. (7) is iterated until the solution converges, and thus the $\{z_k\}$ is estimated. The simplified method was applied to various cases, resulting in a very small difference between the value of u^2 chosen by the simplified calculation method and that obtained by the conventional method. With this method, only a single iteration is needed for minimizing Eq. (7) and estimating $\{z_k\}$, and hence the calculation time can be reduced by several to several tens fold. For all calculated examples to be discussed later, the simplified calculation method was used, where it takes less than one second to estimate the directional spectrum even with a notebook PC.

OBSERVED EXAMPLE

The seabed mounted transducer of the directional wave meter transmits ultrasonic to the upward and inclined three directions every 0.125 seconds, to measure the water surface elevation and three components of the water particle velocity along the direction of each ultrasonic beam.

Fig. 3 shows an example of various waveforms measured by the directional wave meter. Figures from the top to bottom indicate respectively the water surface elevation and three components of the water particle velocity. From these waveform records, representative wave quantities are calculated such as the significant wave height, period and mean wave direction, as well as the current direction and speed at a certain water depth and tidal level fluctuation.

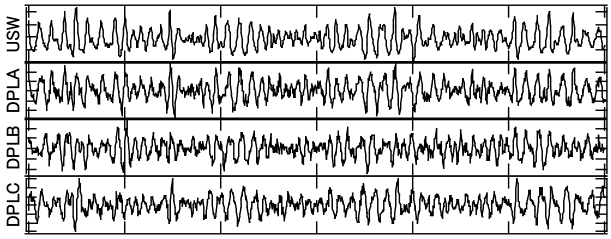

Fig. 3 Example of observed waveform

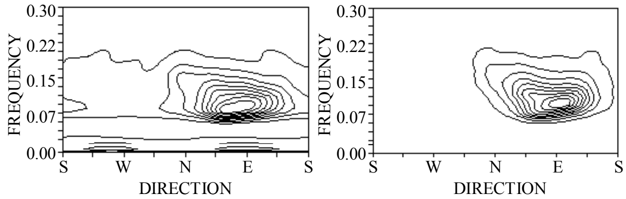

Fig. 4 Annual average directional spectra at Kuji Port
(Left : Upper layer by EMLM, Right : All layers by BDM)

Directional spectrum analysis using multi-layer Information

As shown in Fig. 1, the directional wave meter used at various

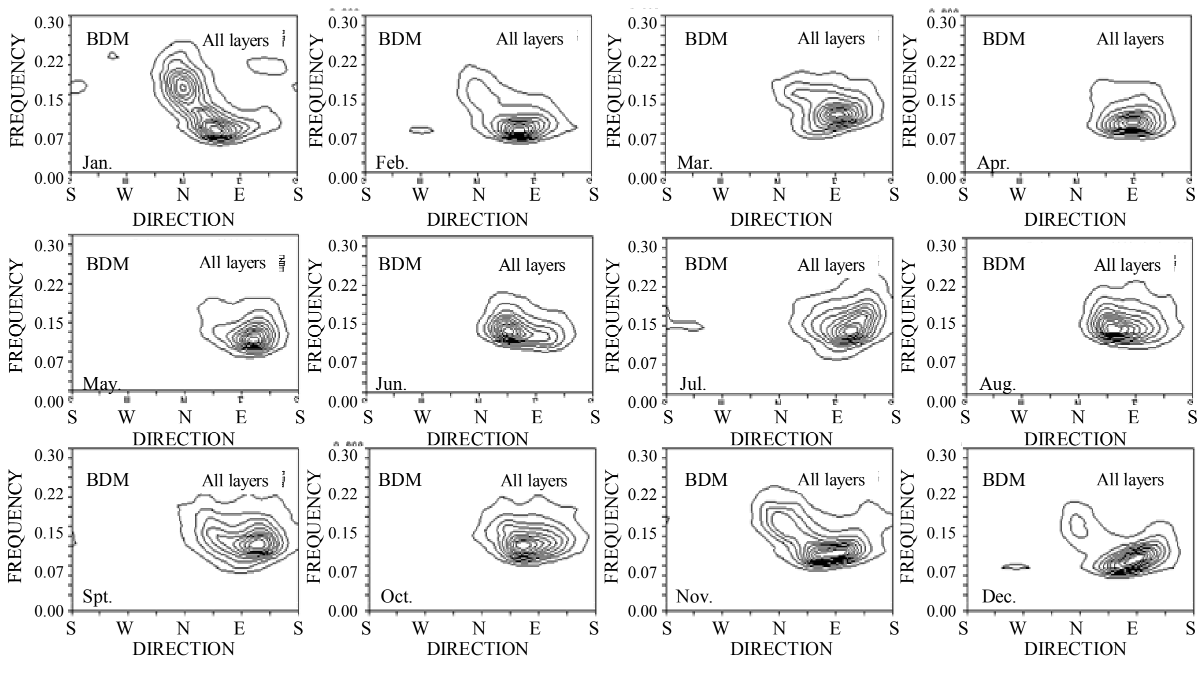

Fig. 5 Monthly average directional spectra observed by the directional wave meter located at Kuji Port

locations generally measures the water particle velocities in the upper, middle and lower layers, as well as the water surface elevation, total up to 10 components simultaneously, which are applicable to the directional spectrum analysis. Until now, however, the information about the water particle velocities in the middle and lower layers were not used for the directional spectrum analysis. Then, we have combined all of the information about the water particle velocities in the three layers to perform the directional spectrum analysis and to study the directional spectra estimated for various cases. For directional spectrum estimation, in addition to the conventional method of Extended Maximum Likelihood Method (EMLM), we also used the Extended Maximum Entropy Principle Method (EMEP) and the Bayesian Directional Method (BDM).

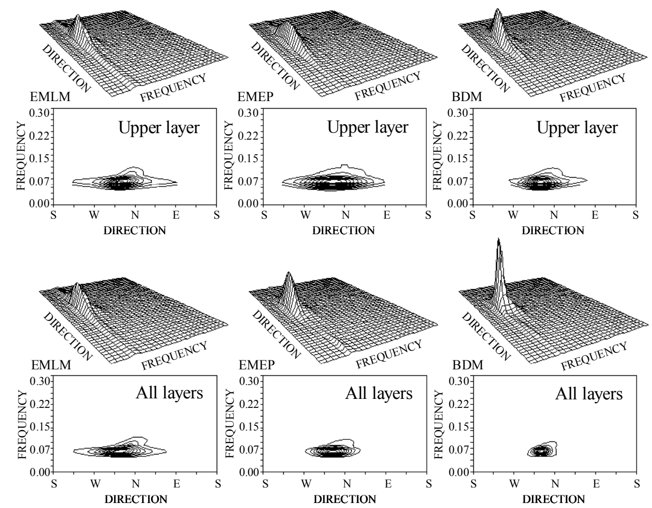

Fig. 6 Example of directional spectra estimated by using the information from the upper layer (top) and all the layers (bottom)

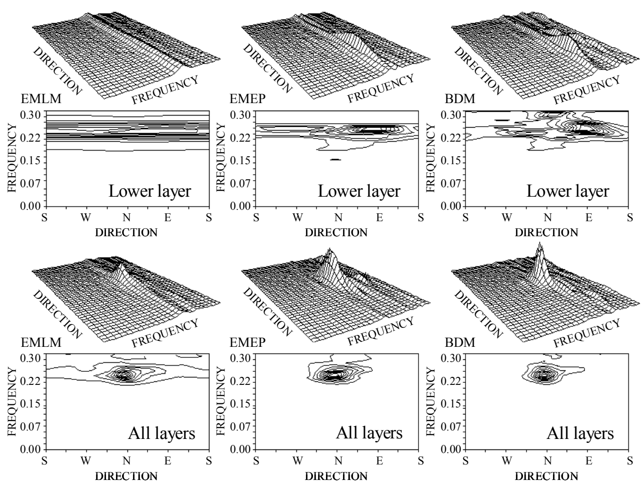

Fig. 7 Example of directional spectra estimated by using the information from the lower layer (top) and all the layers (bottom)

Fig. 4 shows contour maps of the annual average directional spectra, which were observed in 2008 by the directional wave meter located at Kuji Port in Iwate Prefecture. The vertical and horizontal axes indicate the frequency and the wave direction, respectively. The left map shows the directional spectrum estimated by the EMLM in the conventional way using the information about water particle velocities in the upper layer, and the right map shows the directional spectrum estimated by the BDM using all of the water particle velocity data from all the layers (3 layers). According to these results, the estimation by the BDM using the information about the water particle velocities in all the layers, compared with the conventional method, yields a directional spectrum with narrower directional spreading, and thus an accuracy improvement of the directional spectrum estimation can be expected. In order to verify the estimation performance more in detail, instead of the annual average as shown in Fig. 4, estimation of the directional spectra was performed for each month as shown in Fig. 5.

Fig. 5 indicates that in the area at the Kuji Port, waves from easterly directions are predominant throughout the year. In addition, it can be clearly noted that, in the winter season especially in January and in contrast to other seasons, wind waves from north directions are predominant seemingly due to the influence of monsoon.

Fig. 6 shows example of the directional spectra of a swell before the arrival of the typhoon No. 0918 observed by the directional wave meter located at Murotsu Port in Kochi Prefecture. From left to right are the directional spectra estimated by the EMLM, EMEP and BDM, respectively. Top figures are derived from the water particle velocities in the upper layer and bottom figures derived from all the layers, respectively. According to these results, the directional spectra estimated by using all the layers have narrower directional spreading. In addition, the estimation by the EMEP and BDM indicate a clear rise of peak height. Since observed waves are swell-type with long period, these results are inferred to be derived from a slow attenuation of the wave in the direction toward the seabed and a physical filtering action due to an increased water depth. That is, the signal-to-noise-ratio (S/N) is improved for the swell components of the wave and those components data are used in the analysis. These results thus imply that the information from the lower layer would be beneficial in the case of swell-type waves.

Fig. 7 shows example of the directional spectra observed by the directional wave meter located at Kanazawa Port in Ishikawa Prefecture using the water particle velocities in the lower layer (top) and those in all the layers (bottom), respectively. In contrast to Fig. 6, wind waves with short period are dominant in this example. It is worth noting that the shapes of the directional spectra estimated by using the data from the upper layer (not shown here) are nearly the same as those from all the layers. According to Fig. 7, no clear peaks can be seen in the directional spectrum estimated by the EMLM using the water particle velocities in the lower layer. Contrary to this result, an improvement of the directional spreading is seen in the directional spectrum estimated by using the water particle

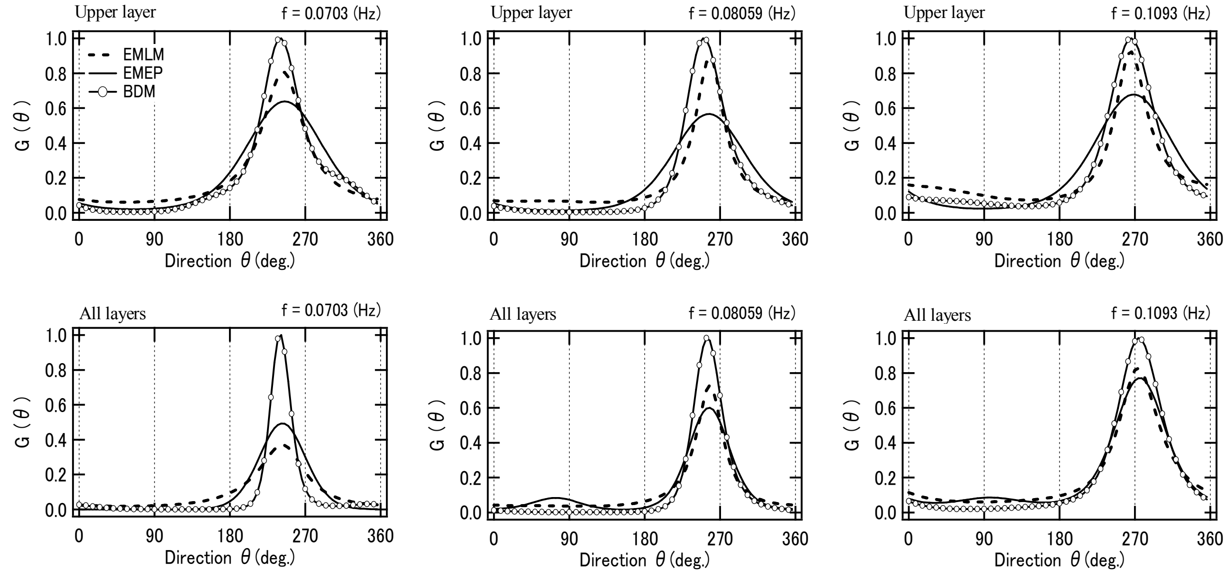

Fig. 8 Directional functions near the peak frequency estimated by using the information from the upper layer (top) and all the layers (bottom)

velocities in all the layers. These results imply that in the case of short period waves, wave phenomena significantly attenuate in the direction toward the seabed, and thus it is difficult to estimate an accurate directional spectrum by using only the information from the lower layer. Also, it is inferred that by adding the information from the upper layer that experiences strong wave phenomena, the directional spreading in the directional spectrum is improved. The information from the upper layer is thus confirmed to be beneficial in the case of wind waves. In addition, directional spectra estimated by the EMEP and BDM have directional spreading narrower compare to the EMLM, and their results confirm that the use of the information from all the layers improves the accuracy and stability.

Considerations on the directional function

Fig. 8 shows the directional functions derived from the directional spectra shown in Fig. 6 at each frequency close to the peak frequency. The top and bottom sets of directional functions are estimated by using the water particle velocities in the upper layer and in all the layers, respectively. The scale of vertical axis is normalized by the directional function estimated by the BDM. In each figure, dashed line, solid line and line with small circles represent the EMLM, EMEP and BDM, respectively. For case 1 through case 3 in Fig. 8, all three estimation methods give nearly the same wave direction, whereas the EMLM and EMEP underestimate the peak value compared to the BDM, implying an advantage of the BDM on the estimation of directional spectrum. It is also clearly shown that the directional function derived from all the layers have narrower spreading compare to the only upper layer. Meanwhile, as the frequency becomes lower, it is distinguishable that the directional spreading becomes narrower in the directional functions estimated by the BDM using the water particle velocities in all the layers. Also, the deviation of the peak energy estimated by the BDM become grater compare to the EMLM or EMEP. In the Fig. 8 for case 1 at the lowest frequency (about 14 seconds), the peak energy value estimated by the BDM reaches double than the other.

Considerations on the wave conditions and S_{max}

From the above considerations, when the directional spectrum is estimated using the water particle velocities, it is confirmed that the use of the information from all the layers yields a higher accuracy than the conventional way using information from only the upper layer. It is also confirmed that the BDM has high stability and accuracy on the estimation of directional spectrum. To consider these findings quantitatively, an analysis is made on the relationship between various wave quantities and the directional spreading parameter S_{max} using the information from only the upper layer or all the layers.

Fig. 9 shows the relationships between the significant wave height and S_{max} (left) and wave steepness and S_{max} (right). At the Kanazawa Port in the Japan Sea, typically on the winter season, the waves were analyzed and were observed by the directional wave meter located during four months, start from January to February and from November to December in 2008. The values of S_{max} were estimated from the directional spectra estimated by the BDM. In these figures, the solid lines represent S_{max} derived from all the layers and dashed lines represent from only the upper layer, respectively. The maximum value of S_{max} during the entire period is about 30, which is an estimate derived by using water particle velocities in all the layers.

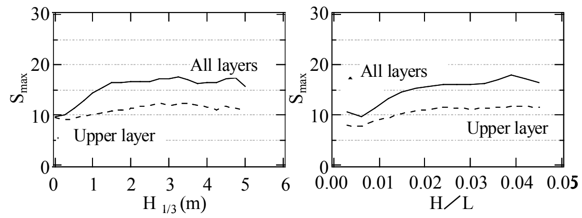

Fig. 9 Relationship between significant wave height/ wave steepness and S_{max} by using BDM

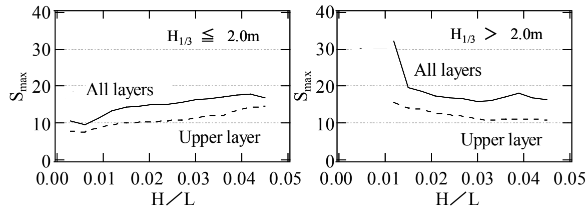

Fig. 10 Relationship between wave steepness and S_{max} by significant wave height by using BDM

Fig. 9 indicates that S_{max} slightly increases as the significant wave height increases, but shows no systematic trend. The values of S_{max} derived from all the layers are around 16, about 50 % higher than S_{max} derived from only the upper layer. As for the change of S_{max} with respect to the wave steepness, contrary to our expectations, it has gradually increased with the increase of the steepness, which is inferred to be the result of the sorting of all data simply by the wave steepness. That is, although we analyzed the waves on winter season in the Japan Sea, they were actually influenced by various mixed wave conditions, which are inferred to have led to such a result. Similar to the left figure, the values of S_{max} derived from all the layers are around 16, about 50 % higher than S_{max} derived from only the upper layer. It is interesting that on the winter season in the Japan Sea side where wind waves become predominant, the average values of S_{max} are around 16, exceeding the typical S_{max} value of 10 that is used as the criterion of wind wave.

In Fig. 10, the data in Fig. 9 are separated in to two groups to perform the relationship between the wave steepness and $S_{max.}$ The left figure is the group for the significant wave height lower or equal than 2 m, and the right figure for the significant wave height higher than 2 m. In the case of significant wave height 2 m or lower, in the same way as Fig. 9, S_{max} increases as the significant wave height increases. While in the case of the significant wave height was greater than 2 m, S_{max} tends to decrease with the increase of the steepness. The relationship between the wave characteristics and S_{max} gives considerable influence on various coastal engineering problems, and thus we intend to perform further research on this subject.

CONCLUSIONS

Up to now, information of the water particle velocity measured in the upper layer has been used for analytical processing of the directional spectrum at the many wave observation locations without the exception. However, it is considered that the directional spectrum with higher reliability is obtained by using information on not only the upper layer but also the lower layer, because its wave period is usually long when thinking about the high wave observation that is one of main purpose on wave observation. On the other hand, when the wave of about 1m or less used as a beginning judgment of construction in the harbors or coast is paid attention, it is considered that information in the upper layer located near the sea surface becomes effective compared with the lower layer.

As mentioned above, selectively using information of the water particle velocity in an appropriate depth layer according to the characteristic of waves that the observation targets might be necessary to estimate the directional spectrum with high reliability. The examination of the algorithm by which the always the best measurement layer under various waves conditions can be selected is not enough at present. For the time being, we think it is effective to use the all information in the upper, middle and lower layers for the estimation of more appropriate directional spectrum.

Finally, we suppose the proposed method accurate and efficient although the actual directional spectra are unknown. The reason is due to the results of the numerical experiments carried out by Hashimoto et, al, (1988, 1994). In the papers, the iterative computations in both BDM and EMEP start with the initial values of the uniform distribution to converge to the optimum estimates of directional spectrum satisfying the two requirements, i.e., the minimization of errors and the smooth of estimates with respect to direction. The optimum estimates are shown to be almost identical to the true solutions in the papers.

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